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Mixed H_2/H-infinity Controller Design for Triple Inverted Pendulum System

Yamama A. Shafeek[1], Hazem I. Ali[2]

[1,2] Control and Systems Engineering Department, University of Technology, Baghdad, Iraq

[1]cse.21.20@grad.uotechnology.edu.iq , [2]Hazem.I.Ali@uotechnology.edu.iq

Abstract *— The steady increase in reliance on robotic systems in industrial applications arises the need to ensure their reliability despite the presence of external disturbances in the environment they work in and their parameters variations due to several factors like: change of temperature, wear and tear effect, and payload variation. Traditional controllers usually fail to maintain the system's stability and performance in the presence of uncertainty, while robust controllers aim to guarantee stability and desired performance across a range of possible uncertainties in the system through incorporating these uncertainties into their design process. This paper investigates the robustness of three controllers namely, H_2 controller, H-infinity controller, and mixed H_2/H-infinity controller, applied to the triple inverted pendulum system which translates to robustly control legged robots and robotic arms. The results show that among the three controllers, only the mixed H_2/H-infinity control system is robustly stable and maintains its performance with stability margin 1.1864 and performance margin 1.1662.*

Index Terms *— H_2, H-infinity, Mixed H_2/H-infinity, Robust Control, triple inverted pendulum.*

I. INTRODUCTION

The triple inverted pendulum system is a challenging benchmark in control engineering due to its complex inherently-unstable nonlinear dynamics. Robust control offers a powerful framework to design controllers for this system that can guarantee desired performance and stability despite the presence of uncertainties.

There are several sources of uncertainty in this system including external disturbance, noise, unmodelled dynamics of actuators, and parameters variations. The external disturbances can be wind gusts, or unexpected applied forces while the noise in potentiometer readings produces fluctuations in measurements. The unmodelled dynamics of actuators are the physical behavioral characteristics of actuator that might not be perfectly captured in the mathematical model. Parameters variations come either from imperfect knowledge of the system's physical parameters (inertia or friction) or from environmental factors or are caused by the manufactoring tolerances of components. Ignoring the presence of uncertainty in the system may lead to performance degredation when the system deviates from its nominal case or even may cause instability leading to oscillations or unexpected behavior.

Several controllers have been designed in the last few years for the triple inverted pendulum system. Masrom *et al.* [1] used Interval Type-2 Fuzzy Logic Control (IT2FLC) as control algorithm for the system and utilized the error and its rate of change as inputs to the controller and applied the output to the motors of the system. Yet, only disturbance rejection on the third link has been tested. Then in [2], they applied particle swarm optimization to obtain the optimal gains of the input and output variables of the IT2FLC without testing system performance under uncertainty. Later in [3], they utilized another

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optimization technique, namely, spiral dynamic algorithm to obtain the controller gains. The study examined only the disturbance effect on the performance.

In [4] Gupta *et al*. applied state feedback to control the fully actuated triple inverted pendulum system, without considering the unmodelled actuators dynamics or parameters variations. Pang *et al*. [5] proposed a data-driven optimistic least-squares-based policy iteration algorithm, to solve the optimal stationary control problem with realtive accuracy for the triple inverted pendulum system perturbed by noise, hence, the functionality of the proposed method was not been validated for perturbations caused by disturbance, unmodelled actuators dynamics, or parameter variations.

The triple inverted pendulum was used by Lippi *et al*. [6] to represent the bio-inspired humanoid for posture control, a distributed control strategy was presented to let balance one module only at a time, keeping other joints fixed. The control system receives real-time estimates of external disturbances. This allows it to adjust the joint torque automatically, hence, other sources of uncertainty in the system were not considered.

Sayer *et al*. [7] have developed and implemented both Discrete Linear Quadratic Regulator (DLQR) and Linear Quadratic Gaussian (LQG) controllers to balance the triple inverted pendulum, Kalman filter was used to reduce the noise. Both controllers could stabilize the system in the vertical position in an appropriate period with better performance achieved by DLQR control system. Though, the work has not study the effect of parameters uncertainty.

H_2 and H-infinity controllers have been used effectively in differenet control systems [8]-[13] to over come the effects of uncertainty. For instance, in [8] H_2 sliding mode controller has been used for mobile inverted pendulum stablization despite parameters variations and applied disturbance. The simulation results have shown that the addition of the H_2 controller to the sliding mode control yields lower control effort and better response.

Model reference control has been integrated with H-infinity technique in [9] to control the magnetic levitation system, the controller was found to be very effective in handling system parameters variations within range of ± 10%. The double pendulum structure has been used by [10] to model human swing leg system, the system has been stabilized by an H-infinity based full state feedback controller within 0.25 sec and 0.21 sec for hip and knee joints, respectively.

A hybrid H-infinity and IT2FLC controller has been proposed by [11] to assure both robust stability and robust performance of human swing lag system. Compared to the classical H-infinity controller, the proposed method enhanced both tracking performance and disturbance rejection. In addition, the system's robustness to parameters changes has been increased to 98%.

Mhmood *et al*. have applied optimal H-infinity model reference control for tail-sitter vertical takeoff and landing unmanned aerial vehicles [12] and nonlinear systems [13], The results indicate that the proposed controller is very powerful in compensating the systems' nonlinearity, parameters variations, rejecting external disturbances, and in achieving asymptotic tracking performance.

Mixed H_2/H-infinity approach has been utilized significantly in many applications. In [14], hard disk drive has been controlled and the disturbance has been rejected successfully through minimizing the H_2 norm of the piezoelectric actuator stroke's variance and constraining the H-infinity norm of the sensitivity function. [15] has developed mixed H2/H-infinity state feedback controller for active suspension system with input delay, the controller could handle parameters uncertainty and increase the ride comfort. Stoica [16] has employed mixed H_2/H-infinity in networked systems with fading communication

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channels for disturbance attenuation, whereas, Chen *et al*. have employed it in elevator active guide shoe and succeeded in suppressing the vibration of elevator car [17].

Bai *et al.* [18] have used sum of squares method to form the mixed H_2/H-infinity control problem by linear matrix inequality (LMI), the approach has enhanced both the disturbance attenuation and the transient response of a mass-spring-damper system compared with the traditional H-infinity control. LMI has been also used in designing mixed H_2/H-infinity controller for the hybrid multi-infeed high-voltage direct current system by Li *et al*. [19] to place the poles in a desired region to avoid the inaccuracy caused by uncertainty.

The literature review shows a gap in complete robustness analysis of triple inverted pendulum and directs towards getting benefit from the effectiveness of H_2, H-infinity and mixed H_2/H-infinity methods to control the system.

On this basis, this paper develops and evaluates the robustness of H_2 controller, H-infinity controller, and mixed H_2/H-infinity controller for triple inverted pendulum system which is a challenging task because of system's high nonlinearity, inherent instability, multi input/multi output nature, under actuation, and uncertainty.

The rest of the paper is organized as follows: Section II investigates the triple inverted pendulum model. Then the control problem is formulated in section III. The results are given and discussed in section IV. Lastly, conclusions are made in section V.

II. TRIPLE INVERTED PENDULUM SYSTEM

The triple inverted pendulum system is composed of of three links hinged by ball bearings as shown in *Fig. 1*. The angles of the three links θ_1, θ_2, and θ_3 are to be controlled by two DC motors attached to the first and third links. The torques produced by motors are provided through timing belts to the second and third hinges. A horizontal bar is added on each link to help in system's balance. In order to measure the three angles, three potentiometers are mounted on the three hinges. The system's nomenclature is defined in Table I [20].

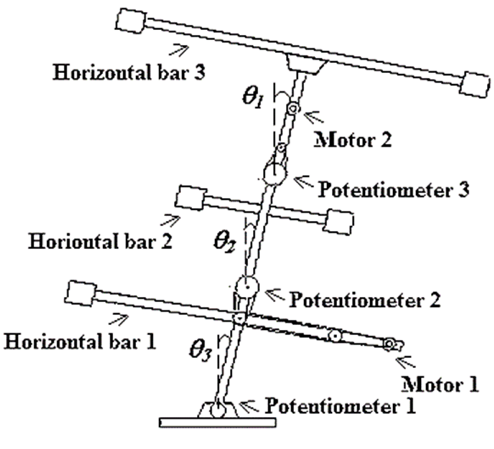

FIG. 1. TRIPLE INVERTED PENDULUM [20].

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The mathematical equations of the model is obtaind by applying Lagrange equations

$$M(\theta)\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} + N_c\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} + \begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix} + G\begin{bmatrix}t_{m1}\\ t_{m2}\end{bmatrix} = T\begin{bmatrix}d_1\\ d_2\\ d_3\end{bmatrix} \tag{1}$$

where:

$$M(\theta) = \begin{bmatrix} J_1 + I_{p1} & l_1\, M_2 \cos(\theta_1 - \theta_2) - I_{p1} & l_1\, M_3 \cos(\theta_1 - \theta_3) \\ l_1\, M_2 \cos(\theta_1 - \theta_2) - I_{p1} & J_2 + I_{p1} + I_{p2} & l_2\, M_3 \cos(\theta_2 - \theta_3) - I_{p2} \\ l_1\, M_3 \cos(\theta_1 - \theta_3) & l_2\, M_3 \cos(\theta_2 - \theta_3) - I_{p2} & J_3 + I_{p2} \end{bmatrix},$$

$$N_c = \begin{bmatrix} C_1 + C_2 + C_{p1} & -C_2 - C_{p1} & 0 \\ -C_2 - C_{p1} & C_{p1} + C_{p2} + C_2 + C_3 & -C_3 - C_{p2} \\ 0 & -C_3 - C_{p2} & C_3 + C_{p2} \end{bmatrix},$$

$$q_1 = l_1 M_2 \sin(\theta_1 - \theta_2)\, \dot{\theta}_2^2 + l_1 M_3 \sin(\theta_1 - \theta_3)\, \dot{\theta}_3^2 - M_1 g \sin(\theta_1),$$

$$q_2 = l_1 M_2 \sin(\theta_1 - \theta_2)\, \dot{\theta}_1^2 + l_2 M_3 \sin(\theta_2 - \theta_3)\, \dot{\theta}_3^2 - M_2 g \sin(\theta_2),$$

$$q_3 = l_1 M_3 \sin(\theta_1 - \theta_3)\,(\dot{\theta}_1^2 - 2\,\dot{\theta}_1\dot{\theta}_3) + l_2 M_3 \sin(\theta_2 - \theta_3)\,(\dot{\theta}_2^2 - 2\,\dot{\theta}_2\dot{\theta}_3) - M_3 g \sin(\theta_3),$$

TABLE I. THE TRIPLE INVERTED PENDULUM MODEL'S NOMENCLATURE [20]

Symbol	Parameter
l_i	i^{th} link length, m
h_i	the distance from the bottom to the center of gravity of the i^{th} link, m
m_i	i^{th} link mass , kg
θ_i	i^{th} link angle from vertical line, rad
α_i	i^{th} potentiometer gain, V. rad $^{-1}$
I_{mj}	j^{th} motor moment of inertia , kg . m^2
$C_{p_i'}$	i^{th} hinge viscous friction coefficient of the belt–pulley system, N . m . s
$I_{p_i'}$	i^{th} hinge moment of inertia of the belt–pulley system, kg . m^2
K_i	ratio of teeth of belt–pulley system of the i^{th} hinge
I_i	i^{th} link moment of inertia around the center of gravity, kg . m^2
C_i	i^{th} hinge viscous friction coefficient, N. M . s
C_{mj}	j^{th} motor viscous friction coefficient, N . m . s
u_j	j^{th} motor input voltage, V
t_{mj}	j^{th} motor control torque, N . m
τ_i	i^{th} lonk disturbance torque, N . m
g	acceleration of gravity, m . s^{-2}

$$G = \begin{bmatrix} K_1 & 0 \\ -K_1 & K_2 \\ 0 & -K_2 \end{bmatrix},\ T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix},$$

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$C_{pi} = C_{p_i'} + K_i^2\, C_{mi}$, $I_{pi} = I_{p_i'} + K_i^2\, I_{mi}$, $M_1 = m_1\, h_1 + m_2\, l_1 + m_3\, l_1$,

$M_2 = m_2\, h_2 + m_3\, l_2$, $M_3 = m_3\, h_3$, $J_1 = I_1 + m_1\, h_1^2 + m_2\, l_1^2 + m_3\, l_1^2$, $J_2 = I_2 + m_2\, h_2^2 + m_3\, l_2^2$, and $J_3 = I_3 + m_3\, h_3^2$.

For controller design, linearized model is required. The linearized system equations are:

$$M_l \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + N_c \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} + P_l \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + G \begin{bmatrix} t_{m1} \\ t_{m2} \end{bmatrix} = T \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \tag{2}$$

where:

$$M_l = \begin{bmatrix} J_1 + I_{p1} & l_1\, M_2 - I_{p1} & l_1\, M_3 \\ l_1\, M_2 - I_{p1} & J_2 + I_{p1} + I_{p2} & l_2\, M_3 - I_{p2} \\ l_1\, M_3 & l_2\, M_3 - I_{p2} & J_3 + I_{p2} \end{bmatrix}, \quad \text{and} \quad P_l = \begin{bmatrix} -M_1 g & 0 & 0 \\ 0 & -M_2 g & 0 \\ 0 & 0 & -M_3 g \end{bmatrix}$$

.

The measured output vector $y_p$is

$$y_p = C_p \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \tag{3}$$

where $C_p = \begin{bmatrix} \alpha_1 & 0 & 0 \\ -\alpha_2 & \alpha_2 & 0 \\ 0 & -\alpha_3 & \alpha_3 \end{bmatrix}$

Lastly, the actuators are modeled as:

$$G_{mj}(s) = \frac{K_{mj}}{T_{mj}\, s + 1} \tag{4}$$

where K_{mj} and T_{mj} are the gain and time constant of the j^{th} actuator.

The uncertainty sources of the triple inverted pendulum system are identified as external disturbance, measuement noise, variations of mements of inertia (±10%), friction coefficients (±15%), actuators' gain coefficients (±10%), and actuators' time constants (±20%).

III. CONTROL PROBLEM FORMULATION

To develop robust control, the control system in the generalized form shown in *Fig. 2* must be used to represent the system and the controller, while accounting for uncertainties. In this form, the generalized plant is represented by *P*, *K* represents the controller, the signals *u*, y, *w*, and *z* represents the control input, the measured output, external disturbances and noise, and the error signal to be minimized; respectively.

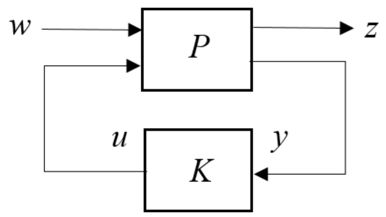

FIG. 2. GENERALIZED SYSTEM FORM.

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The robust control aims to design the controller (K) such that the system becomes robust to uncertainties and maintains good performance even when the actual behavior of the plant deviates from the nominal model due to uncertainties. Then the control input u is:

$$u(s) = K\, y(s) \tag{5}$$

The controller K is to be obtained by H_2 control, H-infinity control, and mixed H_2/H-infinity control as follows:

A. H_2 Control

The transfer matrix P is partitioned as:

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right]$$

so that P_{22} is strictly proper and to guarantee that the H_2 problem is properly posed.

In order to ensure that G is stabilizable by output feedback, and to avoid nonsingularity in the H_2 optimal control problem, the followings are assumed [21]:

a) The pair (A, B_2) is stabilizable.
b) The pair (C_2, A) is detectable.
c) $R_1 = D_{12}^* D_{12} > 0$.
d) $R_2 = D_{21}\, D_{21}^* > 0$.
e) The matrix $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank at all frequencies.
f) The matrix $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full row column at all frequencies.

The stated assumptions also guarantees that the following two Hamiltonian matrices belong to the domain of Riccati:

$$H_2 = \begin{bmatrix} A - B_2 R_1^{-1} D_{12}^* C_1 & -B_2 R_1^{-1} B_2^* \\ -C_1^*(I - D_{12} R_1^{-1} D_{12}^*) C_1 & -(A - B_2 R_1^{-1} D_{12}^* C_1)^* \end{bmatrix} \tag{6}$$

$$J_2 = \begin{bmatrix} (A - B_1 D_{21}^* R_2^{-1} C_2)^* & -C_2^* R_2^{-1} C_2 \\ -B_1(I - D_{21}^* R_2^{-1} D_{21}) B_1^* & -(A - B_1 D_{21}^* R_2^{-1} C_2) \end{bmatrix} \tag{7}$$

The solutions of the Riccati of H_2 and J_2 are X_2 and Y_2; respectively.

The H_2 control problem is to find a proper, real rational controller K_2 that stabilizes P internally and in the same time, minimizes the H_2 norm of the closed loop system. To solve this problem, let:

$$P_c(s) = \begin{bmatrix} A_{F2} & I \\ C_{1F2} & 0 \end{bmatrix} \tag{8}$$

$$P_f(s) = \begin{bmatrix} A_{L2} & B_{1L2} \\ I & 0 \end{bmatrix} \tag{9}$$

where:
$A_{F2} = A + B_2 F_2$, $C_{1F2} = C_1 + D_{12} F_2$, $A_{L2} = A + L_2 C_2$, $B_{1L2} = B_1 + L_2 D_{21}$,
$F_2 = -R_1^{-1}(B_2^* X_2 + D_{12}^* C_1)$, and $L_2 = -(Y_2 C_2^* + B_1 D_{21}^*) R_2^{-1}$
then the optimal controller is:

$$K_2(s) = \left[\begin{array}{c|c} \hat{A}_2 & -L_2 \\ \hline F_2 & 0 \end{array}\right] \tag{10}$$

where $\hat{A}_2 = A + B_2 F_2 + L_2 C_2$

The minimum H_2 norm of the transfer matrix (T_{zw}) is:

$$\min \|T_{zw}\|_2^2 = \|P_c B_1\|_2^2 + \left\| R_1^{1/2} F_2 P_f \right\|_2^2 \tag{11}$$

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B. H-infinity Control

The generalized form given in *Fig. 2* is also used in the H-infinity controller design. The H-infinity control problem is to find all admissible controllers $K_\infty(s)$ such that $\|T_{zw}\|_\infty < \gamma$, where $\gamma > 0$ is the performance level. The realization of the transfer matrix P is the same as used in H_2 control. The followings are assumeptions are made [21]:

a) The pair (A, B_1) is controllable.
b) The pair (C_1, A) is observable.
c) The pair (A, B_2) is stabilizable.
d) The pair (C_2, A) is detectable.
e) $D_{12}^*[C_1 \quad D_{12}] = [0 \quad I]$.
f) $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^* = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

The two Hamiltonian matrices belong to the domain of Riccati are:

$$H_\infty = \begin{bmatrix} A & \gamma^{-2}B_1B_1^* - B_2B_2^* \\ -C_1^*C_1 & -A^* \end{bmatrix} \tag{12}$$

$$J_\infty = \begin{bmatrix} A^* & \gamma^{-2}C_1^*C_1 - C_2^*C_2 \\ -B_1B_1^* & -A \end{bmatrix} \tag{13}$$

The solutions of the Riccati of H_∞ and J_∞ are X_∞ and Y_∞; respectively. The H-infinity controller K_∞ that stabilizes P internally and minimizes $\|T_{zw}\|_\infty$ is then:

$$K_\infty(s) = \left[\begin{array}{c|c} \hat{A}_\infty & -Z_\infty L_\infty \\ \hline F_\infty & 0 \end{array}\right] \tag{14}$$

where $\hat{A}_\infty = A + \gamma^{-2}B_1B_1^*X_\infty + B_2F_\infty + Z_\infty L_\infty C_2$, $F_\infty = -B_2^*X_\infty$, $L_\infty = -Y_\infty C_2^*$, and $Z_\infty = (I - \gamma^{-2}Y_\infty X_\infty)^{-1}$.

C. Mixed H2/H-infinity Control

The generalized form given in *Fig. 2* is slightly modified in the mixed H_2/H-infinity controller design to include two error outputs z_2 and z_∞ as shown in *Fig. 3*.

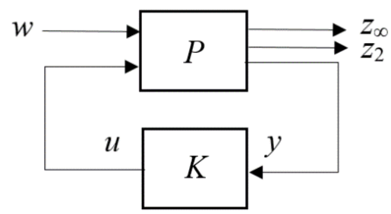

FIG. 3. GENERALIZED SYSTEM FORM FOR MIXED H-2/H-INFINITY CONTROL.

In this configuration, the H-infinity channel is used to improve system's robustness, while the H_2 channel minimizes the output energy in response to uncertainty. The realization of the transfer matrix P is:

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \\ C_y & D_{y1} & D_{y2} \end{array}\right]$$

The mixed H_2/H-infinity control problem is to find a controller $K_{mixed}(s)$ such that $\|T_{z_\infty w}\|_\infty < \gamma_\infty$, $\|T_{z_2 w}\|_2 < \gamma_2$, and minimizes the trade-off criterion:

$$W_1\|T_{z_\infty w}\|_\infty^2 + W_2\|T_{z_2 w}\|_2^2$$

The mixed H_2/H-infinity controller K_{mixed} is [22]:

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$$K_{mixed}(s) = \left[\begin{array}{c|c} A_{ml} + B_2 F_\infty & -L \\ \hline F_\infty & 0 \end{array}\right] \tag{15}$$

where $A_{ml} = A + \gamma^{-2} B_1 B_1^* X_\infty + L(C_2 + \gamma^{-2} D_{21} B_1^* X_\infty)$, and $F_\infty = -(D_{12}^* C_1 + B_2^* X_\infty)$.

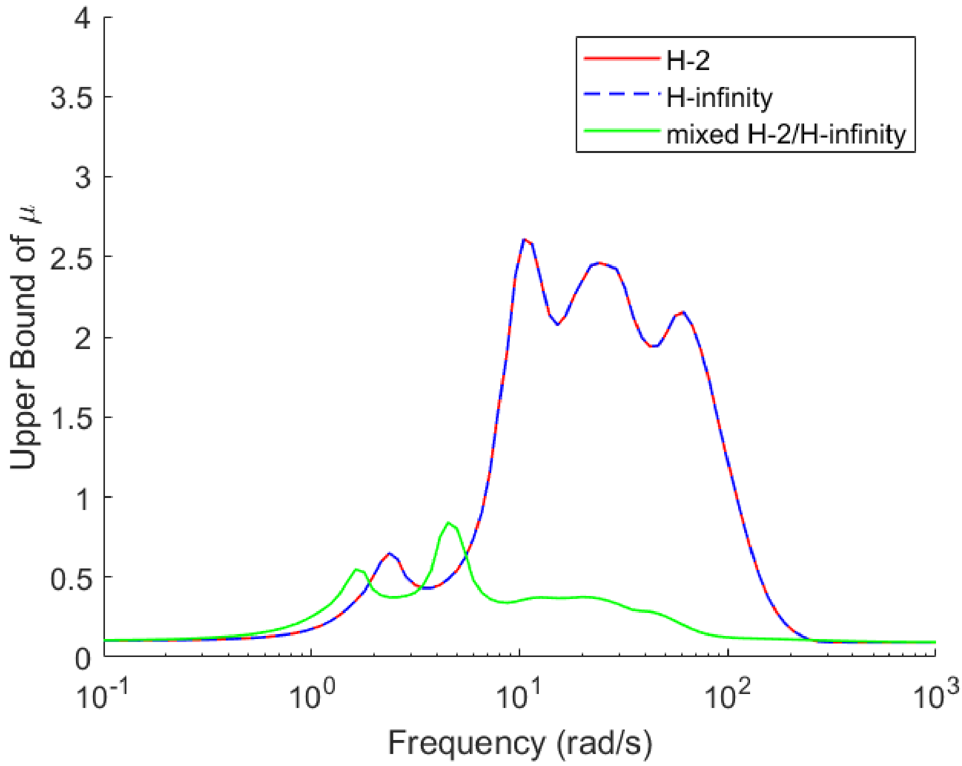
IV. RESULTS AND DISCUSSION

In this section, the three developed controllers are implemented on the triple inverted pendulum system. The control stystems are simulated using Matlab R2023b. The robustness measurements and results are discussed.

The weights W_1 and W_2 of H-infinity norm and H_2 norm in mixed H_2/H-infinity control system are chosen to be 2 and 1; respectively.

Fig. 4 shows the upper bounds of the structured singular values over the frquency range.For robustness, the structured singular value has to be less than one. As the figure shows, the H_2 and H-infinity control systems do not sayisfy the condition for robustness. In fact, the bounds for H_2 and H-infinity control systems are identical as seen. Only the mixed H_2/H-infinity control system satisfies the condition of robustness. Other robustness measurements are given in Table II besides the structured singular value.

For H_2 and H-infinity control systems, the maximum structured singular value (μ) for robust stability is greater than 1 which means that the systems are not robustly stable for the modeled uncertainty. The control systems can handle up to 38.3% of the modeled uncertainty as indicated by the stability margin. The systems also do not achieve performance robustness to modeled uncertainty since μ for robust performance is greater than 1 too. A compromise between model uncertainty and system gain is balanced at a level of 38.2% of the modeled uncertainty as given by the performance margin.



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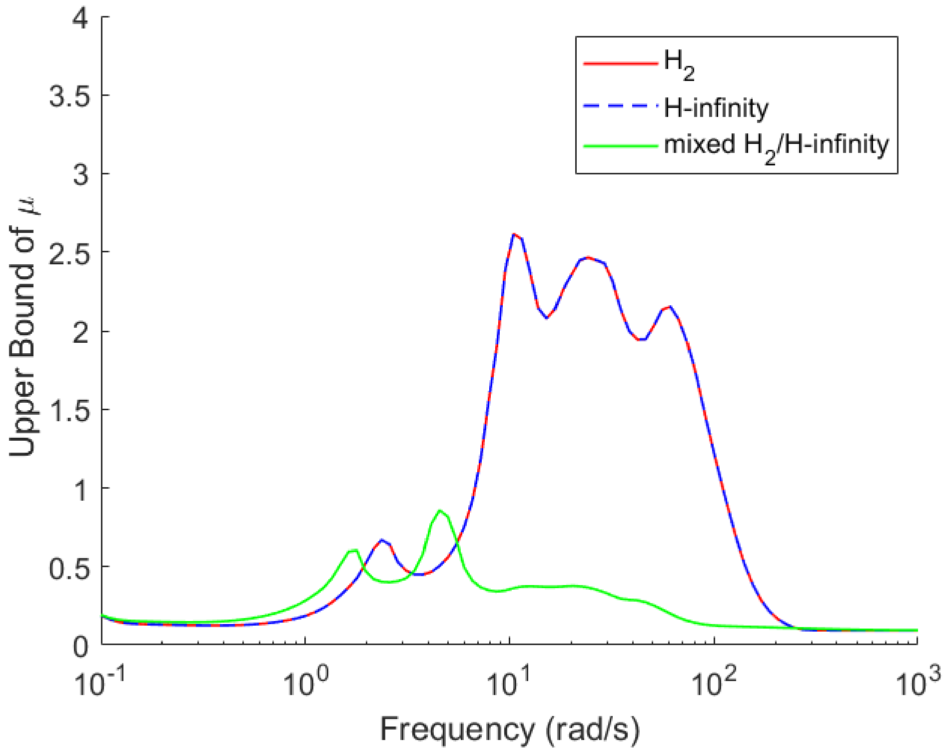

(B)

FIG. 4. UPPER BOUNDS OF STRUCTURED SINGULAR VALUES (A) FOR ROBUST STABILITY (B) FOR ROBUST PERFORMANCE.

TABLE II. CONTROL SYSTEMS' ROBUSTNESS MEASUREMENTS

Measurement	H_2	H-infinity	Mixed H_2/ H-infinity
Maximum μ for robust stability	2.608	2.608	0.842
Maximum μ for robust performance	2.615	2.615	0.857
Performance level	γ_2 =0.0701	γ_∞ =0.4343	γ_∞ =0.4343, γ_2 =0.0114
Stability margin	0.383	0.383	1.1864
Performance margin	0.382	0.382	1.1662

For mixed H_2/H-infinity control system, the maximum structured singular value (μ) for robust stability is 0.842 which means that the system is robustly stable for the modeled uncertainty. The control systems can handle up to 118.6% of the modeled uncertainty. The system also maintains performance robustness to modeled uncertainty with peformance margin 1.1662. The H-infinity norm of the transfer matrix from w to z_∞ , $\|T_{z_\infty w}\|_\infty$, is guaranteed to be less than 0.4343 which means that the effect of the modeled uncertainty on the desired error z_∞ is attenuated. In the same time, the H_2 norm of the transfer matrix from w to z_2 , $\|T_{z_2 w}\|_2$, is guaranteed to be less than 0.0114 which also indicates the attenuation of the uncertainty impact on z_2.

Since the stability and performance margins of the H_2 and H-infinity control systems are less than 1, the response of the systems would detoriorate when deviated from the nominal case due to parameteric variation, disturbance, and noise. On the other hand,

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having stability and performance margins for the mixed H_2/H-infinity control system that are greater than 1 leads to maintaining stability and performance in the presence of uncertainties.

Next, two scenarios are simulated to test the effect of parameters variation, noise and disturbance. The parameter variation arises in control systems due to continuous use and enviromental condition that changes plant physical parameters (moments of enertia and friction coefficients). To imitate variations of these physical values, uncertainty in their values is modeled such that the robust controller results in stable and good performance response when the system deviates from its nominal case. The uncertainty amount of moments of inertia and friction coefficients are considered to be 10% and 15% of their nominal values, respectively. The noise arises in the fed back signals to the controller due to imperfect readings of potentiometers. The considered readings noise is 0.1 V in each potentiometer. Finally, the disturbance that arises from air movement or unexpected force is tested by applying torque equal to 0.1 N.m which is very reasonable amount for this system.

A. First Scenario

The parameters variation effect is tested by simulating 10 system samples of modeled uncertainity. The reference input is $[0\ -5\ 10]^T$ deg. The applied noise is $[0.1\ 0.1\ 0.1]^T$ V. The three control systems respond as shown in figures 5, 6, and 7.

Both H_2 and H-infinity control systems could not maintain robust stability and performance when the systems deviate from the nominal case. While all the 10 samples of uncertainty maintain the system's stability and performance in the mixed H_2/H-infinity control system. As seen in *Fig. 5 and 6*, the three links become unstable for samples of modeled uncertainty which means that the system is not robust according to the stability margin. From the response in *Fig. 7*, it can be seen that θ_1 goes under the desired value by 0.486 deg at 5.58 sec then settles in 21.39 sec on 0.296 deg, θ_2 settles with steady state error 0.197 deg, and θ_3 overshoots 0.511 deg above the desired value.

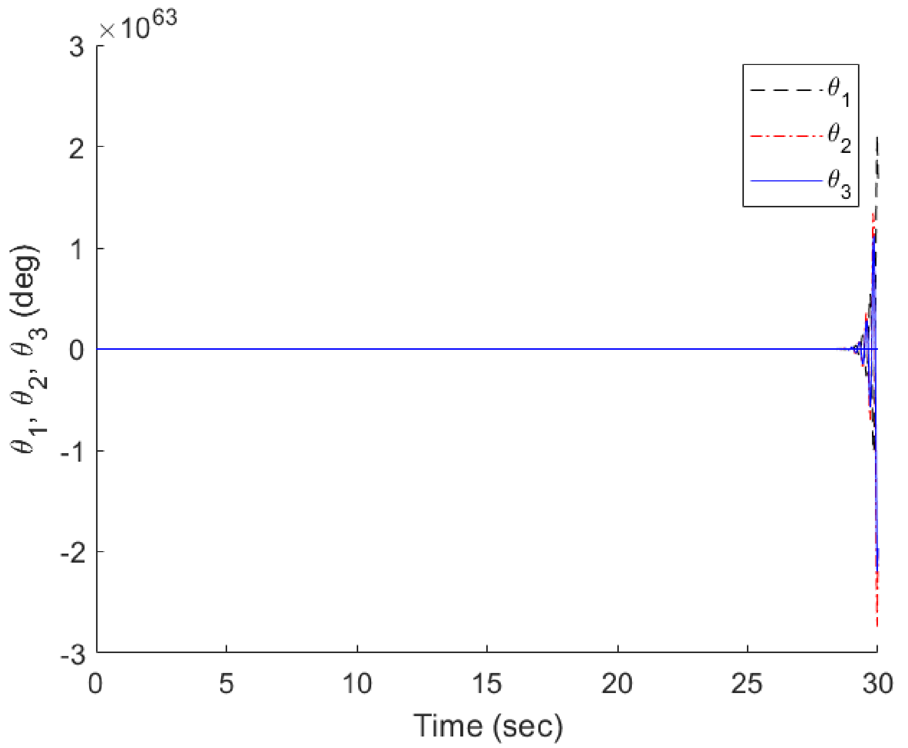

FIG. 5. RESPONSE OF H_2 CONTROL SYSTEM (FIRST SCENARIO).

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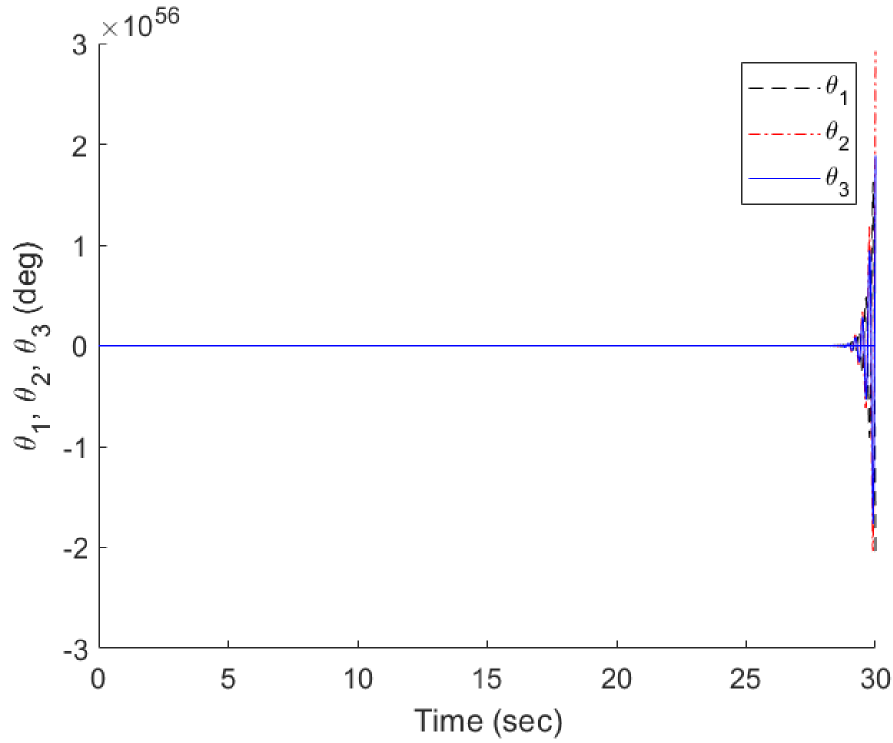

FIG. 6. RESPONSE OF H-INFINITY CONTROL SYSTEM (FIRST SCENARIO).

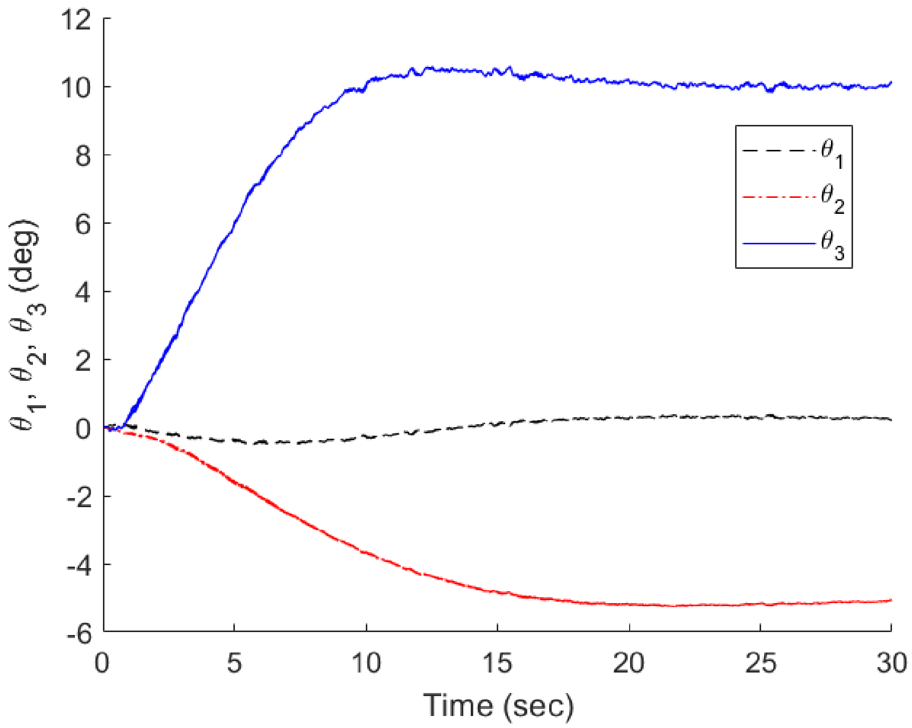

FIG. 7. RESPONSE OF MIXED H2/H-INFINITY CONTROL SYSTEM (FIRST SCENARIO).

B. Second Scenario

To study the robustness against disturbance, the reference input is $[0\ 0\ 0]^T$ deg. The applied disturbance is $[0.1\ 0.1\ 0.1]^T$ N . m. The responses of the three control systems are shown in *Fig. 8, 9, & 10.*

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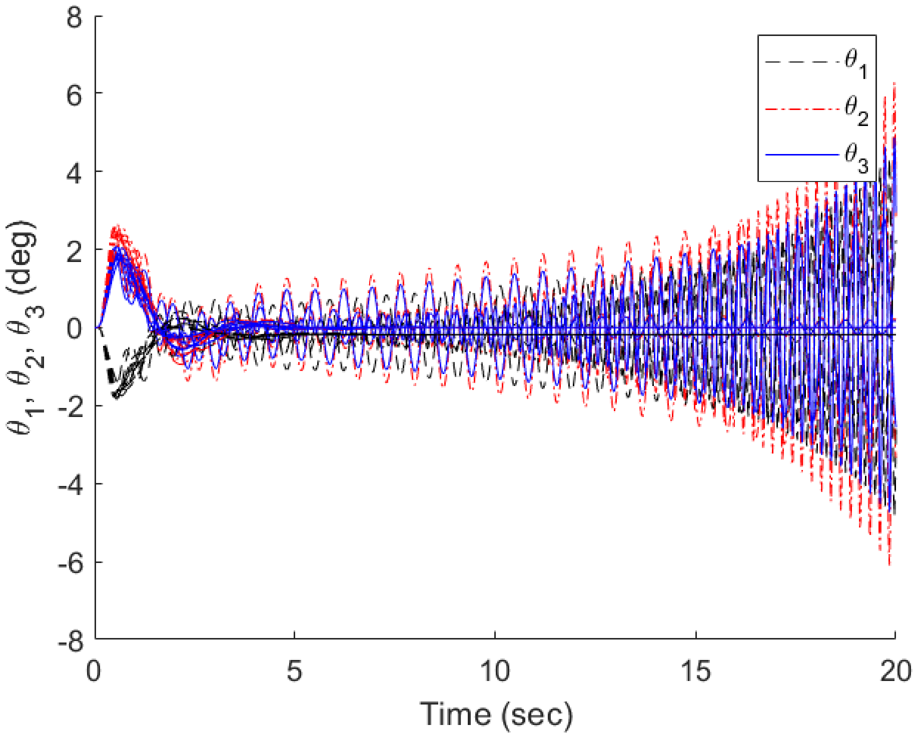

FIG. 8. RESPONSE OF H2 CONTROL SYSTEM (SECOND SCENARIO).

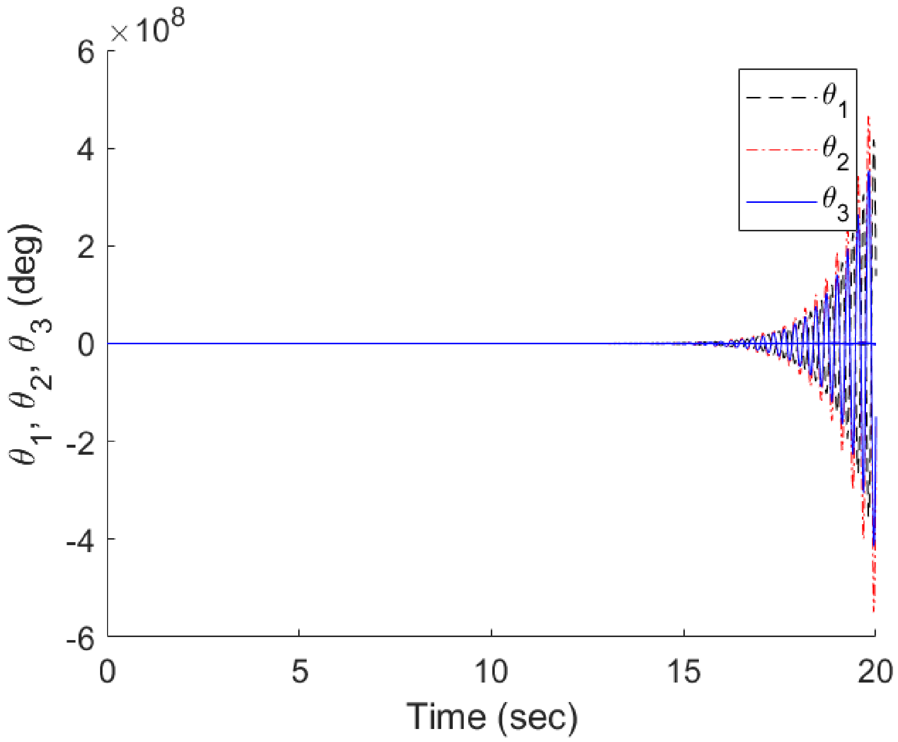

FIG. 9. RESPONSE OF H-INFINITY CONTROL SYSTEM (SECOND SCENARIO).

The H_2 and H-infinity control systems fail to maintain robust stability and performance for the modeled uncertainty as shown in *Fig. 8 and 9.* Both systems diverge from their desired value and exhibits unstable behavior. This follows from the fact that their robust

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stability and performance margins are less than 1. On the other hand, *Fig. 10* shows that all the 10 samples of uncertainty maintain the system's stability and performance in mixed H_2/H-infinity control system.

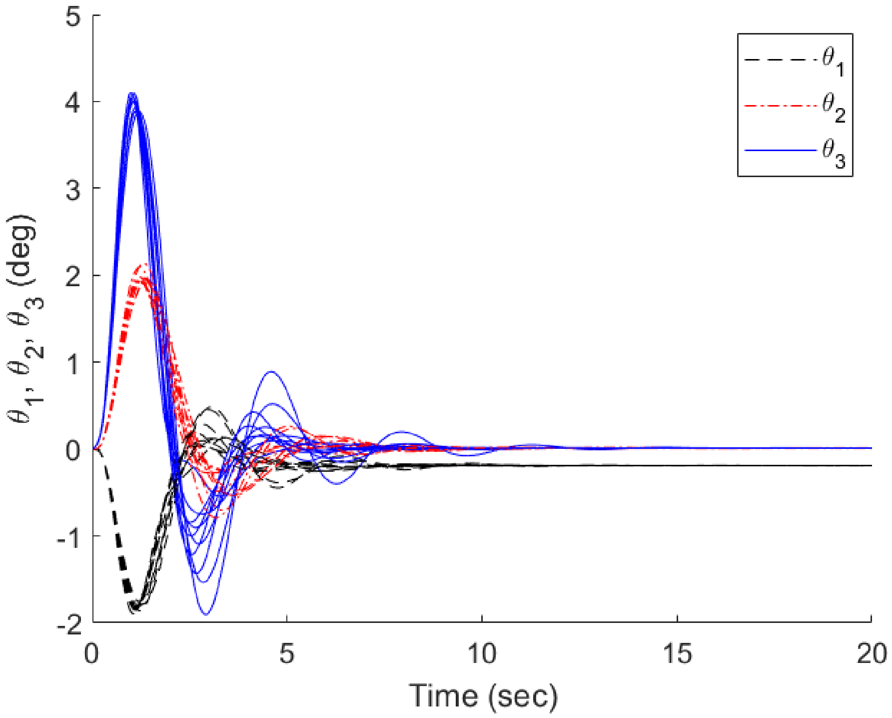

FIG. 10. RESPONSE OF MIXED H_2/H-INFINITY CONTROL SYSTEM (SECOND SCENARIO).

From the response in *Fig. 10*, it can be seen that θ_1 goes under the desired value by 1.89 deg for mixed H_2/H-infinity control system. The steady-state error of θ_1 is 0.188 deg. θ_2 and θ_3 overshoot to 2.18 deg and 4.112 deg; respectively. The steady-state error of θ_2 and θ_3 is 0.0109 deg which is very small.

Based on all the above results, the mixed H_2/H-infinity control system is advised since it maintains robust stability and performance for all modelled uncertainty and succeeded in handling different types of uncertainty.

V. CONCLUSIONS

In this paper, the robustness of H_2 controller, H-infinity controller, and mixed H_2/H-infinity controller for triple inverted pendulum have been developed and tested. It has been shown that the H_2 and H-infinity control systems have stability and performance margins that are less than one, therefore, both control systems could not handle parameter variation, disturbance, nor noise. Conversely, the margins of the mixed H_2/H-infinity control system are greater than one, hence, the control system is able to maintain robustness against parameters variation, disturbances, and noise. This makes the mixed H_2/H-infinity control system reliable in real world applications where the systems parameters changes with use and where external disturbances and readings imperfections are inevitable. In future, it is recommended to apply multi-objective optimization algorithms like genetic algorithms or particle swarm optimization to find best weights of the H_2 and H-infinity norms in order to find good trade-off between system's performance and robustness.

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