Al-Qadisiya Journal For Engineering Sciences, Vol. 5, No. 2, 209-224, Year 2012

NUMERICAL COMPARISON OF TRANSIENT FLOW FOR SIMPLE PIPELINE SYSTEMS USING TIME DOMAIN AND FERQUENCY DOMAIN METHOD

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ABSTRACT

This research is devote to a description of the methods available for the analysis of unsteady flows in pumping stations and their associated hydraulic systems. There are two basic approaches to the solution of unsteady internal flows: solution in the time domain and in the frequency domain. The traditional time domain methods for hydraulic systems are the most important that many unsteady hydraulic system problems can and should be treated by the time domain or "water-hammer" methods. Another approach is frequency domain method, it is sufficient to recognize that one practical advantage of this method is the capability of incorporation of experimentally obtained dynamic information and the greater simplicity of the experiments required to obtain the necessary dynamic data, the disadvantage of frequency domain is that the method are limited to small linear perturbations in the flow rate. Two types of networks are tested in this research, example (1) represents very simple network without any apparatus, and example (2) network containing some complexity and containing intakes, valves, and other apparatus. The results in example (1) identical for both methods, but in example (2) the results showing clear differences for the two approaches.

المقارنة العددية لجريان انتقالي في شبكات الانابيب باستخدام طريقتي المدى الزمني والمدى الترددي

الموجز

كرس ما موجود في هذا البحث للتعبير بشكل نظري عن الطرق المتاحة لتحليل الجريان الانتقالي الذي يحصل في محطات الضخ والنظم الهيدروليكية الملحقة بها. هناك طريقتين اساسيتين لحل مسائل الجريان الداخلي الغير مستقر: الحل بطريقة المدى الزمني وطريقة المدى الترددي. طريقة المدى الزمني التقليدية تعتبر من اهم الطرق الشائعة لحل مشاكل الجريان الانتقالي الذي يحصل في المنظومات الهيدروليكية او ما يعرف بالمطرقة المائية. هناك طريقة اخرى للتعبير عن حالة الجريان الانتقالي وهي طريقة المدى الترددي حيث تتميز هذه الطريقة بكفاءتها على ربط البيانات المختبرية التي يمكن الحصول عليها من تجربة مختبرية وتحليلها نظريا، لكن لهذه الطريقة قصور كونها كفؤة فقط في وصف الجريان الانتقالي الذي يحوي دوامات واضطرابات صغيرة وعادة ما تحصل في الشبكات الصغيرة التي لا تحوي ملحقات كثيرة. تم اختبار نوعين من الشبكات في هذا البحث على شكل مثالين، المثال الاول

يحوي شبكة بسيطة خالية من الملحقات والمثال الثاني يتضمن شبكة اكثر تعقيد وتحوي ملحقات. وجدت النتائج للمثال الاول متطابقة لكلا الطريقتين، بينما في المثال الثاني تم ملاحظة عدم تطابق في النتائج لكلا الطريقتين.

NOMENCLATURE

A : cross-sectional area.
a :radius.
e: specific internal energy.
$e^{[F]l}$: transmission matrix.
[F]: distributed function.
δ :wall thickness of the pipe.
ρ :fluid density.
C : sonic speed.
$C\infty$: sonic speed in the fluid.
E: Young's modulus.
N: order of the system.
P : pressure.
K: bulk modulus.
q^{-n} : vector of fluctuating quantity.
S: coordinate measuredalong the duct.
t : time.
$[T_{ij}]$: transfer matrix elements.
[T] : Transfer matrix based on p^T, $\tilde{m}$.
$[T^*]$:transfermatrix based on $\tilde{p}$, $\tilde{m}$.
u(s, t): volumetric velocity.
gs :acceleration due to gravity.
λ : characteristic factor.
f : friction factor.
h* :piezometric head.
Q : volume flow rate.
ω: frequency.
m: mass flow rate.
p^T: total pressure.
R_e: Reynolds number.
Z: vertical elevation.

INTRODUCTION

Hydraulic transients are the time-varying phenomena that follow when the equilibrium of steady flow in a system is disturbed by a change of flow that occurs over a relatively short time period. The verity of transient pressures must be determined so that the water mains can be properly designed to withstand these additional loads. In fact, pipes are often characterized by their "pressure ratings" that define their mechanical strength and have a significant influence on their cost (**Boulos, 2004**).Transient regimes in water distribution systems are inevitable and will normally be most severe at pump stations and control valves, high elevation areas, locations with low static pressures, and remote locations that are distanced from overhead storage (**Friedman 2003**). All systems will, at some time, be started up, switched off, undergo unexpected flow changes, etc., and will likely experience the effects of human errors, equipment break downs, or other risky disturbances. Although transient conditions can result in many situations, the engineer is most concerned with those that might endanger the safety of a plant and its personnel that have the potential to cause equipment or device damage that results in operational difficulties or pose a risk to the public health. Transient events have significant water quality implications. These events can generate high intensities of fluid shear and may cause suspension of settled particles as well as bio film detachment. So-called red water events have often been associated with transient disturbances. Moreover, a low-pressure transient event, say arising from a power failure or pipe break, has the potential to cause the intrusion of contaminated groundwater into a pipe at a leaky joint or break. Depending on the size of the leaks, the volume of intrusion can range from a few gallons to hundreds of gallons (**Funk 1999, Karim, 2003 and Le Chevallier 2003**). Negative pressures induce back siphon age of no potable water from domestic, industrial, and institutional piping into the distribution system. Dissolved air gas can also be released steel and iron sections with subsequent rust formation and pipe damage. Even some common transient protection strategies, such as relief valves or air/vacuum valves, if not properly designed and maintained, may permit pathogens or other contaminants to find a "back door" route into the potable water distribution system. Engineers must carefully consider all potential dangers for their pipe designs and estimate and eliminate the weak spots. They should then embark upon a detailed transient analysis to make informed decisions on how to best strengthen their systems and ensure safe, reliable operations (**Karney and McInnis 1990**).

THEORETICAL ANALYSIS

1-Time Domain Method

The application of time domain methods to one-dimensional fluid flow normally consists of the following three components. First, one establishes conditions for the conservation of mass and momentum in the fluid. These may be differential equations or they may be jump conditions (as in the analysis of a shock). Second, one must establish appropriate thermodynamic constraints governing the changes of state of the fluid. In almost all practical cases of single-phase flow, it is appropriate to assume that these changes are adiabatic. However, in multiphase flows the constraint scan be much more complicated. Third, one must determine the response of the containing structure to the pressure changes in the fluid. The analysis is made a great deal simpler in those circumstances in which it is accurate to assume that both the fluid and the structure behave bar tropically. By definition, this implies that the change of state of the fluid is such that some thermodynamic quantity (such as the entropy) remains constant, and therefore the fluid density, ρ(p), is a simple algebraic function of just one thermodynamic variable, for example the pressure. In the case of the structure, the assumption is that it deforms quasi statically, so that, for example, the cross-sectional area of a pipe, A (p), is a simple, algebraic function of the fluid pressure, p. Note that this neglects any inertial or damping effects in the structure. The importance of the assumption of a bar tropic fluid and structure lies in the fact that it allows the calculation of a single, unambiguous speed of sound for waves raveling through the piping system. The sonic speed in the fluid alone is given by c_∞ where [3]

$$c_\infty = (d\rho/dp)^{-\frac{1}{2}} \tag{1}$$

In a liquid, this is usually calculated from the bulk modulus, $\kappa = \rho/(d\rho/dp)$,since

$$c_\infty = (\kappa/\rho)^{-\frac{1}{2}} \tag{2}$$

However the sonic speed, c, for one-dimensional waves in a fluid-filled duct isinfluenced by the compressibility of both the liquid and the structure [3]

$$c = \pm \left[\frac{1}{A} \frac{d(\rho A)}{dp} \right]^{-\frac{1}{2}} \tag{3}$$

or, alternatively,

$$\frac{1}{\rho c^2} = \frac{1}{\rho c_\infty^2} + \frac{1}{A} \left(\frac{dA}{dp} \right) \tag{4}$$

The left-hand side is the acoustic impedance of the system, and the equationreveals that this is the sum of the acoustic impedance of the fluid alone, $1/\rho C^2\infty$,plus an "acoustic impedance" of the structure given by (*dA/dp*)/*A*. For example,for a thin-walled pipe made of an elastic material of Young's modulus, E, theacoustic impedance of the structure is 2a/Eδ, where a and δ are the radius andthe wall thickness of the pipe (δ a). The resulting form of equation (4), [3]

$$c = \left[\frac{1}{c_\infty^2} + \frac{2\rho a}{E\delta} \right]^{-\frac{1}{2}} \tag{5}$$

In order to solve unsteady flows in ducts, an expression for the sonic speed iscombined with the differential form of the equation for conservation of mass(the continuity equation),

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial s}(\rho A u) = 0 \tag{6}$$

where u(s, t) is the cross-sectionally averaged or volumetric velocity, s is a coordinatemeasured along the duct, and t is time. The appropriate differentialform of the momentum equation is [3]

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right] = -\frac{\partial p}{\partial s} - \rho g_s - \frac{\rho f u |u|}{4a} \tag{7}$$

where gs is the component of the acceleration due to gravity in the s direction, f is the friction factor, and a is the radius of the duct. Now the barotropic assumption (3) allows the terms in equation (6) to bewritten as [3]

$$\frac{\partial}{\partial t}(\rho A) = \frac{A}{c^2} \frac{\partial p}{\partial t} \quad ; \quad \frac{\partial(\rho A)}{\partial s} = \frac{A}{c^2} \frac{\partial p}{\partial s} + \rho \frac{\partial A}{\partial s}\Big|_p \tag{8}$$

so the continuity equation becomes

$$\frac{1}{c^2} \frac{\partial p}{\partial t} + \frac{u}{c^2} \frac{\partial p}{\partial s} + \rho \left[\frac{\partial u}{\partial s} + \frac{u}{A} \frac{\partial A}{\partial s}\Big|_p \right] = 0 \tag{9}$$

Equations (7) and (9) are two simultaneous, first order, differential equationsfor the two unknown functions, p(s, t) and u(s, t). They can be solved giventhe barotropic relation for the fluid, ρ(p), the friction factor, f, the normalcross-sectional area of the pipe, $A_0(s)$, and boundary conditions which will bediscussed later. Normally the last term in equation (9) can be approximated by$\rho u(dA_0/ds)/A_0$. Note that c may be a function of s.

In the time domain methodology, equations(7) and (9) are normally solvedusing the method of characteristics. This involvesfinding moving coordinate systems in which the equations may be writtenas ordinary rather than partial differential equations. Consider the relation thatresults when we multiply equation (9) by λ and add it to equation (7)[3]

$$\rho\left[\frac{\partial u}{\partial t}+(u+\lambda)\frac{\partial u}{\partial s}\right]+\frac{\lambda}{c^2}\left[\frac{\partial p}{\partial t}+\left(u+\frac{c^2}{\lambda}\right)\frac{\partial p}{\partial s}\right] + \frac{\rho u\lambda}{A_0}\frac{dA_0}{ds}+\rho g_s+\frac{\rho f|u|u}{4}=0 \tag{10}$$

If the coefficients of $\partial u/\partial s$ and $\partial p/\partial s$ inside the square brackets were identical, in otherwords if $\lambda = \pm c$, then the expressions in the square brackets could be writtenas

$$\frac{\partial u}{\partial t}+(u\pm c)\frac{\partial u}{\partial s} \quad \text{and} \quad \frac{\partial p}{\partial t}+(u\pm c)\frac{\partial p}{\partial s} \tag{11}$$

and these are the derivatives du/dt and dp/dt on $ds/dt = u \pm c$. These lines $ds/dt = u \pm c$ are the characteristics, and on them we may write:

1. In a frame of reference moving with velocity $u + c$ or on $ds/dt = u + c$:

$$\frac{du}{dt}+\frac{1}{\rho c}\frac{dp}{dt}+\frac{uc}{A_0}\frac{dA_0}{ds}+g_s+\frac{fu|u|}{4a}=0 \tag{12}$$

2. In a frame of reference moving with velocity $u - c$ or on $ds/dt = u - c$:

$$\frac{du}{dt}-\frac{1}{\rho c}\frac{dp}{dt}-\frac{uc}{A_0}\frac{dA_0}{ds}+g_s+\frac{fu|u|}{4a}=0 \tag{13}$$

A simpler set of equations result if the piezometric head, h*, defined as

$$h^*=\frac{p}{\rho g}+\int\frac{g_s}{g}ds \tag{14}$$

is used instead of the pressure, p, in equations (12) and (13). In almost allhydraulic problems of practical interest $p/\rho Lc^2 \ll 1$ and, therefore, the termρ-$1dp/dt$ in equations (12) and (13) may be approximated by $d(p/\rho)/dt$. It follows that on the two characteristics [3]

$$\frac{1}{\rho c}\frac{dp}{dt} \pm gs \quad \approx \quad \frac{g}{c}\frac{dh^*}{dt} - \frac{u}{c}g_s \tag{15}$$

and equations (12) and (13) become

1. On *ds/dt = u + c*

$$\frac{du}{dt} + \frac{g}{c}\frac{dh^*}{dt} + uc\frac{1}{A_0}\frac{dA_0}{ds} - \frac{ug_s}{c} + \frac{f}{4a}u|u| = 0 \tag{16}$$

3. On *ds/dt = u – c*

$$\frac{du}{dt} - \frac{g}{c}\frac{dh^*}{dt} - uc\frac{1}{A_0}\frac{dA_0}{ds} + \frac{ug_s}{c} + \frac{f}{4a}u|u| = 0 \tag{17}$$

These are the forms of the equations conventionally used in unsteady hydraulic water-hammer problems (**Streeter and Wylie, 1967**). They are typically solved by relating the values at a time t + δt {for example point C of **Figure 1**} to known values at the points A and B at time t. The lines AC and BC are characteristics, so the following finite difference forms of equations (16) and (17) apply [3]

$$\frac{(u_C - u_A)}{\delta t} + \frac{g}{c_A}\frac{(h^*_C - h^*_A)}{\delta t} + u_A c_A\left(\frac{1}{A_o}\frac{dA_0}{ds}\right)_A - \frac{u_A(g_s)_A}{c_A} + \frac{f_A u_A|u_A|}{4a} = 0 \tag{18}$$

And,

$$\frac{(u_C - u_B)}{\delta t} - \frac{g}{c_B}\frac{(h^*_C - h^*_B)}{\delta t} - u_B c_B\left(\frac{1}{A_o}\frac{dA_0}{ds}\right)_B + \frac{u_B(g_s)_B}{c_B} + \frac{f_B u_B|u_B|}{4a} = 0 \tag{19}$$

If $c_A = c_B = c$, and the pipe is uniform, so that *dA0/ds = 0* and $f_A = f_B = f$, then these reduce to the following expressions for u_c and $h*_c$

$$u_C = \frac{(uA + uB)}{2} + \frac{g}{2c}(h^*_A - h^*_B) + \frac{\delta t}{2c}\{u_A(g_s)_A - u_B(g_s)_B\} - b\frac{f\,\delta t}{8a}\{u_A|uA| + u_B|u_B|\} \tag{20}$$

$$h^*_C = \frac{(h_*A + h_*B)}{2} + \frac{c}{2g}(uA - uB) + \frac{\delta t}{2g}\{u_A(g_s)_A + u_B(g_s)B\} - \frac{f\,c\delta t}{8ag}\{u_A|uA| - u_B|u_B|\} \tag{21}$$

1-1-Method of Characteristics

The typical numerical solution by the method of characteristics is depicted graphically in **Figure 2**. The time interval, δt, and the spatial increment, δs, are specified. Then, given all values of the two dependent variables (say u and h*) at one instant in time, one proceeds as follows to find all the value sat points such as C at a time δt later. The intersection points, A and B, of the characteristics through C are first determined. Then interpolation between the known values at points such as R, S and T are used to determine the values of the dependent variables at A and B. The values at C follow from equations such as (20) and (21) or some alternative version. Repeating this for all points at time t + δt allows one to march forward in time. There is, however, a maximum time interval, δt, that will lead to a stable numerical solution. Typically this requires that δt be less than δx/c. In other words, it requires that the points A and B of Figure 2 lie inside of the interval RST. The reason for this condition can be demonstrated in the following way. Assume for the sake of simplicity that the slopes of the characteristics are ±c; then the distances AS = SB = cδt. Using linear interpolation to find u_A and u_B from u_R, u_S and u_T leads to [2]

$$\frac{(uA + uB)}{2} = \frac{(uR + uT)}{2} + u_s \frac{c\delta t}{\delta s} \tag{22}$$

But this is also a principal term in the expression (20) for u_C. Consequently, an error in u_S of, say, δu would lead to an error in u_C (at the same location but δt later) of *δucδt/δs*. Thus the error would be magnified with each time step unless *cδt/δs < 1* and, therefore, the numerical integration is only stable if δt< δx/c. In many hydraulic system analyses this places a quite severe restriction on the time interval δt, and often necessitates a large number of time steps. A procedure like the above will also require boundary conditions to be specified at any mesh point which lies either, at the end of a pipe or, at a junction of the pipe with a pipe of different size (or a pump or any other component).

If the points S and C in **Figure 2** were end points, then only one characteristic would lie within the pipe and only one relation, (18) or (19), can be used. Therefore, the boundary condition must provide a second relation involving u_C or h^*_C (or both). An example is an open-ended pipe for which the pressure and, therefore, h^* is known. Alternatively, at a junction between two sizes of pipe, the two required relations will come from one characteristic in each of the two pipes, plus a continuity equation at the junction ensuring that the values ofuA_0 in both pipes are the same at the junction. For this reason it is sometimes convenient to rewrite equations (16) and (17) in terms of the volume flow rate $Q = uA_0$ instead of u so that [2]

1. On *ds/dt = u + c*

$$\frac{dQ}{dt} + \frac{A_0 g}{c}\frac{dh^*}{dt} + \frac{Qc}{A_0}\frac{dA_0}{ds} - \frac{Qg_s}{c} + \frac{fA_0}{4a}Q|Q| = 0 \tag{23}$$

2. On *ds/dt = u − c*

$$\frac{dQ}{dt} - \frac{A_0 g}{c}\frac{dh^*}{dt} - \frac{Qc}{A_0}\frac{dA_0}{ds} + \frac{Qg_s}{c} + \frac{fA_0}{4a}Q|Q| = 0 \tag{24}$$

In many time domain analyses, turbomachines are treated by assuming thatthe temporal rates of change are sufficiently slow that the turbomachine respondsquasistatically, moving from one steady state operating point to another.Consequently, if points A and B lie at inlet to and discharge from theturbomachine then the equations relating the values at A and B would be

$$QB = QA = Q \tag{25}$$

$$h^*B = h^*A + H(Q) \tag{26}$$

where H(Q) is the head rise across the machine at the flow rate, Q. Data presented later will show that the qua sistatic assumption is only valid for rates of change less than about one-tenth the frequency of shaft rotation. For frequencies greater than this, the pump dynamics become important.

2 Frequency domain methods

When the quai-static assumption for a device like a pump or turbine becomes questionable, or when the complexity of the fluid or the geometry makes the construction of a set of differential equations impractical or uncertain, then it is clear that experimental information on the dynamic behavior of the device is necessary. In practice, such experimental information is most readily obtained by subjecting the device to fluctuations in the flow rate or head for a range of frequencies, and measuring the fluctuating quantities at inlet and discharge. All the dependent variables such as the mean velocity, u, mass flow rate, m, pressure, p, or total pressure, pT , are expressed as the sum of a mean component (denoted by an overbear) and a complex fluctuating component (denoted by a tilde) at a frequency, ω, which incorporates the amplitude and phase of the fluctuation [2]

$$p(s,t) = \bar{p}(s) + Re\left\{\tilde{p}(s,\omega)e^{j\omega t}\right\} \tag{27}$$

$$p^T(s,t) = \bar{p}^T(s) + Re\left\{\tilde{p}^T(s,\omega)e^{j\omega t}\right\} \tag{28}$$

$$m(s,t) = \bar{m}(s) + Re\left\{\tilde{m}(s,\omega)e^{j\omega t}\right\} \tag{29}$$

where*j* is $(-1)^{1/2}$and *Re* denotes the real part. Since the perturbations are assumed linear $(|\tilde{u}| \ll \bar{u}, |\tilde{m}| \ll \bar{m}$, etc.), they can be readily superimposed, so a summation over many frequencies is implied in the above expressions. In general, the perturbation quantities will be functions of the mean flow characteristics as well as position, s, and frequency, ω. We should note that there do exist a number of codes designed to examine the frequency response of hydraulic systems using frequency domain methods.

2-1 Order of the System

The first step in any unsteady flow analysis is to subdivide the system into components; the points separating two (or more) components will be referred to as system nodes. Typically, there would be nodes at the inlet and discharge flanges of a pump. Having done this, it is necessary to determine the order of the system, N, and this can be accomplished in one of several equivalent ways. The order of the system is the minimum number of independent fluctuating quantities which must be specified at a system node in order to provide a complete description of the unsteady flow at that location. It is also equal to the minimum number of independent, simultaneous first order differential equations needed to describe the fluid motion in. In this research, we assume the system includes water-hammer analysis in which the local area depends on the area and the pressure elsewhere, and then the system is of order 3.

2-2 Transfer Matrices

The transfer matrix for any component or device is the matrix which relates the fluctuating quantities at the discharge node to the fluctuating quantities at the inlet node. The earliest exploration of such a concept in electrical networks appears to be due to (**Strecker and Feldtkeller, 1929**). If the quantities at inlet and discharge are denoted by subscripts i = 1 and i = 2, respectively, and, if $\{q_i^{\sim n}\}$, $n = 1, 2 \rightarrow N$ denotes the vector of independent fluctuating quantities at inlet and discharge for a system of order N, then the transfer matrix, [T], is defined as [10]

$$\{\tilde{q}_2^n\} = [T]\{\tilde{q}_1^n\} \tag{30}$$

$$\begin{Bmatrix} \tilde{p}_2^T \\ \tilde{m}_2 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \tilde{p}_1^T \\ \tilde{m}_1 \end{Bmatrix} \tag{31}$$

The most convenient independent fluctuating quantities for a hydraulic system of order two are usually 1. Either the pressure, $\tilde{p}$, or the instantaneous total pressure, $\tilde{p}^{T}$. Note that these are related by

$$\tilde{p}^T = \tilde{p} + \frac{\bar{u}^2}{2}\tilde{\rho} + \bar{\rho}\bar{u}\tilde{u} + gz\tilde{\rho} \tag{32}$$

Where $\bar{\rho}$ is the mean density, $\tilde{\rho}$ is the fluctuating density which is bar tropically connected to $\tilde{p}$, and z is the vertical elevation of the system node. Neglecting the $\tilde{\rho}$ terms as is acceptable for incompressible flows

$$\tilde{p}^T = \tilde{p} + \bar{\rho}\bar{u}\tilde{u} \tag{33}$$

2. Or the velocity, $\tilde{u}$, the volume flow rate, $\{\bar{A}\tilde{u} + \bar{u}A\}$, or the mass flow rate, $\{\tilde{m} = \bar{\rho}\bar{A}\tilde{u} + \bar{\rho}\bar{u}\tilde{A} + \bar{u}\bar{A}\tilde{\rho}\}$. Incompressible flow at a system node in a rigid pipe implies

$$\tilde{m} = \bar{\rho}\bar{A}\tilde{u} \tag{34}$$

The most convenient choices are $\{\tilde{p}, \tilde{m}\}$ or $\{\tilde{p}^{T}, \tilde{m}\}$, and, for these two vectors, we will respectively use transfer matrices denoted by $[T^*]$ and $[T]$, defined as

$$\begin{Bmatrix} \tilde{p}_2 \\ \tilde{m}_2 \end{Bmatrix} = [T^*] \begin{Bmatrix} \tilde{p}_1 \\ \tilde{m}_1 \end{Bmatrix} \quad ; \quad \begin{Bmatrix} \tilde{p}_2^T \\ \tilde{m}_2 \end{Bmatrix} = [T] \begin{Bmatrix} \tilde{p}_1^T \\ \tilde{m}_1 \end{Bmatrix} \tag{35}$$

2-3 Distributed Systems

In the case of a distributed system such as a pipe, it is also appropriate to define a matrix [F] so that [10]

$$\frac{d}{ds}\{\tilde{q}^n\} = -[F(s)]\{\tilde{q}^n\} \tag{36}$$

Note that, apart from the frictional term, the equations (12) and (13) for flow in a pipe will lead to perturbation equations of this form. Furthermore, in many cases the frictional term is small, and can be approximated by a linear term in the perturbation equations; under such circumstances the frictional term will also fit into the form given by equation (37).When the matrix [F] is independent of location, s, the distributed system is called a “uniform system”. For example, in equations (12) and (13), this would require ρ, c, a, f and A_0 to be approximated as constants (in addition to the linearization of the frictional term).

Under such circumstances, equation (37) can be integrated over a finite length, *l*, and the transfer matrix[T] of the form (35) becomes

$$[T] = e^{-[F]\ell} \tag{37}$$

Where $e^{[F]L}$ is known as the "transmission matrix." For a system of order two, the explicit relation between [T] and [F] is [10]

$$\begin{aligned} T_{11} &= jF_{11}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \\ &\quad + \left(\lambda_2 e^{-j\lambda_1\ell} - \lambda_1 e^{-j\lambda_2\ell}\right)/(\lambda_2-\lambda_1) \\ T_{12} &= jF_{12}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \\ T_{21} &= jF_{21}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \\ T_{22} &= jF_{22}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \\ &\quad + \left(\lambda_2 e^{-j\lambda_2\ell} - \lambda_1 e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \end{aligned} \tag{38}$$

Where λ_1, λ_2 are the solutions of the equation

$$\lambda^2 + j\lambda(F_{11}+F_{22}) - (F_{11}F_{22} - F_{12}F_{21}) = 0 \tag{39}$$

2-4 Combinations of Transfer Matrices

When components are connected in series, the transfer matrix for the combination is clearly obtained by multiplying the transfer matrices of the individual components in the reverse order in which the flow passes through them. Thus, for example, the combination of a pump with a transfer matrix, [*TA*], followed by a discharge line with a transfer matrix, [*TB*], would have a system transfer matrix, [*TS*], given by [10]

$$[TS] = [TB]\,[TA] \tag{40}$$

The parallel combination of two components is more complicated and does not produce such a simple result. Issues arise concerning the relations between the pressures of the inlet streams and the relations between the pressures of the discharge streams. Often it is appropriate to assume that the branching which creates the two inlet streams results in identical fluctuating total pressures at inlet to the two components,[$p_1^{\sim T}$]. If, in addition, mixing losses at the downstream junction are neglected, so that the fluctuating total pressure, [$p_2^{\sim T}$] , can be equated with the fluctuating total pressure at discharge from the two components, then the transfer function, [*TS*], for the combination of two components (order two transfer functions denoted by [*TA*] and [*TB*]) become

$$\begin{aligned} TS_{11} &= (TA_{11}TB_{12} + TB_{11}TA_{12})/(TA_{12}+TB_{12}) \\ TS_{12} &= TA_{12}TB_{12}/(TA_{12}+TB_{12}) \\ TS_{21} &= TA_{21} + TB_{21} \\ &\quad -(TA_{11}-TB_{11})(TA_{22}-TB_{22})/(TA_{12}+TB_{12}) \\ TS_{22} &= (TA_{22}TB_{12} + TB_{22}TA_{12})/(TA_{12}+TB_{12}) \end{aligned} \tag{41}$$

On the other hand, the circumstances at the junction of the two discharge streams may be such that the fluctuating static pressures (rather than the fluctuating total pressures) are equal. Then, if the inlet static pressures are also equal, the combined transfer matrix, [*TS**], is related to those of the two components [TA^*] and [TB^*] by the same relations as given in equations (42).Other combinations of choices are possible. Using the above combination rules, as well as the relations (36) between the [*T*] and [T^*] matrices, the transfer functions for very complicated hydraulic networks can be systematically synthesized.

PRACTICAL APPLICATIONS

Case (1)

The first example network was studied earlier by **Streeter and Wylie (1967)** and is shown in **Figure 3**. The network comprises nine pipes, five junctions, one reservoir, three closed loops, and one valve located at the downstream end of the system. The valve is shut to create the transient. **Table 1** summarizes the pertinent pipe system characteristics. The reservoir level is clearly shown in the **Figure 3**, the analysis resulting very identical plots as shown in **Figures 5 and 6** [11]

Case (2)

Using a slightly larger more complex system, the methods were applied to the network shown in **Figure 4**. This represents an actual water system and consists of (7) pipes, (4) junctions, two supply tank, and one surge tank. Reservoir valves (orifices) usually permit flow in both directions. Otherwise, a valve discharging to the atmosphere is equivalent to an infinite area reservoir. All valves (orifices) are considered fully open, expect the control valve at node (7) and pressure relief valve. **Table 2** summarizes the pertinent pipe system characteristics. **Figure 8** compares the transient results obtained using the Time domain method and the Frequency domain method solutionschemes from node (1) to node (7), the demand is changed by reducing the inflow to zero over a period of 6 s. the analysis resulting in a required time step of 0.0139 s. As can be seen from **Figures 7 and 8**, the methods yielded not identical results [3]

CONCLUSION

Transient (water hammer) analysis is essential to good design and operation of piping systems. This important analysis can be done using the mathematically time domain method based on the method of characteristics or the frequency domain method for order three. The two methods are both capable of accurately solving for transient pressures and flows in simple water distribution networks including the effects of pipe friction. The method of characteristics requires calculations at interior points to handle the wave propagation and the effects of pipe friction. The frequency domain method handles these effects by using the transfer matrix, the transfer matrix of order three used in this research. The results showed that for small simple networks without any apparatus the two methods given identical readings, but for large networks with some apparatus the two methods given different results.

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Table 1 Pipe characteristics for case 1.

Pipe number	Length (m)	Diameter (mm)	Darcy friction	Minor loss
1	610	914	0.012	0
2	914	762	0.013	0
3	610	610	0.014	0
4	457	457	0.015	0
5	549	457	0.015	0
6	671	762	0.014	0
7	610	914	0.013	0
8	457	610	0.014	0
9	488	457	0.012	0

Table 2 Pipe characteristics for case 2.

Pipe number	Length (m)	Diameter (mm)	Darcy Friction	Minor losses
1	1,002.2	1.5	0.013	0
2	2,000.0	1.000	0.012	0
3	2,000.0	0.750	0.015	0
4	502.5	0.500	0.013	0
5	502.2	0.500	0.014	0
6	1,001.2	1.000	0.014	0
7	2,000.2	0.750	0.014	0

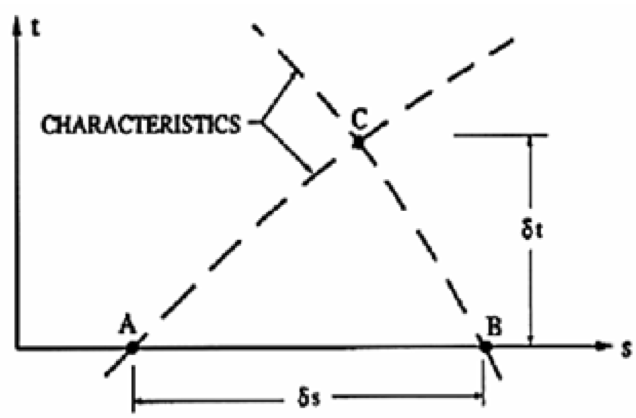

Figure 1 Method of characteristics.

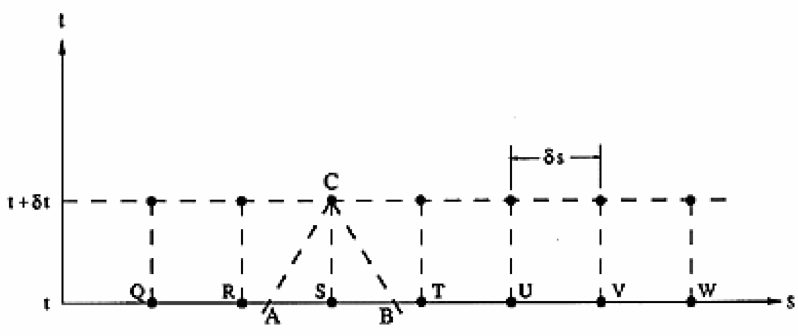

Figure 2 Numerical solution of method of characteristics.

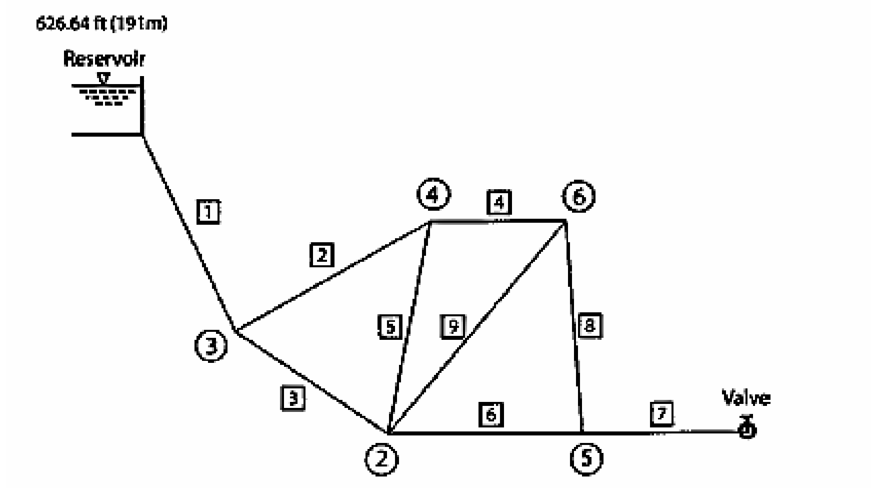

Figure 3 simple pipeline systems.

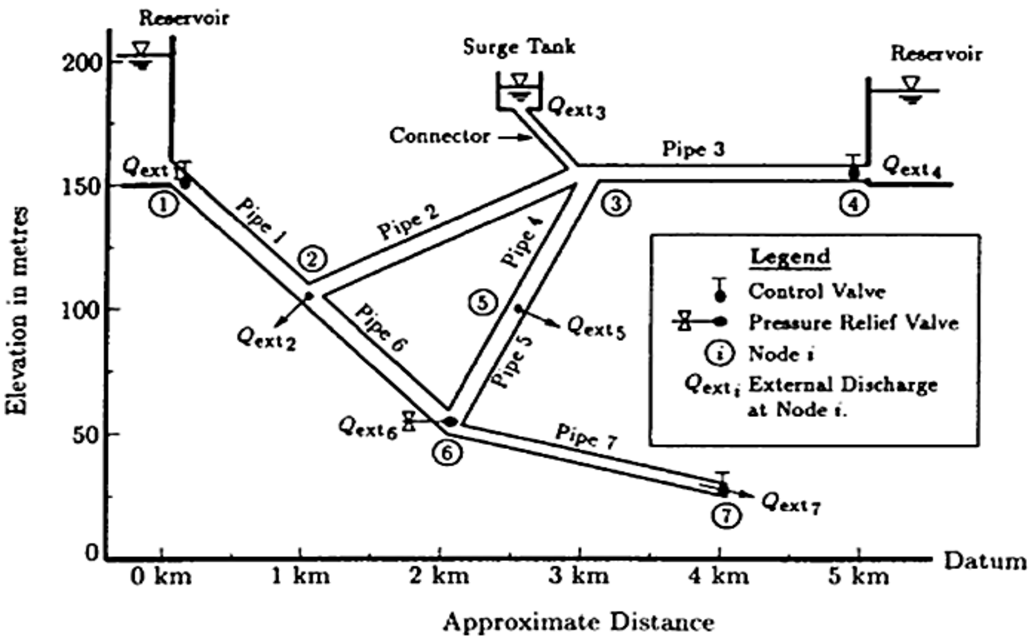

Figure 4 Network with more apparatus.

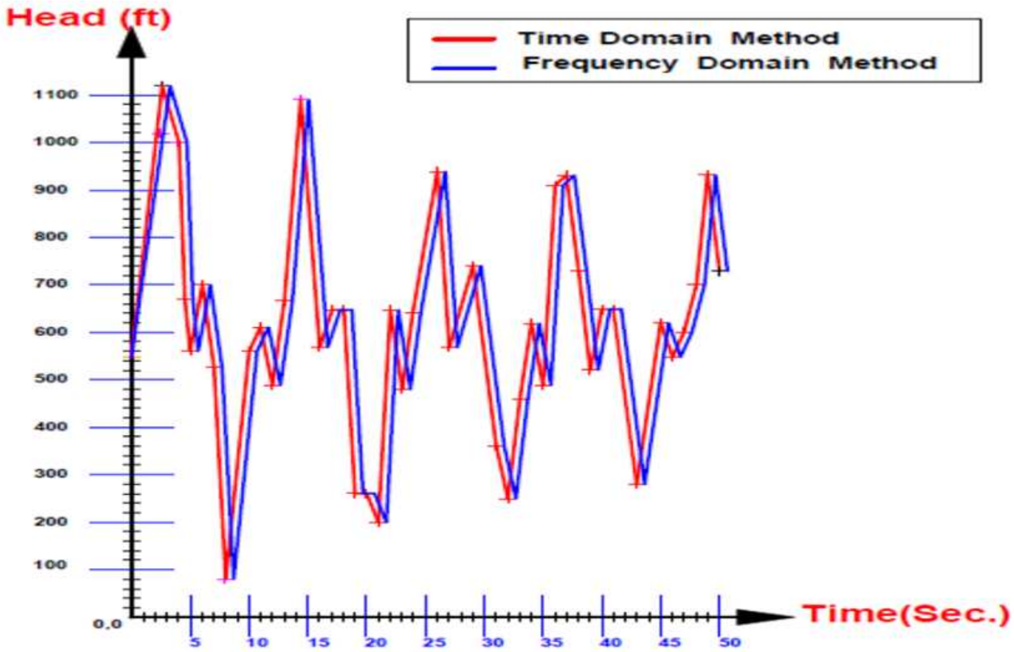

Figure 5 Comparison of results of time domain and frequency domain methods, for case 1, at Junction 4.

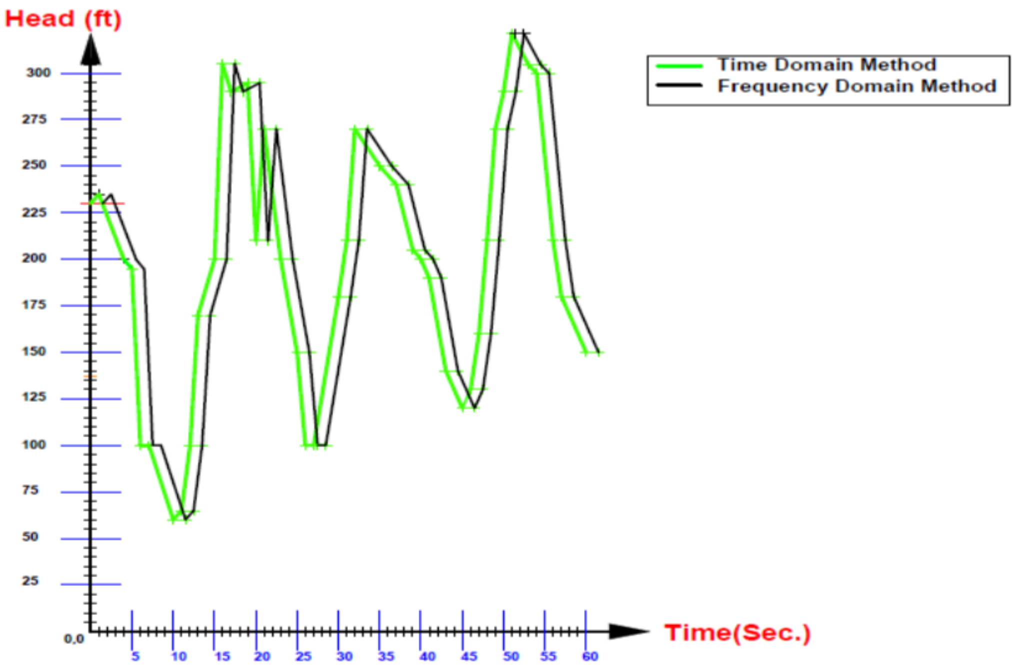

Figure 6 Comparison of results of time domain and frequency domain methods, for case 1, upstream of valve.

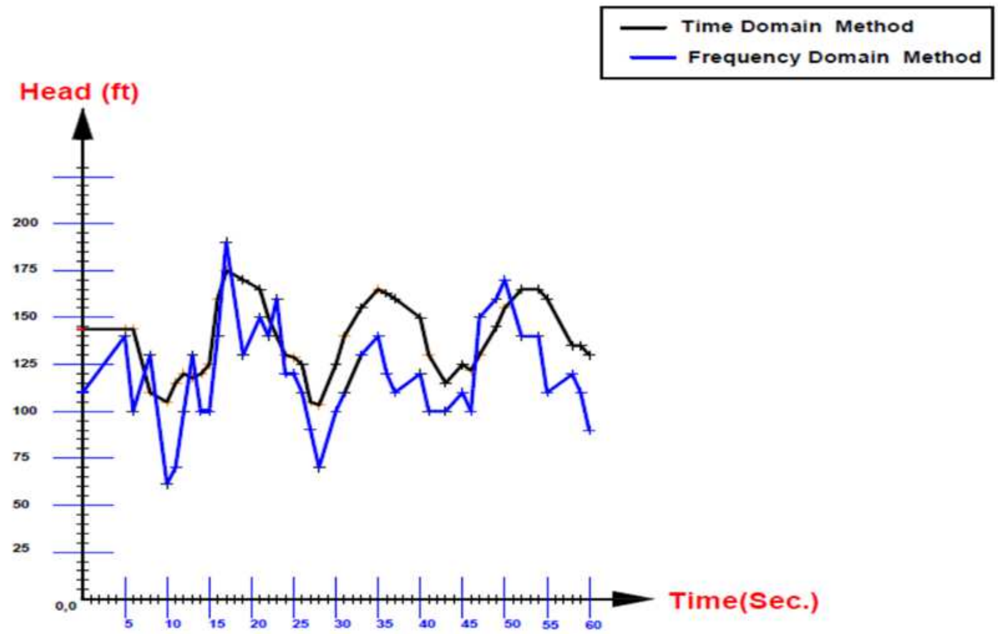

Figure 7 Comparison of results of time domain and frequency domain methods, for case 2, at junction 2.

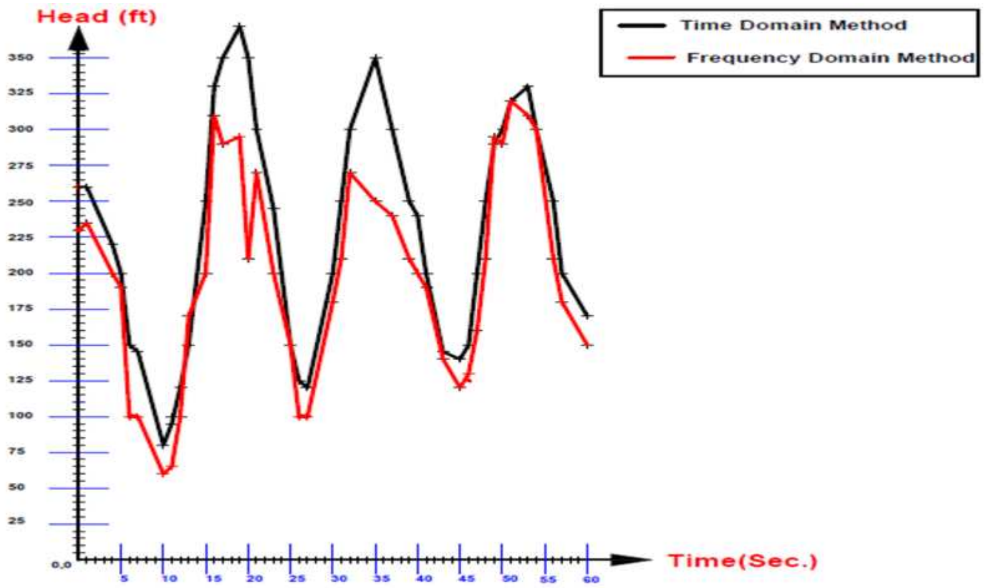

Figure 8 Comparison of results of time domain and frequency domain methods, for case 2, upstream of valve at node 7.