On Semi Feebly Separation Axioms

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Abstract

The goal of this work is to present some new separation axioms based on the concept of defining new types of open sets namely semi-feebly open set. we investigate their fundamental features.

Keywords: semi-feebly-T_0, semi-feebly-T_1

1 Introduction

separation qualities are a standout amongst the most vital and fascinating concepts in topology. In 1963, N.levin[10] proposed concept of a semi-open set. S.N Maheshwari and R. Prasad[9], used semi-open sets to characterize and investigare new partition aphorisms known as semi-detachment aphorisms. In 1975, Maheshwari and et.al.[8] created semi-R_0. P. Bahattacharya B. K. Lahiri [7] summarized up of shut sets to semi-summed up shut sets using semi-receptiveness in 1987 . Cueva M. C characterized the idea of new type of topological space called semi-$T_{1/2}$ in 2000 [6] (i.e. the space where the semi-closed sets and semi-summed up sets classes meet). Although none these applications reversible, it is proved that each semi-T_1 space is semi-$T_{1/2}$ and each semi-$T_{1/2}$ is semi-T_0 . Maheshwari and et. al.[5] initiated the study of feebly open in 1978. Aaad Aziz Hussan Abdulla in [1] presented the idea of semi-feebly open (sf-open) set. "the goal of this study is to provide some characterizations of semi-feebly separation axioms".

2 Preliminaries

Definition 2.1.[1]

Let (X, τ) be a topological space. A subset A of X is said to be

(1) semi-feebly open set if $\overline{A}^{f} \subseteq U$ whenever $A \subseteq U$ and U is semi-open set.

i.e. $\forall$ U is semi-open in X $(A \subseteq U \longrightarrow \overline{A}^{f} \subseteq U)$.

(2) the complement semi-feebly open set is said semi-feebly closed set.

Remark 2.2.[1]

If A is f-closed set, then A is sf-open set.

Proof.

let A be f-closed set in a topological space X. $A \subseteq U$, U s-open,

Since A is (f-closed) set then $A = \overline{A}^{f}$ and $A = \overline{A}^{f} \subseteq U$

Hence A is (sf-open) set.

Remark 2.3[1]

If A is closed set, then A is sf-open set.

Proposition 2.4.[1]

If A_{λ} is a family of sf-open set, then $\cup A_{\lambda}$ is sf-open set.

Proposition 2.5.[1]

Let X is a topological space and $A, B \subseteq X$, then

1. A is an sf-closed set if and only if $A = \overline{A}^{sf}$.

2. $\overline{A}^{sf} \subseteq \overline{A}$.

3. $\overline{A}^{sf} = \overline{(\overline{A}^{sf})}^{sf}$.

4. If $A \subseteq B$ then $\overline{A}^{sf} \subseteq \overline{B}^{sf}$.

Lemma 2.6.

Let X is a topological space and $A \subseteq X$, then

$\overline{A}^{s} \subseteq \overline{A}^{sf}$

Proof.

Let $x \in \overline{A}^{s}$ and A is a s-closed set, then $A = \overline{A}^{s} \Rightarrow x \in A \subseteq \overline{A}^{sf}$.

Then$x \in \overline{A}^{sf}$.

Therefor $\overline{A}^{s} \subseteq \overline{A}^{sf}$.

3 Lower separation axioms

Definition 3.1.

A topological space (X, τ) is sf-R_0 if for each sf-open set U, $x \in U$ implies

that $\overline{\{x\}}^{sf} \subset U$.

Lemma 3.2.

If a space X is sf-R_0, then for every sf-open set U and each $x \in U$,

$\overline{\{x\}}^{\circ} \subset U$.

Proof.

Let X be sf-R_0. Then for every sf-open set U and each $x \in U$, $\overline{\{x\}}^{sf} \subset U$. But by

[**Lemma (2.6)**] $\overline{\{x\}}^{s} \subset \overline{\{x\}}^{sf}$ and $\overline{\{x\}}^{s} = \{x\} \cup \overline{\{x\}}^{\circ}$ by [[11] **Proposition(1.1.19)**],

this implies that $\overline{\{x\}}^{\circ} \subset U$.

Definition 3.3.

A topological space (X, τ) is sf-R_1 if for each $x, y \in X$ with$\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$,there exist disjoint sf-open set U and V such that $\overline{\{x\}}^{sf} \subset U$ and $\overline{\{y\}}^{sf} \subset V$.

Theorem 3.4.

If a topological space (X, τ) is sf-R_1, then (X, τ) is sf-R_0.

Proof.

Let U be a sf-open set such that $x \in U$. If $y \notin U$, then $x \notin \overline{\{y\}}^{sf}$, therefore $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$. So, there exists a sf-open set V such that $\overline{\{y\}}^{sf} \subset V$ and $x \in V$, which implies $y \notin \overline{\{x\}}^{sf}$. Hence $\overline{\{x\}}^{sf} \subset U$. Therefore (X, τ) is sf-R_0.

Theorem 3.5.

A topological space (X, τ) is sf-R_0 if and only if for every sf-closed set F

and $x \notin F$, there exist a sf-open set G such that $F \subset G$, $x \notin G$.

Proof.

Let (X, τ) is sf-R_0 and F is sf-closed set in X and $x \notin F$. Then $X \setminus F$ is sf-open set containing x, since (X, τ) is sf-R_0 implies that $\overline{\{x\}}^{sf} \subset X \setminus F$ and then $F \subset X \setminus \overline{\{x\}}^{sf}$. Now let $G = X \setminus \overline{\{x\}}^{sf}$, then G is sf-open set not contains x and $F \subset G$.

Conversely: Let $x \in G$ where G is sf-open set in X. Then $X \setminus G$ is sf-closed set

and $x \notin X \setminus G$ implies that by hypothesis there exists and sf-open set U such

that$x \notin U$ and $X \setminus G \subset U$. Now $X \setminus U \subset G$ and $x \in X \setminus U$, but $X \setminus U$ sf-closed set then $\overline{\{x\}}^{sf} \subset X \setminus U \subset G$ this implies that (X, τ) is sf-R_0.

Theorem 3.6.

For a space X, the following are equivalent:

1. X is sf-R_0.

2. For any two points x and y in X, $x \in \overline{\{y\}}^{sf}$ if and only if $y \in \overline{\{x\}}^{sf}$.

Proof.

(1) ⇒ (2). Let X is sf-R_0 and $x \in \overline{\{y\}}^{sf}$. To show $y \in \overline{\{x\}}^{sf}$, let V be any sf-open set containing y. Since X is sf-R_0 so $\overline{\{y\}}^{sf} \subset V$ implies that $x \in V$, hence every sf-open set which containing y contains x this implies that $y \in \overline{\{x\}}^{sf}$. By the same way we can prove that if $y \in \overline{\{x\}}^{sf}$, then $x \in \overline{\{y\}}^{sf}$.

(2) ⇒ (1). Let the hypothesis be satisfied and$U$ U be any sf-open set and $x \in U$. To show $\overline{\{x\}}^{sf} \subset U$, let $y \in \overline{\{x\}}^{sf}$ implies that by hypothesis $x \in \overline{\{y\}}^{sf}$, and then $U \cap \{y\} \neq \emptyset$ this implies that $y \in U$. Thus $\overline{\{x\}}^{sf} \subset U$, therefore X is sf-R_0.

Theorem 3.7.

A space X is sf-R_0 if and only if for any x and y in X if $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$,

then $\overline{\{x\}}^{sf} \cap \overline{\{y\}}^{sf} = \emptyset$.

Proof.

Let X be sf-R_0 and $x, y \in X$ such that $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$. Then there exists $z \in \overline{\{x\}}^{sf}$ such that $z \notin \overline{\{y\}}^{sf}$ implies that there exists an sf-open set U containing z but not y, hence $x \in \overline{\{x\}}^{sf}$. Therefore we have $x \notin \overline{\{y\}}^{sf}$ implies that $x \in X \setminus \overline{\{y\}}^{sf}$ which is an sf-open set, but X is sf-R_0 so $\overline{\{x\}}^{sf} \subset X \setminus \overline{\{y\}}^{sf}$ this implies that $\overline{\{x\}}^{sf} \cap \overline{\{y\}}^{sf} = \emptyset$.

Conversely. Let the hypothesis be satisfied and let U be any sf-open set in X and $x \in U$. If $U = X$, then clearly $\overline{\{x\}}^{sf} \subset U$, but if $U \neq X$, then there exists $y \in X$ such that $y \notin U$. Now $x \neq y$ and $x \notin \overline{\{y\}}^{sf}$ implies that $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$, then by hypothesis $\overline{\{x\}}^{sf} \cap \overline{\{y\}}^{sf} = \emptyset$ implies that $y \notin \overline{\{x\}}^{sf}$. Thus if $y \notin U$, then $y \notin \overline{\{x\}}^{sf}$ this implies that $\overline{\{x\}}^{sf} \subset U$. Hence X is sf-R_0.

Definition 3.8.

Let (X, τ) is a topological space. If for each $a, b \in X$ where $a \neq b$ there exists a semi-feebly-open set W of X containing a but not b, we say that X is semi-feebly-T_0 space.

Theorem 3.9.

Let (X, τ) is a topological space. We say that X is sf-T_0-space if and only if

for every $x, y \in X, x \neq y$. Implies $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$

Proof.

Let x,$y \in X$ with $x \neq y$ and X is sf-T_0-space . We shall show that $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$. Since X is sf-T_0-space, there exists a sf-open set U such that $x \in U$ but $y \notin U$. Also $x \notin X \setminus U$ and $y \notin X \setminus U$ where $X \setminus U$ is sf-closed set in X. Now by definition $\overline{\{Y\}}^{sf}$ is the intersection of all sf-closed set which contain y . Hence, $y \in \overline{\{y\}}^{sf}$ but $x \notin \overline{\{y\}}^{sf}$ as

$x \notin X \setminus U$. Therefore, $\overline{\{x\}}^{sf} \neq \overline{\{y\}}^{sf}$.

Conversely, for any $x, y \in X, x \neq y$. And $\overline{\{x\}}^{sf} \neq \overline{\{x\}}^{sf}$. Then there exists at least one point such that $z \in X$ such that $z \in \overline{\{x\}}^{sf}$ but $z \notin \overline{\{y\}}^{sf}$.

We claim that $x \notin \overline{\{y\}}^{sf}$. If $x \in \overline{\{y\}}^{sf}$ then $\{x\} \subseteq \overline{\{y\}}^{sf}$ implies $\overline{\{x\}}^{sf} \subseteq \overline{\{y\}}^{sf}$. So, $z \in \overline{\{y\}}^{sf}$, which is a contradiction. Hence, $x \notin \overline{\{y\}}^{sf}$. Now, $x \notin \overline{\{y\}}^{sf}$

implies $x \in X \setminus \overline{\{y\}}^{sf}$ and $X \setminus \overline{\{y\}}^{sf}$ is sf-open in X but $y \notin X \setminus \overline{\{y\}}^{sf}$ Observe X that is sf-T_0-space.

Proposition 3.10.

Whenever X is sf-T_0-space, then each subspace of X is sf-T_0-space.

Proof.

Consider X as a sf-T_0-space and $Y \subset X$. Take α and β as unequal points of Y. As $Y \subset X$, α and β are also unequal points of X. As per given, X is sf-T_0-space, we have a sf-open set K so that $\alpha \in K$, $\beta \notin K$. Then we have $Y \cap K$ is sf-open in Y having α but not β. Thus Y is sf-T_0-space.

Definition 3.11.

A subset A of a topological space (X, τ) is called to be semi-feebly generalized closed set (written in short as sfg-closed) if, $\overline{A}^{sf} \subset O$ hold whenever $A \subset O$ and O is sf-open.

Proposition 3.12.

Every sf-closed set is sfg-closed set.

Definition 3.13.

A topological space (X, τ) is called sf-$T_{1/2}$ if every sfg-closed set in (X, τ) is sf-closed set in (X, τ).

Definition 3.14.

That for any subset E of (X, τ), $\overline{E}^{sf*} = \cap\{A : E \subset A\ (\in \text{sf}D(X, \tau))\}$, where sf$D(X, \tau)$ = { A : $A \subset X$ and A is sfg-closed in (X, τ)} and

sf$O(X, \tau)^*$ = {B : $\overline{B^c}^{sf*} = B^c$}.

Theorem 3.15.

A topological space (X, τ) is a sf-$T_{1/2}$ space if and only if

sf$O(X, \tau)$ = sf$O(X, \tau)^*$

Proof.

Necessity : Since the sf-open sets and the sfg-closed sets coincide by the assumption, $\overline{E}^{sf} = \overline{E}^{sf*}$ holds for every subset E of (X, τ).

Therefore, we have sf$O(X, \tau)$ = sf$O(X, \tau)^*$

Sufficiency : Let A be a sfg-closed set of (X, τ). Then, we have $A = \overline{A}^{sf*}$ and A^c is sf-open set in (X, τ). Thus A is sf-closed set. Therefore (X, τ) is sf-$T_{1/2}$.

Theorem 3.16.

A topological space is sf-$T_{1/2}$ space if and only if each $x \in X$, $\{x\}$ is sf-open or $\{x\}$ is sf-closed.

Proof.

Necessity: Suppose that for some $x \in X$, $\{x\}$ is not sf-closed. Since X is the only sf-open set containing $\{x^c\}$, the set $\{x^c\}$ is sfg-closed and so it is sf-closed in the sf-$T_{1/2}$ space (X, τ). Therefore $\{x\}$ is sf-open.

Sufficiency: Since sf$O(X, \tau) \subset$ sf$O(X, \tau)^*$ holds,

we show sf$O(X, \tau)^* \subset$ sf$O(X, \tau)$. Let $E \in$ sf$O(X, \tau)^*$

Suppose that $E \notin$ sf$O(X, \tau)$. Then, $\overline{E^c}^{sf*} = E^c$ and $\overline{E^c}^{sf} \neq E^c$ hold. There exists a point x of X such that $x \in \overline{E^c}^{sf}$ and $x \notin E^c = \overline{E^c}^{sf*}$. Since $x \notin \overline{E^c}^{sf*}$ there exists a sfg-closed set A such that $x \notin A$ and $A \supset E^c$ .By the hypothesis, the singleton $\{x\}$ is sf-open or sf-closed.

Case (1). $\{x\}$ is sf-open. Since $\{x^c\}$ is a sf-closed set with $E^c \subset\{x^c\}$,

we have $\overline{E^c}^{sf} \subset\{x^c\}$, i.e., $x \notin \overline{E^c}^{sf}$. This contradicts the fact that $x \in \overline{E^c}^{sf}$.

Therefore $E \in \text{sf}O(X, \tau)$.

Case (2). $\{x\}$ is sf-closed. Since $\{x^c\}$ is a sf-open set containing the sfg-closed set $A \supset E^c$, we have $\{x^c\} \supset \overline{A}^{sf} \supset \overline{E^c}^{sf}$. Therefore $x \notin \overline{E^c}^{sf}$. This is a contradiction. Therefore $E \in \text{sf}O(X, \tau)$.

Hence in both cases, we have $E \in \text{sf}O(X, \tau)$, i.e., $\text{sf}O(X, \tau)^* \subset \text{sf}O(X, \tau)$.

Corollary 3.17.

X is sf-$T_{1/2}$ if and only if every subset of X is the intersection of all sf-open sets and all sf-closed sets containing it.

Proof.

Necessity: let X is sf-$T_{1/2}$ with $B \subset X$ arbitrary. Then $B = \{\{x\}^c, x \notin B\}$, an intersection of sf-open and sf-closed[**Theorem(3.16)**].The result follows.

Sufficiency: for $x \in X$, $\{x\}^c$ is the intersection of all sf-open sets and all sf-closed sets containing it. Thus $\{x\}^c$ is either sf-open or sf-closed and X is sf-$T_{1/2}$.

Definition 3.18.

A space(X, τ) is called a sf-$T_{1/4}$ space if for every finite subset $F \subset X$ and every point $y \notin F$ there exists a subset $A \subset X$ such that $F \subset A$, $y \notin A$ and A is sf-open or sf-closed.

Proposition 3.19.

Let (X, τ) be a sf-$T_{1/4}$ space. Then every subspace of X is a sf-$T_{1/4}$space.

Recall that a subset F of a space (X, τ) sf-locally finite if every point has an sf-open neighborhood U_x such that $F \cap U_x$ is at most finite. $x \in X$

Theorem 3.20.

For a space (X, τ) the following are equivalent:

(1) (X, τ) is a sf-$T_{1/4}$ space,

(2) For every sf-locally finite subset $F \subset X$ and every point $y \notin F$ there

exists a subset $A \subset X$ such that $F \subset A$, $y \notin A$ and A is sf-open or sf-closed.

Definition 3.21.

Let (X, τ) is a topological space. Then X is sf-T_1-space if for each $a, b \in X$ such that

$a \neq b$ there exists a sf-open set W of X containing a but not b and a sf-open set U of X

containing b but not a.

Remark 3.22.

Every sf-T_1space is sf-T_0 space

Proof.

From the definition of sf-T_1space it is follows that it is sf-T_0, since there

exists a sf-open set G such that $x \in G$ but $y \notin G$

the converse is not true.

Corollary 3.23.

Every sf-T_0space is not sf-T_1 space.

The following example supports this.

Example 3.24.

Let $X = \{1, 2, 3\}$, $\tau = \{X, \emptyset, \{1\}\}$ be a topology defined on X. Here

sf-open sets are $\{X, \emptyset, \{2\}, \{3\}, \{2, 3\}\}$. It is clear X is sf-T_0 space but is

not sf-T_1 since $1 \neq 2$ and there exist sf-open set contain 2 but there is not

exist sf-open set such that containing 1 but not 2

Proposition 3.25.

X is sf-T_1 if and only if for all $x \in X$ implies $\{x\}$is sf-closed sets.

Proof.

Let $\{z\}$ sf-closed set for every $z \in X$. Let$x$, $y \in X$ such that $x \neq y$.

Then $x \in \{y\}^c$ and $\{y\}$ is sf-closed set . Therefore $\{y\}^c$ sf-open set containing x but not y and, and $\{x\}^c$ sf-open set containing y but not x.

Then X is sf-T_1

Conversely, let X be a sf-T_1-space and $y \in X$. To prove $\{y\}$ is sf-closed set. Let $x \in \{y\}^c$ then $x \neq y$. Since X sf-T_1, then there exists sf-open set in X, U such that $x \in U$ and $x \notin U$. Then $x \in U \subset \{y\}^c = \cup\{U_x : x \in \{y\}^c\}$ which is sf-open set. Hence$\{x\}$ is sf-closed set.

Theorem 3.26.

A space X is sf-T_1 if and only if it is sf-T_0 and sf-R_0.

Proof.

Let X be sf-T_1 space. Then from [**Remark (3.22)**] X is sf-T_0 and by [**Proposition (3.25**] every singleton set in X is sf-closed. Now X is sf-R_o space since for any $x \in U$, where U is sf-open set, $\overline{\{x\}}^{sf} = \{x\} \subset U$.

Thus the space X is sf-R_o.

Conversely, let x, $y \in X$ be any two distinct points. Since X is sf-T_0 so there exists an sf-open set U such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.

Now let $x \in U$ and $y \notin U$ and since X is sf-R_0 space so $\overline{\{x\}}^{sf} \subset U$ and we have $y \notin U$ implies that $y \notin \overline{\{x\}}^{sf}$, then $y \in X \setminus \overline{\{x\}}^{sf}$ which is sf-open set so take $V = X \setminus \overline{\{x\}}^{sf}$. Thus U and V are sf-open sets in X such that $x \in U$, $y \in V$ and $x \notin V$ and $y \notin U$, implies that X is sf-T_1 space.

Definition 3.27[2]

A subset A of a topological space (X, τ) is called sg-closed set if,

$\overline{A}^{s} \subset O$ hold whenever $A \subset O$ and O is s-open of (X, τ), the complement of a sg-closed set is called a sg-open set.

Definition 3.28[3]

A subset A of a topological space (X, τ) is called Ψ -closed set if,

$\overline{A}^{s} \subset O$ hold whenever $A \subset O$ and O is sg-open of (X, τ).

Theorem 3.29[4]

Let A be a subset of topological space (X, τ), then

1) A is Ψ-closed if and only if $\overline{A}^{s} - A$ does not contain any non-empty sg-closed set.

2) If A is Ψ-closed and $A \subset B \subset \overline{A}^{s}$, then B is Ψ-closed

Definition 3.30.

A space (X, τ) is said to be a sf-$T_{1/3}$ space if every Ψ-closed set in (X, τ) is sf-closed.

Theorem 3.31.

For a topological space (X, τ), the following conditions are equivalent:

(i) (X, τ) is a sf-$T_{1/3}$ space.

(ii) Every singleton of X is either sg-closed or sf-open set.

(iii) Every singleton of X is either sg-closed or open set.

Proof.

(i) ⇒ (ii) let $x \in X$ and suppose that $\{x\}$ is not sg-closed of (X, τ). Then X-$\{x\}$ is not sg-open set. so, X is the only sg-open set containing X-$\{x\}$. Hence X-$\{x\}$ is Ψ-closed set. Since (X, τ) is sf-$T_{1/3}$ space, then X-$\{x\}$ is a sf-closed set or equivalently $\{x\}$ is sf-open set.

(ii) ⇒ (i) let A be a Ψ-closed set. clearly $A \subset \overline{A}^{s}$. let $x \in X$. By Assumption, $\{x\}$ is either sg-closed or sf-open.

Case(1) suppose $\{x\}$is sg-closed.[**Theorem(3.29)**] $\overline{A}^{s} - A$ does not contain any non-empty sg-closed set. Since $x \in \overline{A}^{s}$, then $x \in A$.

Case(2) suppose $\{x\}$ is a sf-open set. Since $x \in \overline{A}^{sf}$, then $\{x\} \cap A \neq \emptyset$.

So $x \in A$. Thus in any case $\overline{A}^{sf} \subset A$.

Therefore $A = \overline{A}^{sf}$ or equivalently A is sf-closed set of (X, τ).

Hence (X, τ) is an sf-$T_{1/3}$ space.

(iii) ⟺ (ii) Follows from the fact that a singleton is sf-open if and only if it is open.

4 Some new separation axioms

Definition 4.1.

Let (X, τ) be a topological space. Let $A \subset X$ we say that A is semi-feebly -Difference (sf-D) set if there exists U, V are sf-open set such that $U \neq X$ and $A = U \setminus V$.

Remark 4.2.

Every sf-open set $U \neq$ X is sf-D-set if A = U and $V = \emptyset$

Corollary 4.3.

Every sf-D-set is not sf-open set.

The following example shows.

Example 4.4.

Let $X = \{1, 2, 3, 4\}$, $\tau = \{ \emptyset, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}, X\}$. So, sf-open set are $\{\emptyset, X, \{1, 3, 4\}, \{1, 2, 4\}, \{3, 4\}, \{1, 4\}, \{4\}, \{1, 3\}, \{1\}, \{3\}, \{1, 2\}\}$, then $U = \{1, 2, 4\} \neq X$ and $V = \{1, 3, 4\}$ are sf-open sets in X and $A = U \setminus V = \{1, 2, 4\} \setminus \{1, 3, 4\} = \{2\}$, then we have $A = \{2\}$ is a sf-D-set but it is not sf-open set.

Definition 4.5.

A topological space (X, τ) is said to be:

1. sf-D_0 if for any pair of distinct points x and y of X there exists a sf-D-set of X containing x but not y or a sf-D-set of X containing y but not x.

2. sf-D_1 if for any pair of distinct points x and y of X there exists a sf-D-set of X containing x but not y and a sf-D-set of X containing y but not x.

3. sf-D_2 if for any pair of distinct points x and y of X there exist disjoint sf-D-set G and E of X containing x and y, respectively.

Remark 4.6.

For a topological space (X, τ), the following properties hold:

1. If (X, τ) is sf-T_k, then it is sf-D_k, for $k = 0, 1, 2$.

2. If (X, τ) is sf-D_k, then it is sf-D_{k-1}, for k = 1, 2.

Proof.

It follows from [**Remark (4.2)**] and [**Definition (4.5)**].

Proposition 4.7.

A space X is sf-D_0 if and only if it is sf-T_0.

Proof.

Suppose that X is sf-D_0. Then for each distinct pair $x, y \in X$, at least one of x, y, say x, belongs to sf-D-set G but $y \notin G$. Let $G = U_1 \setminus U_2$ where $U_1 \neq X$ and U_1, U_2 are sf-open set. Then $x \in U_1$, and for $y \notin G$ we have two cases: (a) $y \notin U_1$, (b) $y \in U_1$ and $y \in U_2$.

In case (a), $x \in U_1$ but $y \notin U_1$.

In case (b), $y \in U_2$ but $x \notin U_2$.

Thus in both the cases, we obtain that X is sf-T_0.

Conversely, if X is sf-T_0, by [**Remark (4.6)** (1)], X is g sf-D_0.

Proposition 4.8.

A space X is sf-D_1 if and only if it is sf-D_2.

Proof.

Necessity. Let $x, y \in X, x \neq y$. Then there exist sf-D-sets G_1, G_2 in X such that $x \in G_1$, $y \notin G_1$ and $y \in G_2, x \notin G_2$. Let $G_1 = U_1 \setminus U_2$ and $G_2 = U_3 \setminus U_4$, where U_1, U_2, U_3 and U_4 are sf-open sets in X. From $x \notin G_2$, it follows that either $x \notin U_3$ or $x \in U_3$and $x \in U_4$. We discuss the two cases separately.

(i) $x \notin U_3$. By $y \notin G_1$ we have two sub-cases:

(a) $y \notin U_1$. Since $x \in U_1 \setminus U_2$, it follows that $x \in U_1 \setminus (U_2 \cup U_3)$, and

since $y \in U_3 \setminus U_4$, we have $x \in U_3 \setminus (U_1 \cup U_4)$.

Therefore $(U_1 \setminus (U_2 \cup U_3)) \cap (U_3 \setminus (U_1 \cup U_4)) = \emptyset$.

(b) $y \in U_1$ and $y \in U_2$. We have $x \in U_1 \setminus U_2$, and $y \in U_2$. Therefore

$(U_1 \setminus U_2) \cap U_2 = \emptyset$.

(ii) $x \in U_3$ and $x \in U_4$. We have $y \in U_3 \setminus U_4$ and $x \in U_4$. Hence

$(U_3 \setminus U_4) \cap U_4 = \emptyset$. Therefore X is sf-D_2.

sufficiency. Follows from [**Remark (4.6)**(2)].

Corollary 4.9.

If (X, τ) is sf-D_1, then it is sf-T_0.

Proof.

Follows from [**Remark (4.6)** (2)] and [**Proposition (4.7)**].

Remark 4.10.

Here is an example which shows that the converse of [**Corollary (4.9)**] is

not true in general.

Example 4.11.

Let $X = \{1, 2\}$, $\tau = \{\emptyset, \{1\}, X\}$ be a topology on X.

Then (X, τ) is sf-T_0, but not sf-D_1, since there is no sf-D-set containing 2 but not 1.

5 Conclusion

In topological space, separation axioms are very important. Through this paper, it was concluded that there is a relationship between semi-feebly-T_1 , semi-feebly-T_0 and the axioms of separation of type semi-feebly-R_0, semi-feebly-R_1 there is also a relationship between semi-feebly-D_0 , semi-feebly-D_1 , semi-feebly-D_2.

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