Dynamic Behavior Analysis of the Slider Crank Linkage using ANSYS Workbench and MATLAB Program

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Abstract:

This paper is concerned with the study of the kinematic and kinetic analysis of a slider crank linkage using D'Alembert's principle. The links of the considered mechanism are assumed to be rigid. The analytical solution to observe the motion (displacement, velocity, and acceleration), reactions at each joint, torque required to drive the mechanism and the shaking force have been computed by a computer program written in MATLAB language over one complete revolution of the crank shaft. The results are compared with a finite element simulation carried out by using ANSYS Workbench software and are found to be in good agreement. A graphical method (relative velocity and acceleration method) has been also applied for two phases of the crank shaft (θ_2 = 10° and 130°). The results obtained from this method (graphical) are compared with those obtained from analytical and numerical method and are found very acceptable. To make the analysis linear the friction force on the joints and sliding interface are neglected. All results, in this work, are obtained when the crank shaft turns at a uniform angular velocity (ω_2 = 188.5 rad/s) and time dependent gas pressure force on the slider crown.

تحليل السلوك الديناميكي لآلية المرفق والمنزلق باستخدام برنامج الـ ANSYS Workbench وبرنامج الـ MATLAB

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الخلاصة:

تم في هذا البحث دراسة وتحليل السلوك الكينماتي والديناميكي لآلية المرفق والمنزلق. حيث شملت هذه الدراسة حساب الحركة (الازاحة، والسرعة، والتعجيل) لجميع اجزاء الآلية وردود

الافعال عند جميع المساند والمفاصل والعزم اللازم لتدوير الآلية وكذلك القوة المسببة في اهتزاز الآلية وذلك عن طريق كتابة برنامج بلغة الـ MATLAB خلال دورة كاملة للمرفق. وقد تم مقارنة النتائج مع تلك التي تم الحصول عليها من برنامج المحاكاة للعناصر المحددة (ANSYS Workbench) ولوحظ ان النتائج متطابقة بشكل كبير جداً. وقد تم كذلك استخدام طريقة السرعة والتعجيل النسبي (طريقة التخطيط) لطورين فقط من اطوار الآلية (θ_2 = 10°, 130°) واظهرت النتائج توافقا كبيرا مع تلك التي تم الحصول عليها في الطريقتين السابقتين (التحليلية والعددية). اجريت جميع الحسابات عند اكتساب المرفق سرعة دورانية منتظمة مقدارها (ω_2 = 188.5 rad/s) وقوة متغيرة مع الوقت ناتجة من ضغط الغازات مع اهمال الاحتكاك بين اجزاء الآلية.

List of Symbols

a_{Gi}, = Acceleration of mass center of the links, (i =2,3,4), (m/s^2)
$\mathbf{a}_B$ = Acceleration vector of the slider
F_{12} = Total force at the pin joint O_2 (N)
F_{12x}, F_{12y} = Force components at the pin joint O_2 (N)
F_{14} = Total force at sliding interface joint (N)
F_{14x}, F_{14y} = Force components at sliding interface joint (N)
F_{23}, F_{32} = Total force at the pin joint A (N)
F_{23x}, F_{23y} or
F_{32x}, F_{32y} = Force components at the pin joint A (N)
F_{34x}, F_{34y} or
F_{43x}, F_{43y} = Force components at the pin joint B (N)
F_{21}, F_{41} = Forces acting on the ground plane or reaction forces (N)
F_{Px}, F_{Py} = Gas force components (N)
F_S = Shaking force (N)
G_1, G_2, G_3 = Center of mass of crank, connecting rod, and slider
I_{G2}, I_{G3}, I_{G4} = Mass moment of inertia of the crank, connecting rod, and slider ($kg\ m^2$)
m_2, m_3, m_4 = Mass of crank, connecting rod, and slider (kg)
r_2, r_3, r_4= length of crank, connecting rod, and slider offset (m)
$\mathbf{r}_B$, $\dot{\mathbf{r}}_B$, $\ddot{\mathbf{r}}_B$ = position, velocity, and acceleration vectors of the slider
$\mathbf{R}_{12}$, $\mathbf{R}_{23}$, $\mathbf{R}_{32}$, $\mathbf{R}_{34}$, $\mathbf{R}_{43}$ = Position vectors
T_{12} = Driving torque (N m)
θ_1 = Angle between slider velocity and frame x axis (deg.)
θ_2, θ_3 = Crank and connecting rod angles (deg.)
ω_2, ω_3 = Crank and connecting rod angular velocities (rad/s)
α_2, α_3 = Crank and connecting rod angular acceleration (rad/s^2)
σ = Classifies assembly mode

Introduction:

The slider crank linkage (SCL) is probably the most commonly used mechanism [1]. It appears in all pumps, compressors, steam engines, feeders, crushers, punches and injectors. This linkage appears also in all diesel and gasoline engines. A general SCL is represented in Figure 1. The SCL chain consists of four bodies (crank shaft, connecting rod, slider, and the frame) linked with three cylindrical joints and one sliding or prismatic joint. It is used to change circular into reciprocating motion, or reciprocating into circular motion.

Kinematic analysis of the SCL comprises the motions (displacement, velocity and accelerations) of various links of the mechanism like the connecting rod and slider or piston. The dynamic analysis of the SCL includes static and inertia force analysis for all the possible phases of the crank shaft which leads to an important aspect of the estimation of the loads carried by different members of this linkage.

Cveticanin and Maretic [2], have studied dynamics of a special type of the crank shaper mechanism. The influence of the cutting force on the motion of the mechanism is considered.

Ha et al [3] have employed Hamilton's principle, Lagrange multiplier, and geometric constraints to drive the dynamic equations of a SCL driven by a servomotor using real-coded genetic algorithm.

Fung et al [4] presented the kinematic and dynamic analysis of the intermittent SCL by connecting the connecting rod with a pneumatic cylinder and a spring model.

Ranjbarkohan et al [5], have considered the influence of engine RPM and downshifting effects on the members of the SCL using ADAMS software.

Jaballi et al [6] have examined the dynamic behavior of the SCL when the gravity is the only external force acting on the mechanism using Maplesim software with Gaussian distribution functions assumed.

Anis [7] has studied the kinematics and dynamics analysis of the SCL using ADAMS software to observe the response of the slider block. The analysis has been performed by applying moment on the joint which connect the connecting rod and crank.

In the present paper, the complete kinematic analysis of the considered SCL is carried out by an analytical method using complex-algebra method as this method is more accurate than the graphical method (GM) and can give results for all the phases of the mechanism.

The complete force analysis of the mechanism shown in Figure 1 for one revolution of the crank shaft using principle of D'Alembert is carried out by summation of the effects of external forces (gas forces) and inertia forces ignoring the effect of friction. This analysis includes, with linkage parameters available, an investigation of variation of crankpin load, piston pin load, main bearing load, driving required torque and total shaking force. Three methods for analyzing of SCL were used; namely: - a MATLAB program to solve the complex-algebra equations, numerical simulation with ANSYS Workbench software and the graphical method.

Analytical approach

The general kinematic sketch for the offset SCL used in this work is shown in Figure 1. The non-offset SCLs are the special cases. The analytical solution procedure follows the major steps below:

a. Position representation:

The SCL could be represented by only three position vectors, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_B$, but one of them ($\mathbf{r}_B$) will be a vector of varying magnitude and angle. It will be easier to use four vectors, $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$ with $\mathbf{r}_1$ arranged inclined to the axis of the sliding and $\mathbf{r}_4$ perpendicular. The position of point B is [8, 9]:

$$\mathbf{r}_B = \mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4 \quad (1)$$

or

$$r_2[\cos\theta_2 \boldsymbol{i} + \sin\theta_2 \boldsymbol{j}] + r_3[\cos\theta_3 \boldsymbol{i} + \sin\theta_3 \boldsymbol{j}] = r_1[\cos\theta_1 \boldsymbol{i} + \sin\theta_1 \boldsymbol{j}] + r_4[\cos\theta_4 \boldsymbol{i} + \sin\theta_4 \boldsymbol{j}] \quad (2)$$

where

$$\theta_4 = \theta_1 + \pi/2 \quad (3)$$

Rewriting Eq.(1) in its component equations gives:

$$r_2\cos\theta_2 + r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4 \quad (4)$$

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_1 \sin\theta_1 + r_4 \sin\theta_4 \quad (5)$$

The base vector, $\mathbf{r}_1$, will vary in magnitude but be constant in direction. The vector $\mathbf{r}_4$ will be constant.

To eliminate θ_3, first isolate it in using in Eqs. (4) and (5) as follows

$$r_3\cos\theta_3 = r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2 \quad (6)$$

$$r_3 \sin\theta_3 = r_1 \sin\theta_1 + r_4 \sin\theta_4 - r_2 \sin\theta_2 \quad (7)$$

Now square both sides of both equations and add. This gives

$$r_3^2 [\cos^2\theta_3 + \sin^2\theta_3] = [r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2]^2 + [r_1 \sin\theta_1 + r_4 \sin\theta_4 - r_2 \sin\theta_2]^2 \quad (8)$$

Expansion and simplification Eq.(8) give:

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 + 2 r_1 r_4[\cos\theta_1\cos\theta_4 + \sin\theta_1 \sin\theta_4] - 2r_1 r_2[\cos\theta_1\cos\theta_2 + \sin\theta_1 \sin\theta_2] - 2r_2 r_4[\cos\theta_2\cos\theta_4 + \sin\theta_2 \sin\theta_4] \quad (9)$$

Equation (9) can be solved for r_1 to gives the following quadratic expression:

$$r_1^2 + Ar_1 + B = 0 \tag{10}$$

where

$$A = 2r_4 [\cos\theta_1\cos\theta_4 + \sin\theta_1 \sin\theta_4] - 2r_2 [\cos\theta_1\cos\theta_2 + \sin\theta_1 \sin\theta_2]$$
$$B = r_2^2 + r_4^2 - r_3^2 - 2r_2r_4 [\cos\theta_2\cos\theta_4 + \sin\theta_2 \sin\theta_4] \tag{11}$$

or

$$r_1 = \frac{-A+\sigma\sqrt{A^2-4B}}{2} \tag{12}$$

where $\sigma = \pm 1$ is a sign variable identifying assembly mode.

Once a value for r_1 is determined, Eqs. (4) and (5) can be solved for θ_3

$$\theta_3 = tan^{-1}\left[\frac{r_1 sin\theta_1 + r_4 sin\theta_4 - r_2 sin\theta_2}{r_1 cos\theta_1 + r_4 cos\theta_4 - r_2 cos\theta_2}\right] \tag{13}$$

Therefore, once the angular quantities are known, it is relatively straightforward to compute the coordinates or positions of any point in the mechanisms. The coordinates of points A, G_3 and B are given by:

$$\mathbf{r}_A = \mathbf{r}_2 = r_2[\cos\theta_2\boldsymbol{i} + \sin\theta_2\boldsymbol{j}] \tag{14}$$

$$\mathbf{r}_{G3} = r_2 [\cos\theta_2\boldsymbol{i} + \sin\theta_2\boldsymbol{j}] + r_{G3} [\cos\theta_3\boldsymbol{i} + \sin\theta_3\boldsymbol{j}] \tag{15}$$

$$\mathbf{r}_B = \mathbf{r}_2 + \mathbf{r}_3 = r_2[\cos\theta_2\boldsymbol{i} + \sin\theta_2\boldsymbol{j}] + r_3[\cos\theta_3\boldsymbol{i} + \sin\theta_3\boldsymbol{j}] \tag{16}$$

where r_{G3} is the length of center of mass of link 3, i.e. $r_{G3} = AG_3$.

b. Velocity representation:

The analytical form of the velocity equations can be developed by differentiating Eq. (1) with respect to time. The result is

$$\dot{\mathbf{r}}_B = \dot{\mathbf{r}}_2 + \dot{\mathbf{r}}_3 = \dot{\mathbf{r}}_1 + \dot{\mathbf{r}}_4 \tag{17}$$

or in matrix form when ω_2 is the input:

$$\begin{bmatrix} cos\theta_1 & r_3 sin\theta_3 \\ sin\theta_1 & -r_3 cos\theta_3 \end{bmatrix} \begin{Bmatrix} \dot{r}_1 \\ \omega_3 \end{Bmatrix} = \begin{Bmatrix} -r_2\omega_2 sin\theta_2 \\ r_2\omega_2 cos\theta_2 \end{Bmatrix} \tag{18}$$

Therefore, the velocity of point B is given by:

$$\dot{\mathbf{r}}_B = (-r_2\omega_2 \sin\theta_2 - r_3\omega_3 \sin\theta_3)\boldsymbol{i} + (r_2\omega_2\cos\theta_2 + r_3\omega_3\cos\theta_3)\boldsymbol{j} \tag{19}$$

c. Acceleration representation:

The acceleration equations can be developed by differentiating Eq. (17) with respect to time. The result is

$$\ddot{\mathbf{r}}_B = \ddot{\mathbf{r}}_2 + \ddot{\mathbf{r}}_3 = \ddot{\mathbf{r}}_1 + \ddot{\mathbf{r}}_4$$
(20)

Or in matrix form when α_2 is input:

$$\begin{bmatrix} cos\theta_1 & r_3 sin\theta_3 \\ sin\theta_1 & -r_3 cos\theta_3 \end{bmatrix} \begin{Bmatrix} \ddot{r}_1 \\ \alpha_3 \end{Bmatrix} = \begin{Bmatrix} -r_2\alpha_2 sin\theta_2 - r_2\omega_2^2 cos\theta_2 - r_3\omega_3^2 cos\theta_3 \\ r_2\alpha_2 cos\theta_2 - r_2\omega_2^2 sin\theta_2 - r_3\omega_3^2 sin\theta_3 \end{Bmatrix}$$
(21)

Once the angular accelerations are known, it is simple to compute the linear acceleration of any point on the mechanism. The accelerations of points G_3 and B are given:

$$\mathbf{a}_{G3} = -(r_2\,\omega_2^2 cos\theta_2 + r_2\,\alpha_2\,sin\theta_2 + r_{G3}\omega_3^2 cos\theta_3 + r_{G3}\,\alpha_3\,sin\theta_3)\boldsymbol{i} + (-r_2\omega_2^2\,sin\theta_2 + r_2\alpha_2 cos\theta_2 - r_{G3}\omega_3^2\,sin\theta_3 + r_{G3}\,\alpha_3 cos\theta_3)\,\boldsymbol{j}$$
(22)

$$\mathbf{a}_{B} = -(r_2\,\omega_2^2 cos\theta_2 + r_2\,\alpha_2\,sin\theta_2 + r_3\omega_3^2 cos\theta_3 + r_3\,\alpha_3\,sin\theta_3)\boldsymbol{i} + (-r_2\omega_2^2\,sin\theta_2 + r_2\alpha_2 cos\theta_2 - r_3\omega_3^2\,sin\theta_3 + r_3\,\alpha_3 cos\theta_3)\,\boldsymbol{j}$$
(23)

d. Force analysis representation:

When all dimensions of link lengths, link positions, locations of the link's CGs, linear acceleration of those CGs, and link angular accelerations and velocities have been done, the forces acting at all the pin joins of the linkage and the driving torque needed on the crank for one complete revolution of the crank shaft can be calculated. D'Alembert's principle provides three equations for any link or rigid body in motion, the first and second represents the summation of forces and the third is for summation of torque or moment. Figure 2 shows the SCL with an external force on the slider block, link 4. The force analysis can be explained for each link as follows [10].

For link2:

$$\begin{aligned} &F_{12_x} + F_{32_x} = m_2 a_{G2_x} \\ &F_{12_y} + F_{32_y} = m_2 a_{G2_y} \\ &T_{12} + (R_{12_x}F_{12_y} - R_{12_y}F_{12_x}) + (R_{32_x}F_{32_y} - R_{32_y}F_{32_x}) = I_{G2}\alpha_2 \end{aligned} \qquad (24a)$$

For link3:

$$\begin{aligned} &F_{43_x} - F_{32_x} = m_3 a_{G3_x} \\ &F_{43_y} - F_{32_y} = m_3 a_{G3_y} \\ &(R_{43_x}F_{43_y} - R_{43_y}F_{43_x}) - (R_{23_x}F_{32_y} - R_{23_y}F_{32_x}) = I_{G3}\alpha_3 \end{aligned} \qquad (24b)$$

For link4:

$$F_{14_x} - F_{43_x} + F_{P_x} = m_3 a_{G3_x}$$
$$F_{14_y} - F_{43_y} + F_{P_y} = m_4 a_{G4_y} \qquad (24c)$$
$$(R_{14_x}F_{14_y} - R_{14_y}F_{14_x}) - (R_{34_x}F_{43_y} - R_{34_y}F_{43_x}) + (R_{P_x}F_{P_y} - R_{P_y}F_{P_x}) = I_{G4}\alpha_4$$

From Figure 2, the slider block, or piston, is in pure translation against the stationary ground plane; thus it can have no angular acceleration or velocity. Also, its linear acceleration has no y component.

$\alpha_4 = 0$; and $a_{G4y} = 0$
(24d)

and $F_{14x} = \pm\mu\, F_{14y}$
(24e)

where$\pm\mu$ is coefficient of friction.
Substituting, Eqs. (24e) and (24d) into Eq. (24c) yields:

$$\mp\mu F_{14_y} - F_{43_x} + F_{P_x} = m_4 a_{G4_x}$$
$$F_{14_y} - F_{43_y} + F_{P_y} = 0 \qquad (24f)$$

The plus and minus signs on the coefficient of friction are to identify the fact that the friction force always opposes motion. The last substitution has reduced the unknowns to eight. Therefore, there are only eight equations need to be solved to obtain all required forces. Putting these equations in matrix form yield:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \mp\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} x \begin{Bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} m_2 a_{G2_x} \\ m_2 a_{G2_y} \\ I_{G2}\alpha_2 \\ m_3 a_{G3_x} \\ m_3 a_{G3_y} \\ I_{G3}\alpha_3 \\ m_4 a_{G4_x} - F_{P_x} \\ -F_{P_y} \end{Bmatrix} \qquad (25)$$

e. Shaking force:

The sum of all forces acting on the ground plane is called the shaking force [11].

$\mathbf{F}_s = \mathbf{F}_{21} + \mathbf{F}_{41} - \mathbf{F}_{Px}$
(26)

The shaking force produces linear vibration of the frame in the horizontal direction [12].

It is clear that the above system of equations, Eqs. 25, provides force and torque information for one position or phase of the linkage. To do a complete force analysis for multiple positions of a machine requires that these computations be repeated with new input data for each position. Therefore, a computer program has been developed in MATLAB language to compute the kinematic parameters over change in time or driver angle plus the forces and torques associated with the linkage kinematics and link geometry. Also, these equations do not account for the gravitational force (weight) on a link. When it is considered, the weight must be treated as an external force acting on the CG of the link at a constant angle and then the right side of Eq. 25 converted to [13]:

$$\begin{Bmatrix} m_2 a_{G2_x} \\ m_2(a_{G2_y}+g) \\ I_{G2}\alpha_2 \\ m_3 a_{G3_x} \\ m_3(a_{G3_y}+g) \\ I_{G3}\alpha_3 \\ m_4 a_{G4_x} - F_{P_x} \\ m_4 g - F_{P_y} \end{Bmatrix} \quad (27)$$

Figure 3 represents the summary flow chart of the MATLAB program.

Numerical approach

The 3D model of the mechanism, Figure 4, was modeled and analyzed in ANSYS Workbench (WB) finite element software [14]. This model consists of a revolute joint, O_2, which is attached to the crank shaft link. This link is attached to a connecting rod by a second revolute joint, A. The connecting rod is then joined to a sliding block by a third revolute joint, B. The sliding block is connected to frame by a prismatic joint, and the frame of the model in turns is fixed to the ground via a fixed joint.
The aim of the analysis was to study all kinematics and kinetics factors. The kinematic factors were displacement, velocity and acceleration of the sliding block and connecting rod while the kinetic parameters were the static and dynamic forces on all joints, shaking force and driving torque required.

a. Loading and boundary conditions:

The boundary conditions and loading applied on the mechanism at the initial position of the crank shaft (top dead center) are shown in Figure 5. The positive sense of the driving angular velocity is taken in the counter-clockwise direction about the Z axis as shown in the figure.

b. Dimension and mass properties of the mechanism:

The important dimensions, masses and mechanical properties of the SCL used in this work are listed in Table 1.

c. Assumptions:

The results of the numerical simulation have been carried out assuming the followings:

- The mechanism components are built entirely from structural steel and, hence for practical purposes, are assumed to be perfectly rigid. Therefore, the type of analysis is taken as Rigid Dynamic.
- The friction forces between the joints are ignored.
- The gravity force is neglected and the gas pressure force is assumed to be the only external force, acting along the negative X axis, as shown in Figure 5. This force is time dependent and its magnitude is taken from the simulated force curve shown in Figure 6, [15]. The load curve is divided into eight sub-steps during one operation cycle of the mechanism.
- The shaking force is not directly obtained in ANSYS WB.
- The masses of the links are translated to their center of mass.
- The crank shaft is originally rotating at a constant angular velocity.

Table 1: Dimension, mass and mechanical properties of the SCL [11].

	Length (m)	Mass (kg)	I_G (kg m^2)	CG (m)
Link 2 Crank shaft	0.0762	2.26	0.00544	0.0508 from O_2
Link 3 Connecting rod	0.286	3.63	0.0408	0.127 from A
Link 4 Slider block	-	2.72	-	At point B
Young's Modulus = 2.07 $*10^{11}$ N/m^2				
Poisson ratio = 0.3				
ω_2 = 188.5 rad/s (uniform) ccw, one revolution, t = 0.0333325 sec.				
$F_{Px} = f(t)$				

Graphical velocity and acceleration analysis

Before programmable calculators and computers became universally available to engineer, graphical methods (GMs) were the only practical way to solve these velocity and acceleration analysis problems. With some practices and with proper tools such as drafting machines or CAD package, one can fairly rapidly solve for the velocities and accelerations of particular points in a mechanism for any one input position or phase by drawing vector diagrams. However, it is a tedious process if velocities or accelerations for many positions of the mechanism are to be found, because each new position requires a completely new set of vector diagrams be drawn. Though, this method can provide a quick check on the results from a computer program solution. In this work, the dynamic behavior of the SCL has been performed, in GM, only at two positions of the crank shaft (θ_2 = 10° and 130°, ω_2 = 1800 rpm, α_2 = 0) using AutoCAD software.
The kinematics and kinetics vector polygons for entire analysis of the GM for this mechanism, when the crank is at θ_2 = 130°, are detailed in Figure 13.

Results and discussions

Once the mechanism has been constructed and the inputs have been defined, the kinematics and kinetics quantities have been calculated from the ANSYS WB, MATLAB program, and the GM. The results were stored in a separate output data file. This file in turn, was sent to one of grapher software (Grapher 9) to plot the x/y graphs as follows:

1. Figure 7 and 8 represent the changes of linear position, velocity and acceleration of the slider or piston against the required time of one revolution of the crank shaft.
2. The display of changes of angular velocity and acceleration of the connecting rod with respect to time are represented in Figure 9.
3. Figure 10 illustrates the variations of total force F_{12} and F_{32} on pin joints O_2 and A with time over one revolution of the crank shaft.
4. The distribution behavior of the total force F_{14} or reaction at the sliding interface and the required total torque T_{12} on link 2 (crank shaft) to drive the mechanism against time is shown in Figure 11.
5. The fluctuation of shaking force that vibrates the frame in the horizontal direction through the operation of the linkage over one cycle is covered in Figure 12.
6. Table 2 and 3 summarize the maximum and minimum values of the all kinematics and kinetics parameters obtained from the numerical and analytical methods. Note that the small variance in the results especially in ANSYS WB (numerical method) is due to the time interval increment.

Table 2: Maximum and minimum values of the kinematics parameters

Method \ Parameters		r_B (m)	V_B (m/s)	a_B (m/s^2)	ω_3 (rad/s)	α_3 (rad/s^2)
MATLAB Program	Max	**0.3622**	**14.8644**	**1986.32**	**50.2227**	**9822.02**
	Min	**0.20980**	**-14.8644**	**-3428.94**	**-50.2227**	**-9822.02**
ANSYS WB	Max	**0.3622**	**14.86445**	**1986.317**	**50.22278**	**9822.024**
	Min	**0.209799**	**-14.86435**	**-3428.942**	**-50.22273**	**-9822.023**

Table 3: Maximum and minimum values of the kinetics parameters

Method \ Parameters		F_{12} (N)	F_{32} (N)	F_{14} (N)	T_{12} (N m)	F_S (N)
MATLAB Program	Max	**18763.4**	**16718.6**	**1057.54**	**620.786**	**24397.4**
	Min	**1580.5**	**1978.84**	**-1284.88**	**-564.023**	**9421.46**
ANSYS WB	Max	**18763.41**	**16718.52**	**1057.571**	**620.7912**	**24397.357**
	Min	**1578.745**	**1978.027**	**-1284.875**	**-564.0177**	**9422.6467**

7. The kinematics and kinetics parameters obtained by the GM are illustrated in Figure 13.
8. The comparison of the results obtained by the mentioned three methods are listed in Table 4 when $\theta_2 = 130°$.

Table 4: Comparison of the results

	r_B (m)	V_B (m/s)	a_B (m/s^2)	ω_3 (rad/s)	α_3 (rad/s^2)	F_{12} (N)	F_{14} (N)	T_{12} (N m)	F_S (N)
ANSYS	0.231	-9.07851	1855.099	32.97592	7181.497	16947.4	970.266	-457.5332	15945.182
MATLAB	0.23099	-9.07830	1855.11	32.9767	7181.35	16947.5	970.26	-457.527	15945.3
GM	0.23099	-9.0940	1854.925	32.9654	7180.8566	16930.4	958.7	-457.967	15934.04

9. The results of combined forces (gas force and inertia forces) analysis show variation of the bearing loads F_{12}, F_{14}, F_{32}, F_{34}, F_S and driving torque T_{12} during crank shaft rotation. Therefore, the knowledge of these forces is essential for design of the bearings.

Conclusion:

Slider crank linkage has been analyzed using three methods: MATLAB program, ANSYS WB, and graphical method GM.

The kinematics and kinetics quantities (position, linear velocity and acceleration, angular velocity and acceleration, forces, driving torque) of all members (links and joints) of the linkage have been studied and plotted due to the constant angular velocity of the crank shaft and variable or time dependent gas pressure force acting on the piston crown. The fluctuation of shaking force which makes the mechanism vibrate in the horizontal direction due to the forces acting on the ground plane (F_{41}, F_{21} and F_{Px}) has also been performed. It has been observed that all results obtained by the three methods agree with each other. The present investigation can be applied to multi-cylinder in-line engines, V-engines and radial engines. Therefore, the author would like to suggest that any rigid dynamics problem can be easily analyzed with less time using ANSYS Workbench software.

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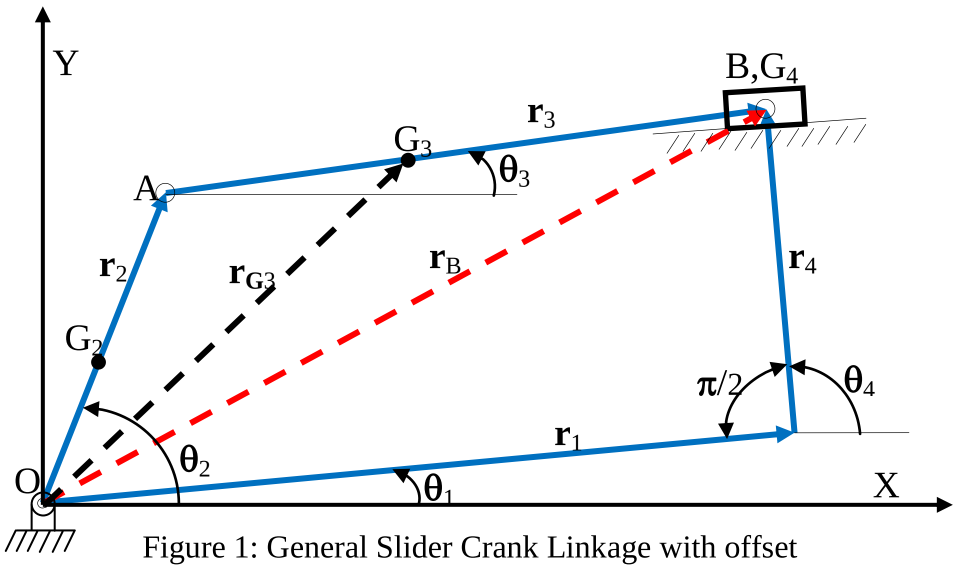


Figure 1: General Slider Crank Linkage with offset

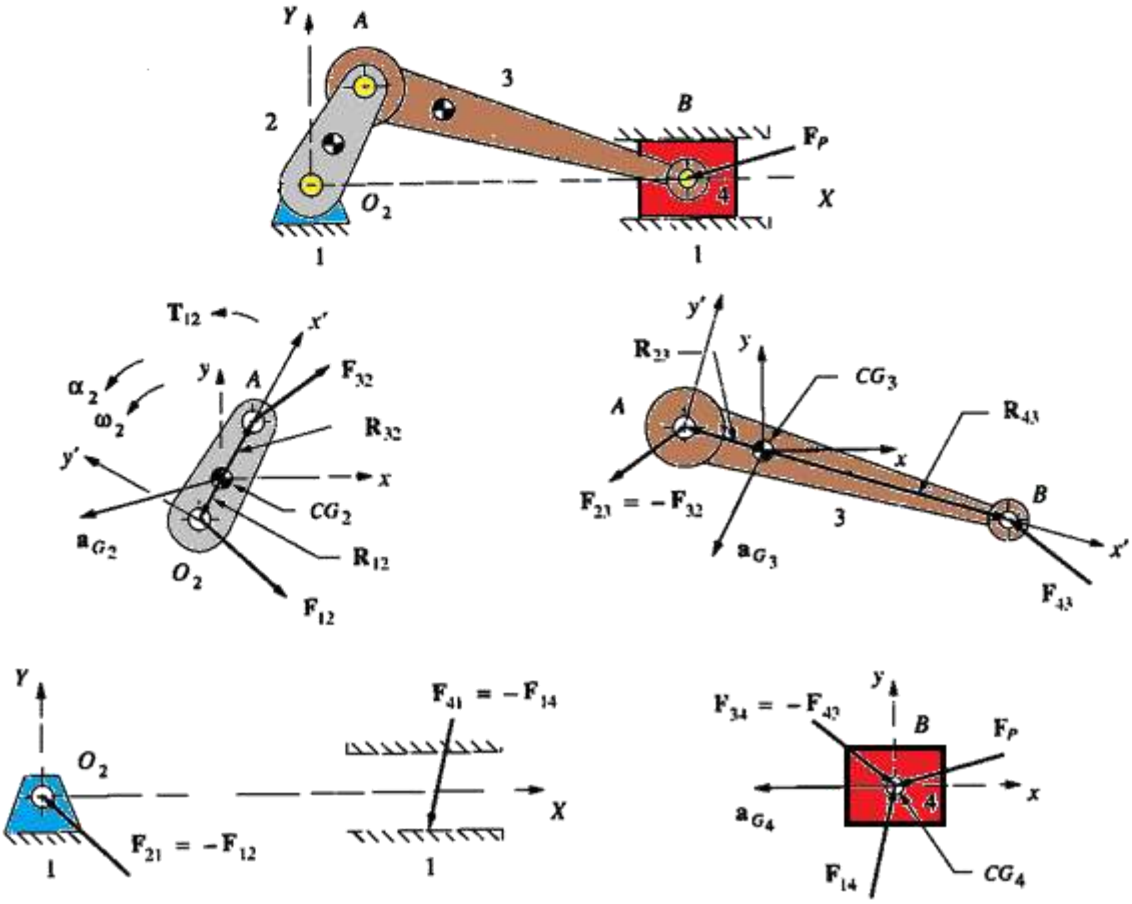


Figure 2: Slider Crank Linkage with forces vector

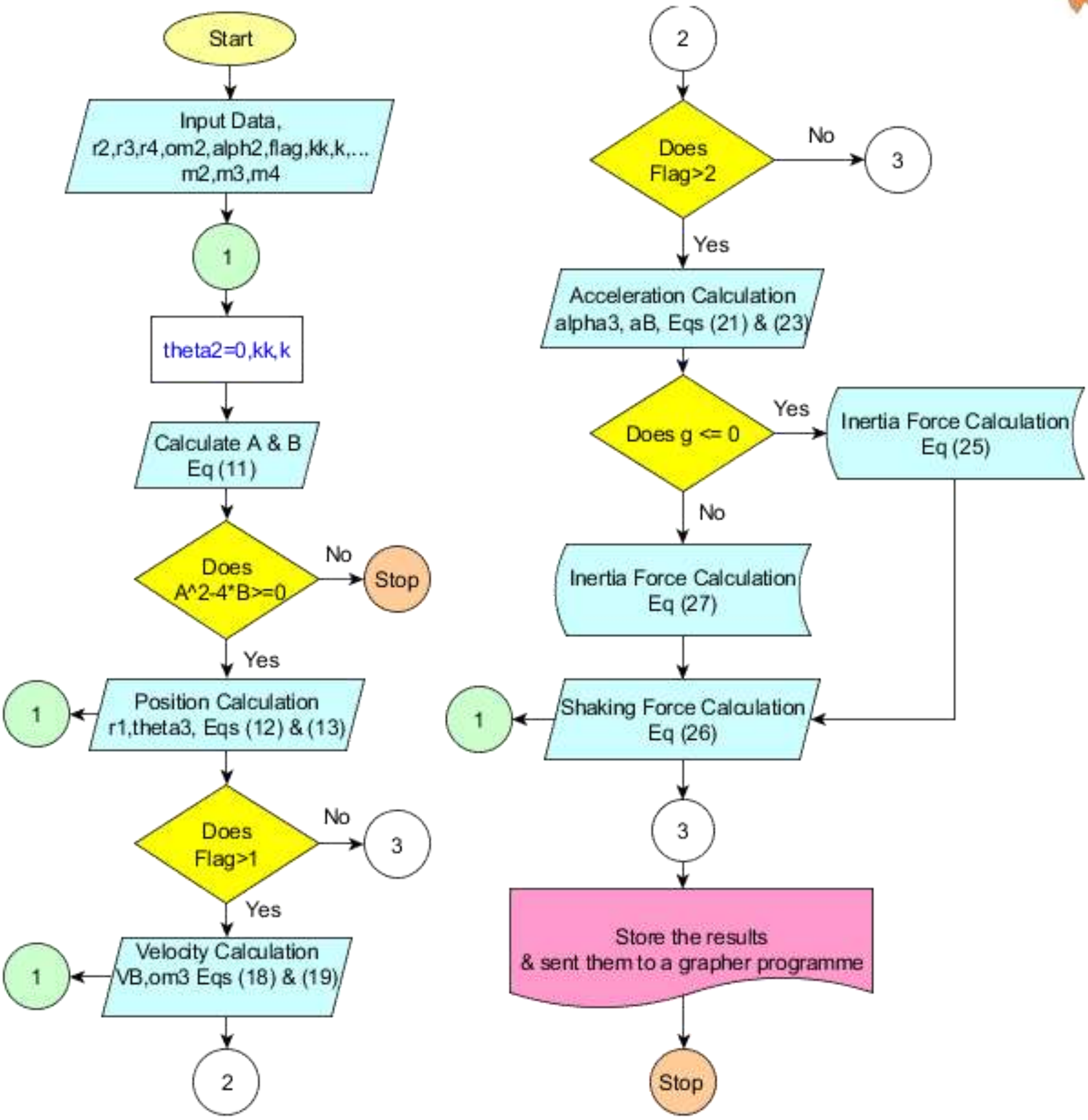


Figure 3: Flowchart of the MATLAB program.

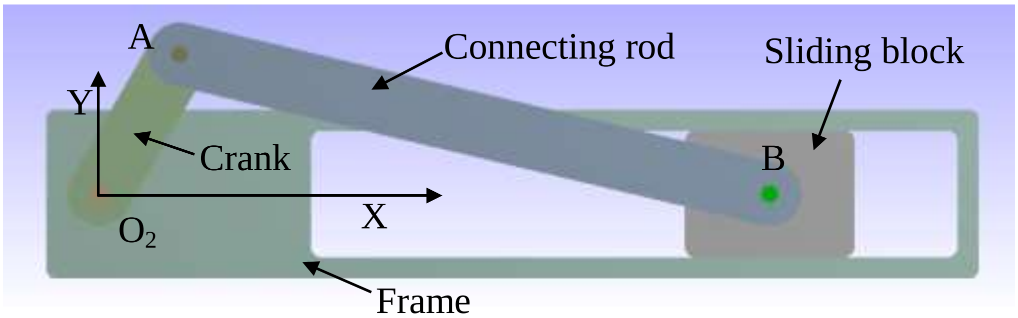


Figure 4: 3D Model of the slider crank mechanism

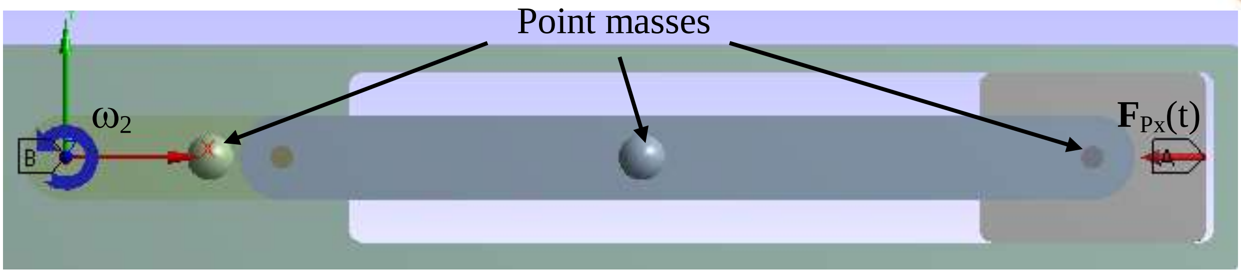


Figure 5: Boundary conditions and loading

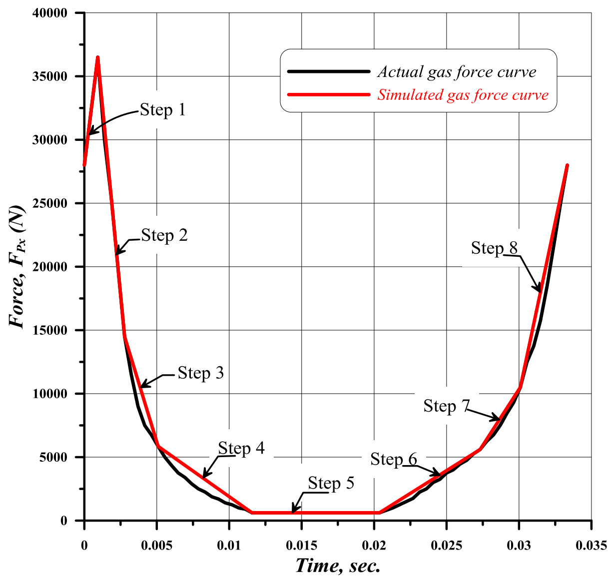
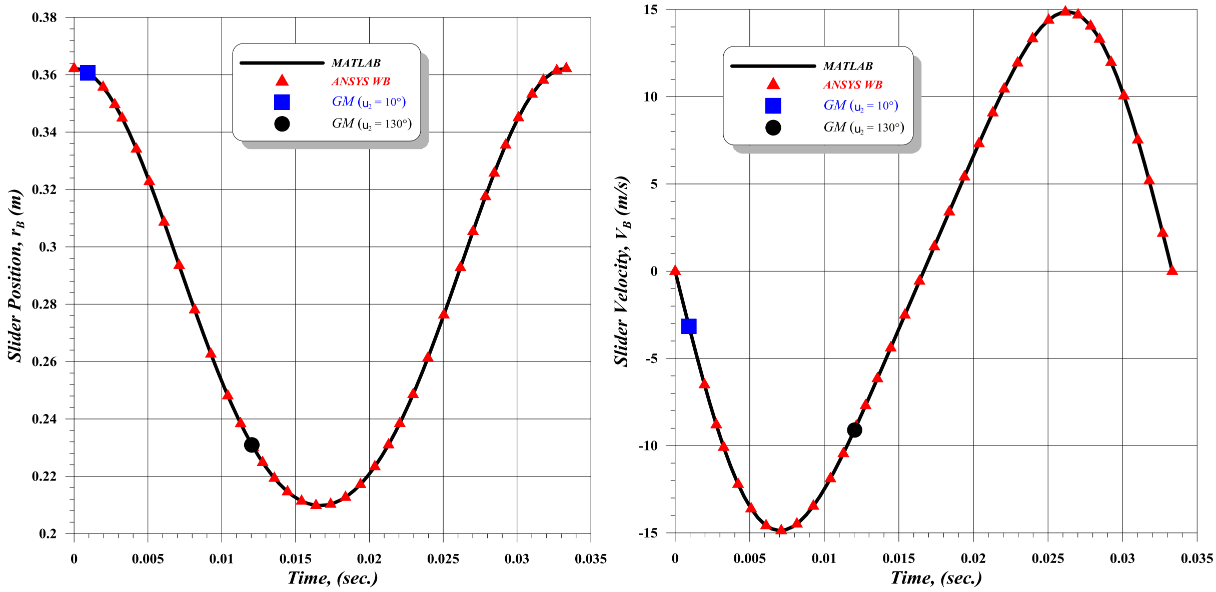


Figure 6: Variation of gas force against time.



(a) (b)

Figure 7: Changes of: (a) piston position and (b) piston velocity against time over one revolution.

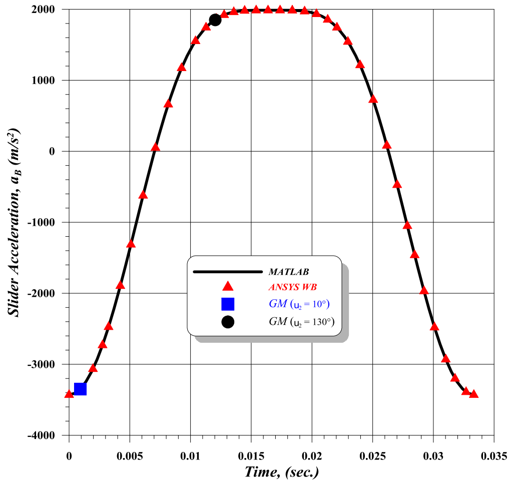
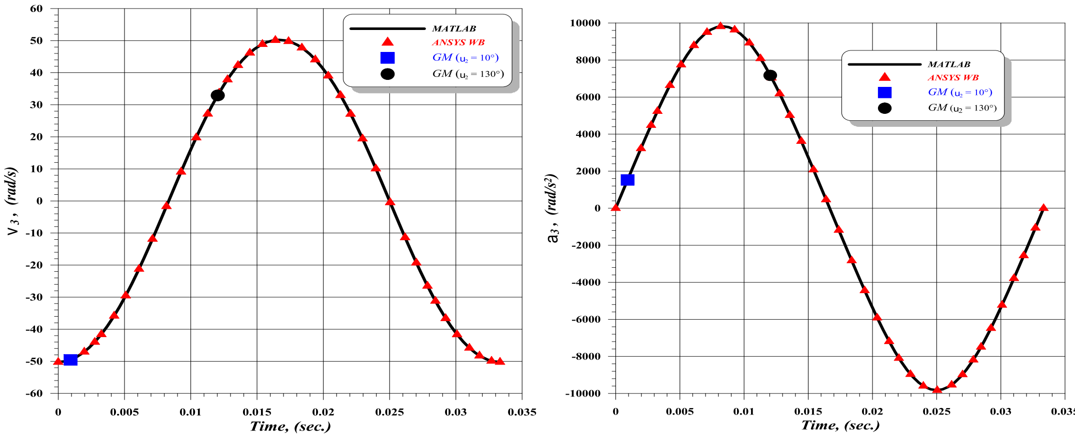
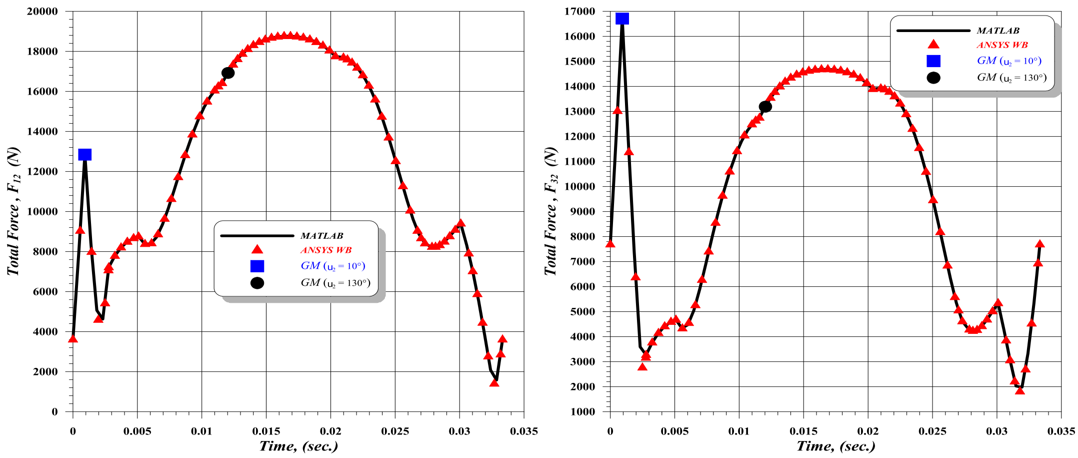


Figure 8: Changes of piston acceleration against time over one revolution.



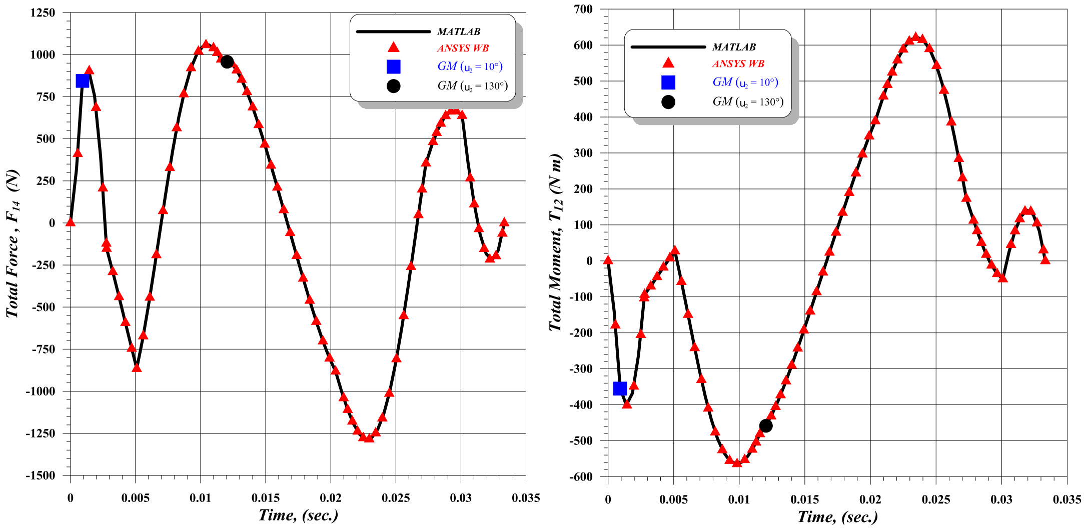
(a) (b)

Figure 9 Changes of: (a) rod speed and (b) acceleration against time over one revolution.



(a) (b)

Figure 10: Changes of: (a) total force F_{12} and (b) total force F_{32} against time over one revolution.



(a) (b)

Figure 11: Changes of: (a) total force F_{14} and (b) total driving torque T_{12} against time over one revolution.

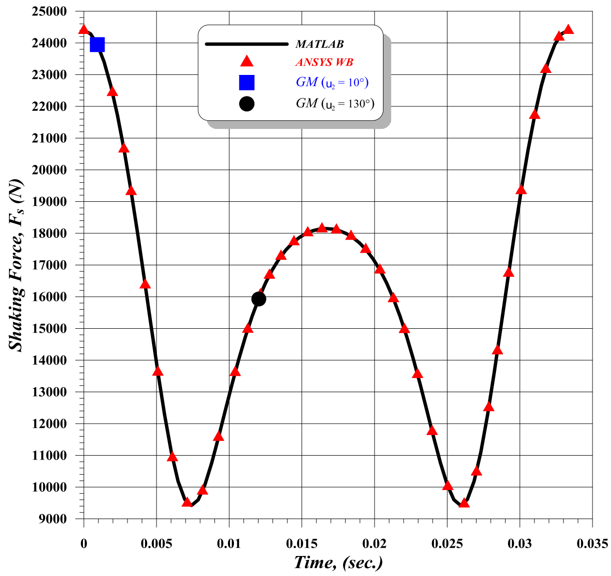


Figure 12: Fluctuations of total shaking force F_S against time over one revolution.

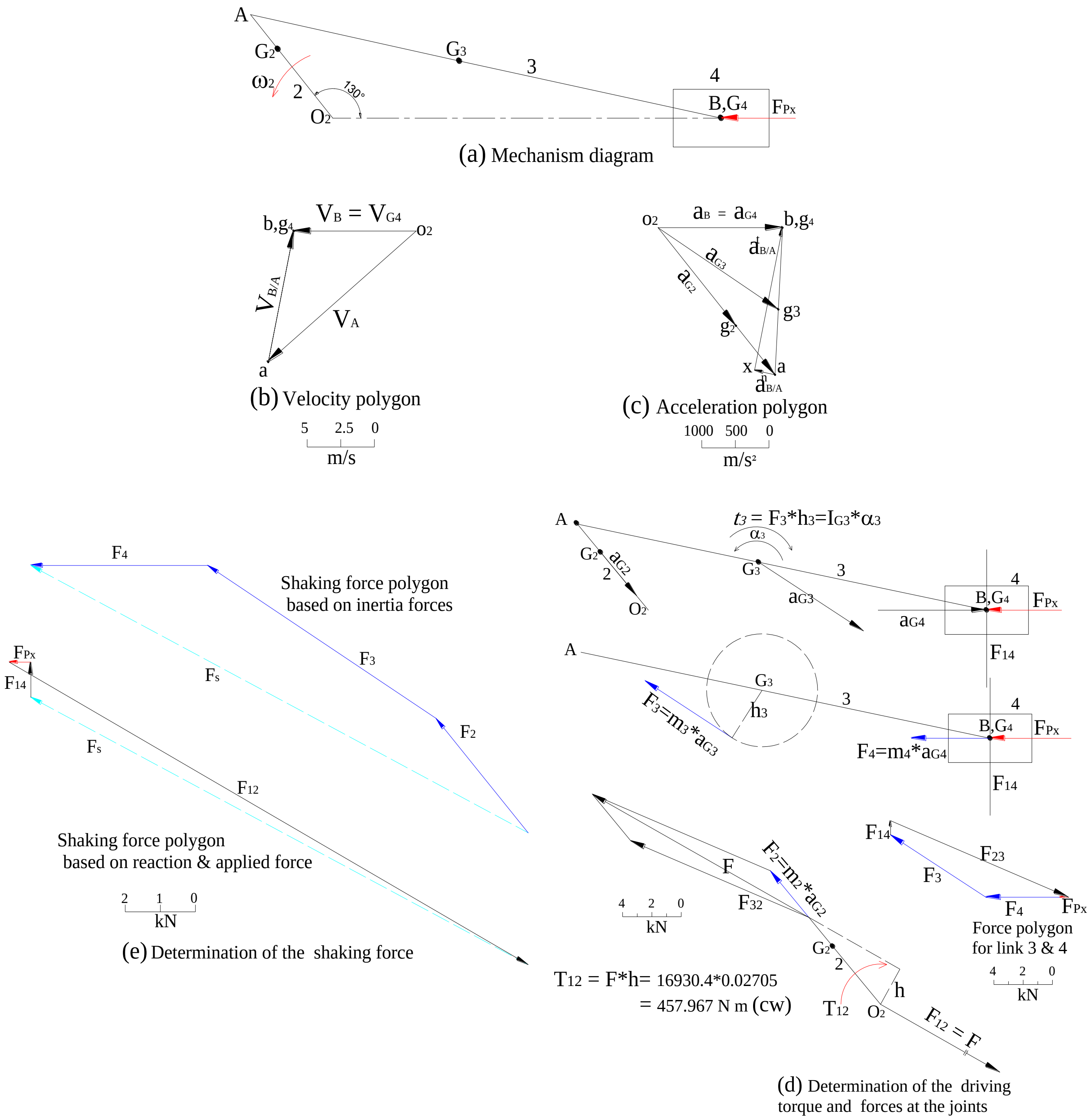


Figure 13: Graphical analysis method when the crank is at $\theta_2 = 130°$:
a) Mechanism diagram: $r_B = 0.23099$ m;
b) Velocity diagram: $V_A = \omega_2 * O_2A$, $V_B = -9.094$ m/s, $\omega_3 = V_{B/A}/AB = 32.9654$ rad/s (ccw);
c) Acceleration polygon: $a_B = 1851.925$ m/s^2, $\alpha_3 = 7180.8566$ rad/s^2 (ccw);
d) steps for calculation the driving torque and forces at joints: $F_{14} = 958.7$ N, $F_{12} = 16930.4$ N, $T_{12} = 457.967$ N (cw);
e) Shaking force polygons: $F_S = 15934.04$ N.