Mathematical Modelling of Airflow in an Inclined Main Trachea: Using Adomain Decomposition Method

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Abstract: The Inclined bed therapy is an ideal natural remedy for several health problems without introducing harmful chemicals or other substances into the patients' body.The mathematical model of the effect of inclination angle position on the airflow in human trachea under resting and normal breathing condition is presented. The governing equations in 2D cylindrical coordinate system of motion consisting of unsteady, nonlinear, non-homogenous, Navier-Stokes equations with initial and boundary conditions are analytically solved using the Adomian Decomposition method. The main advantage of this method is that it can be applied directly to all types of differential equations linear or non-linear, homogeneous or inhomogeneous, with constant or variable coefficients. Another important advantage is that, the method is capable of greatly reducing the size of computational work. Results obtained are validated by comparison with theoretical and experimental results for the axial velocity. Furthermore, the result is increase of volumetric flow rate leads is found to reduce the resistance to the flow. Consequently, the resistance to airflow is reduced with the increase of inclination angle position, Reynold's numbers and increase of gravitation (lower Froude number) respectively.Thus, it was justified that sleeping in horizontal position may cause a negative effect on the patient, whereas the inclined position is better and depending on the slope angle, and improved breathing.

Keywords: Inclination bed theraby, Adomain Decomposition Method, Airflow, Main Trachea.

1.Introduction

Newly, in-depth prediction and understanding of phenomena in life sciences and physiology demanded precise mathematical models and subsequent numerical simulation tools. This is especially true for achieving inclined bed therapy (IBT), which is introduced by Andrew Fletcher [5] appeared promising. This therapy maximizes the ability of the body to perform on its own in the absence of externally injected chemicals. It is advantageous for several illnesses related to breathing problems such as mild sleep apnea, snoring, asthma and chronic obstructive pulmonary disease (COPD) [3].

A few researchers have tried to find analytical solutions for airflow velocity in the human respiratory instance.

Li et al. [16,17] studied the transient and steady laminar air flow field and microparticle deposition in a realistic trachea with inlet Reynolds number (1201) for the resting breathing conditions. The simulated results on axial velocities are compared with experimental values of Zhao and Lieber [15]. Liu et al. [18] employed a child model in the upper respiratory tracts to scrutinize the effects of physiological features on the airflow patterns and particle deposition. Kongnuan et al. [12,13] described a mathematical model of the airflow in the human upper respiratory tract by assuming the fluid incompressible and Newtonian in the absence of external force having constant density and viscosity. An effective computational method of finite element analysis (COMSOL software) used to compare simulation results with the one obtained from an analytical calculation based on Fourier series [12].

Alnussairy et al. [3] investigated the inclination angle dependence on the unsteady airflow in human trachea by developing a 2D mathematical model. The exact solutions are achieved using Bromwich integral method and the result for axial velocity at horizontal position of the trachea ($\theta = 0^0$) is compared with the observation of Kongnuan [12].

Many types of research [3] achieved results worth using commercial simulation software and using a given inlet velocity and pressure with proper boundary conditions. These software programs are often very expensive and not easily available. However, some common numerical methods give weak points such a low accuracy, slow convergence and limitation stability [9]. A powerful method which is called Adomian decomposition method (ADM) was firstly introduced by George

Adomian in 1981. The main advantage of this method is that it can be applied directly to all types of differential and integral equations, linear or non-linear, homogeneous or inhomogeneous, with constant or variable coefficients. Another important advantage is that, the method is capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution (S. Somali and G. Gokmen) [14]. The non-linear problems are solved easily and elegantly without linearizing the problem by using ADM. It also avoids linearization, perturbation and discretization unlike other classical techniques (Mustafa Inc [8]).

2. Mathematical Model Formulation

Consider the unsteady airflow in a two-dimensional, incompressible (low Mach number, M = 0.1($\Delta p/\rho = 0.5M^2 \leq 1$)). Following Calay *et al.* (2002)[11], laminar, Newtonian fluid of constant density and viscosity in along straight trachea with constant a kinematic viscosity ($\nu = \mu/\rho$) . Let (r,θ,z) be the cylindrical coordinates of any point in the airflow domain in trachea with body force (g) (gravitation parameter).

2.1 The Governing Equations

The governing continuity and momentum (Navier-Stocks Equations) conservation equations in the presence of external force for axisymmetric airflow in the cylindrical coordinate system ($\bar{r},0,\bar{z}$) are written as:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\overline{ru}) + \frac{\partial}{\partial \bar{z}}(\bar{w}) = 0 \tag{1}$$

$$\rho\left(\frac{\partial \bar{w}}{\partial \bar{t}} + \frac{\partial(\overline{wu})}{\partial \bar{r}} + \frac{\partial \bar{w}^2}{\partial \bar{z}} + \frac{\overline{wu}}{\bar{r}}\right) = -\frac{\partial \bar{p}}{\partial \bar{z}} + \mu\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\frac{\partial \bar{w}}{\partial \bar{r}}) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) + \rho g \sin(\theta) \tag{2}$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{u}^2}{\partial \bar{r}} + \frac{\partial(\overline{uw})}{\partial \bar{z}} + \frac{\bar{u}^2}{\bar{r}}\right) = -\frac{\partial \bar{p}}{\partial \bar{r}} + \mu\left(\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\frac{\partial \bar{u}}{\partial \bar{r}}) - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) - \rho g \cos(\theta) \tag{3}$$

2.2 Initial and Boundary Conditions

The components of velocity of air stream at the rigid wall (the usual no slip condition) must be zero . The fully developed parabolic velocity profile that is assumed to achieve maximum velocity (U_{max}) at the inlet corresponding to the Poiseuille's flow (White, 2011) [6] of the trachea human yields:

$$\bar{w}(\bar{r},\bar{z},\bar{t}) = U_{\max}\left(1-\frac{\bar{r}^2}{\bar{R}_0^2}\right), \bar{u}(\bar{r},\bar{z},\bar{t}) = 0, \text{ at } \bar{t} = 0,\ \bar{z} \geq 0, \tag{4}$$

where

$$U_{max} = 2U_0\ , at\ \bar{r} = 0, \bar{u}(\bar{r},\bar{z},\bar{t}) = 0,\ \frac{\partial \bar{w}}{\partial \bar{r}} = 0, \tag{5}$$

At downstream (outlet) conditions by treating the gradient velocity as zero one gets:

For

$$\bar{z} = L \qquad \frac{\partial \bar{w}(\bar{r},\bar{z},\bar{t})}{\partial \bar{z}} = \frac{\partial \bar{u}(\bar{r},\bar{z},\bar{t})}{\partial \bar{z}} = 0, \tag{6}$$

By introducing the following dimensionless of quantities:

$$u = \frac{\bar{u}}{U_0}, w = \frac{\bar{w}}{U_0}, p = \frac{\bar{p}}{\rho U_0^2}, R = \frac{\bar{R}}{\bar{R}_0}, t = \frac{\bar{t}U_0}{\bar{R}_0}, r = \frac{\bar{r}}{\bar{R}_0}, z = \frac{\bar{z}}{\bar{R}_0},$$

the conservation equations in dimensionless form are written as:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(w) = 0 \tag{7}$$

$$\frac{\partial w}{\partial t} + \frac{\partial (wu)}{\partial r} + \frac{\partial w^2}{\partial z} + \frac{wu}{r} = -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial w}{\partial r}) + \frac{\partial^2 w}{\partial z^2}\right) + \frac{\sin(\theta)}{Fr} \tag{8}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial r} + \frac{\partial (uw)}{\partial z} + \frac{u^2}{r} = -\frac{\partial p}{\partial r} + \frac{1}{\text{Re}}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\cos(\theta)}{Fr} \tag{9}$$

Where u and w are the components of the velocity along, r and z directions respectively, p stands for the pressure, while $Re = \rho U_0 R_0 / \mu$,, and $Fr = U_0^2 / gR_0$,are the Re and the Fr number, respectively. Here, t, p, L, R_0 are the time, pressure and the length ,radius of the trachea, respectively.

And the pressure gradient $\frac{\partial p}{\partial z}$ has been taken following and Alnussairy at. el. (2016) [2] for a human being who is given by:

$$\frac{\partial p}{\partial z} = \frac{P}{L}\sin(\omega t), \text{ for } t > 0, \tag{10}$$

Where P/L is the constant amplitude of the pressure gradient, ω the cyclic frequency of the oscillating pressure gradient.

The momentum equation for Eq. (8) axial and radial Eq. (9) flow are imposed with a gravitational force parameter g. The angle θ is the inclination between the direction of the trachea and the horizontal direction as shown in Fig. 1.

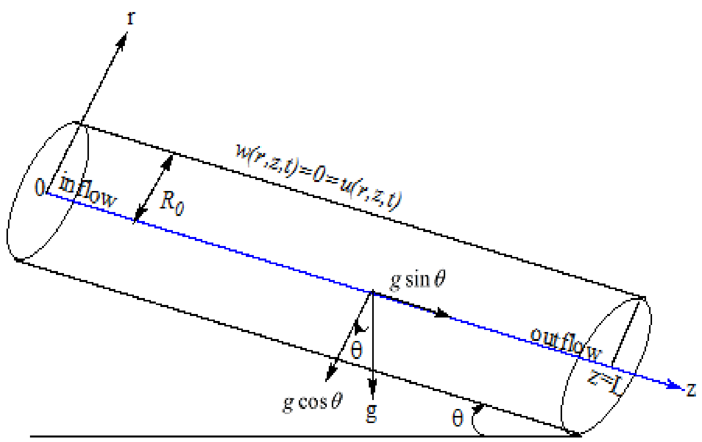

Figure 1: An inclination main trachea [3].

And the initial and boundary conditions in dimensionless Eqs.(4)-(6) becomes:

$$w(r,z,t)=U_{\max}\left(1-r^2\right), u(r,z,t)=0 \text{ at } t=0, z\geq 0 \text{, where } U_{\max}=2U_0, \tag{11}$$

$$\text{at } r=0 \quad u(r,z,t)=0, \ \frac{\partial w}{\partial r}=0 \tag{12}$$

For

$$z=L \qquad \frac{\partial w(r,z,t)}{\partial z}=\frac{\partial u(r,z,t)}{\partial z}=0, \ , \tag{13}$$

2.3 Analytical Method using ADM

For Eqs. (7)-(9), we see that three partial differential equations have three unknown *u*, *w* and *p* functions of three independent variables *r*, *z* , *t* and .Assuming fully developed airflow in these regions, *u*(*r,z,t*) = 0 and *w* = *w*(*r,z,t*) the continuity Eq. (7) is satisfied and when developed flow conditions Eq. (9) can be omitted (the radial velocity being very small can be neglected (see Schroter, and Sudlow [14])). Hence, we can reduce Eqs. (7)-(9) and Eq. (10) to

$$\frac{\partial w}{\partial t}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{p}{L}\sin\omega t+\frac{\sin\theta}{Fr} \tag{14}$$

or

$$w_t=\frac{1}{r\,\mathrm{Re}}(rw_{rr}+w_r+rw_{zz})-\frac{p}{L}\sin\omega t+\frac{\sin\theta}{Fr} \tag{15}$$

$$\Rightarrow w_t=\frac{1}{r\,\mathrm{Re}}(rw_{rr}+w_r+rw_{zz})-\frac{p}{L}\sin\omega t+\frac{\sin\theta}{Fr} \tag{16}$$

By using Adomian decomposition method (ADM) (t-solution) (refer Wazwaz [1])
Take *L* operator to both sides Eq. (16)

$$L_t(w)=\frac{1}{r\,\mathrm{Re}}\left[rL_r(w)+w_r+rL_z(w)\right]-\frac{P}{L}\sin\omega t+\frac{1}{Fr}\cos\theta \tag{17}$$

where $L_t=\frac{\partial}{\partial t}, L_r=\frac{\partial^2}{\partial r^2}, L_z=\frac{\partial^2}{\partial z^2},$

Applying L_t^{-1} to both sides Eq. (17) becomes

$$L_t^{-1}(L_t(w)) = \frac{1}{r\,\mathrm{Re}} L_t^{-1}(rL_r(w) + w_r + rL_z(w)) - \frac{P}{L} L_t^{-1}(\sin \omega t) + \frac{1}{Fr} L_t^{-1}(\sin\theta) \qquad (18)$$

where $L_t^{-1} = \int_0^t (.)dt$

$$w(r,z,t) - w(r,z,0) = \frac{1}{r\,\mathrm{Re}} L_t^{-1}(rL_r(w) + w_r + rL_z(w)) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta \qquad (19)$$

Substituting Eq. (11) into Eq. (19) we obtain

$$w(r,z,t) = U_{\max}(1-r^2) + \frac{1}{r\,\mathrm{Re}} L_t^{-1}(rL_r(w) + w_r + rL_z(w)) + \frac{P}{\omega L}\cos\omega t - \frac{P}{\omega L} + \frac{t}{Fr}\sin\theta \qquad (20)$$

The linear terms $w(r,z,t)$ can be represented by the decomposition series

$$w(r,z,t) = \sum_{k=0}^{\infty} w_k(r,z,t) = w_0(r,z,t) + w_1(r,z,t) + w_2(r,z,t) + \ldots.. \qquad (21)$$

Substituting Eq. (21) into Eq. (20) we get

$$\sum_{k=0}^{\infty} w_k(r,z,t) = U_{\max}(1-r^2) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta$$

$$+ \frac{1}{r\,\mathrm{Re}} L_t^{-1}(rL_r(\sum_{k=0}^{\infty} w_k(r,z,t) + (\sum_{k=0}^{\infty} w_{rk}(r,z,t)) + rL_z(\sum_{k=0}^{\infty} w_k(r,z,t))) \qquad (22)$$

Let $$w_0 = U_{\max}(1-r^2) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta \qquad (23)$$

$$\mathrm{w}_{k+1} = \frac{1}{\mathrm{rRe}} L_t^{-1}(rL_r(w_k) + (w_{rk}) + rL_z(w_k)), \quad k \geq 0 \qquad (24)$$

If $k = 0$

$$w_1(r,z,t) = \frac{1}{r\,\mathrm{Re}}\left(L_t^{-1}(rL_r(w_0) + (w_{r0}) + rL_z(w_0))\right),$$

$$= \frac{1}{r\,\mathrm{Re}} L_t^{-1} \begin{bmatrix} rL_r(U_{\max}(1-r^2) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta \;) + (\frac{\partial}{\partial r}(U_{\max}(1-r^2) \\ -\frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta) + rL_z(U_{\max}(1-r^2) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta) \end{bmatrix},$$

$$\Rightarrow w_1(r,z,t) = \frac{1}{r\,\mathrm{Re}}\left(L_t^{-1}(-2rU_{\max} - 2rU_{\max})\right) = \frac{-4U_{\max}}{\mathrm{Re}} t \qquad (25)$$

If $k = 1$

$$w_2(r,z,t) = \frac{1}{r\,\mathrm{Re}} L_t^{-1}(rL_r(w_1) + (w_{r1}) + rL_z(w_1))$$

$$= \frac{1}{r\,\mathrm{Re}} L_t^{-1}\left(rL_r(\frac{-4U_{\max}}{\mathrm{Re}}t) + \frac{\partial}{\partial r}(\frac{-4U_{\max}}{\mathrm{Re}}t) + rL_z(\frac{-4U_{\max}}{\mathrm{Re}}t)\right) = 0$$

So, $\Rightarrow w_2(r,z,t) = 0$ (26)

Therefore, $\Rightarrow w_k(r,z,t) = 0$ for all $k \geq 2$ (27)

Substituting Eqs. (23), (25) and Eq. (27) into Eq.(21)
we obtain

$$\Rightarrow w(r,z,t) = U_{\max}(1-r^2) - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta - \frac{4U_{\max}}{\mathrm{Re}}t \qquad (28)$$

Hence Eq. (28) is an exact solution.
The volumetric flow rate (Q) through main trachea in dimensionless is given by

$$\Rightarrow Q = 2\pi\int_0^1 rw(r,z,t)dr \qquad (29)$$

$$Q = 2\pi\left[U_{\max}(\frac{r^2}{2} - \frac{r^4}{4}) - \frac{\mathrm{P}r^2}{2\omega L} + \frac{\mathrm{P}r^2}{2\omega L}\cos\omega t + \frac{tr^2}{2Fr}\sin\theta - \frac{4r^2U_{\max}}{2\,\mathrm{Re}}t\right]_0^1 \qquad (30)$$

$$= 2\pi\left[U_{\max}(\frac{1}{2} - \frac{1}{4}) - \frac{P}{2\omega L} + \frac{P}{2\omega L}\cos\omega t + \frac{t}{2Fr}\sin\theta - \frac{2U_{\max}}{\mathrm{Re}}t\right] \qquad (31)$$

$$\text{Hence} \Rightarrow Q = \pi\left[\frac{U_{\max}}{2} - \frac{P}{\omega L} + \frac{P}{\omega L}\cos\omega t + \frac{t}{Fr}\sin\theta - \frac{4U_{\max}}{\mathrm{Re}}t\right] \qquad (32)$$

we can calculated the resistance to flow (λ), obtain

$$\Rightarrow \lambda = \frac{\Delta p}{Q}; \text{ where } \Delta p = p - p_0 \qquad (33)$$

and p_0 is an initial pressure of airflow.

3. Results and Discussion

The computational domain has been confined to a finite-nondimensional tracheal, *ρ = 1.225 kg.m−3, μ= 1.79×10-5 pa.s.* , *U0 = 85×10-2 m.s*$^{-1}$. Moreover, *L = 0.10 m* and R_0 *= 0.01085 m*, are length and width of trachea respectively (Alnussairy et al. 2017, 2019) [3,4]. And also, Reynolds number of light conditional breathing is considered in the range of 800 to 2000. While, the amplitude of the oscillating pressure P= -133.32 Pa has been taken from the literature (Alnussairy et al., 2016) [2], with normal body temperature (37C^o). When the breathing period is about 1Sec of the breathing cycle, the cyclic frequency $\omega = \pi/ 2$, g=9.8 m/s^2 gravity acceleration. Therefore, from Eqs. (28), (32) and Eq. (33) Fig. 3,4,5 and Fig. 6 has been determined after substituting the values of t, θ , Re and Fr. The accuracy of proposed method is validated with existing experimental data and numerical studies is shown in Fig.2. The w-velocity when Re=1200 , Fr=0.27 and $\theta=0^0$ revealed parabolic pattern at difference time t. In addition, the laminar flow exhibited maximum velocity in the central area and decreased to zero close to the walls. This is in good agreement with the findings of experimental and numerical results (Schroter and Sudlow,[10] and other numerical studies Alnussairy et al. [3,4]).

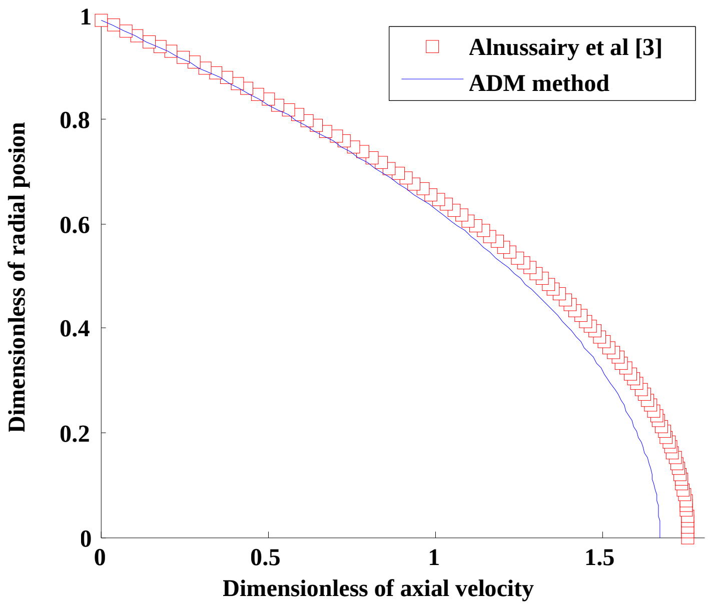

Figure 2. Comparison of axial velocity profiles in the horizontal straight trachea with Alnussairy *et al*. [3] at *Re* = 1200

The effect of the inclination angle position on the (u-velocity) velocity profile is displayed in Fig. 3 when Fr=0.27 and Re=1200, which shows that the increasing of θ (0^o, 15^o, 30^o, 45^o) leads to increasing the velocity of airflow. These results close to Alnussairy et al. [3,4] when they established the inclination angle had reasonable effect.

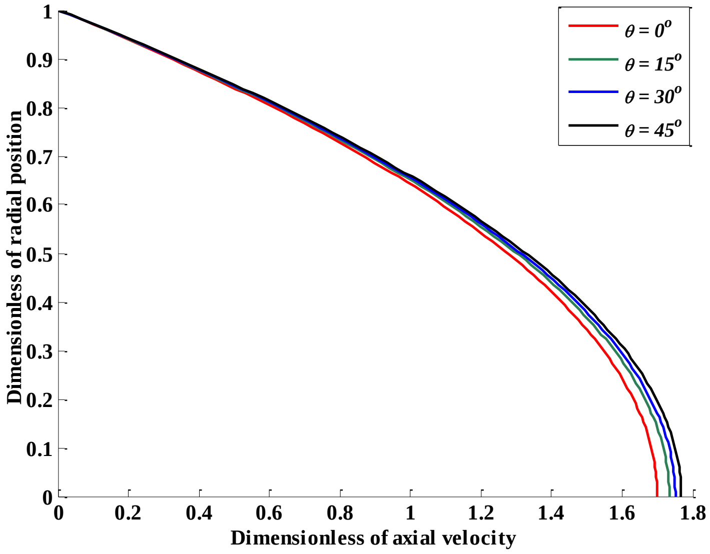

Figure 3. Variation of axial velocity profiles for different angle position at t=1 and *Re* = 1200 with Fr=0.27

The *u*-velocity at different Reynolds number (800, 1000, 1200, 1500, and 2000) at at t=1, Fr=0.27 and θ = 0° is shown in Fig. 3. The maximum value of axial velocity (2.42) is appeared at Re= 2000 with the initial velocity of U_0= 0.85. Conversely, the minimum value of axial velocity (1.32) is obtained at Re= 800 with the initial velocity

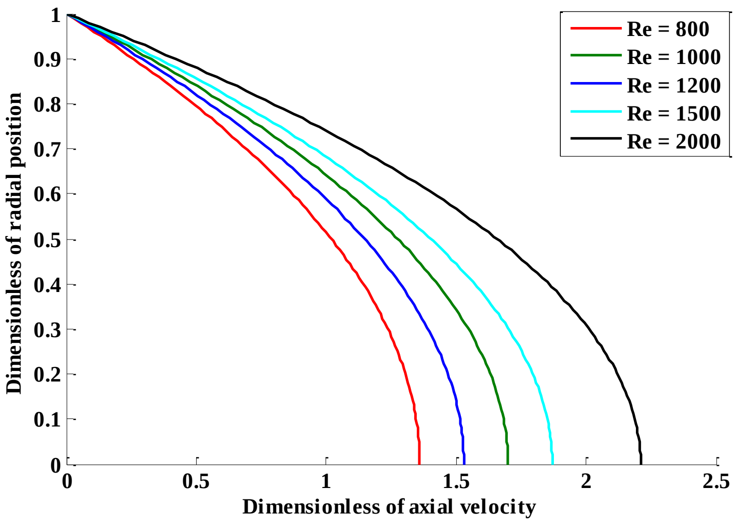

Figure 4. Variation of axial velocity profiles for different *Re* at *t=1* and *θ = 0°*.

Fig. 5 illustrates that the increasing of Froud number's (Fr) at t=1 and Re= 1200 leads to decreasing of the airflow velocity when θ=30°. Therefore, it is supported the phenomenon that increasing of gravity g may lead to reducing the flow resistance, also the velocity of air flow increases with the increase in gravity g.

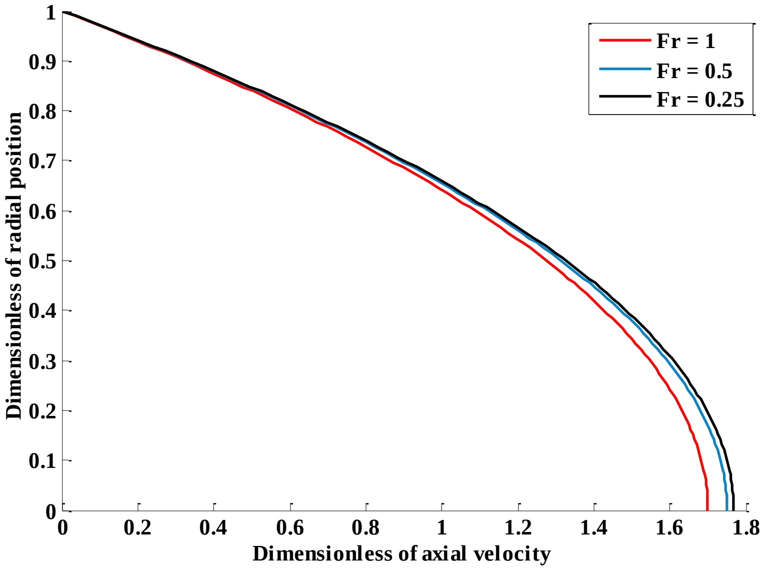

Figure 5. Axial velocity profile for different Fr at Re = 1200 and θ = 30°

Fig. 6. presents the volumetric flow rate (Q). It is increased for higher tracheal inclination angle position (θ= 45°) and Reynold's numbers (Re=2000) compared with the horizontal angle position (θ=0°) and low (Re =800). According to Boyle's law, the increase of flow rate volume of air reduces the resitance to the flow in the trachea, which is extremely important in diving physiology. This is because an increase in the fow resistance (the pressure drop) can collapse the air chambers of the diver's body, especially the lungs, and often causes serious damage (Alnussairy [3]).

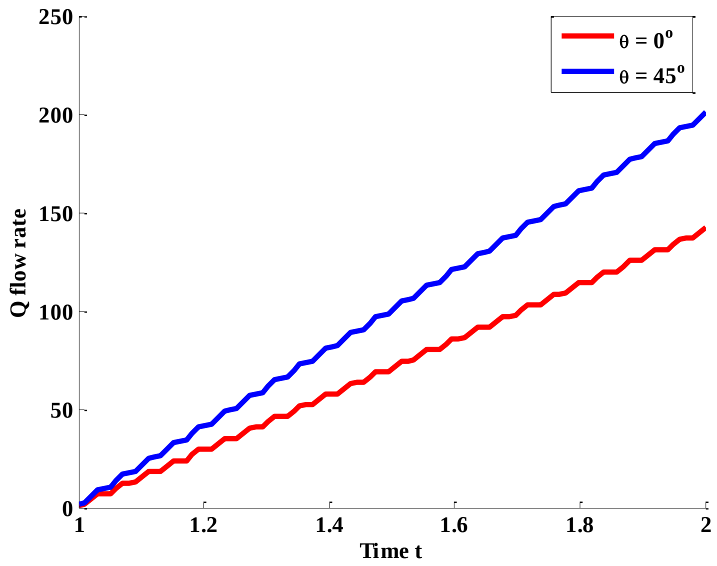

Figure 6. A variation of Q for different θ at Re=1200 and Fr=0.25 a t=1

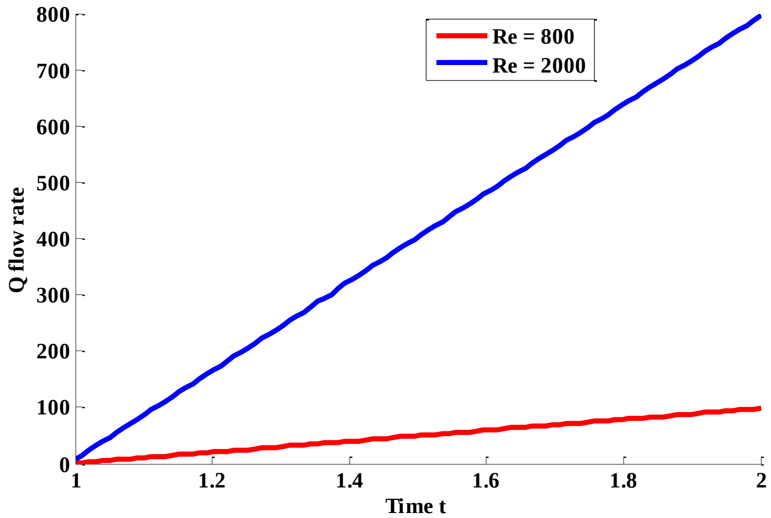

Figure 7. A variation of Q for different Re and θ=0° at t=1

Moreover, the volumetric flow rate (Q). It is increased for lower Froud numbers (Fr=0.25) compared with the higher (Fr =1) when θ=30° and Re=1200 at t=1 is shown in Fig. 8. Therefore, it is supported the phenomenon that increasing of gravity g may lead to reducing the flow resistance, also the velocity of air flow increases with the increase in gravity g(lower Fr).

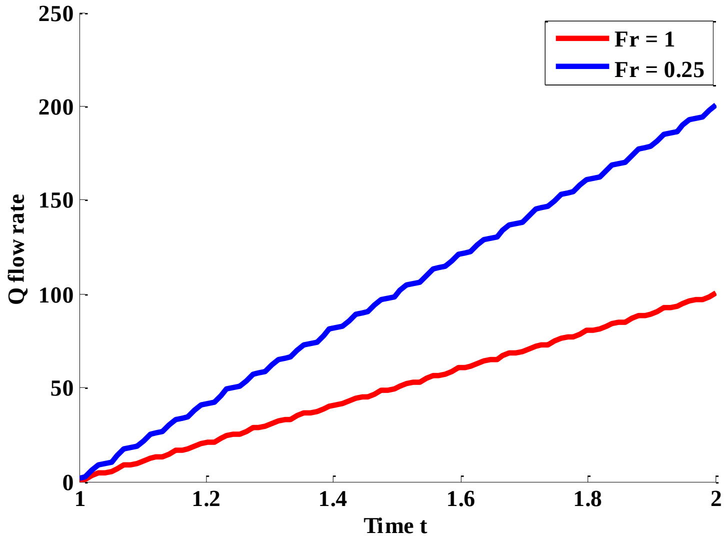

Figure 8. A variation of Q for different Fr when Re=1200 and θ=30º at t=1

Resistance to airflow (λ) is a key mechanical factor contributing to airflow into and out of the lungs [7]. The increase of volumetric flow rate leads is found to reduce the resistance to the flow (Fig. 9,Fig. 10 and Fig. 11). Consequently, the resistance to airflow is reduced with the increase of inclination angle position, Reynold's numbers and increase of gravitation (lower Froud number) respectively

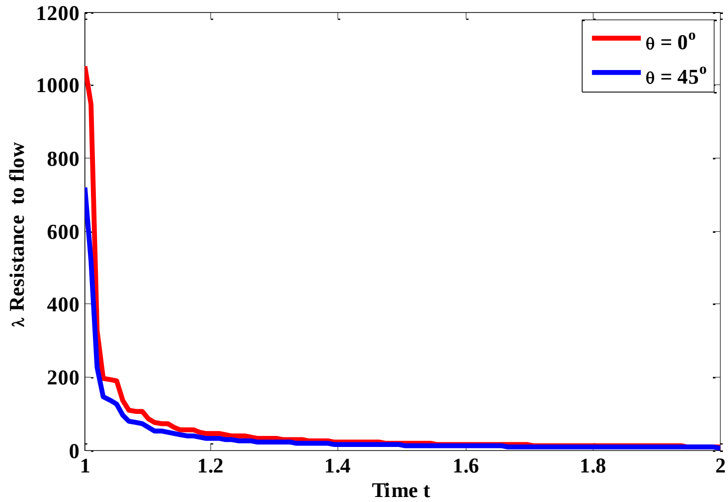

Figure 9. Variation of λ for a different angle at *Re* = 1200 and Fr=0.25 at t=1.

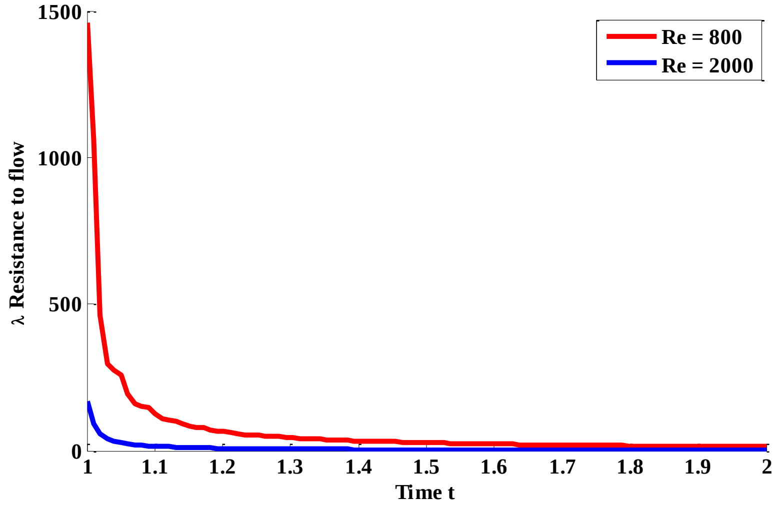

Figure 10. Variation of λ for different Re when θ=0° at t=1.

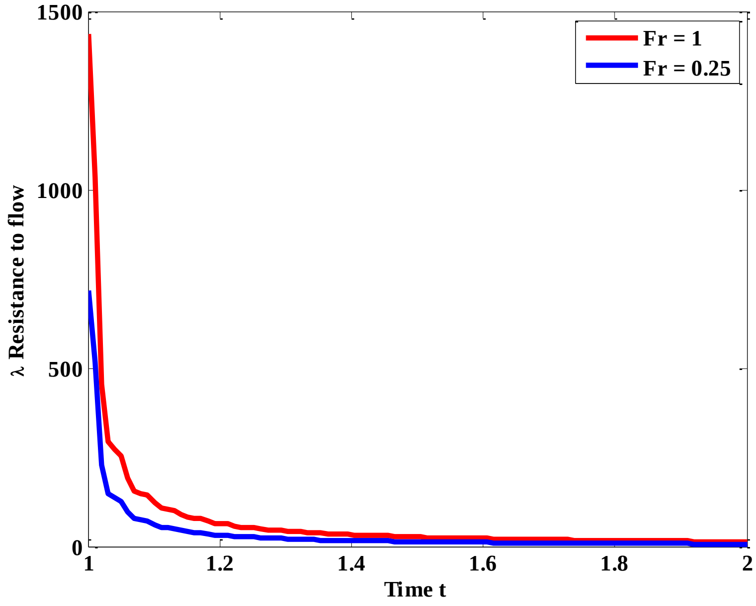

Figure 11. Variation of λ for a different Fr when Re = 1200 and θ=30° at t=1.

4. Concolusion

A symmetric mathematical model based on biomechanics that consists of a set of nonlinear partial differential equations that known as Navier Stokes Equations was developed to investigate the influence of inclination angle on flow characteristics (axial velocity, volumetric flow rate, and resistance to flow). The analytical solution are obtained using Adomian decomposition method (ADM) and the results for axial velocity in horizontal position of the tracheal tube (θ = 0^o) are compared with the findings of (Alnussairy *et al*. [3]). Increasing of inclination angle, Reynold's numbers and gravity force (lower Fr) led to increase of axial velocity and volumetric flow rate in the lung, while, decreasing to the resistance to flow. Thus, it was justified that sleeping in horizontal position may cause a negative effect on the patient, whereas the inclined position is better and depending on the slope angle, and improved breathing. The admirable features of the results suggest that the present model could be applied to validate other experimental and numerical data, which could ultimately assist the diagnostic of patients with chronic obstructive pulmonary and respiratory diseases.

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