Mathematical Model of the Effect of Hyphal Death on (Y-F-H) Types of fungi with Energy

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Abstract— The mathematical model depicts the behavior of dichotomous branching, lateral branching, Tip-hypha anastomosis and Hyphal death, as well as energy consumption. Although there is an error rate, mathematical modeling reduces the amount of work, time, and money required to obtain the desired result. In this work, we will look at a branched mathematical model based on solving the system of partal equation (PDEs). The rsults of this solution, will descrbe the succes or failure of the fungi species tested in terms of growth, and we will used some codes in the numerical solution (pplane7, pdepe), since it was difficult to obtain direct mathematical solutions.

Keywords—Dichotomous branching, Lateral branching, Tip-hypha anastomosis, Hyphal death.

1. Introduction

We established new model, for the growth of mycorrhizal fungi, at this scale. (PDEs) describing the intraction of biomass, with the substrate are the best option. The mathematical structure of these models is complex, with parabolic and hyperbolic elements. As a result, their analytical and numerical features are complex, and a combenation for any number, of the species can be expresed during of the growth stages of partecular fungus. To facilitate the explanation of these species, acronyms are used for each species, as shown in Table (1), which shows several of the biological species that have been mathematically examined, and gives a explanation in addition to descrption of the parameters. And in this paper, we will be combining various types of fungi. [1,7].

Biological type	Symbol	Version	Parameters description
Dichotomous branching	Y	$\delta = \alpha_1 n$	α_1 is the number of tips produced per tip per unit time
Lateral branching	F	$\delta = \alpha_2 \rho$	$\alpha_2$is the number of branches produced per unit length hypha per unit time
Tip-hypha anastomosis	H	$\delta = -\beta_2 n\rho$	$\beta_2$is the rate of tip re-connections per unit length hypha per unit time
Hyphal death	D	d=$\gamma_1\rho$	$\gamma_1$is the loss rate of hyphal (constsnt for hyphal death)

Table 1: Biological type of fungi, symbol of this type, version and description of these parameters.

1 Mathematical Model

We will study a new type of branching of fungal growth with hyphal death and Consumption of whole vegetarian food. We can call it energy $\boldsymbol{\Psi}$**(x)**, **0≤$\boldsymbol{\Psi}$(x)≤1**. Here if $\boldsymbol{\Psi}$**(x)=1** that mean is the growth is very good when the fungi consume all the energy if $\boldsymbol{\Psi}$**(x)=0** means when the grow die it is not consume energy[2 , 3].

We can describe hyphal growth by system below:

$$\frac{\partial\rho}{\partial t} = nv - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial nv}{\partial x} + e^{[\sigma(\rho,n)]} - \Psi \qquad (1)$$

Where: $\sigma(\rho,n)$=$\alpha_1 n$+$\alpha_2\rho - \beta_2 n\rho$ and Ψ=1. Then the system (1) becomes:

$$\frac{\partial\rho}{\partial t} = nv - d\rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial nv}{\partial x} + e^{[\alpha_1 n + \alpha_2\rho - \beta_2 n\rho]} - 1 \qquad (2)$$

2 Non-dimensionlision and Stability

Leah-Keshet (1982) and Ali H. Shuaa Al-Taie (2011) clear up how can put these parameters as dimensionlision [4,8].

$$\frac{\partial\rho}{\partial t} = n - \rho$$
$$\frac{\partial n}{\partial t} = -\frac{\partial n}{\partial x} + e^{[\beta n + \alpha\rho(1-n)]} - 1 \qquad (3)$$

Where:=$\frac{\alpha_2 v}{\gamma^2}$, and β=$\frac{\alpha_1}{\gamma}$and d=1

Now, to find steady states when take from system (3):

n-ρ=0 → n=ρ

And on the other hand

$e^{[\beta n+\alpha\rho(1-n)]} - 1$=0

$e^{[\beta n+\alpha\rho(1-n)]}$=1

$ln e^{[\beta n+\alpha\rho(1-n)]} = ln 1$

$\beta n + \alpha\rho(1-n)$=0

After solving the above equation. We get values of (ρ, n) the steady state are:(0,0), ($\frac{\alpha+\beta}{\alpha}$,$\frac{\alpha+\beta}{\alpha}$) therefor, we take Jacobain of these equation.

$$J_{(\rho,n)}=\begin{bmatrix} -1 & 1 \\ \alpha(1-n) & \beta-\alpha\rho \end{bmatrix}$$

Now, determent the eigenvalues as λ_i; i=1,2

In this case we will take this point (0,0) is saddle point and the point ($\frac{\alpha+\beta}{\alpha}$,$\frac{\alpha+\beta}{\alpha}$) stable spiral for all α,β≥0 and α>β.

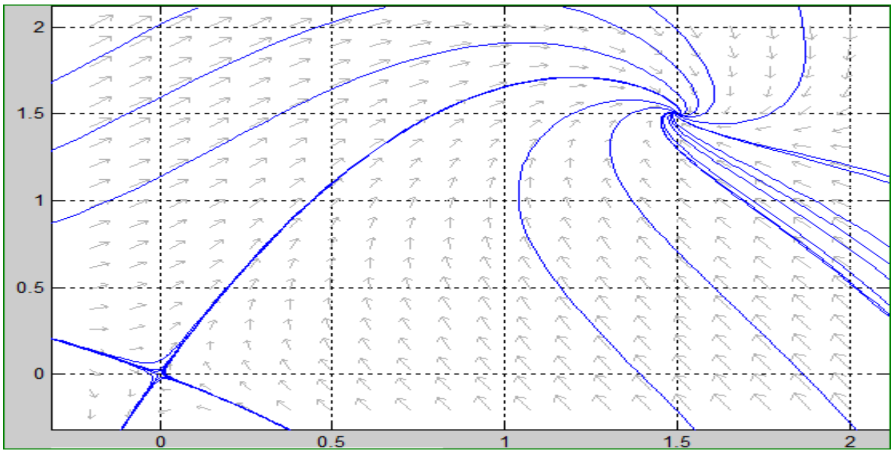

Figure 1: The (ρ,n) plane note that the trajectory connects in the point (0,0) is the saddle point and in the point ($\frac{\alpha+\beta}{\alpha}$,$\frac{\alpha+\beta}{\alpha}$) the stable spiral.

3 Traveling wave solution

Now, we will discuss the traveling wave solution. Here we assume that: ρ(x,t)=P(z), and n(x,t)=N(z) where z=x-ct, P(z) and N(z) are density profile and c rate of propagation of colony edge. P(z) and N(z) non-negative function of z, the function ρ(x,t), n(x,t) are traveling waves and are moves at constant speed c in positive x-direction. Where c>0, Ψ(x)=1, and α=2, and β=1, to find the traveling wave solution of equations in x and t in the form (3) [9]

$$\frac{d\rho}{dt} = -c\frac{dP}{dz}, \frac{dn}{\partial t} = -c\frac{dN}{dz}. \text{ And } \frac{dn}{dx} = \frac{dN}{dz}$$

See [5], therefore the above equation becomes:

$$\frac{dP}{dz} = \frac{-1}{c}[N-P]$$
$$\frac{dN}{dz} = e^{\frac{1}{1-c}[\beta N+\alpha P(1-N)]} \qquad , c \neq 1, -\infty < z < \infty \qquad (4)$$

Then the steady state of equation (4) are:

$$\frac{-1}{c}[N-P] = 0$$

$$\frac{1}{1-c}[\beta N+\alpha P(1-N)] = 0 \qquad , c \neq 1, -\infty < z < \infty \qquad (5)$$

To determent steady state of above system we get (N,P)=(0,0), unstable spiral and (N,P)=($\frac{\alpha+\beta}{\alpha}$,$\frac{\alpha+\beta}{\alpha}$), saddle point, for all $\alpha>\beta$, and 0<c<1. See Fig. 2 Using **MATLAB** pplane7.

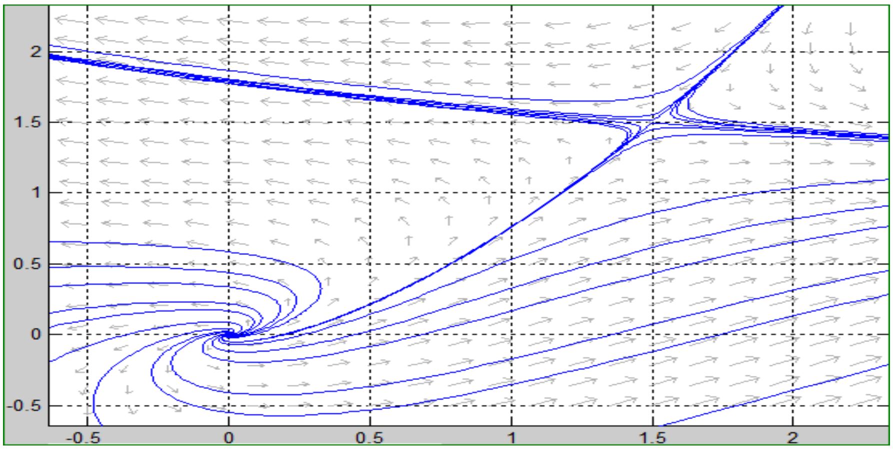

Figure 2: The (N,P) plane note that a trajectory connects the from saddle point at ($\frac{\alpha+\beta}{\alpha}$,$\frac{\alpha+\beta}{\alpha}$) to unstable spiral (0,0).

4 Numerical Solution

Because the system (3) cannot be solved directly, so we resort to numerical solution, and we will using pdepe code in Matlap [6,10-17].

That is clear the initial condition start from 1 to 0 for ρ and n, See Fig. 3

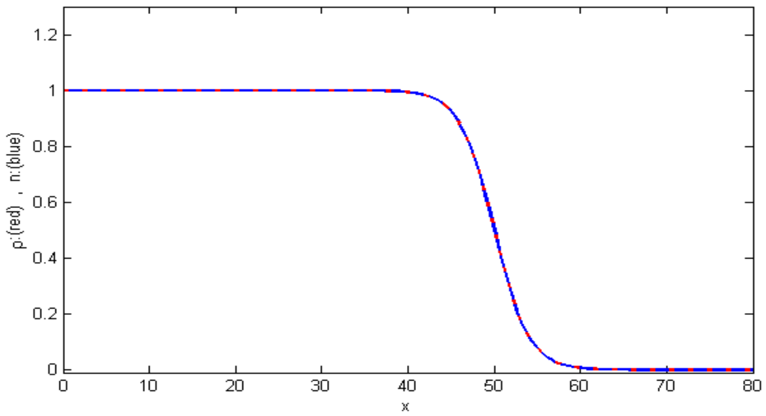

Figure 3: The initial condition of (n), and (p) from 1 to 0, with parameter α=1 and β=0.

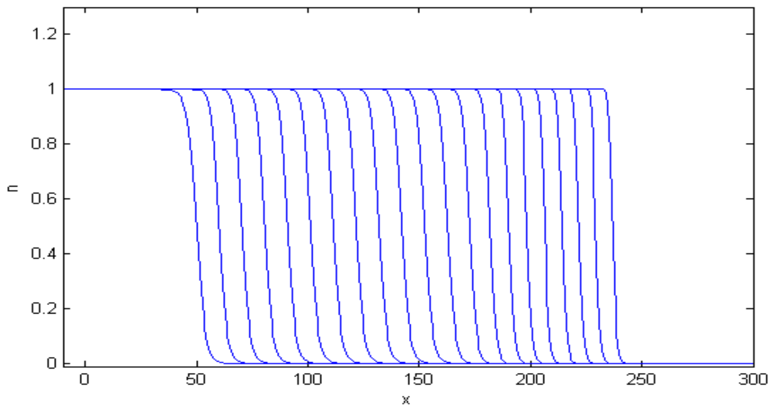

Figure 4: The blue line represented tips n, for all α =1 and β =0, and c=2.0421 ,for time t=1,10,20,....,300.

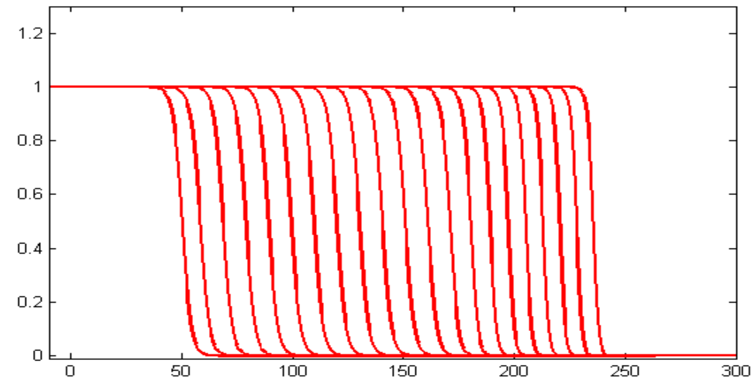

Figure 5: The red line represented branches p, for all α =1 and β =0, and c=2.0421 ,for time t=1,10,20,....,300.

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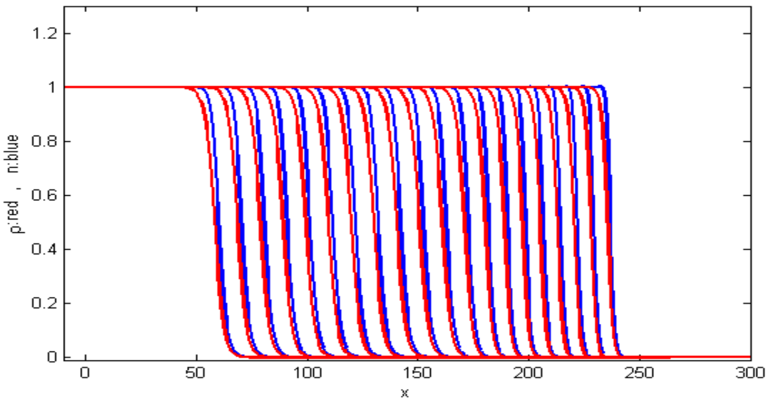

Figure 6: The blue line represented tips N, with the red line represented branches p.

In this paper it was concluded the relationship between traveling wave solution c and parameter α where traveling wave decreasing whenever the values of α increase. See Fig (7).

α	0.5	1	2	3	4	5	6	7	8	9	10
c	10.32	7.56	6.55	6.27	5.71	5.07	4.57	4.26	3.71	3.01	2.56

Table 2: The relation between values α and waves speed c with taking v=β=d=1.

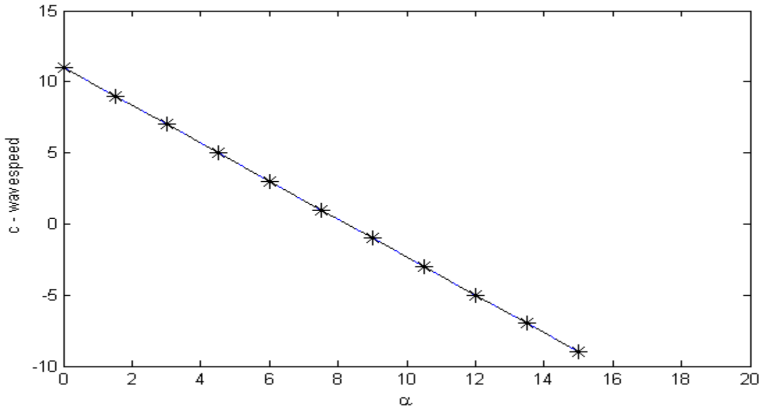

Figure 7: The relation between the wave speed c and parameter α.

Now, we take the relationship between the wave speed c and parameter β, such that we note the traveling wave increasing whenever the values of β increase. See Fig (8).

β	0.5	1	2	3	4	5	6	7	8	9	10
c	4.33	7.56	9.78	13.01	17.23	20.46	22.68	25.90	28.13	31.35	34.57

Table 3: The relation between values α and waves speed c with taking v=α=d=1.

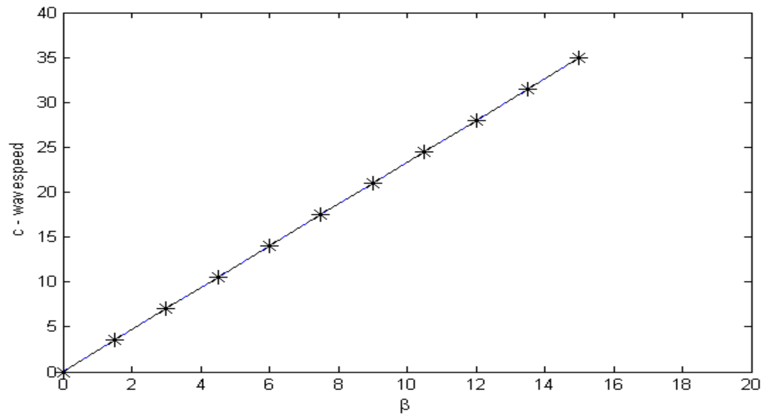

Figure 8: The relation between the wave speed c and parameter β.

Now, we take the relationship between the wave speed c and values v, such that we note the traveling wave increasing when the values of v increase. See Fig. (9).

v	0.5	1	2	3	4	5	6	7	8	9	10
c	5.15	7.56	13.1	18.79	22.74	25.31	27.28	29.83	31.80	33.23	35.73

Table 4: The relation between values v and waves speed c with taking $\beta=\alpha=$d=1.

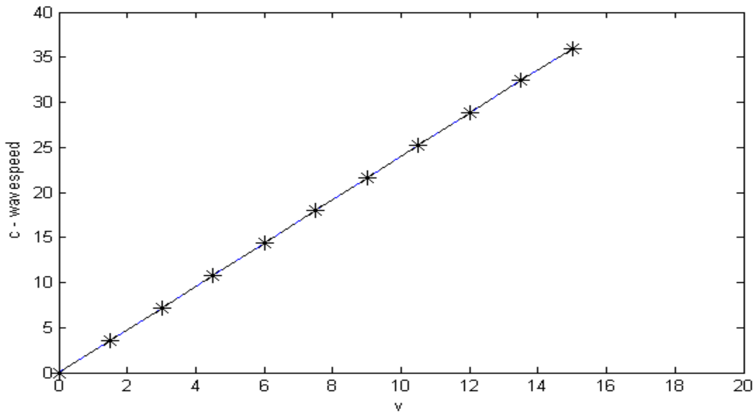

Figure 9: The relation between the wave speed c and values of v.

Now, take the relationship between the wave speed c and the values of d, then we note the wave speed decreasing when the value of d increase. See Fig (10).

d	0.5	1	2	3	4	5	6	7	8	9	10
c	11.73	7.56	5.48	4.66	4.27	4.04	3.88	3.64	3.25	2.89	2.41

Table 5: The relation between values d and waves speed c with taking $\beta=\alpha=$v=1.

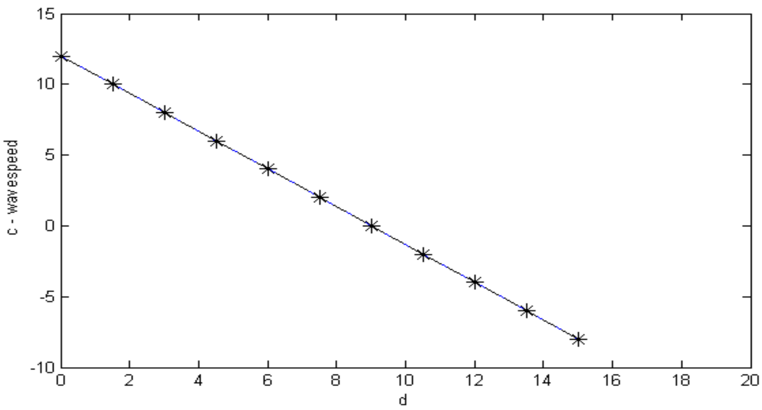

Figure 10: The relation between the wave speed c and the values of d.

5 Conclusion

We plot relationship between c and α, see Fig. (7), that is clear the wave speed c is decreasing when α an increase function . And we plot relationship between c and β, see Fig. (8), such that the wave speed c is increasing when β an increase function. Also, we plot relationship between c and v, see Fig. (9), that is clear the wave speed c is increasing when v an increase function, and in Fig. (10) we note the wave speed deceasing when the values of d increasing. Since (α=$\frac{\alpha_2 v}{\gamma^2}$, $and\ \beta = \frac{\alpha_1}{\gamma}$), therefore the growth rate is increasing with $\alpha_2 v$ while keeping γ^2 are fixed, while the growth is decreasing with γ^2 increases while keeping $\alpha_2 v$, and the growth rate is increasing with α_1 while keeping γ are fixed, while the growth is decreasing with γ increases while keeping α_1 are fixed.

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