Investigation on Soft Semi-Compact Maps

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Abstract— This paper holds to establish a soft semi-compact map and to investigate its associations with soft semi-compact maps, almost soft semi-compact maps, besides mildly soft semi-compact maps which are utilized from the relations between their spaces under some conditions. Consequently, the composition factors of soft compact maps with soft semi-compact maps, almost soft semi-compact maps, and mildly soft semi-compact maps are studied based on the previous association between them. Many examples are given to explain the relationships between the topologies and relations of the soft set.

Keywords— soft semi-compact maps, almost soft semi-compact maps, mildly soft semi-compact maps.

1 Introduction

Molodtsov at the end of the twentieth century presented the soft set with indeterminate information [1]. Afterward, Maji et al. [2] demonstrated numerous novel concepts on soft sets for instance equality, subset, and the complement of a soft set. In 2010, Babitha and Sunil gave the concept of a soft set relation and function, and they explained the composition of functions [3]. Shabir and Naz [4] 2011 originated soft topology and demonstrated some features of soft separation axioms. Aygünoğlu and Aygün [5] established the conception of soft compact spaces. Hida [6] is equipped more powerful explanation for soft compact spaces than space as long as in [5]. Al-Shami et. al. [7] studied unprecedented forms of covering features known as almost soft compact.

Kharal and Ahmad [8] characterized soft maps and instituted principal features. Subsequently, Zorlutuna and Çakir [9] investigated the notion of soft continuous maps.

In continuation of their work, Addis et. al. in 2022 proposed a new definition for soft maps and investigate their features [8].

The principal intent of this work is to create a soft semi-compact map and to investigate its correlation between soft semi-compact maps, almost soft semi-compact maps, with mildly soft semi-compact maps, which are utilized from the relations between their spaces under some conditions. Consequently, the composition factors of soft semi-compact maps with soft semi-compact maps, almost soft semi-compact maps, with mildly soft semi-compact maps are studied based on the previous association between them. Many examples are given to explain the relationships between the topologies and relations of the soft set.

2 Preliminaries

Definition 2.1 [1]: Let $\mathbb{W}$ be an initial universal set, $\mathbb{E}$ be a set of parameters, and Let $\mathbb{P}(\mathbb{W})$ sign as long as the power set of $\mathbb{W}$. A pair $(\mathbb{F},\mathbb{E})$ ($\mathbb{F}_{\mathbb{E}}$ for short) is known as a soft set as long as $\mathbb{F}$ is a map of $\mathbb{E}$ into the set of all subsets of the set $\mathbb{W}$.

Definition 2.2 [2]: Let $\mathbb{F}_{\mathbb{E}}$ be a soft set over $\mathbb{W}$. Subsequently:

1) As long as $\mathbb{F}(e) = \phi$, for all $e \in \mathbb{E}$, so $\mathbb{F}_{\mathbb{E}}$ is known as a null soft set and we symbolize it by $\widetilde{\emptyset}$.
2) As long as $\mathbb{F}(e) = \mathbb{W}$, for all $e \in \mathbb{E}$, so $\mathbb{F}_{\mathbb{E}}$ is known as an absolute soft set and we symbolize it by $\widetilde{\mathbb{W}}$.

Definition 2.3 [8]: Let $S(\mathbb{W},\mathbb{E})$ with $S(\mathbb{M},\mathbb{K})$ are families of all soft sets over $\mathbb{W}$ and $\mathbb{M}$, one by one. The map φ_ψ is known as a soft map from $\mathbb{W}$ to $\mathbb{M}$, indicated by φ_ψ: $S(\mathbb{W},\mathbb{E}) \to S(\mathbb{M},\mathbb{K})$, where φ: $\mathbb{W} \to \mathbb{M}$ and ψ: $\mathbb{E} \to \mathbb{K}$ are two maps.

1) Let $\mathbb{F}_{\mathbb{E}} \in S(\mathbb{W},\mathbb{E})$, therefore the image of $\mathbb{F}_{\mathbb{E}}$ under the soft map φ_ψ is the soft set over $\mathbb{M}$ indicated by $\varphi_\psi \mathbb{F}_{\mathbb{E}}$ and defined by

$$\varphi_\psi(\mathbb{F}_{\mathbb{E}})(\Bbbk) = \begin{cases} \bigcup_{e \in \psi^{-1}(\Bbbk) \cap \mathbb{E}} \varphi\big(\mathbb{F}(e)\big), \text{as long as } \psi^{-1}(\mathbb{K}) \cap \mathbb{E} \neq \emptyset; \\ \emptyset \quad , \quad othrewise. \end{cases}$$

2) Let $\mathbb{G}_{\mathbb{K}} \in S(\mathbb{M},\mathbb{K})$, therefore the pre-image of $\mathbb{G}_{\mathbb{K}}$ under the soft map φ_ψ is the soft set over $\mathbb{W}$ indicated by ${\varphi_\psi}^{-1}\, \mathbb{G}_{\mathbb{K}}$ and defined by

$$\varphi_{\psi}{}^{-1}(\mathbb{G}_{\mathbb{K}})(e)=\begin{cases}\varphi^{-1}(\mathbb{G}_{\mathcal{K}}(\psi(e))), & \text{as long as } \psi(e)\in\mathbb{K};\\ \emptyset, & \text{otherwise.}\end{cases}$$

The soft map φ_{ψ} is known as injective, as long as φ and ψ are injective. The soft map φ_{ψ} is known as surjective, as long as φ and ψ are surjective.

Definition 2.4 [4]: Let $\mathbb{T}$ be a family of soft sets over $\mathbb{W}$, $\mathbb{E}$ be a set of parameters. So $\mathbb{T}$ is known as a soft topology on $\mathbb{W}$ as long as the subsequent is satisfied:

1) $\tilde{\phi}$ and $\widetilde{\mathbb{W}}$ are in $\mathbb{T}$.
2) the union of any number of soft sets in $\mathbb{T}$ is in $\mathbb{T}$.
3) the intersection of any two soft sets in $\mathbb{T}$ is in $\mathbb{T}$.

The triple $(\mathbb{W},\mathbb{T},\mathbb{E})$ is known as a soft topological space ($\mathcal{STS}$ for short) over $\mathbb{W}$. The members of $\mathbb{T}$ are known as the soft open sets in $\mathbb{W}$. A soft set $\mathbb{F}_{\mathbb{E}}$ over $\mathbb{W}$ is known as a soft closed set in $\mathbb{W}$, as long as its relative complement $\mathbb{F}'_{\mathbb{E}}$ belongs to $\mathbb{T}$.

Definition 2.5 [4]: Let $\mathbb{F}_{\mathbb{E}}$ be a non-null soft subset of $(\mathbb{W},\mathbb{T},\mathbb{E})$ subsequently $\mathbb{T}_{\mathbb{F}}=\{\mathbb{F}_{\mathbb{E}}\cap\mathbb{G}_{\mathbb{E}}, \forall\mathbb{G}_{\mathbb{E}}\in\mathbb{T}\}$ is known as relative $\mathcal{STS}$ on $\mathbb{F}_{\mathbb{E}}$ and $(\mathbb{F}_{\mathbb{E}}, \mathbb{T}_{\mathbb{F}}, \mathbb{E})$ is known as a soft subspace of $(\mathbb{W},\mathbb{T},\mathbb{E})$.

Definition 2.6 [9]: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$ be a $\mathcal{STS}$ over $\mathbb{W}$, $\mathbb{G}_{\mathbb{E}}$ be a soft set over $\mathbb{W}$, and $x\in\mathbb{W}$. Subsequently, $\mathbb{G}_{\mathbb{E}}$ is known as a soft neighborhood of $x_{\mathbb{E}}$, as long as there exists a soft open set $\mathbb{F}_{\mathbb{E}}$ such that $x_{\mathbb{E}}\in\mathbb{F}_{\mathbb{E}}\subseteq\mathbb{G}_{\mathbb{E}}$.

Definition 2.7 [10]: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$ and $(\mathbb{M},\mathbb{T}',\mathbb{E})$ be two $\mathcal{STS}$, $\mathcal{L}:(\mathbb{W},\mathbb{T},\mathbb{E})\to(\mathbb{M},\mathbb{T}',\mathbb{E})$ be a soft map. For each soft neighborhood $\mathbb{G}_{\mathbb{E}}$ of $\mathcal{L}(x_{\mathbb{E}})$, as long as there exists a soft neighborhood $\mathbb{F}_{\mathbb{E}}$ of $x_{\mathbb{E}}$, such that $\mathcal{L}(\mathbb{F}_{\mathbb{E}})\subseteq\mathbb{G}_{\mathbb{E}}$, subsequently, $\mathcal{L}$ is known as a soft continuous map at $x_{\mathbb{E}}$. As long as $\mathcal{L}$ is a soft continuous map for all $x_{\mathbb{E}}$, subsequently, $\mathcal{L}$ is known as a soft continuous map.

Definition 2. 8 [5]: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$ be a $\mathcal{STS}$. A subcollection β of $\mathbb{T}$ is known as a base for $\mathbb{T}$ as long as each member of $\mathbb{T}$ can be uttered as a union of memberships of β.

Proposition 2.9 [11]: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$ be a $\mathcal{STS}$ and $\mathbb{F}_{\mathbb{E}}$ be any soft set over $\mathbb{W}$. Subsequently (1) $\mathbb{G}_{\mathbb{E}}\,\tilde{\subseteq}\,\mathbb{F}_{\mathbb{E}}$ is closed in $\mathbb{T}_{\mathbb{F}_{\mathcal{E}}}$ as long as f $\mathbb{G}_{\mathbb{E}}=\mathbb{F}_{\mathbb{E}}\,\tilde{\cap}\,\mathbb{H}_{\mathbb{E}}$ where $\mathbb{H}_{\mathbb{E}}$ is closed in $(\mathbb{W},\mathbb{T},\mathbb{E})$; (2) β be an open base of $\mathbb{T}$ subsequently $\beta_{\mathbb{F}_{\mathbb{E}}}=\{\mathbb{G}_{\mathbb{E}}\,\tilde{\cap}\,\mathbb{F}_{\mathbb{E}}:\mathbb{G}_{\mathbb{E}}\in\beta\}$ is an open base of $\mathbb{T}_{\mathbb{F}_{\mathbb{E}}}$.

Definition 2. 10 [12]: A soft subset $\mathbb{F}_{\mathbb{E}}$of $(\mathbb{W}, \mathbb{T}, \mathbb{E})$is known as soft semi-open as long as $\mathbb{F}_{\mathbb{E}} \tilde{\subseteq} cl(int\mathbb{F}_{\mathbb{E}})$ with its relative complement is known as soft semi-closed.

Theorem 2. 11 [13]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ be a $\mathcal{STS}$ each open soft set is semi-open soft.

Definition 2. 12 [15]: A collection β of soft semi-open sets is known as a soft semi-base of $(\mathbb{W}, \mathbb{T}, \mathbb{E})$as long as each soft semi-open subset of $\mathbb{W}$ can be written as a soft union of members of β.

Proposition 2. 13 [16]: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$be $\mathcal{STS}$ over $\mathbb{W}$ and for each $x_{\mathbb{E}} \in \mathbb{F}_{\mathbb{E}}$. $\mathbb{G}_{\mathbb{E}} \subseteq \mathbb{F}_{\mathbb{E}}$ is soft semi-open as long as and only as long as $\mathbb{G}_{\mathbb{E}}$contains a soft semi-basic neighborhood of each of its points.

Proposition 2. 14: Each soft open base is a soft semi-open base.

Proof: Let $(\mathbb{W}, \mathbb{T}, \mathbb{E})$be a $\mathcal{STS}$ and Let βbe a soft open base. thus,$\mathcal{V}$ is a soft open set, $\forall\ \mathcal{V} \in \beta$. Theorem (2. 11) $\mathcal{V}$ is a soft semi-open set, $\forall\ \mathcal{V} \in \beta$. consequently, $\mathcal{V}$ is a soft neighborhood for each of its points. Thus, β is a soft semi-open base by Theorem 2. 13.

Definition 2.15 [15]:

1) The collection $\{(\mathbb{F}_{\mathbb{E}\ i}: i \in I\}$ of soft semi-open sets is known as a soft semi-open cover of an $\mathcal{STS}$ $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ as long as $\mathbb{W} = \tilde{\cup}_{i \in I}\ \mathbb{F}_{\mathbb{E}\ i}$.
2) An STS $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is known as a soft semi-compact space ($\mathcal{SS}$e-compact space for short) as long as each soft semi-open cover of $\mathbb{W}$ has a finite sub-cover of $\mathbb{W}$.

Example 2.16 [14]: Let $\mathbb{E} = \{e_1, e_2\}$be a set of parameters and consider the following two soft sets over

$\mathbb{W}=\{\ h_1, h_2\ \}, \mathbb{F}_{\mathbb{E}} =\{\ (e_1, \{h_2\}), (e_2, \{h_1\})\ \}$, $\mathbb{G}_{\mathcal{E}} = \{(e_1, \{\ h_1\}), (e_2\ , \{h_2\})\}$ subsequently $\mathbb{T}= \{\ \tilde{\phi}, \widetilde{\mathbb{W}}, \mathbb{F}_{\mathbb{E}}, \mathbb{G}_{\mathbb{E}}\}$ is $\mathcal{STS}$ on $\mathbb{W}$. $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is $\mathcal{SS}$e-compact

Definition 2.17 [14]: A $\mathcal{STS}$ $(\mathbb{W}, \mathbb{T}, \mathbb{E})$is known as almost $\mathcal{SS}$e-compact as long as each soft semi-open cover of $\mathbb{W}$ has a finite sub-cover such that the soft semi-closures whose members cover $\mathbb{W}$.

Example 2.18 [14]: Let $\mathcal{E}$ be a set of parameters and $\mathbb{T} = \{\ \mathbb{F}_{\mathbb{E}\ i} \tilde{\subseteq}\ \mathbb{R}: 1 \in \mathbb{F}_{\mathbb{E}\ i}\ \}$ be a soft topology on the set of real numbers $\mathbb{R}$. since the semi closure of any soft semi-open set is $\mathbb{R}$ Subsequently $(\mathbb{R}, \mathbb{T}, \mathbb{E})$ is almost $\mathcal{SS}$e-compact.

Definition 2.20 [14]: An $\mathcal{STS}$ $(\mathbb{W}, \mathbb{T}, \mathbb{E})$is known as mildly $\mathcal{SS}$e-compact as long as each soft semi-clopen cover of $\mathbb{W}$ has a finite soft subcover $\mathbb{W}$.

Example 2.21 [14]: Let $\mathbb{E}=\{\mathbb{e}_1,\mathbb{e}_2\}$ be a set of parameters and $\mathbb{T}$={$\mathbb{G}_{\mathbb{E}}\tilde{\subseteq}\mathbb{R}$: either [1∈ $\mathbb{G}_{\mathbb{E}}$and $\mathbb{G}^c_{\mathbb{E}}$is finite]or1∉ $\mathbb{G}_{\mathbb{E}}$} be a $\mathcal{STS}$ on $\mathbb{R}$. The relative complement of any soft open set containing {1} is finite. Subsequently $(\mathbb{R},\mathbb{T},\mathbb{E})$ is mildly $\mathcal{SS}$e-compact.

Proposition 2.22 [14]: Each $\mathcal{SS}$e-compact space is almost $\mathcal{SS}$e-compact.

Proposition 2.23 [14]: Each almost $\mathcal{SS}$e-compact space is mildly $\mathcal{SS}$e-compact.

Theorem 2.24 [14]: Consider $(\mathbb{W},\mathbb{T},\mathbb{E})$has a soft semi-base consisting of soft semi-clopen sets. Subsequently $(\mathbb{W},\mathbb{T},\mathbb{E})$is $\mathcal{SS}$e-compact as long as and only as long as it is mildly $\mathcal{SS}$e-compact.

Theorem 2.25 [15]: As long as $\mathbb{G}_{\mathbb{E}}$ is $\mathcal{SS}$e-compact subset of $\mathbb{W}$ and $\mathbb{F}_{\mathbb{E}}$is a soft semi-closed subset of $\mathbb{W}$ subsequently $\mathbb{G}_{\mathbb{E}}\cap\ \mathbb{F}_{\mathbb{E}}$ is $\mathcal{SS}$e-compact.

Theorem 2.26 [14]: As long as $\mathbb{G}_{\mathbb{E}}$is an almost (resp. a mildly) $\mathcal{SS}$e-compact subset of $\mathbb{W}$ and $\mathbb{F}_{\mathbb{E}}$is a soft semi-clopen subset of $\mathbb{W}$, subsequently $\mathbb{G}_{\mathbb{E}}\cap\ \mathbb{F}_{\mathbb{E}}$is an almost (resp. a mildly) $\mathcal{SS}$e-compact.

3 $\mathcal{SS}$e-compact map

Definition 3.1: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$and $(\mathbb{M},\mathbb{T}',\mathbb{E})$be two $\mathcal{STS}$ and Let, $\mathcal{L}:(\mathbb{W},\mathbb{T},\mathbb{E})\longrightarrow(\mathbb{M},\mathbb{T}',\mathbb{E})$be a soft map. Subsequently, $\mathcal{L}$ is known as a $\mathcal{SS}$e-compact map, as long as it is a soft surjective continuous map, and as long as the pre-image of each $\mathcal{SS}$e-compact subset of $\mathbb{M}$ is a $\mathcal{SS}$e-compact subset of $\mathbb{W}$.

Example 3.2: Let $\mathbb{W}=\mathbb{M}=\{x,y,z\}$, $\mathcal{E}=\{\mathbb{e}_1,\mathbb{e}_2\}$and $\mathbb{T}=\{\tilde{\emptyset},\widetilde{\mathbb{W}},\mathbb{F}_{\mathbb{E}},\mathbb{G}_{\mathbb{E}}\}$where $\mathbb{F}_{\mathbb{E}}$={($\mathbb{e}_1$,{x}),($\mathbb{e}_2$,{∅})},$\mathbb{G}_{\mathbb{E}}$={($\mathbb{e}_1$,{y, z}),($\mathbb{e}_2$,$\mathbb{W}$)}also$\mathbb{T}'$={$\tilde{\emptyset}$,$\widetilde{\mathbb{W}}$,$\mathbb{H}_{\mathbb{E}}$ },$\mathbb{H}_{\mathbb{E}}$ ={($\mathbb{e}_1$,{y }),($\mathbb{e}_2$,{∅})}. A map $\mathcal{L}:(\mathbb{W},\mathbb{T},\mathbb{E})\longrightarrow(\mathbb{M},\mathbb{T}',\mathbb{E})$ defined by
$\mathcal{L}$($\mathbb{e}_1$,{x})=($\mathbb{e}_1$,{y}), $\mathcal{L}$($\mathbb{e}_1$,{y})= ($\mathbb{e}_1$,{x}), $\mathcal{L}$($\mathbb{e}_1$,{z})=($\mathbb{e}_1$,{z}), $\mathcal{L}$($\mathbb{e}_2$,{x}) = ($\mathbb{e}_2$,{x}), $\mathcal{L}$($\mathbb{e}_2$,{y})=($\mathbb{e}_2$,{z}), $\mathcal{L}$($\mathbb{e}_2$,{z})=($\mathbb{e}_2$,{y}), Subsequently $\mathcal{L}$ soft surjective continuous and $\mathcal{SS}$e-compact map.

Example 3.3: Let $\mathbb{W}=\mathbb{R}$, $x\in\mathbb{W}$, $\mathbb{E}=\{\mathbb{e}_1,\mathbb{e}_2\}$and $\mathbb{T}$={$\tilde{\emptyset}$,$\widetilde{\mathbb{W}}$,$\mathbb{F}_{\mathbb{E}}$}where $\mathbb{F}_{\mathbb{E}}$ = {($\mathbb{e}_1$, {x}), ($\mathbb{e}_2$, {∅})}, consider a map $\mathcal{L}$:($\mathbb{W}$,$\mathbb{T}$,$\mathbb{E}$)→ ($\mathbb{W}$,$\mathbb{T}_{ind}$, $\mathbb{E}$), $\mathcal{L}(x)=x$ Subsequently $\mathcal{L}$ is not a semi-compact map since ($\mathbb{W}$,$\mathbb{T}_{ind}$,$\mathbb{E}$) is semi-compact, but $\mathcal{L}^{-1}(\widetilde{\mathbb{W}})=\widetilde{\mathbb{W}}$ is not semi-compact.

Definition 3.4: Let $(\mathbb{W},\mathbb{T},\mathbb{E})$and $(\mathbb{M},\mathbb{T}',\mathbb{E})$ be two $\mathcal{STS}$ and Let $\mathcal{L}:(\mathbb{W},\mathbb{T},\mathbb{E})\longrightarrow(\mathbb{M},\mathbb{T}',\mathbb{E})$ be a soft map. Subsequently, $\mathcal{L}$ is known as a soft almost $\mathcal{SS}$e-compact map, as long as it is a soft surjective continuous map and as long as the pre-image of each almost $\mathcal{SS}$e-compact subset of $\mathbb{M}$is an almost $\mathcal{SS}$e-compact subset of $\mathbb{W}$.

Example 3.5 Let $\mathbb{E} = \{e_1, e_2\}$ be a set of parameters and $\mathbb{T}$ ={$\mathbb{F}_{\mathbb{E}} \tilde{\subseteq} \mathbb{R}$: either [1 $\in \mathbb{F}_{\mathbb{E}}$and ($\mathbb{F}_{\mathbb{E}}^c$is finite]or1$\notin \mathbb{F}_{\mathbb{E}}$} be $\mathcal{STS}$ on $\mathbb{R}$ it is clear that ($\mathbb{R}$,$\mathbb{T}$, $\mathbb{E}$) is almost soft and compact. Now, define a soft map $\mathcal{L}$: ($\mathbb{R}$,$\mathbb{T}$, $\mathbb{E}$) → ($\mathbb{R}$,$\mathbb{T}$, $\mathbb{E}$) such that $\mathcal{L}(x) = x, \forall x \in \mathbb{R}$, $\mathcal{L}$ is an almost soft compact map.

Example 3.6 Let $\mathbb{E} = \{e_1, e_2\}$be a set of parameters and ($\mathbb{R}$,$\mathbb{T}'$,$\mathbb{E}$) such that $\mathbb{T}'$= $\mathbb{T}_{ind}$ also ($\mathbb{R}$,$\mathbb{T}$,$\mathbb{E}$) in Example3.5 with a map $\mathcal{L}$:($\mathbb{R}$,$\mathbb{T}$,$\mathbb{E}$) → ($\mathbb{R}$, $\mathbb{T}_{ind}$, $\mathbb{E}$), $\mathcal{L}(x) = x, \forall x \in \tilde{\mathbb{R}}$ subsequently, $\mathcal{L}$ is not an almost soft compact map.

Definition3.7: Let $\mathcal{L}$: ($\mathbb{W}, \mathbb{T}, \mathbb{E}$) and ($\mathbb{M}$,$\mathbb{T}'$,$\mathbb{E}$) be two $\mathcal{STS}$ and Let $\mathcal{L}$: ($\mathbb{W}, \mathbb{T}, \mathbb{E}$) ⟶ ($\mathbb{M}$,$\mathbb{T}'$,$\mathbb{E}$) be a soft map. Subsequently, $\mathcal{L}$ is known as a mildly $\mathcal{SS}$e-compact map, as long as it is a soft surjective continuous map and as long as the pre-image of each mildly $\mathcal{SS}$e-compact subset of $\mathbb{M}$ is a mildly $\mathcal{SS}$e-compact subset of $\mathbb{W}$.

Example 3.8: Let $\mathbb{W}$=[0,1], $\mathbb{E}$={0,1} ,define a map $\mathcal{L}$: ([0,1], $\mathbb{T}_{\boldsymbol{u}}$, $\mathbb{E}$) → ($\mathcal{S}^1$, $\mathbb{T}_{\boldsymbol{u}}$, $\mathbb{E}$) where $\mathcal{S}^1(r, \epsilon) = \{s \in \mathbb{R}: d(r, s) = 1\}$ defined by $\mathcal{L}(x) = e^{2\pi i x}$. Subsequently, $\mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map.

Example 3.9: Let $\mathbb{W}$= $\mathbb{R}$, with any parameter set $\mathbb{E}$ subsequently a map $\mathcal{L}$: ($\mathbb{W}$, $\mathbb{T}_{ind}$, $\mathbb{E}$) → ($\mathbb{W}$, $\mathbb{T}_{ind}$, $\mathbb{E}$), $\mathcal{L}(x) = x, \forall x \in \mathbb{W}$ is not a mildly $\mathcal{SS}$e-compact map.

Theorem 3.10: Each $\mathcal{SS}$e-compact map is a mildly $\mathcal{SS}$e-compact map when the co-domain has a soft semi-base consisting of soft semi-clopen sets.

Proof: Let $\mathcal{L}$: ($\mathbb{W}, \mathbb{T}, \mathbb{E}$) ⟶ ($\mathbb{M}$, $\mathbb{T}'$, $\mathbb{E}$) be a $\mathcal{SS}$e-compact map such that $\mathbb{M}$ has a semi base consisting of soft semi-clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ is a mildly $\mathcal{SS}$e-compact in $\mathbb{M}$. Since $\mathbb{M}$ has a soft semi-base consisting of soft semi-clopen sets. Subsequently, $\mathbb{G}_{\mathbb{E}}$ has a soft semi-base consisting of soft semi-clopen sets by Proposition 2.14. Thus, $\mathbb{G}_{\mathbb{E}}$is a $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Theorem 2.24. So, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a $\mathcal{SS}$e-compact set in $\mathbb{W}$ by definition of the $\mathcal{SS}$e-compact map. As a result of Proposition 2.22 and proposition 2.23, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\mathcal{L}$is a mildly $\mathcal{SS}$e-compact map. ■

Theorem 3.11: Each mildly $\mathcal{SS}$e-compact map is a $\mathcal{SS}$e-compact map when the domain has a soft semi-base consisting of soft semi-clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ be a mildly $\mathcal{SS}$e-compact map such that $\mathbb{W}$ has a semi-base consisting of soft semi-clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{M}$. Subsequently, $\mathbb{G}_{\mathbb{E}}$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Proposition 2.22 and Proposition 2.23, subsequently that $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$ by definition of a mildly$\mathcal{SS}$e-compact map. Since $\mathbb{W}$ has a semi-base consisting of soft semi-clopen sets subsequently that $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$has semi base consisting of soft semi-clopen sets by Proposition 2.14. by Theorem 2.24, that $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$is a $\mathcal{SS}$e-compact set in $\mathbb{W}$.Therefore, $\mathcal{L}$ is a $\mathcal{SS}$e-compact map. ∎

Theorem 3.12: Each $\mathcal{SS}$e-compact map is an almost $\mathcal{SS}$e-compact map when the co-domain has a soft semi-base consisting of soft semi-clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$be a $\mathcal{SS}$e-compact map such that $\mathbb{M}$ has a semi base consisting of soft semi-clopen sets. Suppose that $\mathbb{G}_{\mathbb{E}}$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}$, so $\mathbb{G}_{\mathbb{E}}$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Proposition 2.23. Since $\mathbb{M}$ has semi base consisting of soft semi-clopen sets, subsequently $\mathbb{G}_{\mathbb{E}}$ has a soft semi-base consisting of soft semi-clopen sets Proposition 2.14. Thus, $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Theorem 2.24. Subsequently, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a $\mathcal{SS}$e-compact set in $\mathbb{W}$ due to $\mathcal{L}$ is a $\mathcal{SS}$e-compact map. Proposition 2.22 implies that $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\mathcal{L}$ is an almost $\mathcal{SS}$e-compact map. ∎

Theorem 3.13: Each almost $\mathcal{SS}$e-compact map is a $\mathcal{SS}$e-compact map when the domain has a soft semi-base consisting of soft semi-clopen sets.

Proof: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map such that $\mathbb{W}$ has a semi-base consisting of soft semi-clopen sets. Let $\mathbb{G}_{\mathbb{E}}$ $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Proposition 2.22. $\mathbb{G}_{\mathbb{E}}$is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}$. $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$ by defection almost $\mathcal{SS}$e-compact map. $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$ by proposition 2.23. $\mathbb{W}$ has a soft semi-base consisting of soft semi-clopen sets subsequently $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ has a soft semi-base consisting of soft semi-clopen sets by Proposition 2.14. As a result of Theorem 2.24 $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$is a $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\mathcal{L}$ is a $\mathcal{SS}$e-compact map. ∎

Theorem 3.14: Each almost $\mathcal{SS}$e-compact map is a mildly $\mathcal{SS}$e-compact map When the co-domain has a semi-base of soft semi-clopen sets.

Proof: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$be an almost $\mathcal{SS}$e-compact map such that $\mathbb{M}$ has a semi-base of a soft semi-clopen set. Suppose that $\mathbb{G}_{\mathbb{E}}$ be a mildly $\mathcal{SS}$e-compact set in $\mathbb{M}$. Thus,$\mathbb{G}_{\mathbb{E}}$ has a semi-base of soft semi-clopen sets because $\mathbb{M}$ has a semi-base of soft semi-clopen sets Proposition 2.14. So, $\mathbb{G}_{\mathbb{E}}$is a $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Theorem 2.24, and as a result of Proposition 2.22, $\mathbb{M}$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}$. Thus, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$ since $\mathcal{L}$is an almost $\mathcal{SS}$e-compact map. Therefore, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$ by Proposition 2.23 Therefore, $\mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map. ■

Theorem 3.15: Each mildly $\mathcal{SS}$e-compact map is an almost soft semi- compact map when the domain has a semi-base of soft semi-clopen sets.

Proof: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ be a mildly $\mathcal{SS}$e-compact map such that $\mathbb{W}_{\mathcal{E}}$ has a semi-base of a soft semi-clopen set. Suppose that $\mathbb{G}_{\mathbb{E}}$is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}_{\mathcal{E}}$. $\mathbb{G}_{\mathbb{E}}$is a mildly $\mathcal{SS}$e-compact set in $\mathbb{M}$ by Proposition 2.23. Subsequently $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$ by definition of a mildly $\mathcal{SS}$e-compact map. $\mathbb{W}$ has a semi-base of soft semi-clopen sets, subsequently $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ has a semi-base of soft semi-clopen sets by Proposition 2.14. As a result of Theorem 2.24, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is a $\mathcal{SS}$e-compact set in $\mathbb{W}$ by Proposition 2.22. $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$, Therefore, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact map. ■

4 Restriction of types $\mathcal{SS}$e-compact maps

Theorem 4.1: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ be a $\mathcal{SS}$e-compact map. As long as $\mathbb{A}_{\mathbb{E}}$is a soft semi-closed subset of $\mathbb{W}$subsequently the restriction $g = \mathcal{L}|(\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E})$: $(\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is a $\mathcal{SS}$e-compact map.

Proof: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ a $\mathcal{SS}$e-compact map,$\mathbb{A}_{\mathbb{E}}$ is a soft semi-closed subset of $\mathbb{W}$, the relative topology on $\mathbb{A}_{\mathbb{E}}$ is $\mathbb{T}_{\mathbb{A}} = \{\mathbb{A}_{\mathbb{E}}^* = \mathbb{A}_{\mathbb{E}} \cap \mathbb{F}_{\mathbb{E}}, \forall \mathbb{F}_{\mathbb{E}} \in \mathbb{T}\}$. Suppose $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{M}$,$\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$is a $\mathcal{SS}$e-compact set in$\mathbb{W}$ since $\mathcal{L}$ a $\mathcal{SS}$e-compact map. Subsequently $\mathbb{A}_{\mathbb{E}} \cap \mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}}) \in \mathbb{T}_{\mathbb{A}}$ is a $\mathcal{SS}$e-compact set by Theorem 2.25. Therefore $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$is a $\mathcal{SS}$e-compact map. ■

Corollary 4.2: Let $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{W}, \mathbb{T}, \mathbb{E})$ is a $\mathcal{SS}$e-compact map and $\mathbb{A}_{\mathbb{E}} \subseteq \mathbb{W}$ subsequently $\mathbb{A}_{\mathbb{E}}$ is soft semi-closed.

Proof: Let $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow (\mathbb{W}, \mathbb{T}, \mathbb{E})$be a $\mathcal{SS}$e-compact map, the relative topology on $\mathbb{A}_{\mathbb{E}}$ is $\mathbb{T}_{\mathbb{A}} = \{\mathbb{A}_{\mathbb{E}}^* = \mathbb{A}_{\mathbb{E}} \cap \mathbb{G}_{\mathbb{E}}, \forall \mathbb{G}_{\mathbb{E}} \in \mathbb{T}\}$. Let $\mathbb{G}_{\mathbb{E}}$ be a $\mathcal{SS}$e-compact set in $\mathbb{W}$,

$g^{-1}(\mathbb{G}_{\mathbb{E}})$= $\mathbb{A}_{\mathbb{E}} \cap \mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set since g is a $\mathcal{SS}$e-compact map, but $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set. As a result Theorem 2.25 $\mathbb{A}_{\mathbb{E}}$ is soft semi-closed. ■

Theorem 4.3: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$be an almost(one by one a mildly) $\mathcal{SS}$e-compact map. As long as $\mathbb{A}_{\mathbb{E}}$ is a soft semi-clopen subset of $\mathbb{W}$subsequently the restr-iction $g = \mathcal{L}|(\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E})$: $(\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E})\rightarrow$ $(\mathbb{M},\mathbb{T}',\mathbb{E})$ is an almost(one by one a mildly) $\mathcal{SS}$e-compact map.

Proof: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E}) \longrightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$be an almost(one by one a mildly) $\mathcal{SS}$e-compact map,$\mathbb{A}_{\mathbb{E}}$ is a soft semi-clopen subset of $\mathbb{W}_{\mathcal{E}}$, the relative topology on $\mathbb{A}_{\mathbb{E}}$ is $\mathbb{T}_{\mathbb{A}} = \{\mathbb{A}_{\mathbb{E}}{}^* = \mathbb{A}_{\mathbb{E}} \cap \mathbb{F}_{\mathbb{E}}, \forall \mathbb{F}_{\mathbb{E}} \in \mathbb{T}\}$. Suppose $\mathbb{G}_{\mathbb{E}}$ is an almost(one by one a mildly) $\mathcal{SS}$e-compact set in $\mathbb{M}$,$\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost (one by one a mildly) $\mathcal{SS}$e-compact set in $\mathbb{W}$ since $\mathcal{L}_{\mathcal{E}}$ a soft almost (one by one a mildly) compact map. Subsequently, $\mathbb{A}_{\mathbb{E}} \cap \mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}}) \in \mathbb{T}_{\mathbb{A}}$ is an almost (one by one a mildly) $\mathcal{SS}$e-compact set by Theorem 2.26. Therefore, $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E})\rightarrow (\mathbb{M}, \mathbb{T}', \mathbb{E})$ is an almost (one by one a mildly) $\mathcal{SS}$e-compact map. ■

Corollary 4.4: Let $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow(\mathbb{W}, \mathbb{T}, \mathbb{E})$ is an almost(one by one a mildly) $\mathcal{SS}$e-compact map and $\mathbb{A}_{\mathbb{E}} \subseteq \mathbb{W}$ subsequently $\mathbb{A}_{\mathbb{E}}$ is soft semi-clopen.

Proof: Let $g = (\mathbb{A}_{\mathbb{E}}, \mathbb{T}_{\mathbb{A}}, \mathbb{E}) \rightarrow(\mathbb{W}, \mathbb{T}, \mathbb{E})$ be an almost(one by one a mildly) $\mathcal{SS}$e-compact map, the relative topology on $\mathbb{A}_{\mathcal{E}}$ is $\mathbb{T}_{\mathbb{A}} = \{\mathbb{A}_{\mathbb{E}}{}^* = \mathbb{A}_{\mathbb{E}} \cap \mathbb{F}_{\mathbb{E}}, \forall\ \mathbb{F}_{\mathbb{E}} \in \mathbb{T}\}$. Let $\mathbb{G}_{\mathbb{E}}$ be an almost(one by one a mildly) $\mathcal{SS}$e-compact set in $\mathbb{W}$, $\mathcal{L}^{-1}(\mathbb{G}_{\mathbb{E}})$= $\mathbb{A}_{\mathbb{E}} \cap \mathbb{G}_{\mathbb{E}}$ is an almost (one by one a mildly) $\mathcal{SS}$e-compact set since g is an almost(one by one a mildly) $\mathcal{SS}$e-compact, but $\mathbb{G}_{\mathbb{E}}$ is an almost(one by one a mildly) $\mathcal{SS}$e-compact set. As a result Theorem 2.26 $\mathbb{A}_{\mathbb{E}}$ is soft semi-clopen. ■

5 Composition of certain types of $\mathcal{SS}$e-compact maps

Theorem 5.1: The composition of $\mathcal{SS}$e-compact maps (one by one almost $\mathcal{SS}$e-compact maps, mildly $\mathcal{SS}$e-compact maps) is also a $\mathcal{SS}$e-compact map (one by one almost $\mathcal{SS}$e-compact maps, mildly $\mathcal{SS}$e-compact map).

Proof: Let $\mathcal{L}$: $(\mathbb{W}, \mathbb{T}, \mathbb{E})\rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ and $\hbar$: $(\mathbb{J}, \mathbb{T}', \mathbb{E})\rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ be two $\mathcal{SS}$e-compact(one by one almost $\mathcal{SS}$e-compact, mildly $\mathcal{SS}$e-compact) maps. To prove that $\hbar \circ \mathcal{L}$ is also $\mathcal{SS}$e-compact (one by one almost $\mathcal{SS}$e-compact, mildly $\mathcal{SS}$e-compact) maps. Suppose that $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact(resp. an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) set in $\mathbb{M}$. (to show that $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) set in $\mathbb{W}_{\mathcal{E}}$. We have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) set in $\mathbb{J}$ since $\hbar$ is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) map. Subsequently $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) set in $\mathbb{W}$ because $\mathcal{L}$ is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) map. We have $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}= \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. so $(\hbar \circ \mathcal{L})^{-1}$is a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is also a $\mathcal{SS}$e-compact(one by one an almost $\mathcal{SS}$e-compact, a mildly $\mathcal{SS}$e-compact) map. ■

Theorem 5.2: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ be a soft map semi-compact map and $\hbar : (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map. As long as $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ has a soft semi-base of soft semi-clopen sets, subsequently, $\hbar \circ \mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{M}$ (to show that $\hbar \circ \mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{J}$since $\hbar$ is a mildly $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is a $\mathcal{SS}$e-compact map with a co-domain that has a soft semi-base of soft semi-clopen sets. As a result of Theorem 3.10**.** we get $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$, because of $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ $= \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. so, $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is also a mildly $\mathcal{SS}$e-compact map. ■

Corollary 5.3: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ have a soft semi-basis of soft semi-clopen sets, subsequently, $\hbar \circ \mathcal{L}$ is a $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.2 and Theorem 3.11. ■

Theorem 5.4: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map and $\hbar : (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$has a soft semi-base of a soft semi-clopen set Subsequently $\hbar \circ \mathcal{L}$ is a $\mathcal{SS}$e-compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{M}$. (to show that $\hbar \circ \mathcal{L}$ is a $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is a $\mathcal{SS}$e-compact set in $\mathbb{J}$ since $\hbar$ is a $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map with a domain that has a soft semi-base of a soft semi-clopen set. As a result of Theorem 3.11 $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. is $\mathcal{SS}$e-compact set in $\mathbb{W}$. Because of $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is also a $\mathcal{SS}$e-compact map. ■

Corollary 5.5: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map and $\hbar : (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}'', \mathbb{E})$ have a soft semi-basis of soft semi-clopen sets, subsequently $\hbar \circ \mathcal{L} : (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.4and Theorem 3.10. ■

Theorem 5.6: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ be a $\mathcal{SS}$e-compact map and $\hbar : (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map. As long as $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ has a soft semi-base of soft semi-clopen. Subsequently, $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}$. (to show that $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{J}$ since $\hbar$ is an almost $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is a $\mathcal{SS}$e-compact map with a codomain that has a semi-base soft semi-clopen set. As a result of Theorem 3.12, we get $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$ is an almost soft compact set in $\mathbb{W}$. Because $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathcal{E}}$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is also an almost $\mathcal{SS}$e-compact map. ■

Corollary 5.7: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map**.** As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{J}, \mathbb{T}', \mathbb{E})$ have a soft semi-basis of a soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.6 and Theorem 3.13. ■

Theorem 5.8: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ has a semi-base soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is $\mathcal{SS}$e-compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{M}$ (to show that $\hbar \circ \mathcal{L}$ is $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is $\mathcal{SS}$e-compact set in $\mathbb{J}$ since $\hbar$ is a $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is an almost $\mathcal{SS}$e-compact map with a domain that has a semi-base soft semi-clopen set. As a result, to Theorem 3.13**.** we get $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. is a $\mathcal{SS}$e-compact set in $\mathbb{W}$. Because of $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is a $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is also a $\mathcal{SS}$e-compact map. ■

Corollary 5.9: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$and $(\mathbb{M},\mathbb{T}'', \mathbb{E})$ have a soft semi-base of a soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.8 and Theorem 3.12. ■

Theorem 5.10: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map. As long as $(\mathbb{J}, \mathbb{T}', \mathbb{E})$has a soft semi-base of a soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is a mildly soft compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$is a mildly $\mathcal{SS}$e-compact set in $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$. (to show that $\hbar \circ \mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$is a mildly $\mathcal{SS}$e-compact set in $\mathbb{J}$since $\hbar$ is a mildly $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is an almost $\mathcal{SS}$e-compact map with a co-domain that has a soft semi-base of a soft semi-clopen set. As a result, of Theorem 3.14 we get $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$. Because of $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} = \mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$is a mildly $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\hbar \circ \mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map. ■

Corollary 5.11: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$is an almost $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$is a mildly $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$and $(\mathbb{W}, \mathbb{T}, \mathbb{E})$have a soft semi-base of a soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.10 and Theorem 3.15. ■

Theorem 5.12: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \to (\mathbb{J}, \mathbb{T}', \mathbb{E})$is a mildly $\mathcal{SS}$e-compact map and $\hbar: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \to (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$has a soft semi-base soft semi-clopen set. Subsequently, $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map.

Proof: Suppose $\mathbb{G}_{\mathbb{E}}$is an almost $\mathcal{SS}$e-compact set in $\mathbb{M}$. (to show that $\hbar \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map). we have $\hbar^{-1}(\mathbb{G}_{\mathbb{E}})$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{J}_{\mathcal{E}}$since $\hbar_{\mathcal{E}}$ is an almost $\mathcal{SS}$e-compact map. Subsequently, $\mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map with a domain that has a semi-base soft semi-clopen set. From Theorem 3.15 we get $\mathcal{L}^{-1}(\hbar^{-1}(\mathbb{G}_{\mathbb{E}}))$is an almost soft compact set in $\mathbb{W}$. Because of $(\hbar \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}} =$

$\mathcal{L}^{-1}(\mathcal{h}^{-1}(\mathbb{G}_{\mathbb{E}}))$. So $(\mathcal{h} \circ \mathcal{L})^{-1}\mathbb{G}_{\mathbb{E}}$ is an almost $\mathcal{SS}$e-compact set in $\mathbb{W}$. Therefore, $\mathcal{h} \circ \mathcal{L}$ is an almost $\mathcal{SS}$e-compact map. ■

Corollary 5.13: Let $\mathcal{L}: (\mathbb{W}, \mathbb{T}, \mathbb{E}) \rightarrow (\mathbb{J}, \mathbb{T}', \mathbb{E})$ is a mildly $\mathcal{SS}$e-compact map and $\mathcal{h}: (\mathbb{J}, \mathbb{T}', \mathbb{E}) \rightarrow (\mathbb{M}, \mathbb{T}'', \mathbb{E})$ is an almost $\mathcal{SS}$e-compact map. As long as $(\mathbb{W}, \mathbb{T}, \mathbb{E})$ and $(\mathbb{M}, \mathbb{T}'', \mathbb{E})$ have a soft semi-base of a soft semi-clopen set. Subsequently, $\mathcal{h} \circ \mathcal{L}$ is a mildly $\mathcal{SS}$e-compact map.

Proof: By Theorem 5.12 and Theorem 3.14. ■

6 Conclusion

To sum up, we create in this paper a soft semi-compact map and investigate its associations with soft semi-compact maps, almost soft semi-compact maps, besides mildly soft semi-compact maps which are utilized from the relations between their spaces under some conditions. Moreover, the composition factors of soft compact maps with soft semi-compact maps, almost soft semi-compact maps, and mildly soft semi-compact maps are studied based on the previous association between them.

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