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Some Results on Fuzzy ω -Local Covering Dimension Function in Fuzzy Topological Space

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A B S T R A C T

For a fuzzy topological space, we aim to investigate a class of fuzzy covering function which we call the ω-local covering dimension function. We also examine the relationships between fuzzy ω-covering dimension function under the conditions of fuzzy ω-separation axioms and fuzzy ω-connected spaces. Finally, we give a number of characterizations for **ω**-covering dimension and fuzzy **ω**- local covering dimension by the use fuzzy countable sets provided a number of properties

Keywords: fuzzy ω disconnected space, fuzzy ω-open set, $\omega\dim_f(X)$, $\omega\text{loc}\dim_f(X)$, fuzzy locally countable set

1. Introduction

The notion, that will be studied in this work, is referred to as fuzzy sets which differ from the standard crisp sets. This idea was first presented by Zadeh (1965) [14], where he defined fuzzy sets in the form of a class consisting of objects having a variety degree of memberships. These sets are distinguished via membership function assigning each object a membership grade which range from zero to one. Chang (1968) [3] defined fuzzy topological spaces extending the crisp concept of a topological space. In (1976) Lowen [9] defined the stratified fuzzy topology. Wong (1973) [12] gave some generalizations to a number of properties regarding fuzzy spaces. For the fuzzy topological space $(X, \tilde{T})$, Zougdani 1984 [15] defined the fuzzy covering dimension using the concept of a shading family which was defined in the literature in (1978) by Gantner. et.al. [4] in 1978, while studying compactness in a fuzzy space. Zougdani defined a fuzzy topological space $(X,\tilde{T})$ in the sense of chang. Jackson in (1998) [6] debunked Zougdani's paper by giving a number counter-examples. Ajmal, and Kohli[1] in 1994, defined the fuzzy covering dimension of fuzzy topological space, this notion is an extension to that of a covering dimension in topological spaces. Th work shown a deviation form the standard topological approach by using the quasi-coincidence notion which was used by Ming, p.p. and Ming, L.Y. (1980) [8], to define the order for a collection of fuzzy sets, which vital to define the dimension function. Ajmal, and Kohli in (1994) [1] limited some of their results to the definition of fuzzy topological space $(X,\tilde{T})$ given by Lowen.

The concepts of fuzzy open set, fuzzy covering dimension function, and fuzzy local covering dimension function on fuzzy topological space are introduced in this work. Also, the study looked at the connection between fuzzy separation axioms and fuzzy -connected spaces, from which we deduced a number of properties.

2. Preliminaries

Definition 2.1 [14] :

Given $X \neq \phi$. A fuzzy set (F-set) $\tilde{A}$ in X can be defined by means of a function $\mu_{\tilde{A}}$, called the membership function, from X into $I = [0,1]$. So that $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : \forall x \in X\}$. The collection consisting of F-sets in X is written as I^X, i.e. $I^X = \{\tilde{A}: \tilde{A} \text{ is a F-set in } X\}$

Definition 2.2 [12] :

A fuzzy point (F-point) x_r is a F-set s.t. :

$\mu_{x_r}(y) = r > 0$ if $x = y, \forall y \in X$ and

$\mu_{x_r}(y) = 0$ if $x \neq y, \forall y \in X$

We denote the collection of all F-points of $\tilde{A}$ by $FP(\tilde{A})$"

Definition 2.3 [12] :

If x_r is a F-point and $\tilde{A}$ a F-set. Then x_r is said to belong to $\tilde{A}$ or (in $\tilde{A}$) or (contained in $\tilde{A}$) and is written $x_r \in \tilde{A}$ iff $\mu_{x_r}(x) < \mu_{\tilde{A}}(x), \forall x \in X$.

Proposition 2.4 [12] :

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two F-sets in X having memberships $\mu_{\tilde{A}_1}$ and μ_{A_2}, then $\forall x \in X$: -

1. $\tilde{A}_1 \subseteq \tilde{A}_2$ iff $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.
2. $\tilde{A}_1 = \tilde{A}_2$ iff $\mu_{\tilde{A}}(x) = \mu_{\tilde{A}_2}(x)$.
3. $\tilde{C} = \tilde{A}_1 \cap \tilde{A}_2$ iff $\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}_2}(x)\}$.
4. $\tilde{D} = \tilde{A}_1 \cup \tilde{A}_2$ iff $\mu_{\tilde{D}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}_2}(x)\}$.

Definition 2.5 [8]:

For a F-set $\tilde{A}$, we define the support, $S(\tilde{A})$, to be $S(\tilde{A}) = \{x \in X: \mu_{\tilde{A}}(x) > 0\}$.

Definition 2.6[3]

A fuzzy topology (F-topology) is a family $\tilde{T}$ of F-sub sets of X, that satisfy:

i. $\tilde{\emptyset}, 1_X \in \tilde{T}$.

ii. If $\widetilde{A}_1, \widetilde{A}_2 \in \tilde{T}$, then $\min\{\mu_{\widetilde{A}_1}(x), \mu_{\widetilde{A}_2}(x)\} \in \tilde{T}$.

iii. If $\widetilde{G}\alpha \in \tilde{T}$ then $\text{sux}\{\mu_{\widetilde{G}_\alpha}(x): \alpha \in \Lambda\} \in \tilde{T}$.

The pair $(X,\widetilde{T})$ is called a fuzzy topological space (FT-space). Members of $\widetilde{T}$ are fuzzy open sets ($\widetilde{T}$-F-open set). The F-set $\tilde{C}$ in 1_X is called fuzzy closed set ($\widetilde{T}$-F-closed set) iff its complement, $\tilde{C}^c$,is $\widetilde{T}$- F-open set.

($\widetilde{T}$- F-closed set) iff its complement $\tilde{C}^c$ is $\widetilde{T}$- F-open set"

3. Properties Of Fuzzy *ω*-Open Set In Fuzzy Topological Space

Definition 3.1 [13]:

The F-set $\widetilde{A}$ in a FT-space $(X, \widetilde{T})$ is fuzzy uncountable iff $S(\widetilde{A})$ is an uncountable set in X.

Definition 3.2 :

A F-point x_r in a FT-space $(X, \widetilde{T})$ is a fuzzy condensation point of $\widetilde{A}_1 \subseteq 1_X$ if $\min\{\mu_{\widetilde{A}_2}(x), \mu_{\widetilde{A}_1}(x)\}$is fuzzy uncountable for every F-open set $\widetilde{A}_2$ which contains x_r. We denote the set consisting of the fuzzy condensation points of $\widetilde{A}_1$ by $\text{Cond}(\widetilde{A}_1)$.

Definition 3.3 :

The F-set $\widetilde{A}$ in a FT-space $(X, \widetilde{T})$ is said to be fuzzy ω-closed (Fω-closed) if $\text{Cond}(\widetilde{A}) \subseteq \widetilde{A}$. Complements of F$\omega$-closed sets are called fuzzy ω-open (Fω-open).

Theorem 3.4 :

A F-subset $\widetilde{G}$ of a FT-space $(X, \widetilde{T})$ is Fω-open set iff $\mu_{x_r}(x) < \mu_{\widetilde{G}}(x)$ $\exists$ a F-open set $\widetilde{U}$ s.t. $\mu_{x_r}(x) < \mu_{\widetilde{U}}(x)$ and that $\mu_{\widetilde{U}}(x) - \mu_{\widetilde{G}}(x)$ is fuzzy countable

Proof:

$\widetilde{G}$ is Fω-open set iff $\widetilde{G}^c$ is Fω-closed set, and

$\widetilde{G}^c$ is Fω-closed set iff $\text{Cond}(\widetilde{G}^c) \subseteq \widetilde{G}^c$,

and $\text{Cond}(\widetilde{G}^c) \subseteq \widetilde{G}^c$ iff each $\mu_{x_r}(x) < \mu_{\widetilde{G}}(x)$,

$x_r \notin \text{Cond}(\widetilde{G}^c)$, thus $x_r \notin \text{Cond}(\widetilde{G}^c)$ $\exists$ a F-open set $\widetilde{U}$ s.t. $\mu_{x_r}(x) < \mu_{\widetilde{U}}(x)$ and also $\min\{\mu_{\widetilde{U}}(x), \mu_{\widetilde{G}^c}(x)\} = \mu_{\widetilde{U}}(x) - \mu_{\widetilde{G}}(x)$ is fuzzy countable ■

Theorem 3.5 :

A F-subset $\widetilde{G}$ in a FT-space $(X, \widetilde{T})$ is ω-open set iff for each $x_r \in \widetilde{G}$ $\exists$ an F-open set $\widetilde{U}$ that contain x_r and a countable F-set $\tilde{C}$ in 1_X s.t. $\widetilde{U} - \tilde{C} \subseteq \widetilde{G}$

Proof:

($\Longrightarrow$) let $\widetilde{G}$ be Fω-open and with $x_r \in \widetilde{G}$

Then ∃ a F-open set $\widetilde{U}$ and $x_r \in \widetilde{U}$ and $\widetilde{U}$- $\widetilde{G}$ is countable. Put $\tilde{C} = \widetilde{U} - \widetilde{G}$, then $\tilde{C}$ is countable and $x_r \in \widetilde{U} - \tilde{C} = \widetilde{U} - (\widetilde{U} - \widetilde{G}) \subseteq \widetilde{G}$

(⟸) let $x_r \in \widetilde{G}$ then form the assumption ∃ F-open set $\widetilde{U}$ that contain x_r and a countable F-set $\tilde{C}$ of 1_X s.t. $\widetilde{U} - \tilde{C} \subseteq \widetilde{G}$. since $\widetilde{U} - \widetilde{G} \subseteq \tilde{C}$ then $\widetilde{U} - \widetilde{G}$ is countable, so that $\widetilde{G}$ is Fω-open ■

Proposition 3.6 :

In a FT-space F-open sets are Fω-open.

Proof:

Suppose $\widetilde{G}$ is F-open with $x_r \in \widetilde{G}$, Set $\widetilde{U} = \widetilde{G}$, $\tilde{C} = \emptyset$, then $\widetilde{U}$ is F-open and $\tilde{C}$ is a countable set, s.t. $x_r \in \widetilde{U} - \tilde{C} \subseteq \widetilde{G}$, thus $\widetilde{G}$ is Fω-open.

Remark 3.7 :

The conversed statement of the above need not generally hold as can be seen below:

Examples 3.8 :

Suppose $X = \{x_1, x_2, x_3\}$ and $\widetilde{A}$, $\widetilde{B}$ are F-subset in 1_X where:

$1_X = \{(x_1, 1), (x_2, 1), (x_3, 1)\}$

$\widetilde{A} = \{(x_1, 0.7), (x_2, 0.6), (x_3, 0.6)\}$

$\widetilde{B} = \{(x_1, 0.4), (x_2, 0.5), (x_3, 0.5)\}$

Let $T = \{\emptyset, 1_X, \widetilde{A}\}$ be a fuzzy topology on X,

Then the F-set $\widetilde{B}$ is a Fω – open which isn't F-open.

Definition 3.9 :

Suppose $\widetilde{B}$ is a F-subset of the FT-space $(X, \tilde{T})$. **The fuzzy ω-interior** of $\widetilde{B}$ written as

$\omega - Int(\widetilde{B})$ and is given by $\omega - Int(\widetilde{B}) = \cup \{\widetilde{G}: \widetilde{G}$ is Fω-open in 1_X , $\widetilde{G} \subseteq \widetilde{B}\}$x

Definition 3.10 :

Suppose $\widetilde{B}$ is a F-subset of the FT-space $(X, \tilde{T})$. **The fuzzy ω-closure** of $\widetilde{B}$ is denoted by

$\omega - cl(\widetilde{B})$ and defined by $\omega - cl(\widetilde{B}) = \cap\{\tilde{G}: \tilde{G}$ is Fω-closed in 1_X , $\widetilde{B} \subseteq \tilde{G}\}$

Definition 3.11 :

Suppose $\widetilde{B}$ is a F-subset of the FT-space $(X, \tilde{T})$. **The fuzzy ω-boundary** of $\widetilde{B}$ written as $\omega - b(\widetilde{B})$ and is defined by $\omega - b(\widetilde{B}) = \omega - cl(\widetilde{B}) - \omega - Int(\widetilde{B})$

Theorem 3.12 :

Suppose $\widetilde{A}$ is a F-subset of the FT-space $(X,\widetilde{T})$, we have $(\widetilde{T}_{\tilde{A}})^{\omega} = \widetilde{T}_{\tilde{A}}^{\omega}$

Proof:

For $(\widetilde{T}_{\tilde{A}})^{\omega} \subseteq \widetilde{T}_{\tilde{A}}^{\omega}$, let $\widetilde{B} \in (\widetilde{T}_{\tilde{A}})^{\omega}$ and $x_r \in \widetilde{B}$ **,** by **Theorem 3.5** ∃ F-open set $\widetilde{V}$ of $\widetilde{T}_{\tilde{A}}$ and $\tilde{C}$ fuzzy countable subset of $\widetilde{T}_{\tilde{A}}$ s.t. $x_r \in \widetilde{V} - \tilde{C} \subseteq \widetilde{B}$, choose $\widetilde{U} \in \widetilde{T}$ s.t. $\widetilde{V} = \widetilde{U} \cap \widetilde{A}$,Then $\widetilde{U} - \tilde{C} \in \widetilde{T}^{\omega}$, $x_r \in \widetilde{U} - \tilde{C}$ and $\widetilde{U} - \tilde{C} \cap \tilde{A} = \widetilde{V} - \tilde{C} \subseteq \widetilde{B}$

Therefore $\widetilde{B} \in \widetilde{T}_{\tilde{A}}^{\omega}$, To prove $\widetilde{T}_{\tilde{A}}^{\omega} \subseteq (\widetilde{T}_{\tilde{A}})^{\omega}$, let $\widetilde{G} \in \widetilde{T}_{\tilde{A}}^{\omega}$ then ∃ $\widetilde{H} \in \widetilde{T}^{\omega}$ s.t.

$\widetilde{G} = \widetilde{H} \cap \tilde{A}$ if $x_r \in \widetilde{G}$ then $x_r \in \widetilde{H}$ and ∃ F-open set $\widetilde{U}$ of

$\widetilde{T}$ and $\widetilde{D}$ countable subset of $\widetilde{T}$ s.t. $x_r \in \widetilde{U} - \widetilde{D} \subseteq \widetilde{H}$

We put $\widetilde{V} = \widetilde{U} \cap \tilde{A}$, then $\widetilde{V} \in \widetilde{T}_{\tilde{A}}$ and $x_r \in \widetilde{V} - \widetilde{D} \subseteq \widetilde{G}$ It follows that $\widetilde{G} \in (\widetilde{T}_{\tilde{A}})^{\omega}$ ■

Definition 3.13:

The fuzzy collection $\{\widetilde{B}_{\alpha} : \alpha \in \Lambda\}$ of sets in the FT-space $(X,\widetilde{T})$ is:

1- Fω-locally finite if $\forall x_r \in 1_X$ ∃ a Fω-open set $\widetilde{G}$ that contain x_r s.t. the set $\{\widetilde{G} \cap \widetilde{B}_{\alpha} \neq \emptyset : \alpha \in \Lambda\}$ is finite.
2- Fω-discrete if $\forall x_r \in 1_X$ ∃ a Fω-open set $\widetilde{G}$ that contain x_r s.t. the set $\{\widetilde{G} \cap \widetilde{B}_{\alpha} \neq \emptyset : \alpha \in \Lambda\}$ contains at most one element.

Proposition 3.14 :

A Fuzzy discrete family (resp.fuzzy locally finite family) in a FT-space $(X,\widetilde{T})$ is Fω- discrete (resp.Fω- locally finite)

Proof : holds using (F-open sets are Fω-open)

Definition 3.15 :

A FT-space $(X,\widetilde{T})$ is fuzzy antilocally-countable if every non-empty F-open set in 1_X is uncountable.

Definition 3.16 :

A FT-space $(X,\widetilde{T})$ is ω-$\widetilde{T}_0$ if for any two distinct F-points x_r and y_t of 1_X ∃ Fω-open set $\widetilde{G}$ s.t. either" $x_r \in \widetilde{G}$ and $y_t \notin \widetilde{G}$ or $y_t \in \widetilde{G}$ and $x_r \notin \widetilde{G}$.

Definition 3.17 :

A FT-space $(X,\widetilde{T})$ is ω-$\widetilde{T}_1$ if for any two distinct F-point x_r and y_t of 1_X ∃ Fω-open sets $\widetilde{G}$ and $\widetilde{H}$ s.t. $x_r \in \widetilde{G}$ and $y_t \notin \widetilde{G}$ and $y_t \in \widetilde{H}$ and $x_r \notin \widetilde{H}$.

Definition 3.18 :

A FT-space $(X,\widetilde{T})$ is ω-$\widetilde{T}_2$ if for every pair of distinct F-point x_r and y_t of 1_X ∃ disjoint Fω-open sets $\widetilde{G}$ and $\widetilde{H}$ that respectively contain x_r and y_t.

Definition 3.19 :

A FT-space $(X,\widetilde{T})$ is Fω-regular if for every Fω-closed set $\tilde{B}$ in 1_X and x_r in 1_X s.t. $x_r \notin \tilde{B}$, ∃ disjoint Fω-open sets $\widetilde{U}$ and $\widetilde{V}$ that contain respectively contain x_r and $\tilde{B}$.

Definition 3.20 :

A FT-space $(X,\widetilde{T})$ is called a Fω-Normal space if for every pair of disjoint Fω-closed sets $\widetilde{A}$ and $\tilde{B}$ in 1_X ∃ disjoint Fω-open sets $\widetilde{U}$ and $\widetilde{V}$ containing $\widetilde{A}$ and $\tilde{B}$ respectively

Theorem 3.21 :

A FT-space $(X,\widetilde{T})$ is Fω-Normal if for every pair of Fω-open sets $\widetilde{G}$ and $\widetilde{H}$ in 1_X s.t. $1_X = \widetilde{G} \cup \widetilde{H}$ there are Fω-closed sets $\widetilde{U}$ and $\widetilde{V}$contained in $\widetilde{G}$ and $\widetilde{H}$ respectively s.t.

$1_X = \widetilde{U} \cup \widetilde{V}$

Proof : Clear

Theorem 3.22 :

Fω-closed subspaces of Fω-Normal spaces are Fω-Normal.

Proof : Clear

4. Properties Of Fuzzy ω-Covering Dimension Function

Definition 4.1:

The Fω-covering dimension function of the FT-space $(X,\tilde{T})$ which we denote as $\omega\dim_f(X)$, is the smallest $n \in \mathbb{Z}$ with the property for each finite ω-open covering of 1_X possesses a finite ω-open refinement having an order at most n.

Then we immediately have $\omega\dim_f(X) = -1$ iff $X = \varnothing$. Also $\omega\dim_f(X) \le n$ if each finite ω-open covering of 1_X possesses a finite ω-open refinement with an order at most n. Finally, $\omega\dim_f(X) = +\infty$ if ∀ positive integer $\omega\dim_f(X) > n$.

Propositions 4. 2 :

For the closed fuzzy subspace $(\widetilde{B},\widetilde{T}_B)$ in the FT-space $(X,\tilde{T})$. then $\omega\dim_f(\widetilde{B}) \le \omega\dim_f(X)$.

Proof :

If $\omega\dim_f(X) = \infty$ or -1 we are done. So we only have to prove that whenever $\omega\dim_f(\mathbf{X}) = n$ we have $\omega\dim_f(\widetilde{B}) \le n$. to that end, suppose $\{\widetilde{U}_i\}_{i=1}^{t}$ is a fuzzy finite cover of $\widetilde{B}$ by Fω-open sets in $\widetilde{B}$, then using Theorem 3.12 ∃ Fω-open sets $\widetilde{V}_i$ in 1_X s.t. $\mu_{\widetilde{U}_i}(x) = \min\{\mu_{\widetilde{V}_i}(x), \mu_{\widetilde{B}}(x)\} \forall\, i = 1,2,\ldots\ldots,t$, hence $\max\{\mu_{\widetilde{V}_i}(x), \mu_{\widetilde{B}^c}(x)\}$ is fuzzy finite ω-open cover of $1_X \forall\, i = 1,2,\ldots\ldots,t$, since $\omega\dim_f(X) = n$,we get that

∃ Fω-open refinement $\{\widetilde{G}_\lambda\}_{\lambda\in\Lambda}$ of $\max\{\mu_{\widetilde{V}_i}(x), \mu_{\widetilde{B}^c}(x)\} \forall\, i = 1,2,\ldots\ldots,t$ with an order at most , thus $\min\{\mu_{\widetilde{G}_\lambda}(x), \mu_{\widetilde{B}}(x)\} \forall\, \lambda \in \Lambda$ is Fω-open refinement of $\{\widetilde{U}_i\}_{i=1}^{t}$ with order at most n , which means that $\omega\dim_f(\widetilde{B}) \le n$ ■

Propositions 4. 3 :

Suppose $(X,\widetilde{T})$ is fuzzy locally countable we conclude that $\omega\dim_f(X) = 0$.

Proof : Clear.

Propositions 4. 4 :

A FT-space $(X, \tilde{T})$ s.t. $\omega\dim_f(X) = 0$is fuzzy ω-Normal space.

Proof :

Let $\omega\dim_f(\mathbf{X}) = 0$ and $\widetilde{G}$, $\widetilde{H}$ be two Fω-open sets of 1_X s.t. $\widetilde{G} \cup \widetilde{H} = 1_X$ therefore ∃ Fω-open refinement $\{\tilde{Z}_\lambda\}_{\lambda\in\Lambda}$ of the fuzzy cover $\{\widetilde{G}, \widetilde{H}\}$ of order at most 0. This means that the members of $\{\tilde{Z}_\lambda\}_{\lambda\in\Lambda}$ are fuzzy pairwise disjoint then $\widetilde{N} = \cup_{\lambda\in\Lambda}\{\tilde{Z}_\lambda : \tilde{Z}_\lambda \subseteq \widetilde{G}\}$ and $\widetilde{M} = \cup_{\lambda\in\Lambda}\{\tilde{Z}_\lambda : \tilde{Z}_\lambda \subseteq \widetilde{H}\}$ are fuzzy disjoint and $\widetilde{N} \cup \widetilde{M} = 1_X$ therefore $(X,\tilde{T})$ is Fω-Normal space ■

Propositions 4. 5 :

A FT-space $(X,\tilde{T})$ with $\dim_f(\mathbf{X}) = 0$ is fuzzy Normal space.

Proof : Clear.

Propositions 4. 6:

A Fω-$\tilde{T}_1$ space $(X, \tilde{T})$ having two or more F-points with $\omega\dim_f(\mathbf{X}) = 0$ is Fω-disconnected

Proof :

Suppose $\omega\dim_f(\mathbf{X}) = 0$. Take the distinct F-points $\mu_{x_r}(x)$ and $\mu_{q_t}(x)$ in 1_X. We have $\mu_{1_X}(x) - \{\mu_{x_r}(x)\}, \mu_{1_X}(x) - \{\mu_{q_t}(x)\}$ is a fuzzy finite ω-open cover of 1_X. Put $\mu_{\widetilde{G}}(x) = \mu_{1_X}(x) - \{\mu_{x_r}(x)\}$ and

$\mu_{\widetilde{G}}(x) = \mu_{1_X}(x) - \{\mu_{q_t}(x)\}$ in the proof of Propositions 4.4 there are fuzzy disjoint Fω-clopen sets $\mu_{\widetilde{N}}(x) \leq \mu_{\widetilde{G}}(x)$ and $\mu_{\widetilde{M}}(x) \leq \mu_{\widetilde{H}}(x)$ s.t. $\mu_{1_X}(x) = \max\{\mu_{\widetilde{N}}(x), \mu_{\widetilde{M}}(x)\}$, thus $\widetilde{N}$ and $\widetilde{M}$ are proper fuzzy maximal ω-clopen sets in 1_X , so $(X, \tilde{T})$ is Fω-disconnected ■

5. Properties Of Fuzzy *ω*- local Covering Dimension Function

Definition 5.1:

"The Fω-local dimension, ωloc-$\dim_f(\mathbf{X})$ of a FT-space X is defined as follows. If X is fuzzy empty, then ωloc-$\dim_f(\mathbf{X}) = -1$. Otherwise ωloc-$\dim_f(\mathbf{X})$ is the least integer n s.t. for every F-point x o X there is some ω-open set U containing x s.t." dim $\overline{U} \leq n$ "

Definition 5.2:

"The Fωlocal inductive dimension ωloc-$\mathrm{Ind}_f(\mathbf{X})$ is defined as. If X is fuzzy empty, then ωloc-$\mathrm{Ind}_f(\mathbf{X}) = -1$. Otherwise ωloc-$\mathrm{Ind}_f(\mathbf{X})$ is the least integern s.t. for every F-point x of X there is some Fω-open set U containing x s.t. ωloc-$\mathrm{Ind}_f(\overline{U}) \leq n$"

Proposition 5.3:

for every FT-space X

1- ωloc-$\dim_f(\mathbf{X}) \leq \omega$ -$\dim_f(\mathbf{X})$

Proof :

If ωloc-dim_f(**X**)≤ n, x ∈ X and U is an Fω-open set s.t. x ∈ U, then ∃ an Fω-open set V s.t. x ∈ V ⊂ U and dim $\overline{V}$ ≤ n. For ∃ an Fω-open set W s.t. x ∈ W and dim $\overline{W}$ ≤ n. If V = U∩W, then x ∈ W and $\overline{V}$ is a Fω-closed subset of $\overline{W}$, so that dim $\overline{V}$ ≤ n. Hence ωloc-dim_f(**X**)≤ n iff every Fω-open covering of the space X has an Fω-open refinement {U_λ}, say, s.t. ωdim_f($\overline{U}_\lambda$ ≤ n), for" each $_\lambda$. ■

Definition 5.4:

A "continuous function f: X → Y is said to be a Fωlocal homeomorphism if each F-point x of X has an Fω-open neighbourhood U_X s.t. f (U_X) is Fω-open in Y and the mapping of U_X onto f (U_X) given by restriction of f is a homeomorphism".

Proposition 5.5:

"If X and Y are Fω-regular spaces and f : X → Y is a surjectuve Fωlocal homeomorphism, then"

ωloc-dim_f(**X**) = ωloc-dim_f(**Y**)

Proof :

Suppose that loc dim Y ≤ n, and let x be a F-point of X. ∃ a Fω-open neighbourhood U of a s.t. f(U) is Fω-open in Y and f maps U homeomorphically onto f(U). ∃s an Fω-open set H of Y s.t. f(x) ∈ H ⊂ $\overline{H}$ ⊂ f(U) and dim $\overline{H}$ ≤ n. If $V = f^{-1}(H) \cap U$,"

"then V is an Fω-open neighbourhood of x and $\overline{V}$ is homeomorphic with $\overline{H}$ so that ω-dim $\overline{V}$ ≤ n.Thus ωloc-dim_f(**X**)≤ n. Now let $\omega loc - dimf(\mathbf{X}) \leq n$ and let y be a F-point of Y. ∃ x in X s.t. f(x)=y, and an Fω-open neighbourhood U of x s.t. f maps U homeomorphically onto f(U), which is an Fω-open neighbourhood of y; and ∃ an Fω-open set V of X s.t. x ∈ V ⊂ $\overline{V}$ ⊂ U and dim $\overline{V}$ ≤ n. If H = f(V), then H is an Fω-open neighbourhood of y and $\overline{H}$ is homeomorphic with $\overline{V}$ so that ωdim $\overline{H}$ ≤ n. Thus ωloc-dim_f(**X**)≤ n. Hence ωloc-dim_f(**X**) = ωloc-dim_f(**Y**) ■

Proposition 5.6:

If A is a F-closed set of a space X, then $\omega loc - dimf(\mathbf{A}) \leq \omega loc - dimf(\mathbf{X})$

Proof :

Let $\omega loc - dimf(\mathbf{X}) \leq n$ and let x be a F-point of A. Then there is an Fω-open set U of X s.t. x ∈ U and ωdim $\overline{U}$ ≤ n. Then U ∩ A is an F-open neighbourhood of x in A and the closure of U ∩ A in A is a F-closed set of $\overline{U}$and hence has dimension at most n. Hence $\omega loc - dimf(\mathbf{A}) \leq n$. Thus $\omega loc - dimf(\mathbf{A}) \leq \omega loc - dimf(\mathbf{X})$ as was to be shown" ■

Proposition 5.7:

If Y is an Fω-open set of a Fω-regular space X, $\omega\text{loc} - \text{dimf}(\mathbf{Y}) \leq \omega\text{loc} - \text{dimf}(\mathbf{X})$

Proof :

Suppose that ω-loc $\dim X \leq n$ and let y be a F-point of Y. There is an Fω-open neighbourhood U of y in X s.t. $\omega\dim \overline{U} \leq n$. Since X is a Fω-regular space ∃ an Fω-open set V s.t. $y \in V \subset \overline{V} \subset U \cap Y$. Then V is an Fω-open neighbourhood of y in Y and $\overline{V}$ is the fuzzy closure of V in Y. Since $\overline{V}$ is a Fω-closed subset of $\overline{U}$, it follows that $\dim \overline{V} \leq n$. Thus $\omega\text{loc} - \text{dimf}(\mathbf{Y}) \leq n$. Hence" ωloc-$\dim_f(\mathbf{Y}) \leq \text{loc}\dim X$ ■

Proposition 5.8:

If A is a subset of a totally normal space X, then $\omega\text{loc} - \text{dimf}(\mathbf{A}) \leq \omega\text{loc} - \text{dimf}(\mathbf{X})$

Proof :

Suppose that ωloc-$\dim_f(\mathbf{X}) \leq n$ and let x be a F-point of A. There is a neighborhood U of x in X s.t. $\dim \overline{U} \leq n$. Then $U \cap A$ is an Fω-open neighborhood of x in A and its fuzzy closure in A is a subset of the fuzzy totally ω-normal space $\overline{U}$ and hence has ω-covering dimension at most n by Theorem 3.6.4. Thus ωloc-$\dim_f(\mathbf{A}) \leq n$. Hence" ωloc-$\dim_f(\mathbf{A}) \leq \omega\text{loc} - \text{dimf}(\mathbf{X})$ ■

Theorem 5.9:

If a Fω-normal space X is the union of a pair of Fω-closed sets A and B s.t. ωloc-$\dim_f(\mathbf{A}) \leq n$ and ωloc-$\dim_f(\mathbf{B}) \leq n$, then ωloc-$\dim_f(\mathbf{X}) \leq n$.

Proof :

If $x \in X\backslash A$, then $x \in B$ and there is an Fω-open set $U \cap B$ of B, where U is Fω-open in X s.t. $\dim\ (U \cap B)^{-} \leq n$. If $W = U \cap (X\backslash A)$, then W is Fω-open in X, $x \in W$ and since $\overline{W} \subset (U \cap B)^{-}$, it follows that ω-$\dim \overline{W} \leq n$. Similarly if $x \in W \backslash B$, there is an Fω-open neighborhood of x in X which has fuzzy closure of Fω-covering dimension at most n. If $x \in A \cap B$, then ∃ Fω-open sets U and V containing x s.t. $\omega\dim\ (U \cap A)^{-} \leq n$ and $\omega\dim\ (V \cap B)^{-} \leq n$. Let $W = X\backslash(A\backslash U)\backslash(B\backslash V)$.

Then W is Fω-open and $x \in W$. And $W \subset (U \cap A) \cup (V \cap B)$ for if $y \in W$, then $y \notin A\backslash U$ and $y \notin B\backslash V$, but either $y \in A$ or $y \in B$, so that eithe $y \in A \cap U$ or $y \in B \cap V$. Thus ω-$\dim \overline{W} \leq \omega - \dim((U \cap A)^{-} \cup (V \cap B)^{-}) \leq n$, by the sum theorem in the Fω-normal space $(U \cap A)^{-} \cup (V \cap B)^{-}$. Thus ωloc-$\dim_f(\mathbf{X}) \leq n$ ■

Reference

1. Ajmal N., and Kohli K. J., "Overlopping Families and Covering Dimension in Fuzzy Topological Spaces" F-sets and Systems, 64 (1994), 257- 263.
2. Bai Shi – Zhong, Wang Wan – Liang "Fuzzy non – continuous mapping and fuzzy pre – semi – separation axioms" F-sets and systems Vol.94, pp.261 – 268(1998).

3. Chang, C. L., "Fuzzy Topological Spaces", J. Math. Anal. Appl., Vol.24, PP. 182-190, 1968.
4. Gantner T.E. and Steinlage R.S. “Compactness in Fuzzy Topological Spaces” J. Math Anal. Appl., 62 (1978), 547- 562.
5. Hutton, B., 1975, Normality in Fuzzy topological Spaces, J. Math. Anal. Appl. 50, 74-79.
6. Jackson M. “Dimension Theory in the Context of Fuzzy Topological Spaces” M.Sc., D. Thesis, University of Canterbury (1998).
7. Lou, S. P. and Pan, S. H., “Fuzzy Structure”, J. Math. Analy. and Application, Vol.76, PP.631-642, 1980.
8. Ming, P. P. and Ming, L. Y., "Fuzzy Topology I. Neighborhood Structure of a Fuzzy point and Moor-smith Convergence", J. Math. Anal. Appl., Vol.76, PP. 571-599, 1980.
9. R. Lowen, Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl.,56 (1976) 621-633.
10. Sarkar M. “On Fuzzy Topological Spaces” J. Math. Anal. Appl. 79 (1981) 384- 394.
11. Sunil,J.J " fuzzy topological games and related topics" Ph.D Thesis, cochin university of science and Technology,(2002).
12. Wong, C. K., "Fuzzy Points and Local Properties of Fuzzy Topology", J. Math. Anal. Appl., Vol.46, PP. 316-328, 1973.
13. XuzhuWang, Da Ruan and Etienne E. Kerre " Mathematics of Fuzziness – Basic Issues" Volume 245,pp.185-186(2009)
14. Zadeh, L.A., "F-sets", Inform. Control, Vol.8, PP. 338-353, 1965.
15. Zougdani H. “Covering Dimension of Fuzzy Spaces”, Mat. Vesnik, 36 (1984), 104- 118.