Mathematical Model of the Effect the Energy on (Y-W-D) Types of Fungi, with Exponential function

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Abstract

The phenomenon of dysplasia describes the mathematical model. The model that shows the behaviour of the growth of bilateral branching, lateral branching, filament tip anastomosis, limb anastomosis, and limb death due to overcrowding with thread death. The study shows energy consumption; in general that the growth of fungi needs to be resolved until its goal becomes a correction. Moreover the study reduced the cost, effort by predicting the best class of plants for cultivation according to the results. Herein we suggest mathematical solution using the solution of Partial Differential Equations (PDEs). Furthermore, Matlab software codes were utilized numerical analysis because of some of the difficulties we face in the direct mathematical solution. Finally, the study models shows the success or failure of the growth of the studied fungi

Keywords: Dichotomous branching, Tip-tip anastomosis.

1   Introduction

In 1982, Leah-Keshed [1, 2] denoted that Dichotomous branching (Y), Tip-tip anastomosis (W), and Hyphal death (D). Table (1) it is illustrate these types.

There are many papers of the mathematical models which have been proposed by various researchers in order to explain the Mathematical Model, for example:

-In (2011) Shuaa [2], Studied to develop a model for the growth of fungi which can be used to create a source term in a single root model to account for nutrient uptake by the fungi. Therefore, focus on the hyphal loss or death.

 -In (2012) Brian Ingalls [3], offered an introduction to mathematical con-cepts and techniques needed for the construction and interpretation of models in molecular systems biology.

-In (2013) Walter [7], Studied independent sections that illustrate the most important principles of mathematical modeling, a variety of applica-tions, and classic models...

-In (2014) Mudhafar [8], Proposed the different modelling procedures, with a special emphasis on their ability to reproduce the biological system and to predict measured quantities which describe the overall processes. A comparison between the different methods is also made, highlighting their specific features.

Table 1.Illustrate branching, Biological type, symbol of this type and version

Biological type	Symbol	Version	Parameters Description
Dichotomous branching	Y	$\delta = \alpha_1 \eta$	$\alpha_{1=}$ Is the number of tips produced per tip per unit time
Tip-tip anostomosis	W	$\delta = -\beta_2 \eta^2$	$\beta_{2=}$Is the rate of tip reconnections per unit time
Hypal death	D	$d = \gamma_1 p$	γ_1 =Is the loss rate of hyphal (constant for hyphal death)

2   Mathematical Model

We will study a new type of branching of fungal growth with hyphal death and Consumption of whole vegetarian food, we can call it energy $E(\psi)$, this energy function lies between one and zero as $0 \leq E(\psi) \leq 1$ means if the grow die if it is not consume energy but E (ψ), that mean is the growth is very good if the fungi consume all the energy [1, 2, 3].

We can describe hyphal growth by the system below:

$$\frac{\partial \rho}{\partial t} = nv - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial(nv)}{\partial x} + e^{[\delta(p,n)]} - E(\psi) \qquad (1)$$

Where: $\delta(p,n) = \alpha_1 \eta - \beta_1 \eta^2$ that is dented above and E (ψ) = 1. Then this system (1) becomes: [2]

$$\frac{\partial \rho}{\partial t} = nv - \gamma\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial(nv)}{\partial x} + e^{[\alpha_1 - \beta_1 \eta^2]} - 1 \qquad (2)$$

Where:$\alpha = \frac{\alpha_1}{\gamma_1}$

3   Non-dimensionlision and Stability

Leah-keshet (1982) and Ali H. Shuaa Al-Taie (2011) clear up how can put these parameters as dimensionlision less.

$$\frac{\partial \rho}{\partial t} = nv - \gamma \rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + e^{[\alpha \eta (1-n)]} - 1 \quad (3)$$

Where: $\alpha = \frac{\alpha_1}{\gamma_1}$ is represented rate of hyphal branching per unit hyphal per unit length hypha per unit time, and $\alpha \eta (1 - n)$ thus represented the number of branches produced per unit time per unit length of hyphae [2, 3].

Now, to find steady states when take from system (2):

$$\frac{\partial \rho}{\partial t} = n - \rho = 0 \quad \rightarrow n = \rho \quad (4)$$

And on the other hand

$$\frac{\partial n}{\partial t} = e^{[\alpha \rho (1-n)]} - 1 = 0 \rightarrow \ e^{[\alpha \rho (1-n)]} = 1$$

$$\rightarrow \alpha \rho (1 - n) = 0 \qquad\qquad (5)$$
$$\rho = 0, \ then \quad \rightarrow (p,n) = (0,0) \ \ and \ \ (1 - n) = 0$$

$$n = 1, \quad then \ (p,n) = (1,1)$$

So that is clear the steady state are (p,n) = (0,0) and, (p,n) =(1,1) therefor, we take Jacobain of these equations. [5, 8, 6]

$$J_{(p,n)} = \begin{bmatrix} -1 & 1 \\ 0 & \alpha(1 - 2n) \end{bmatrix}$$

We can classify the critical point according to the eigenvalues of this matrix. Jacobain at (0, 0):

$$J_{(0,0)} = \begin{bmatrix} -1 & 1 \\ 0 & \alpha \end{bmatrix}$$

Thus, $|A - \lambda I| = 0$ we get two values of (λ):
$$\lambda_1 = -1, \qquad \lambda_2 = \alpha$$

Then we take the Jacobain at (1, 1):

$$J_{(1,1)} = \begin{bmatrix} -1 & 1 \\ 0 & -\alpha \end{bmatrix}$$

Thus$|A - \lambda I| = 0$, then we get two values of (λ):

$$\lambda_1 = -1, \qquad \lambda_2 = -\alpha$$

We notes the probabilities of the α.

If α is negative, we get the point (0, 0) stable spiral and the point (1, 1) is saddle point. See Fig (1).Using (MATLAB pplane7). [3, 7, 6]
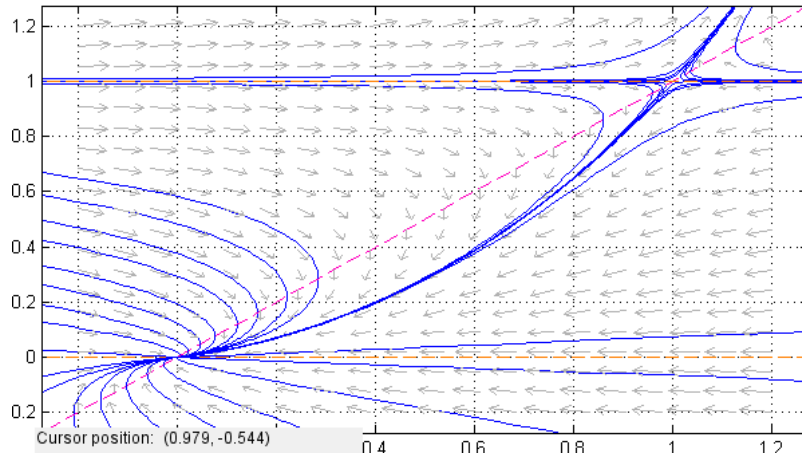
Figure 1. The (p ,n) plane : note that a trajectory connects in the point (1,1) is the saddle point and in the point (0,0) the stable node in point (0,0)Document title and meta-data

4   Traveling wave solution

We will now discuss the traveling wave solution, we assume that: $\rho(x,t) = \rho(z)$ and, $n(x,t) = N(z)$ where $z = x - ct$, $P(z)$ profile density and propagation rate c of edge of the colony. $P(z)$ and, $N(z)$ is a non-negative function of z. The function, $p(x,t)$, $n(x,t)$ are moving waves, moving with a constant speed c in a positive x direction, where $c > 0$, $E(\psi) = 1$, and $\alpha = 1$ to search for a traveling wave solution to the equations in x and t of the system (3).

$$\frac{d\rho}{dt} = -c\,\frac{d\rho}{dx}, \quad \frac{dn}{dt} = -\frac{dN}{dx}, \quad \text{And,}\ \frac{dn}{dt} = \frac{dN}{dx}$$

See [3] therefor, the above equation becomes:

$$\frac{dP}{dz} = \frac{-1}{c}[N - P]$$

$$\frac{dN}{dZ} = \frac{1}{1-c}\,e^{[\alpha\eta(1-N)]} \qquad , c \neq 1, \ -\infty < z < \infty \qquad (6)$$

To deter the steady states of the above system, we get $(N, P) = (0,0)$ saddle point and $(1,1)$ stable node constant for negative c and $\alpha = 1$. This helps us to determine the initial conditions of p and n which is the above system (3) See Fig. (2).
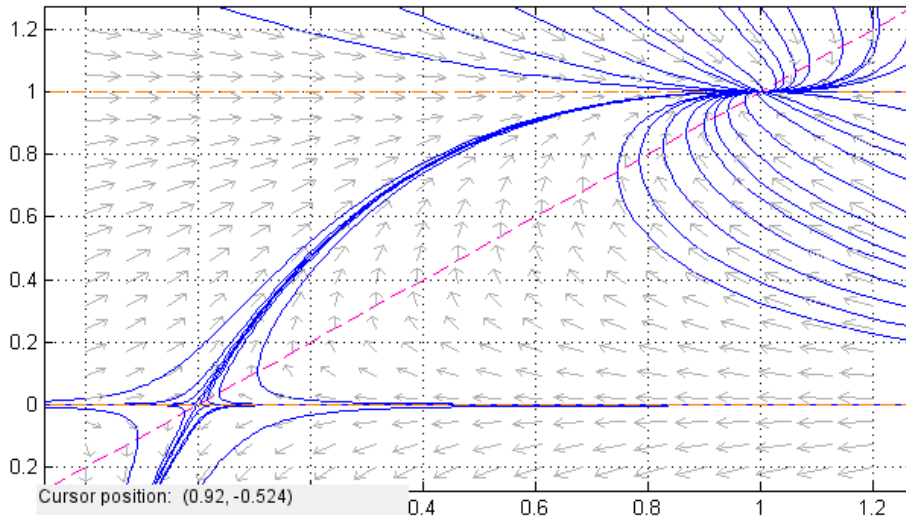
Figure 2. The (n, p) plane note that a trajectory connects the saddle point in (0,0) and stable node in point (1,1)

5   Numerical Solution

Because the system (3) cannot be solved exactly, so we resort to numerical solutions, and here we using pdepe code in MATLAB that is clear the initial condition start from 1 to zero for ρ and η, Figure(3) illustrate the behavior for ρ and η that is very clear the traveling waves are regular for time.

Illustrate solution to the system (3) with the parameters $\alpha = 0.5$
And c =2.0411 for time t=1, 10, 20, 30...300.
In (Figure 4) where the blue line represented tips (n),
In (Figure 5) the red line represented branches (p),
In (Figure 6) where the blue line represented tips (n), with the red line represented branches (p), and illustrate the solution of p and n numerically with take values of $\alpha = 0.5$, that is very clear the traveling wave solution start from left to right and still the same wave.

From this operation we get the relationship between traveling wave solution (c) and parameter α where α increasing the traveling wave solution (c) is increasing, using Matlab [2,7,8].
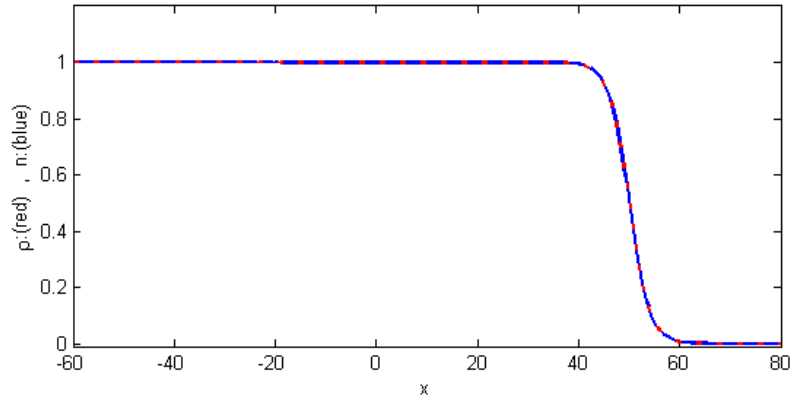
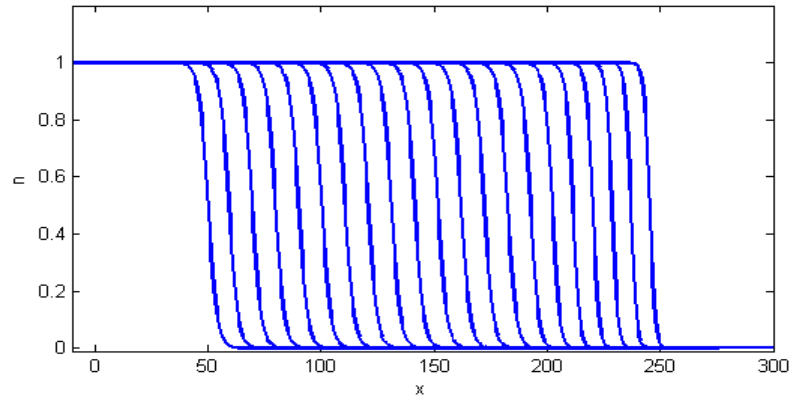**Figure 3**.The initial condition of solution to the system (3) with the parameters, $\alpha=1$.

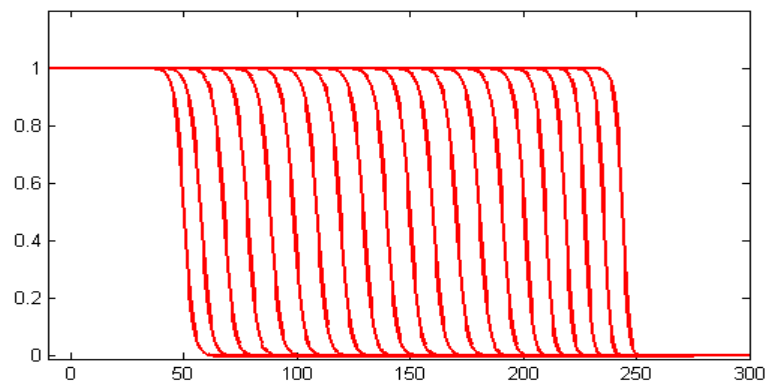**Figure 4.** The blue line represents tips (n)

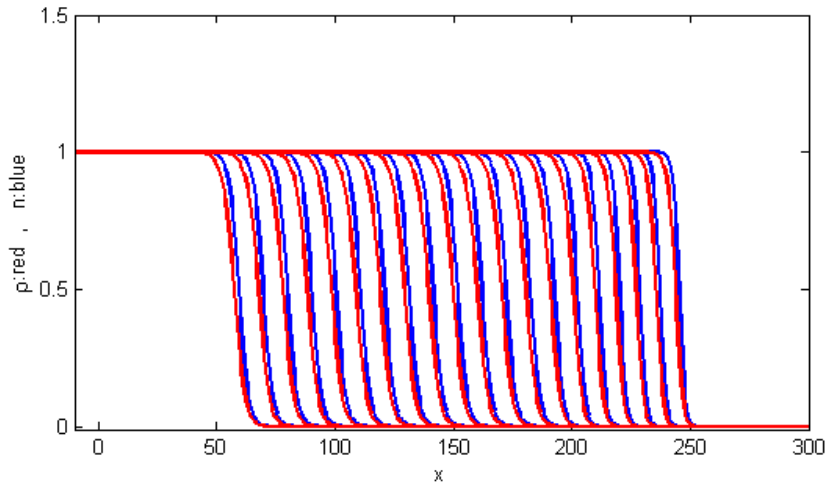**Figure 5.** The red line represents the branches (p)

Figure 6. The initial condition of solution to the system (3) with the parameters, $\alpha = 1$

From this operations, We get the relation between traveling waves solution c and α values with taking $v = d = 1$, we can show that the (2) table. Where α increasing the traveling waves solution C is increasing, (See Fig.7)

Table 2.The relation between waves speed c and α values with taking $v = d = 1$

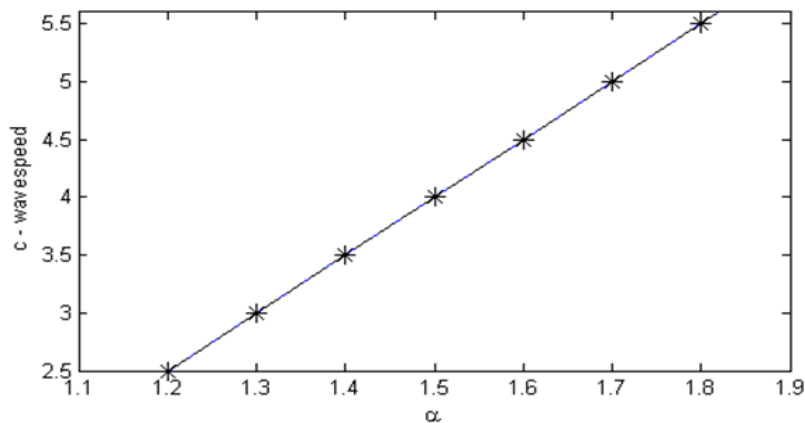| α	0.5	1	2	3	4	5	6	7	8
C	0.5	1.13	1.53	2.05	2.53	3.09	3.50	4.11	5.21

Figure 7. The relation between waves speed c and α values with taking v=d=1

Now, we get the relation between traveling waves solution c and υ values with taking α=d=1, we can show that the table (3). Where υ. increasing the traveling waves solution c is increasing (See fig.8).

Table 3.The relation between waves speed c and α values with taking $\alpha = d = 1$

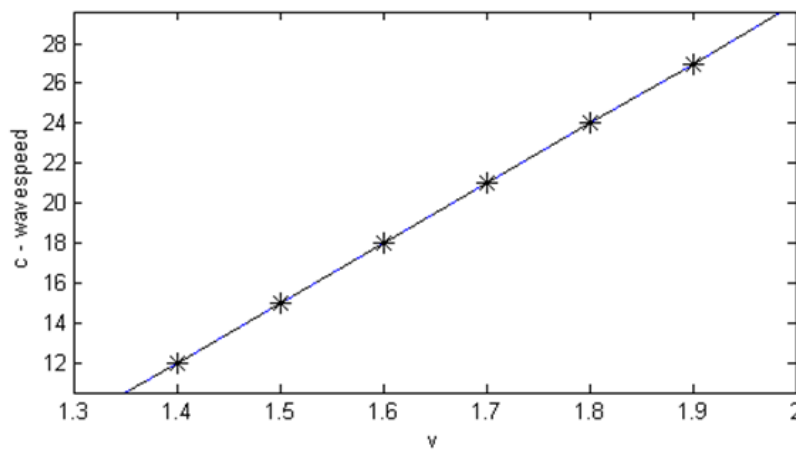| υ	0.5	1	2	3	4	5	6	7	8
c	0.5	1	1.6	2.43	3.1	4.2	4.4	5.3	6.3

Figure 8. The relation between waves speed c and υ values with taking $\alpha = d = 1$

Then, we get the relation between traveling waves solution c and d values with taking $\alpha = \upsilon = 1$,we can show that the (4) table. Where d increasing the traveling waves solution c is decreasing, See fig (9).

Table 4.The relation between waves speed c and α values with taking $\alpha = \upsilon = 1$

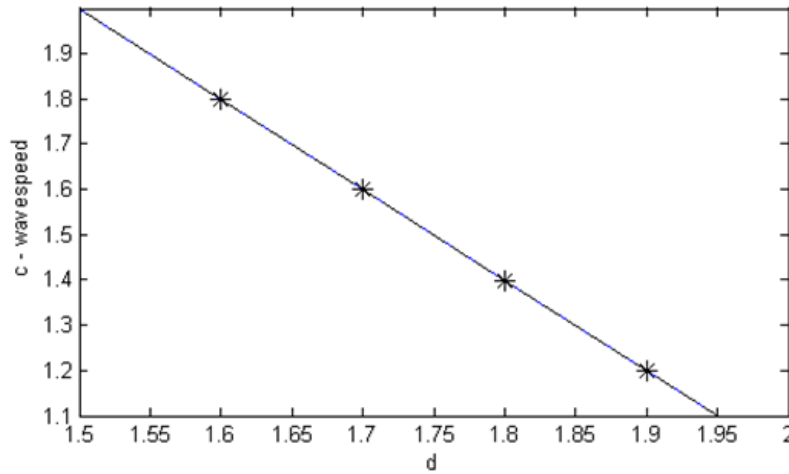| d	0.5	1	2	3	4	5	6	7	8
c	4.8	4.4	3.8	2.7	1.6	1.01	0.8	0.6	0.2

Figure 9. The relation between waves speed c and d values with taking $v = \alpha = 1$

6   Conclusion

In conclusion we plot relationship between C and α, See Fig. (6), that is clear the wave speed C is increasing when α increase function of α. Since ($\alpha = \frac{\alpha_1}{\gamma_1}$), therefore the growth rate is always increasing with α_1 while keeping and γ_1 is fixed, also the growth rate is always decreasing with γ_1 increases while keeping α_2 and v are fixed. We note that changing the rate of anastomosis β_2, cannot alter the colony growth rate, since the growth parameter α has no dependence on β_2. However, increasing β_2 word decrease the density levels accumulated in the interior see table (1) [3, 2].

α_1 = Is the number of tips produced per tip per unit time.

γ_1 = Is the loss rate of hyphal (constant for hyphal death).

β_2 = Is the rate of tip reconnections per unit length hypha per unit time.

7   References

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