

Certain Types of Soft Compact Maps

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Abstract— This paper holds to establish a soft compact map and to investigate its associations with soft compact maps, almost soft compact maps, besides mildly soft compact maps which are utilized from the relations between their spaces under some conditions. Consequently, the composition factors of soft compact maps with soft compact maps, almost soft compact maps, and mildly soft compact maps are studied based on the previous association between them. Many examples are given to explain the relationships between the topologies and relations of the soft set.

Keywords— soft compact maps, almost soft compact maps, mildly soft compact maps.

1 Introduction

Molodtsov at the end of the twentieth century presented the soft set with indeterminate information [1]. Afterward, Maji et. al.[2] demonstrated various novel concepts on soft sets for instance equality, subset, and the complement of a soft set. In 2010, Babitha and Sunil gave the concept of a soft set relation and function, and they explained the composition of functions [4]. In 2011, Shabir and Naz [5] originated soft topology and demonstrated some characteristics of soft separation axioms. Aygünoğlu and Aygün [6] established a conception of soft compact spaces. Hida [7] is equipped more powerful explanation for soft compact spaces than specified in [6]. Al-Shami et. al. [8] studied unprecedented forms of covering characteristics known as almost soft compact.

Kharal and Ahmad [3] characterized soft maps and instituted principal characteristics. Therefore, Zorlutuna and Çakir [9] investigated the notion of soft continuous maps. In continuation of their work, Addis et. al. in 2022 proposed a new definition for soft maps and investigate their characteristics [10].

The principal intent of this work is to create a soft compact map and to investigate its correlation between soft compact maps, almost soft compact maps, and mildly soft compact maps, which are utilized from the relations between their spaces under some conditions. Consequently, the composition factors of soft compact maps with soft compact maps, almost soft compact maps, and mildly soft compact maps are studied based on the previous association between them. Many examples are given to explain the relationships between the topologies and relations of the soft set.

2 Preliminaries

Definition 2.1 [1]: Let \mathbb{W} is an initial universal set, \mathbb{E} is a set of parameters, and Let $\mathbb{P}(\mathbb{W})$ indicates the power set. A couple (\mathbb{F}, \mathbb{E}) ($\mathbb{F}_{\mathbb{E}}$ for short) is known as a soft set if \mathbb{F} is a map of \mathbb{E} toward the set of all subsets of the set \mathbb{W} .

Definition 2.2 [2]: Let $\mathbb{F}_{\mathbb{E}}$ is a soft set over \mathbb{W} . Thus:

1) If $\mathbb{F}(\mathfrak{e}) = \emptyset$, for all $\mathfrak{e} \in \mathbb{E}$, so $\mathbb{F}_{\mathbb{E}}$ is known as a null soft set and we symbolize it by $\tilde{\emptyset}$.

2) If $\mathbb{F}(\mathfrak{e}) = \mathbb{W}$, for all $\mathfrak{e} \in \mathbb{E}$, so $\mathbb{F}_{\mathbb{E}}$ is known as an absolute soft set and we symbolize it by $\widetilde{\mathbb{W}}$.

Definition 2.3 [3]: Let $S(\mathbb{W}, \mathbb{E})$ and $S(\mathbb{M}, \mathbb{K})$ are families of all soft sets over \mathbb{W} and \mathbb{M} , one by one. The map φ_{ψ} is known as a soft map from \mathbb{W} to \mathbb{M} , indicated by $\varphi_{\psi}: S(\mathbb{W}, \mathbb{E}) \rightarrow S(\mathbb{M}, \mathbb{K})$, where $\varphi: \mathbb{W} \rightarrow \mathbb{M}$ and $\psi: \mathbb{E} \rightarrow \mathbb{K}$ are two maps.

1) Let $\mathbb{F}_{\mathbb{E}} \in S(\mathbb{W}, \mathbb{E})$, therefore the image of $\mathbb{F}_{\mathbb{E}}$ under the soft map φ_{ψ} is the soft set over \mathbb{M} indicated by $\varphi_{\psi}\mathbb{F}_{\mathbb{E}}$ and defined by

$$\varphi_{\psi}(\mathbb{F}_{\mathbb{E}})(\mathbb{k}) = \begin{cases} \bigcup_{\mathfrak{e} \in \psi^{-1}(\mathbb{k}) \cap \mathbb{E}} \varphi(\mathbb{F}(\mathfrak{e})), & \text{if } \psi^{-1}(\mathbb{k}) \cap \mathbb{E} \neq \emptyset; \\ \emptyset, & \text{otherwise.} \end{cases}$$

2) Let $\mathbb{G}_{\mathbb{K}} \in S(\mathbb{M}, \mathbb{K})$, therefore the pre-image of $\mathbb{G}_{\mathbb{K}}$ under the soft map φ_{ψ} is the soft set over \mathbb{W} indicated by $\varphi_{\psi}^{-1} \mathbb{G}_{\mathbb{K}}$ and defined by

$$\varphi_{\psi}^{-1}(\mathbb{G}_{\mathbb{K}})(\mathfrak{e}) = \begin{cases} \varphi^{-1}(\mathbb{G}_{\mathbb{K}}(\psi(\mathfrak{e}))), & \text{if } \psi(\mathfrak{e}) \in \mathbb{K}; \\ \emptyset, & \text{otherwise.} \end{cases}$$

The soft map φ_{ψ} is known as injective, if φ and ψ are injective. The soft map φ_{ψ} is known as surjective, if φ and ψ are surjective.

Definition 2.4 [5]: Let \mathbb{T} be a family of soft sets over \mathbb{W} , \mathbb{E} be a set of parameters. So \mathbb{T} is known as a soft topology on \mathbb{W} if the subsequent is satisfied:

- 1) $\tilde{\emptyset}$ and $\tilde{\mathbb{W}}$ are in \mathbb{T} .
- 2) the union of any number of soft sets in \mathbb{T} is in \mathbb{T} .
- 3) the intersection of any two soft sets in \mathbb{T} is in \mathbb{T} .

The triple $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is known as a soft topological space over \mathbb{W} . The members of \mathbb{T} are known as the soft open sets in \mathbb{W} . A soft set $\mathbb{F}_{\mathbb{E}}$ over \mathbb{W} is known as a soft closed set in \mathbb{W} , if its relative complement $\mathbb{F}'_{\mathbb{E}}$ belongs to \mathbb{T} .

Definition 2.5 [11]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ be a soft topological space over \mathbb{W} , $\mathbb{G}_{\mathbb{E}}$ be a soft set over \mathbb{W} , and $x \in \mathbb{W}$. Thus, $\mathbb{G}_{\mathbb{E}}$ is known as a soft neighborhood of x , if there exists a soft open set $\mathbb{F}_{\mathbb{E}}$ such that $x \in \mathbb{F}_{\mathbb{E}} \subseteq \mathbb{G}_{\mathbb{E}}$.

Definition 2.6 [12]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}', \mathbb{E})$ are two soft topological spaces, $\mathcal{L} : (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a soft map. For each soft neighborhood $\mathbb{G}_{\mathbb{E}}$ of $\mathbb{G}_{\mathbb{E}}(x)$, if there exists a soft neighborhood $\mathbb{F}_{\mathbb{E}}$ of x , such that $\mathcal{L}(\mathbb{F}_{\mathbb{E}}) \subseteq \mathbb{G}_{\mathbb{E}}$, therefore \mathcal{L} is known as a soft continuous map at x . If \mathcal{L} is a soft continuous map for all x , therefore \mathcal{L} is known as a soft continuous map.

Theorem 2.7 [12]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}', \mathbb{E})$ are two soft topological spaces, $\mathcal{L} : (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ be a soft map. Thus the subsequent conditions are equivalent:

- 1) \mathcal{L} is a soft continuous map.
- 2) For each soft open set $\mathbb{G}_{\mathbb{E}}$ over \mathbb{M} , $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a soft open set over \mathbb{W} .
- 3) For each soft closed set $\mathbb{H}_{\mathbb{E}}$ over \mathbb{M} , $\mathcal{L}^{-1}(\mathbb{H}_{\mathbb{E}})$ is a soft closed set over \mathbb{W} .

Definition 2.8 [8]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a soft topological space. A subcollection β of \mathbb{T} is known as a base for \mathbb{T} if every member of \mathbb{T} can be expressed as a union of members of β .

Proposition 2.9 [13]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a soft topological space and $\mathbb{F}_{\mathbb{E}}$ be any soft set over \mathbb{W} thus:

- 1) $\mathbb{G}_{\mathbb{E}} \subseteq \mathbb{F}_{\mathbb{E}}$ is closed in $\mathbb{T}_{\mathbb{F}_{\mathbb{E}}}$ iff $\mathbb{G}_{\mathbb{E}} = \mathbb{F}_{\mathbb{E}} \tilde{\cap} \mathbb{H}_{\mathbb{E}}$ where $\mathbb{H}_{\mathbb{E}}$ is closed in $(\mathbb{W}, \mathbb{T}, \mathbb{E})$.
- 2) β be an open base of \mathbb{T} therefore $\beta_{\mathbb{F}_{\mathbb{E}}} = \{\mathbb{G}_{\mathbb{E}} \tilde{\cap} \mathbb{F}_{\mathbb{E}} : \mathbb{G}_{\mathbb{E}} \in \beta\}$ is an open base of $\mathbb{T}_{\mathbb{F}_{\mathbb{E}}}$.

Definition 2.10 [14]: Let soft topological space $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is :

- 1) A family $\mathfrak{F} = \{F_{\mathbb{E} i} : i \in \Delta\}$ of open soft sets in $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is known as a soft open cover of \mathbb{W} , if it satisfies $\bigcup_{i \in \Delta} F_{\mathbb{E} i} = \mathbb{W}$ a finite subfamily of a soft open cover \mathfrak{A} of \mathbb{W} is known as a finite subcover of \mathfrak{A} , if it is also a soft open cover of \mathbb{W} .
- 2) \mathbb{W} is known as a soft compact space (for short, a \mathcal{SC} -space) if every soft open cover of \mathbb{W} has a finite subcover.

Proposition 2.11 [15]:

- 1) In a soft topology $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ if the sets \mathbb{W} and \mathbb{E} are finite therefore $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a \mathcal{SC} -space.
- 2) If the soft sets $G_{\mathbb{E} i}$ in a soft topological space $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ are finite therefore $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a \mathcal{SC} -space.

Example 2.12[15] The soft cofinite topology (where \mathbb{E} is a finite set) is a \mathcal{SC} -space.

Definition 2.13 [8]: A soft topological space $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is known as an almost \mathcal{SC} -space if every soft open cover of \mathbb{W} has a finite soft sub-cover the soft closure of whose members cover \mathbb{W} .

Example 2.14 [8]: Let $(\mathbb{R}, \mathbb{T}, \mathbb{E})$ is a soft topological space such that $\mathbb{E} = \{e_1, e_2\}$ and $\mathbb{T} = \{G_{\mathbb{E}} \subseteq \tilde{\mathbb{R}} \text{ such that } 1 \in G_{\mathbb{E}}\}$. Therefore $(\mathbb{R}, \mathbb{T}, \mathbb{E})$ is an almost \mathcal{SC} -space.

Definition 2.15 [8]: A soft topological space $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ known as is a mildly \mathcal{SC} -space if every soft clopen cover of \mathbb{W} has a finite soft sub-cover.

Example 2.16 [8]: Let $\mathbb{E} = \{e_1, e_2\}$ is a set of parameters and $\mathbb{T} = \{G_{\mathbb{E}} \subseteq \mathbb{R} : \text{either } [1 \in G_{\mathbb{E}} \text{ and } (G_{\mathbb{E}}^c \text{ is finite)} \text{ or } 1 \notin G_{\mathbb{E}}]\}$ be a soft topology on \mathbb{R} . $(\mathbb{R}, \mathbb{T}, \mathbb{E})$ is a mildly \mathcal{SC} -space.

Proposition 2.17 [8]: Every \mathcal{SC} -space is an almost \mathcal{SC} -space.

Proposition 2.18 [8]: Every almost \mathcal{SC} -space is a mildly \mathcal{SC} -space.

Theorem 2.19 [8]: Consider $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ has a soft base that comprises soft clopen sets. Therefore $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a \mathcal{SC} -space if and only if it is mildly \mathcal{SC} -space.

Theorem 2.20 [16]: If $G_{\mathbb{E}}$ is a \mathcal{SC} -subset of \mathbb{W} and $F_{\mathbb{E}}$ is a soft closed subset of \mathbb{W} thus $G_{\mathbb{E}} \cap F_{\mathbb{E}}$ is a \mathcal{SC} -set.

Theorem 2.21 [8]: If $G_{\mathbb{E}}$ is an almost (one by one a mildly) \mathcal{SC} -subset of \mathbb{W} and $F_{\mathbb{E}}$ is a soft clopen subset of \mathbb{W} , therefore $G_{\mathbb{E}} \cap F_{\mathbb{E}}$ is an almost (one by one a mildly) \mathcal{SC} -set.

3 Soft compact map

Definition 3.1: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}', \mathbb{E})$ are two soft topological spaces, and Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a soft map. Thus, \mathcal{L} is known as a \mathcal{SC} -map, if it is a soft surjective continuous map, and if the pre-image of every \mathcal{SC} -subset of \mathbb{M} is a \mathcal{SC} -subset of \mathbb{W} .

Example 3.2: Let $\mathbb{W} = \mathbb{R}, \mathbb{E} = \{0\}$ and $\mathbb{T} = \{\tilde{\emptyset}, \widetilde{\mathbb{W}}, \mathbb{G}_{\mathbb{E}}\}$ are soft topological space on \mathbb{W} such that $\mathbb{G}(0) = (-1, 1)$. A map $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ such that $\mathcal{L}(\mathcal{X}_{\mathbb{e}}) = -\mathcal{X}_{\mathbb{e}}, \forall x \in \mathbb{W}$, therefore \mathcal{L} is a \mathcal{SC} - map.

Example 3.3: Let $\mathbb{W} = \mathbb{E}$ be an uncountable set and $\mathcal{L}: (\mathbb{W}, \mathbb{T}_{\text{dis}}, \mathbb{E}) \rightarrow (\mathbb{W}, \mathbb{T}_{\text{dis}}, \mathbb{E})$ is a soft map such that $\mathcal{L}(\mathcal{X}_{\mathbb{e}}) = \mathcal{X}_{\mathbb{e}}$, therefore \mathcal{L} is not a \mathcal{SC} - map.

Definition 3.4: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}', \mathbb{E})$ are two soft topological spaces, and Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a soft map. Thus, \mathcal{L} is known as a mildly \mathcal{SC} - map, if it is a soft surjective continuous map, and if the inverse of every mildly \mathcal{SC} - set in \mathbb{M} is a mildly \mathcal{SC} - set in \mathbb{W} .

Example 3.5: Consider $\mathbb{W} = \mathbb{Z}^+$ and $\mathbb{E} = \{\mathbb{e}_1, \mathbb{e}_2\}$ is a set of parameters and $\mathbb{T} = \{\tilde{\emptyset}, \widetilde{\mathbb{W}}, \mathbb{F}_{\mathbb{E}}, \mathbb{G}_{\mathbb{E}}\}$ such that $\mathbb{F}_{\mathbb{E}} = \{(\mathbb{e}_1, \mathbb{Z}_0^+), (\mathbb{e}_2, \mathbb{Z}_0^+)\}, \mathbb{G}_{\mathbb{E}} = \{(\mathbb{e}_1, \mathbb{Z}_e^+), (\mathbb{e}_2, \mathbb{Z}_e^+)\}$. A map $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}, \mathbb{E})$ defined by $\mathcal{L}(1_{\mathbb{e}_1}) = 2_{\mathbb{e}_1}, \mathcal{L}(x_{\mathbb{e}_1}) = x_{\mathbb{e}_1}, \forall x \in \mathbb{Z}, x > 1$ and $\mathcal{L}(x_{\mathbb{e}_2}) = x_{\mathbb{e}_2}, \forall x \in \mathbb{Z}^+$ therefore $\mathcal{L}_{\mathcal{E}}$ is a mildly \mathcal{SC} - map.

Example 3.6: Let $\mathbb{W} = \{0\} \cup \{\frac{1}{x}: x \in \mathbb{N}\}$, therefore for any parameter finite \mathbb{E} when \mathbb{W} with soft usual topology a map $\mathcal{L}: (\mathbb{W}, \mathbb{T}_u, \mathbb{E}) \rightarrow (\mathbb{W}, \mathbb{T}_u, \mathbb{E})$ indicated by $\mathcal{L}(\tilde{0}) = \tilde{0}, \mathcal{L}(\frac{1}{x}) = \frac{1}{x+1}, \forall x \in \mathbb{N}$, thus \mathcal{L} is not a mildly \mathcal{SC} - map.

Definition 3.7: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}', \mathbb{E})$ are two soft topological spaces, and Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a soft map. Therefore, \mathcal{L} is known as an almost \mathcal{SC} - map, if it is a soft surjective continuous map, and if the inverse image of every almost \mathcal{SC} - set in \mathbb{M} is an almost \mathcal{SC} - set in \mathbb{W} .

Example 3.8: Let $\mathbb{E} = \{\mathbb{e}_1, \mathbb{e}_2\}$ is a set of parameters and $\mathbb{T} = \{\mathbb{G}_{\mathbb{E}} \cong \mathbb{R} \text{ such that } 1 \in \mathbb{G}_{\mathbb{E}}\}$ and $\mathbb{T}' = \{\mathbb{G}_{\mathbb{E}} \cong \mathbb{R}: \text{either } [1 \in \mathbb{G}_{\mathbb{E}} \text{ and } (\mathbb{G}_{\mathbb{E}}^c \text{ is finite]} \text{ or } 1 \notin \mathbb{G}_{\mathbb{E}}\}$ are an almost \mathcal{SC} -space [21]. A map $\mathcal{L}: (\mathbb{R}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{R}, \mathbb{T}', \mathbb{E})$ such that $\mathcal{L}(x_{\mathbb{e}}) = 2x_{\mathbb{e}}, \forall x \in \mathbb{R}$, \mathcal{L} is an almost \mathcal{SC} -map.

Example 3.9: Let $\mathcal{L}: (\mathbb{R}, \mathbb{T}_u, \mathbb{E}) \rightarrow (\mathbb{R}, \mathbb{T}_{co}, \mathbb{E})$ is a soft map, and $\mathbb{E} = \{\mathbb{e}_1, \mathbb{e}_2\}$ is a set of parameters $\mathcal{L}(x_{\mathbb{e}}) = x_{\mathbb{e}}, \forall x \in \mathbb{R}$, therefore, \mathcal{L} is not an almost \mathcal{SC} - map.

Theorem 3.10: Every \mathcal{SC} -map is a mildly \mathcal{SC} - map, when the co-domain has a base is comprises soft clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map such that \mathbb{M} has a base comprises soft clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ be a mildly \mathcal{SC} set in \mathbb{M} . Since \mathbb{M} has a soft base consisting of soft clopen sets. Thus, $\mathbb{G}_{\mathbb{E}}$ has a soft base consisting of soft clopen sets by Proposition 2.9. Thus, $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} by Theorem 2.19 So, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} by definition of the \mathcal{SC} - map. As a result of Proposition 2.17 and Proposition 2.18 $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{W} . Therefore, \mathcal{L} is a mildly \mathcal{SC} - map. ■

Theorem 3.11: Every mildly \mathcal{SC} - map is a \mathcal{SC} - map, when the domain has a base comprises soft clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a mildly \mathcal{SC} - map such that \mathbb{W} has a base comprises soft clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} . Therefore, $\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{M} by Proposition 2.17 and Proposition 2.18, thus $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{W} by definition of a mildly \mathcal{SC} - map. Since \mathbb{W} has a base comprises soft clopen sets and $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ a mildly \mathcal{SC} - subset of \mathbb{W} , therefore $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ has a base comprises of soft clopen sets by Proposition 2.9. As a result of Theorem 2.19, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} . Therefore, \mathcal{L} is a \mathcal{SC} - map. ■

Theorem 3.12: Every \mathcal{SC} -map is an almost \mathcal{SC} - map, when the co-domain has a base comprises soft clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map such that \mathbb{M} has a base comprises soft clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ be an almost \mathcal{SC} - set in \mathbb{W} , so $\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{M} by Proposition 2.18. Since \mathbb{M} has a base comprises soft clopen sets, therefore $\mathbb{G}_{\mathbb{E}}$ has a soft base consisting of soft clopen sets by Proposition 2.9. Thus, $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} by Theorem 2.19. Therefore, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} due to \mathcal{L} is a \mathcal{SC} - map. Proposition 2.17 implies that $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost \mathcal{SC} - set in \mathbb{W} . Therefore, \mathcal{L} is an almost \mathcal{SC} - map. ■

Theorem 3.13: Every almost \mathcal{SC} - map is a \mathcal{SC} - map, when the domain has a base comprises soft clopen sets.

Proof: $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} - map such that \mathbb{W} has a base comprises soft clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} by Proposition 2.17 $\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} - set in $\mathbb{M}_{\mathcal{E}}$. $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost \mathcal{SC} - set in \mathbb{W} by definition almost \mathcal{SC} -

map. So, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{W} by Proposition 2.18. $\mathbb{W}_{\mathcal{E}}$ has a base comprises soft clopen sets therefore $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ has a soft base consisting of soft clopen sets by Proposition 2.9. As a result of Theorem 2.19. $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} . Therefore, \mathcal{L} is a \mathcal{SC} - map. ■

Theorem 3.14: Every almost \mathcal{SC} -map is a mildly \mathcal{SC} -map. When the co-domain has a base of soft clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} -map such that \mathbb{M} has a base clopen set. Suppose that $\mathbb{G}_{\mathbb{E}}$ be a mildly \mathcal{SC} - set in \mathbb{M} . Thus, $\mathbb{G}_{\mathbb{E}}$ has a base of soft clopen sets by Proposition 2.9. So, $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} and as a result of Proposition 2.17, $\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} - set in \mathbb{M} . Thus, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost \mathcal{SC} - set in \mathbb{W} because \mathcal{L} is an almost \mathcal{SC} - map. Therefore, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{W} by Proposition 2.18. Therefore, \mathcal{L} is a mildly \mathcal{SC} - map. ■

Theorem 3.15: Every mildly \mathcal{SC} - map is an almost \mathcal{SC} - map. When the domain has a base of soft clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is mildly \mathcal{SC} - map such that \mathbb{W} has a base of a soft clopen set. Suppose that $\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} - set in \mathbb{M} . $\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{M} by Proposition 2.18. Therefore $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{W} by definition of a mildly \mathcal{SC} - map. Therefore $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ has a base of soft clopen sets because of Proposition 2.9. As a result of Theorem 2.19, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} by Proposition 2.17 $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a soft and almost \mathcal{SC} - set in \mathbb{W} , Therefore, \mathcal{L} is an almost \mathcal{SC} - map. ■

4 Restriction of types of soft compact maps

Theorem 4.1: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map. If $\mathbb{A}_{\mathbb{E}}$ is a soft closed subset of \mathbb{W} the restriction $g = \mathcal{L}|(\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}): (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map, $\mathbb{A}_{\mathbb{E}}$ is a soft closed subset of \mathbb{W} , the relative topology on $\mathbb{A}_{\mathbb{E}}$ is $\mathbb{T}_{\mathbb{A}} = \{\mathbb{A}_{\mathbb{E}}^* = \mathbb{A}_{\mathbb{E}} \cap \mathbb{F}_{\mathbb{E}}, \forall \mathbb{F}_{\mathbb{E}} \in \mathbb{T}\}$. Suppose $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{M} , $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{W} since \mathcal{L} a \mathcal{SC} - map. Therefore $\mathbb{A}_{\mathbb{E}} \cap \mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}}) \in \mathbb{T}_{\mathbb{A}}$ is a \mathcal{SC} - set by Theorem 2.20. Therefore $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map. ■

Corollary 4.2: Let $g = (A_E, T_A, E) \rightarrow (W, T, E)$ is a \mathcal{SC} - map and $A_E \subseteq W$ therefore A_E is a soft closed set.

Proof: Let $g = (A_E, T_A, E) \rightarrow (W, T, E)$ is a \mathcal{SC} - map, the relative topology on A_E is $T_A = \{A_E^* = A_E \cap F_E, \forall F_E \in T\}$. Suppose that G_E is a \mathcal{SC} -set in W , $\mathcal{L}^{-1}(G_E) = A_E \cap G_E$ is a \mathcal{SC} - set since g is a \mathcal{SC} - map, but G_E is a \mathcal{SC} - set. As a result Theorem 2.20 A_E is a soft closed set. ■

Theorem 4.3: Let $\mathcal{L}: (W, T, E) \rightarrow (M, T', E)$ is an almost (one by one a mildly) \mathcal{SC} - map. If A_E is a soft clopen subset of W the restriction $g = \mathcal{L}|(A_E, T_A, E): (A_E, T_A, E) \rightarrow (M, T', E)$ is an almost (one by one a mildly) \mathcal{SC} - map.

Proof: Let $\mathcal{L}: (W, T, E) \rightarrow (M, T', E)$ is an almost (one by one a mildly) \mathcal{SC} - map, A_E is a soft clopen subset of W , the relative topology on A_E is $T_A = \{A_E^* = A_E \cap F_E, \forall F_E \in T\}$. Suppose G_E is an almost (one by one a mildly) \mathcal{SC} - set in M , $\mathcal{L}^{-1}(G_E)$ is an almost (one by one a mildly) \mathcal{SC} - set in W since \mathcal{L} is an almost (one by one a mildly) \mathcal{SC} -map. Therefore $A_E \cap \mathcal{L}^{-1}(G_E) \in T_A$ is an almost (one by one a mildly) \mathcal{SC} -set by Theorem 2.21. Therefore $g: (A_E, T_A, E) \rightarrow (M, T', E)$ is an almost (one by one a mildly) \mathcal{SC} - map. ■

Corollary 4.4: Let $g = (A_E, T_A, E) \rightarrow (W, T, E)$ is an almost(one by one a mildly) \mathcal{SC} - map and $A_E \subseteq W$ therefore A_E is soft clopen.

Proof: Let $g = (A_E, T_A, E) \rightarrow (W, T, E)$ is an almost (one by one a mildly) \mathcal{SC} - map, the relative topology on A_E is $T_A = \{A_E^* = A_E \cap F_E, \forall F_E \in T\}$. Suppose that G_E is an almost (one by one a mildly) \mathcal{SC} - set in W , $\mathcal{L}^{-1}(G_E) = A_E \cap G_E$ is an almost (one by one a mildly) \mathcal{SC} -set since g is an almost (one by one a mildly) \mathcal{SC} -map, but G_E is an almost (one by one a mildly) \mathcal{SC} - set. As a result Theorem 2.21 A_E is soft clopen. ■

5 Composition of certain types of soft compact maps

Theorem 5.1: The composition of \mathcal{SC} - maps (one by one almost \mathcal{SC} - maps, mildly \mathcal{SC} - maps) is also a \mathcal{SC} - map (one by one almost \mathcal{SC} - maps, mildly \mathcal{SC} - map).

Proof: Let $\mathcal{L}: (W, T, E) \rightarrow (J, T', E)$ and $\mathcal{h}: (J, T', E) \rightarrow (M, T'', E)$ are two \mathcal{SC} - maps (one by one almost \mathcal{SC} - maps, mildly \mathcal{SC} - maps). To verify that $\mathcal{h} \circ \mathcal{L}$ is also \mathcal{SC} - maps (one by one almost \mathcal{SC} - maps, mildly \mathcal{SC} - maps). Suppose that G_E is a \mathcal{SC} - (one by one an almost \mathcal{SC} -, a mildly \mathcal{SC} -) set in M . (to show that $(\mathcal{h} \circ \mathcal{L})^{-1}G_E$ is a \mathcal{SC} - set (one by

one an almost \mathcal{SC} -set, a mildly \mathcal{SC} - set) in \mathbb{W} . We have $\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set (one by one an almost \mathcal{SC} -set, a mildly \mathcal{SC} - set) in \mathbb{J} since \mathcal{h} is a \mathcal{SC} - map (one by one an almost \mathcal{SC} -map, a mildly \mathcal{SC} - map). Therefore $\mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is a \mathcal{SC} - set (one by one an almost \mathcal{SC} -set, a mildly \mathcal{SC} - set) in \mathbb{W} because \mathcal{L} is a \mathcal{SC} - map (one by one an almost \mathcal{SC} -map, a mildly \mathcal{SC} - map). We have $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So, $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set (one by one an almost \mathcal{SC} -set, a mildly \mathcal{SC} - set) in \mathbb{W} . Therefore, $\mathcal{h} \circ \mathcal{L}$ is also a \mathcal{SC} - map (one by one an almost \mathcal{SC} -map, a mildly \mathcal{SC} - map). ■

Theorem 5.2: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} - map and $\mathcal{h}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly \mathcal{SC} - map. If $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ has a soft base of soft clopen sets, therefore $\mathcal{h} \circ \mathcal{L}$ is a mildly \mathcal{SC} - map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{M} (to show that $\mathcal{h} \circ \mathcal{L}$ is a mildly \mathcal{SC} - map). we have $\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} - set in \mathbb{J} since \mathcal{h} is a mildly \mathcal{SC} - map. Therefore \mathcal{L} is a \mathcal{SC} -map with a co-domain that has a soft base of soft clopen sets. As a result of Theorem 3.10, we get $\mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is a mildly \mathcal{SC} - set in \mathbb{W} , because of $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$.so, $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{W} . Therefore, $\mathcal{h} \circ \mathcal{L}$ is also a mildly \mathcal{SC} - map. ■

Corollary 5.3: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} -map and $\mathcal{h}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ have a soft basis of soft clopen sets, therefore $\mathcal{h} \circ \mathcal{L}$ is a \mathcal{SC} - map.

Proof: By Theorem 5.2 and Theorem 3.11. ■

Theorem 5.4: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly \mathcal{SC} - map and $\mathcal{h}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ has a soft base of a soft clopen set Therefore $\mathcal{h} \circ \mathcal{L}$ is a \mathcal{SC} - map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} -set in \mathbb{M} .(to show that $\mathcal{h} \circ \mathcal{L}$ is a \mathcal{SC} -map). We have $\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} - set in \mathbb{J} since \mathcal{h} is a \mathcal{SC} - map. Therefore \mathcal{L} is a mildly \mathcal{SC} - map with a domain that has a soft base of a soft clopen set. As a result of Theorem 3.11 $\mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is a \mathcal{SC} - set in \mathbb{W} . Because of $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} - set in \mathbb{W} . Therefore, $\mathcal{h} \circ \mathcal{L}$ is also a \mathcal{SC} - map. ■

Corollary 5.5: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly \mathcal{SC} - map and $\mathcal{h}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ with $(\mathbb{M}, \mathbb{T}'', \mathbb{E})$ have a soft basis of soft clopen sets, therefore $\mathcal{h} \circ \mathcal{L} (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly \mathcal{SC} - map.

Proof: By Theorem 5.4 and Theorem 3.10. ■

Theorem 5.6: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ a \mathcal{SC} -map and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost \mathcal{SC} -map. If $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ has a soft base of soft clopen. Therefore $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} -map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} -set in $\mathbb{M}_{\mathcal{E}}$. (to show that $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} -map). We have $\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost \mathcal{SC} -set in \mathbb{J} since \mathcal{H} is an almost \mathcal{SC} -map. Therefore $\mathcal{L}_{\mathcal{E}}$ is a \mathcal{SC} -map with co-domain has a base soft clopen set. As a result of Theorem 3.12, we get $\mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is an almost \mathcal{SC} -set in \mathbb{W} . Because $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} -set in \mathbb{W} . Therefore, $\mathcal{H} \circ \mathcal{L}$ is also an almost \mathcal{SC} -map. ■

Corollary 5.7: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a \mathcal{SC} -map, and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost \mathcal{SC} -map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ with $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ have a soft basis of soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is a \mathcal{SC} -map.

Proof: By Theorem 5.6 and Theorem 3.13. ■

Theorem 5.8: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} -map, and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a \mathcal{SC} -map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ has a base soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is a \mathcal{SC} -map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} -set in \mathbb{M} (to show that $\mathcal{H} \circ \mathcal{L}$ is a \mathcal{SC} -map). we have $\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a \mathcal{SC} -set in \mathbb{J} since \mathcal{H} is a \mathcal{SC} -map. Therefore \mathcal{L} is an almost \mathcal{SC} -map with a domain that has a base soft clopen set. As a result, of Theorem 3.13 we get $\mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is a \mathcal{SC} -set in \mathbb{W} . Because of $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a \mathcal{SC} -set in \mathbb{W} . Therefore, $\mathcal{H} \circ \mathcal{L}$ is also a \mathcal{SC} -map. ■

Corollary 5.9: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} -map, and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a \mathcal{SC} -map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}'', \mathbb{E})$ have a soft base of a soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} -map.

Proof: By Theorem (5.8) and Theorem (3.12). ■

Theorem 5.10: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} -map, and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly \mathcal{SC} -map. If $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ has a soft base of a soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is a mildly \mathcal{SC} -map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} -set in \mathbb{M} . We have $\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly \mathcal{SC} -set in \mathbb{J} since \mathcal{H} is a mildly \mathcal{SC} -map. Therefore \mathcal{L} is an almost \mathcal{SC} -map with co-domain has

a soft base of a soft clopen set. As a result, of Theorem 3.14 we get $\mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$. is a mildly \mathcal{SC} - set in \mathbb{W} . Because of $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a mildly \mathcal{SC} - set in \mathbb{W} . Therefore, $\mathcal{H} \circ \mathcal{L}$ is a mildly \mathcal{SC} -map. ■

Corollary 5.11: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost \mathcal{SC} - map, and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ with $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ have a soft base of a soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} - map.

Proof: By Theorem 5.10 and Theorem 3.15. ■

Theorem 5.12: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly \mathcal{SC} -map and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ has a soft base soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} - map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} - set in \mathbb{M} . We have $\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost \mathcal{SC} - set in \mathbb{J} since \mathcal{H} is an almost \mathcal{SC} -map. Therefore \mathcal{L} is a mildly \mathcal{SC} - map with a domain that has a base soft clopen set. As a result, of Theorem 3.15 we get $\mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$ is an almost \mathcal{SC} -set in \mathbb{W} . Because of $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\mathcal{H}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{H} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is an almost \mathcal{SC} - set in \mathbb{W} . Therefore, $\mathcal{H} \circ \mathcal{L}$ is an almost \mathcal{SC} - map. ■

Corollary 5.13: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly \mathcal{SC} -map and $\mathcal{H}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost \mathcal{SC} - map. If $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ with $(\mathbb{M}, \mathbb{T}'', \mathbb{E})$ have a soft base of a soft clopen set. Therefore $\mathcal{H} \circ \mathcal{L}$ is a mildly \mathcal{SC} -map.

Proof: By Theorem 5.12 and Theorem 3.14. ■

6 Conclusion

To summarize, we launch in this work a soft compact map and to investigate its associations with soft compact maps, almost soft compact maps, besides mildly soft compact maps which are utilized from the relations between their spaces under some conditions. Consequently, the composition factors of soft compact maps with soft compact maps, almost soft compact maps, and mildly soft compact maps are studied based on the previous association between them.

7 References

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