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Graph representation of planetary gear trains: A review

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ABSTRACT

The various graphical representations of planetary gear trains (PGTs), including basic assumptions and graph rules, are first reviewed in this paper. They revealed that they had close relationships. Their similarity has stimulated the development of a unified graphical representation. When there are multiple joints in a graph, it can be reconfigured without affecting the information contained in the graph. To accomplish this, a rooted graph in which a multiple joint represents either the housing of the mechanism and/or a transfer vertex of a multi-planet fundamental geared entity is employed.

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1. Introduction

A kinematic chain is a collection of links joined together by joints. Structural analysis is the study of the nature of the connection between the various links of the kinematic chain. The kinematic structure can be represented in a variety of ways. Functional schematic representation, graph representation, and various matrix representations are all examples of kinematic structure representation.

To represent the relationship between the links and joints of a PGT, a conventional schematic diagram is commonly used. Fig. 1 (a) depicts the functional schematic representation of an automatic transmission designed by Hyundai [1]. It includes six shifting elements and an eleven-link 2-DOF PGT. Fig. 1(b) depicts the PGT only. The links are numbered 1 through 11, and the locations of the axis in space are labeled a, b, c, d, and e.

A more abstract representation of the link and joint assemblage is the graph representation. PGTs can be represented graphically in a variety of ways. In a graph, the vertices denote links, and the edges denote joints. Double-line edges denote gear pairs, while single-line edges denote revolute pairs. Furthermore, the level of the rotation axis can be used to label the revolute edges.

The following are some of the advantages of using graphs to represent PGTs [2]:

1. Many graph properties are directly applicable.
2. The structural topology can be identified uniquely.
3. Graphs are used to create computerized PGT analysis.

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Nomenclatures

DOF Degree Of Freedom
 PGT Planetary Gear Train
 f The number of DOFs
 E_r Revolute edges
 E_g Geared edges

E The total number of edges
 L Number of fundamental circuits
 n number of links of the PGT
 v number of vertices

- 4. PGTs can be systematically enumerated using graph theory.
- 5. PGTs can be classified using graphs in a systematic manner.
- 6. PGTs are automatically sketched using graphs.

7. The graph representation will greatly simplify the kinematic analysis.

- c. Avoid making graphs that are nearly identical to one another.
- d. obtains computerizable graphs.

In general, the primary objective is to propose a unified graph representation for PGTs.

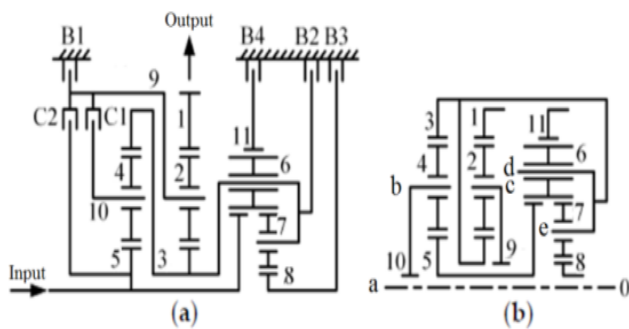


Figure 1. Hyundai automatic transmission [1]

Buchsbaum and Freudenstein [3] were the first to demonstrate that a PGT can be represented graphically. Freudenstein [4] introduced the concepts of rotation and displacement graphs to the representation of PGTs.

Fig. 2 shows the conventional graphs of the eleven-link two-DOF PGT shown in Fig. 1. all the joints are considered to be simple joints. Fig. 2(a) shows the conventional graph without the revolute edges at the common level a. Edges 1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-8 and 6-11 are geared edges, and edges 4-10, 2-9, 3-6, and 3-7 are the revolute edges at levels b, c, d, and e, respectively.

One of the major difficulties in the structural synthesis of PGTs is that there are several methods for arranging the revolute edges on a common level for a single PGT, resulting in a large number of pseudo-isomorphic graphs. For example, Figs. 2 (b), (c), (d), (e), (f), (g), and (h) shows seven conventional graphs that are pseudo-isomorphic. Several researchers [5-11] have proposed several graph representations to avoid such graphs.

The definitions for the graph representation were chosen for maximum generality and conformity with standard concepts. The purpose of the next section is to show that the existing graph representations, although different in appearance, are similar in content. A representative example that confirms this conclusion is given in Fig. 3.

The objectives of this paper are to:

- a. establishes the connection between the PGT and the graph representation
- b. develops a unified graph representation for PGTs.

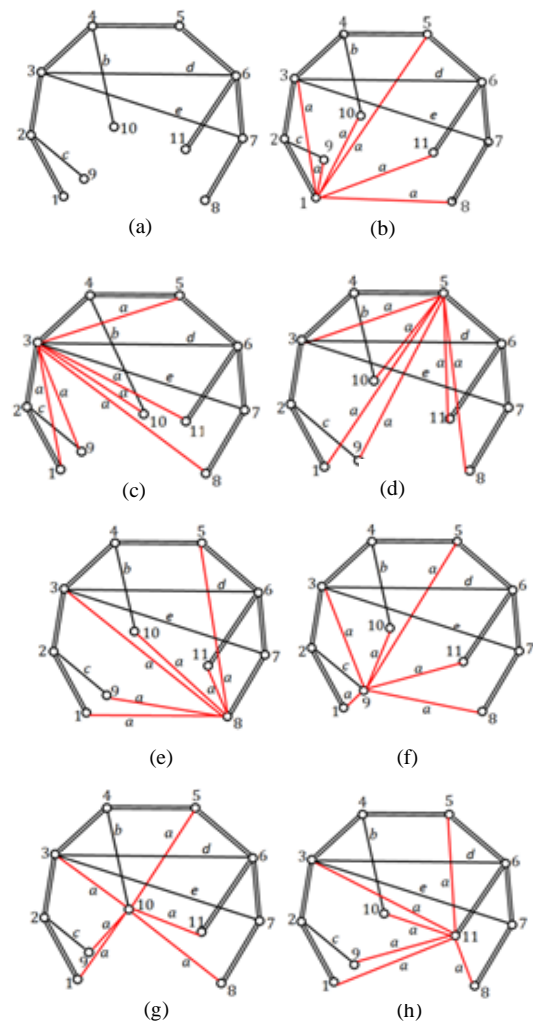


Figure 2. The conventional graphs of the example PGT shown in Fig.1

2. Common revolute joint

When there are several edges with the same labels, they may have originated from a common revolute joint. For example, the edges of links 1, 3, 5, 8, 9, 10, and 11 in Fig. 2 can be originated from a multiple joint. The multiple joints is known as solid polygon [5], hollow vertex [7], the

root [6], or is deleted from the graph [8]. According to [5],[8],[6], and [7], the PGT in Fig. 1 can be graphically represented by Figs. 3(a), (b), (c), and (d), respectively.

Because the planetary gear train is depicted in more than one graphical method, it is critical to recognize the similarities between them. When there is a multiple joint in a graph, it can be reconfigured as in Figs. 3 (a), (c) and (d) without affecting the information contained in it. These equivalent graphs are called pseudo isomorphic graphs.

If a multiple joint of a rooted graph is removed, then a subgraph containing all the second-level edges of the graph is obtained. The removal of the multiple joints implies the removal of all the first level edges from the graph.

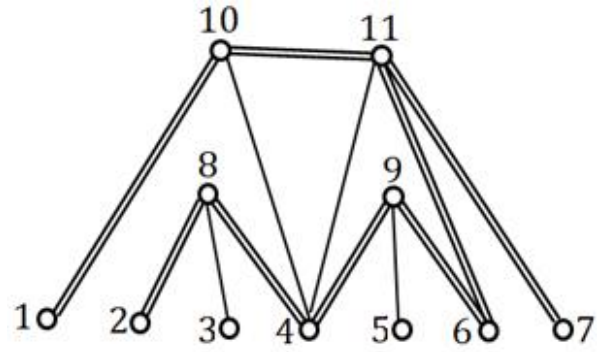


Figure 4. Second level edges

3. Canonical graph

A canonical graph representation was defined by [6, 9, and 12]. A canonical graph is a rooted graph in which the common multiple joints usually represent the housing of the PGT or the carrier of several planets.

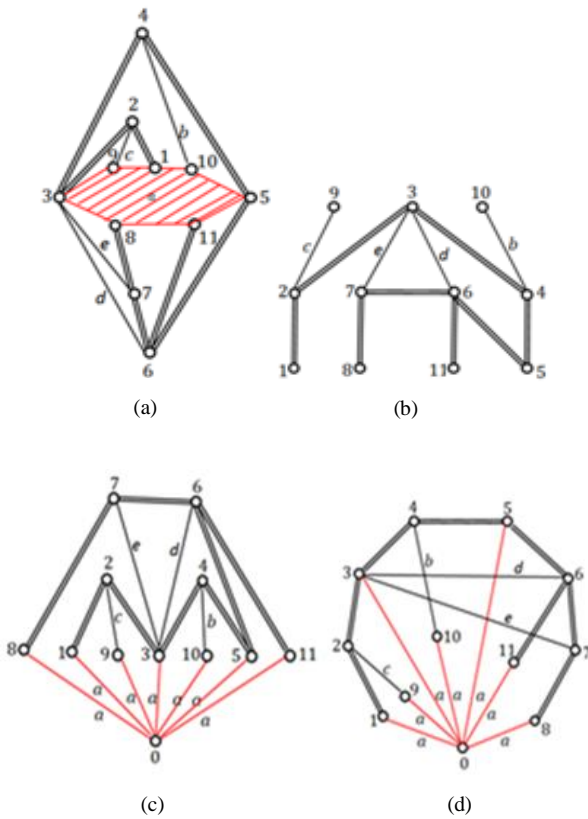


Figure 3. Various representations of the PGT in Fig. 1 (a) with a multiple joint according to [5] (b) without a multiple joint according to [8] (c) with a root according to [6], and (d) with a hollow vertex according to [7]

Fig. 4 shows the corresponding subgraph for the example PGT when the multiple joints is deleted. When compared with Fig. 3 (b), it is completely identical to it. If the multiple joints is deleted from the graphs of Fig. 3, and when compared to each other, they are completely identical.

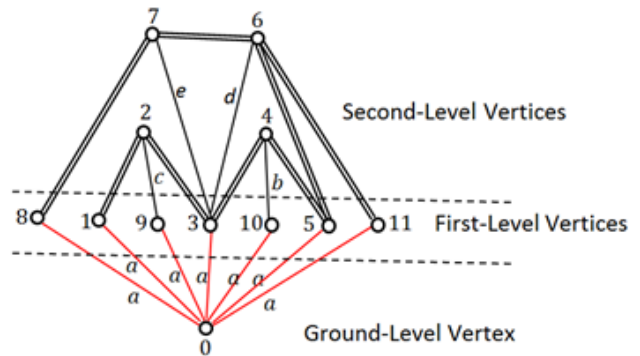


Figure 5. A proper classification of vertex levels

In Fig. 5, The housing will be represented by a vertex called the root. The revolute edges with the same rotation axis level or with the same label (a) can be rearranged around the root without affecting the functionality of the PGT. In the graphs of Fig. 2, the revolute edges with the same label (a) join vertices 1, 3, 5, 8, 9, 10, and 11. If the revolute edges with label-a are joined to the root, the graph in Fig. 5 can be formed from the graphs in Fig. 2. The resulting rooted graph is a unique graph representation with distinct edge labels for all single-line edged paths.

Vertices are classified into levels. The ground-level vertex represents the root. The first-level vertices are one single-line edge away from the root, and the second-level vertices are two single-line edges away from the root.

A spanning tree is obtained by deleting all gear pair edges from the graph. Fig. 6 depicts the spanning tree for the PGT depicted in Fig. 3 (c). A fundamental circuit (FC) is formed by adding a gear edge to the tree. The number of FCs corresponds to the number of gear edges.

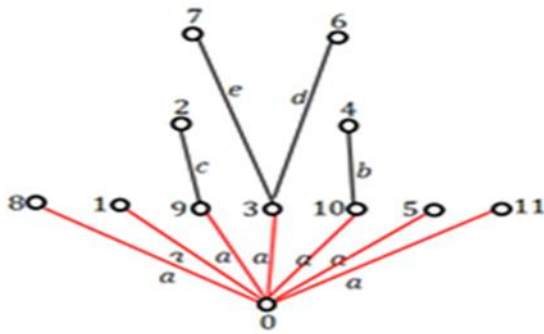


Figure 6. Spanning tree for the PGT shown in Fig. 3 (c)

4. Fundamental rules of the graphs of PGT

The graph of PGT satisfies the following:
 For n-Link PGT, the DOFs (f) is given by

$$f = 3(n - 1) - 2E_r - 1 \times E_g \tag{1}$$

$$E_r = n - 1 \tag{2}$$

$$E_g = n - 1 - f \tag{3}$$

The number of FCs equals E_g .

The total number of edges E is given by

$$E = 2n - 2 - f \tag{4}$$

Also,

$$E = E_r + E_g = 2n - 2 - f \tag{5}$$

Therefore,

$$E_g = E - E_r = E - n + 1 \tag{6}$$

Substituting Eqs. (2) and (6) into Eq. (1), we get

$$f = 2n - E - 2 \tag{7}$$

Since a rooted tree contains $n + 1$ vertices and $E_r + 1$ revolute edges, we further conclude that: $v = n + 1$, $e_r = E_r + 1$, and $e = E + 1$. As a result, in terms of v and e, Eq. (7) becomes:

$$f = 2v - e - 3 \tag{8}$$

For the PGT shown in Fig. 5, we have $v = 12$, and $e = 18$. Substituting these values into Equation (8), yields: $f = 2 \times 12 - 19 - 3 = 2$.

5. Three rows graph representation

Salgado and Del Castillo [11] used a three-row graph representation. The vertices are organized into three rows: the first represents the planet carriers, the second the planet gears, and the third the central gears. It is implicitly assumed that the vertices in the upper and lower rows are linked by a multiple joint. Fig. 7 depicts the graph representation of the example PGT according to Salgado and Del Castillo [11].

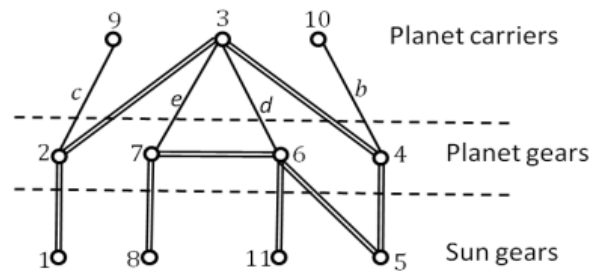


Figure 7. The three-row graph representation for the PGT shown in Fig. 1 (b)

6. Graph representation of solid polygon

Using Hsu and Lam's graph representation [5], the revolute edges with the same label (a) can be rearranged around to form a solid polygon without affecting the functionality of the PGT. The graph in Fig. 8 can be formed from the graphs of Fig. 2, if the revolute edges of the same label (a) joining vertices 1, 3, 5, 8, 9, 10, and 11 are replaced by a solid polygon joining the vertices 1, 3, 5, 8, 9, 10, and 11. The shaded polygon represents that the revolute pairs of the same label have a common axis that indicates its position in space [13, 14].

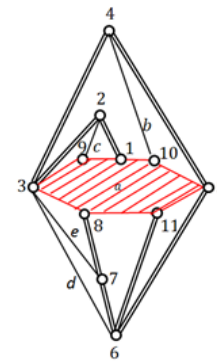


Figure 8. Graph representation of solid polygon

7. Rotation graph

The procedure for obtaining the rotation graph by Yang and Ding [7, 15] is as follows.

Step 1: Meshing gears are represented by vertices, and each gear joints are denoted by a double-line edges between vertices. Revolute edges are used to connect the vertices of the double-line edges to the associated transfer vertex.

Step 2: If a loop is entirely made up of revolute edges, remove them and connect the vertices to a common vertex (0).

8. Displacement graph

The following is the procedure for obtaining the displacement graph from the rotation graph, which is dependent on the edge levels.

Step 1: Remove all revolute edges and common vertices (if any) with the same label.

Step 2: All the vertices should be connected to a new common vertex. The first graph in Fig. 10 is the last graph of Fig. 9. Vertices 1, 3, 5, 8, 9, 10, and 11 in the derived displacement graph are joined to a new common vertex labeled "0"

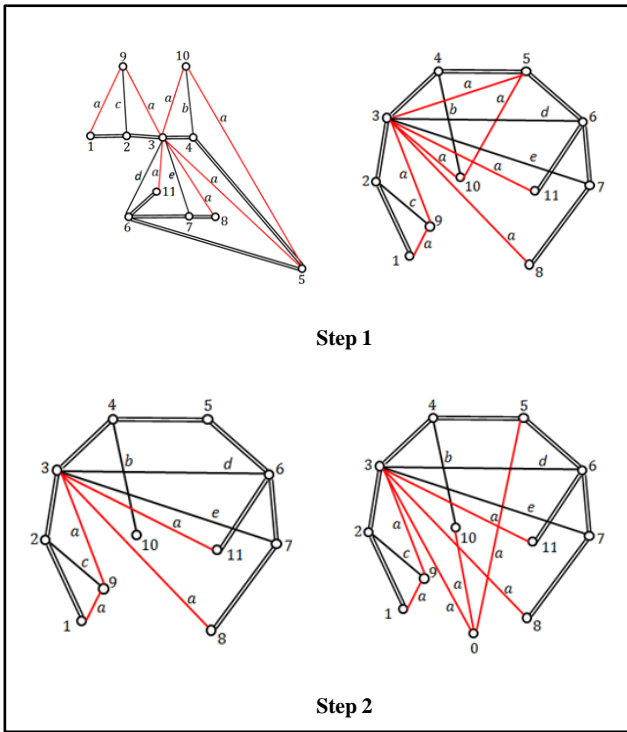


Figure 9. The procedure for obtaining the rotation graph shown in Fig. 1 (b)

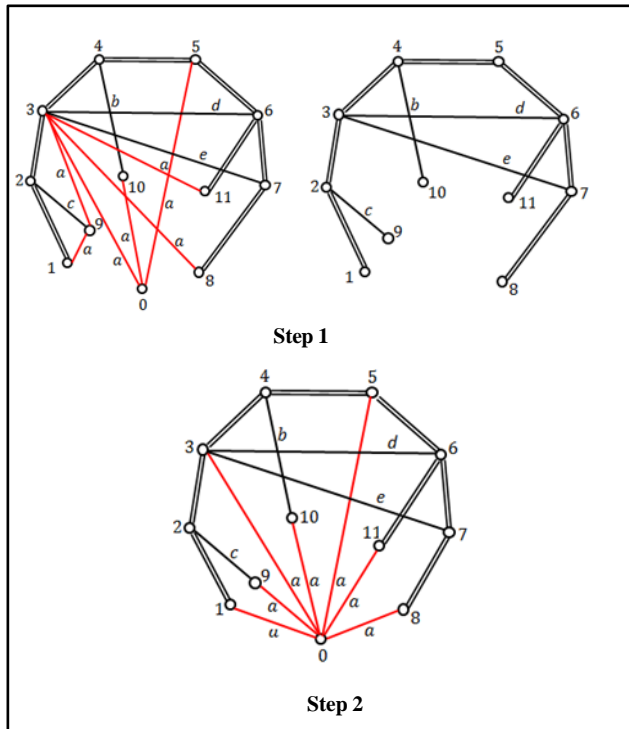


Figure 10. The procedure for obtaining the displacement graph from Fig. 9

9. Conclusions

To automate the synthesis of PGTs, graph-based approaches are being developed. This paper examines the graph representation of PGTs. The fundamental rules of rotation, displacement, and spanning tree graphs are presented. The paper is hoped to be useful in achieving a unified graphical representation of PGTs. The unified graph is created by reconfiguring the multiple joints without changing the information contained in the graph.

Authors’ contribution

All authors contributed equally to the preparation of this article.

Declaration of competing interest

The authors declare no conflicts of interest.

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