Mustansiriyah Journal of Pure and Applied Sciences
https://mjpas.uomustansiriyah.edu.iq
Print ISSN: 2957-9910 Online ISSN: 2957-9929

Orthogonal Generalized Higher (σ,τ)-k Derivations on Semi-prime Γ-Rings

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ABSTRACT

In this paper we study the concept of orthogonal generalized higher (σ,τ)-k-derivations and present some of result obtained from orthogonality.

Keywords: semiprime rings, (σ,τ)-derivations, k-derivations, orthogonal, generalized (σ,τ)-derivation.

1. Introduction

The concept of Γ-ring introduced in [5] and develop by Barans in [2]. Kandamar introduce the definition of k-derivations in [3] and evolution the in [4]. The definition of orthogonal presented in [1]. In [9] the concept of (σ,τ)-derivations was introduce and M.Ashraf study the higher (σ,τ)-derivations in [8]. Some important definitions such as,prime,semiprime and 2-torison introduced in [6]. We studied orthogonal on higher (σ,τ)-k-derivations and developed it in this paper to orthogonal generalized higher (σ,τ)-k-derivations on semiprime Γ-Rings.

And the most import result we obtain in our study.

Let M be 2-torsion free semiprime Γ-ring, $D=(D_n)_{i\in N}$ and $G=(G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d=(d_i)_{i\in N}$ and $g=(g_i)_{i\in N}$ respectively then $D_n\ and\ G_n\ are\ orthogonal$ if and only if for all x,y ∈M

The following lemma is one of important result that we need in our study.

Lemma 1.1: [8]

Let M be a 2-torsion free semi-prime Γ-ring and a, b ∈ M, α,β∈Γ, then the following conditions are equivalent.

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(1) $a\Gamma x\Gamma b$ = 0, for all $x \in$ M.

(2) $b\Gamma x\Gamma a$ = 0, for all $x \in$ M.

(3) $a\Gamma x\Gamma b$ + $b\Gamma x\Gamma a$ = 0, for all $x \in$ M.

If one of the above conditions is fulfilled then $a\Gamma b$ = $b\Gamma a$ = 0.

2. Orthogonal (σ,τ)-k-derivations on semiprime Γ-Rings

Definition 2.1

Let M be Γ-ring, two generalized higher (σ,τ)-k-derivations $D=(D_i)_{i\in N}$ and $G=(G_i)_{i\in N}$ on M where $K=(K_i)_{i\in N}$ family of additive mappings on Γ,then D_n *and* G_n are called orthogonal if for every n$\in N$, x,y $\in$M $D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)MK_n(\Gamma)D_n(x)$

Where $D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y)=\sum_{i=1}^{n}D_i(x)K_i(\Gamma)MK_i(\Gamma)G_i(y)$

Example 2.2

Let D=$(D_i)_{i\in N}$ and G=$(G_i)_{i\in N}$ be two generalized higher (σ,τ)-k-derivations on Γ- ring M associated with (σ,τ)-k-derivations d=$(d_i)_{i\in N}$ and g=$(g_i)_{i\in N}$ on M. Let S=M×M we define $D_n^{`}=(D_i^{`})_{i\in N}$, $G_n^{`}=(G_i^{`})_{i\in N}$ are generalized higher (σ,τ)-k-derivations on S we defined by $D_n^{`}(x,y)=(D_n(x),0)$

$G_n^{`}(x,y)=(0,G_n(y))$

Then $D_n^{`}$ and $G_n^{`}$ are orthogonal.

Theorem 2.3

Let D=$(D_i)_{i\in N}$ *and* $G=(G_i)_{i\in N}$ be two generalized higher (σ,τ)-k-derivations with associated higher (σ,τ)-k-derivations d=$(d_i)_{i\in N}$ and g=$(g_i)_{i\in N}$ respectively where $D_n and G_n$ are commutative , if D_n *and* G_n are orthogonal then the following hold

1) $D_n(x)k_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)D_n(x)$ hence

$D_n(x)K_n(\Gamma)G_n(y)+G_n(y)K_n(\Gamma)D_n(x)=0$

Proof

$$D_n \text{ and } G_n \text{ are orthogonal then } D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y)=0$$
$$=G_n(y)K_n(\Gamma)MK_n(\Gamma)D_n(x)$$

By lemma (1.1) we get:

$D_n(x)k_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)D_n(x)$

Hence $D_n(x)K_n(\Gamma)G_n(y)+G_n(y)K_n(\Gamma)D_n(x)=0$

2) d_n *and* G_n *are orthogonal higher* $(\sigma,\tau)-k-derivation\ and$

$d_n(x)K_n(\Gamma)G_n(y) = G_n(y)K_n(\Gamma)d_n(x) = 0$

Proof

By (1) $D_n(x)K_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)D_n(x)$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

Replace x by mβx

$\sum_{i=1}^{n} D_i(m\beta x)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} D_i\left(\sigma^{n-i}(m)\right)K_i(\beta)d_i(\tau^{n-i}(x))K_i(\alpha)G_i(y) = 0$

Replace $D_i\left(\sigma^{n-i}(m)\right)$ by $d_i(\sigma^{n-i}(x))K_i(\alpha)G_i\left(\sigma^{n-i}(y)\right)$

$\sum_{i=1}^{n} d_i(\sigma^{n-i}(x))K_i(\alpha)G_i\left(\sigma^{n-i}(y)\right)K_i(\beta)d_i(x)K_i(\alpha)G_i(y) = 0$

Replace $K_i(\beta)$ by $K_i(\beta)mK_i(\beta)$

$\sum_{i=1}^{n} d_i(\sigma^{n-i}(x))K_i(\alpha)G_i\left(\sigma^{n-i}(y)\right) K_i(\beta)mK_i(\beta)d_i(x)K_i(\alpha)G_i(y) = 0$

Since M is semiprime $\sum_{i=1}^{n} d_i(\sigma^{n-i}(x))K_i(\alpha)G_i\left(\sigma^{n-i}(y)\right) = 0$

$d_n(x)K_n(\alpha)G_n(y) = 0$

G_n is commutative $G_n(y)K_n(\Gamma)d_n(x) = 0$

3) D_n *and* g_n *are orthogonal higher* $(\sigma,\tau)-k-derivations$ and $g_n(x)K_n(\Gamma)D_n(y) = D_n(y)K_n(\Gamma)g_n(x) = 0$

Proof

By (1) $D_n(x)k_n(\Gamma)G_n(y) = 0 = G_n(y)K_n(\Gamma)D_n(x)$

$G_n(y)K_n(\Gamma)D_n(x) = 0$

$\sum_{i=1}^{n} G_i(x)K_i(\alpha)D_i(y) = 0$

Replace x by mβx

$\sum_{i=1}^{n} G_i(m\beta x)K_i(\alpha)D_i(y) = 0$

$\sum_{i=1}^{n} G_i\left(\sigma^{n-i}(m)\right)K_i(\beta)g_i(\tau^{n-i}(x))K_i(\alpha)D_i(y) = 0$

Replace $G_i(m)$ by $g_i(\sigma^{n-i}(x))K_i(\alpha)D_i\left(\tau^{n-i}(y)\right)$ and $g_i(\tau^{n-i}(x))$ by $g_i(\sigma^{n-i}(x))$

$\sum_{i=1}^{n} g_i(\sigma^{n-i}(x))K_i(\alpha)D_i(\tau^{n-i}(y))K_i(\beta)g_i(\sigma^{n-i}(x))K_i(\alpha)D_i(y)=0$

Replace $K_i(\beta)$ by $K_i(\beta)mK_i(\beta)$ and $D_i(y)$ by $D_i(\tau^{n-i}(y))$

$\sum_{i=1}^{n} g_i(\sigma^{n-i}(x))K_i(\alpha)D_i(\tau^{n-i}(y))\, K_i(\beta)mK_i(\beta)g_i(\sigma^{n-i}(x))K_i(\alpha)D_i(\tau^{n-i}(y))=0$

Since M is semiprime $\sum_{i=1}^{n} g_i(x)K_i(\alpha)D_i(y)=0$

$g_n(x)K_n(\alpha)D_n(y)=0$

D_n is commutative $D_n(y)K_n(\Gamma)g_n(x)=0$

4) $d_n\ and\ g_n\ are\ orthogonal\ higher\ (\sigma,\tau)-K-derivations$,

Proof

By (1) $D_n(x)K_n(\Gamma)G_n(y)=0=G_n(y)K_n(\Gamma)D_n(x)$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y)=0$

Replace x by mβz and y by wβy

$\sum_{i=1}^{n} D_i(m\beta x)K_i(\alpha)G_i(w\beta y)=0$

$\sum_{i=1}^{n} D_i\left(\sigma^{n-i}(m)\right)K_i(\beta)d_i(\tau^{n-i}(x)K_i(\alpha)G_i\left(\sigma^{n-i}(w)\right)K_i(\beta)g_i(\tau^{n-i}(y))=0$

$\sum_{i=1}^{n} d_i\left(\tau^{n-i}(x)\right)K_i(\beta)D_i(\sigma^{n-i}(m)K_i(\alpha)G_i\left(\sigma^{n-i}(w)\right)K_i(\beta)g_i(\tau^{n-i}(y))=0$

Replace $d_i\left(\tau^{n-i}(x)\right)$ by $d_i\left(\sigma^{n-i}(x)\right)$

$\sum_{i=1}^{n} d_i\left(\sigma^{n-i}(x)\right)K_i(\beta)D_i(\sigma^{n-i}(m)K_i(\alpha)G_i\left(\sigma^{n-i}(w)\right)K_i(\beta)g_i(\tau^{n-i}(y))=0$

By lemma (1.1) we get

$\sum_{i=1}^{n} g_i\left(\tau^{n-i}(y)\right)K_i(\beta)D_i(\sigma^{n-i}(m)K_i(\alpha)G_i\left(\sigma^{n-i}(w)\right)K_i(\beta)d_i(\sigma^{n-i}(x))=0$

Hence $d_n\ and\ g_n\ are\ orthogonal$

5) $d_nG_n=G_nd_n=0 \quad and \quad g_nD_n=D_ng_n=0$

Proof

By (2) $d_n(x)K_n(\Gamma)G_n(y)=0$

$\sum_{i=1}^{n} d_i(x)K_i(\alpha)G_i(y)=0$

$\sum_{i=1}^{n} G_i(d_i(x)K_i(\alpha)G_i(y))=0$

$\sum_{i=1}^{n} G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) g_i\left(G_i\left(\tau^{n-i}(y)\right)\right) = 0$

$\sum_{i=1}^{n} G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) G_i\left(g_i\left(\tau^{n-i}(y)\right)\right) = 0$

Replace $g_i(\tau^{n-i}(y))$ by $d_i(\sigma^{n-i}(y))$

$\sum_{i=1}^{n} G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha) m K_i(\alpha)$

$\sum_{i=1}^{n} G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) m K_i(\alpha) G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} G_i\left(d_i\left(\sigma^{n-i}(x)\right)\right) = 0$

$G_n d_n = 0$

By (2) $G_n(y) K_n(\Gamma) d_n(x) = 0$

$\sum_{i=1}^{n} G_i(x) K_i(\alpha) d_i(y) = 0$

$\sum_{i=1}^{n} d_i(G_i(x) K_i(\alpha) d_i(y)) = 0$

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) d_i\left(d_i\left(\tau^{n-i}(y)\right)\right) = 0$

Replace $d_i(\tau^{n-i}(y))$ by $G_i(\sigma^{n-i}(x))$

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha) m K_i(\alpha)$

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) m K_i(\alpha) d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(x)\right)\right) = 0$

$d_n G_n = 0$

By (3) $g_n(x) K_n(\Gamma) D_n(y) = 0$

$\sum_{i=1}^{n} g_i(x) K_i(\alpha) D_i(y) = 0$

$\sum_{i=1}^{n} D_i(g_i(x) K_i(\alpha) D_i(y)) = 0$

$\sum_{i=1}^{n} D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) d_i\left(D_i\left(\tau^{n-i}(y)\right)\right) = 0$

$\sum_{i=1}^{n} D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) D_i\left(d_i\left(\tau^{n-i}(y)\right)\right) = 0$

Replace $d_i(\tau^{n-i}(y))$ by $g_i(\sigma^{n-i}(x))$

$\sum_{i=1}^{n} D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha) m K_i(\alpha)$

$\sum_{i=1}^{n} D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) m K_i(\alpha) D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} D_i\left(g_i\left(\sigma^{n-i}(x)\right)\right) = 0$

$D_n g_n = 0$

By (3) $D_n(x) K_n(\Gamma) g_n(y) = 0$

$\sum_{i=1}^{n} D_i(x) K_i(\alpha) g_i(y) = 0$

$\sum_{i=1}^{n} g_i(D_i(x) K_i(\alpha) g_i(y)) = 0$

$\sum_{i=1}^{n} g_i\left(D_i\left(\sigma^{n-i}(x)\right)\right) K_i(\alpha) g_i\left(g_i\left(\sigma^{n-i}(y)\right)\right) = 0$

Replace $g_i(\sigma^{n-i}(y))$ by $D_i(\sigma^{n-i}(y))$

$\sum_{i=1}^{n} g_i\left(D_i\left(\sigma^{n-i}(y)\right)\right) K_i(\alpha) g_i\left(D_i\left(\sigma^{n-i}(y)\right)\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha) m K_i(\alpha)$

$\sum_{i=1}^{n} g_i\left(D_i\left(\sigma^{n-i}(y)\right)\right) K_i(\alpha) m K_i(\alpha) g_i\left(D_i\left(\sigma^{n-i}(y)\right)\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} g_i\left(D_i\left(\sigma^{n-i}(y)\right)\right) = 0$

$g_n D_n = 0$

(6) $D_n G_n = G_n D_n = 0$

Proof

$D_n(x) K_n(\Gamma) G_n(y) = 0$

$\sum_{i=1}^{n} D_i(x) K_i(\alpha) G_i(y) = 0$

$\sum_{i=1}^{n} G_i(D_i(x)K_i(\alpha)G_i(y)) = 0$

$\sum_{i=1}^{n} G_i\left(D_i(\sigma^{n-i}(x))\right)K_i(\alpha)g_i\left(G_i(\tau^{n-i}(y))\right) = 0$

$\sum_{i=1}^{n} G_i\left(D_i(\sigma^{n-i}(x))\right)K_i(\alpha)G_i\left(g_i(\tau^{n-i}(y))\right) = 0$

Replace $g_i(\tau^{n-i}(y))$ by $D_i(\sigma^{n-i}(x))$

$\sum_{i=1}^{n} G_i\left(D_i(\sigma^{n-i}(x))\right)K_i(\alpha)G_i\left(D_i(\sigma^{n-i}(x))\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} G_i\left(D_i(\sigma^{n-i}(x))\right)K_i(\alpha)mK_i(\alpha)G_i\left(D_i(\sigma^{n-i}(x))\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} G_i\left(D_i(\sigma^{n-i}(x))\right) = 0$

$G_nD_n = 0$

$G_n(x)K_n(\Gamma)D_n(y) = 0$

$\sum_{i=1}^{n} G_i(x)K_i(\alpha)D_i(y) = 0$

$\sum_{i=1}^{n} D_i(G_i(x)K_i(\alpha)D_i(y)) = 0$

$\sum_{i=1}^{n} D_i\left(G_i(\sigma^{n-i}(x))\right)K_i(\alpha)d_i\left(D_i(\tau^{n-i}(y))\right) = 0$

$\sum_{i=1}^{n} D_i\left(G_i(\sigma^{n-i}(x))\right)K_i(\alpha)D_i\left(d_i(\tau^{n-i}(y))\right) = 0$

Replace $d_i(\tau^{n-i}(y))$ by $G_i(\sigma^{n-i}(x))$

$\sum_{i=1}^{n} D_i\left(G_i(\sigma^{n-i}(x))\right)K_i(\alpha)D_i\left(G_i(\sigma^{n-i}(x))\right) = 0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} D_i\left(G_i(\sigma^{n-i}(x))\right)K_i(\alpha)mK_i(\alpha)D_i\left(G_i(\sigma^{n-i}(x))\right) = 0$

Since M is semiprime

$\sum_{i=1}^{n} D_i\left(G_i(\sigma^{n-i}(x))\right) = 0$

$D_nG_n = 0$

Theorem 2.4

Let M be 2-torsion free semiprime Γ-ring, $D = (D_n)_{i\in N}$ and $G = (G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ respectively then $D_n\ and\ G_n\ are\ orthogonal$ if and only if for all x,y ∈M

(1) $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$

(2) $d_n(x)K_n(\Gamma)G_n(y) + g_n(y)K_n(\Gamma)D_n(x) = 0$

Where $D_n\ and\ G_n\ are\ commutative\ mappings$

Proof

Suppose $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$

Replace x by xαy

$\sum_{i=1}^{n} D_i(x\alpha y)K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)D_i(x\alpha y) = 0$

$$\sum_{i=1}^{n} D_i\big(\sigma^{n-i}(x)\big)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)D_i(\sigma^{n-i}(x))K_i(\alpha)d_i(\tau^{n-i}(y)) = 0$$

$$\sum_{i=1}^{n} D_i\big(\sigma^{n-i}(x)\big)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)G_i(y) + G_i(y)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)D_i(\sigma^{n-i}(x)) = 0$$

By lemma (1.1) we get

$\sum_{i=1}^{n} D_i\big(\sigma^{n-i}(x)\big)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} G_i(y)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)D_i(\sigma^{n-i}(x)) = 0$

Hence $D_n\ and\ G_n\ are\ orthogonal$

Conversely:-

Let $D_n\ and\ G_n\ are\ orthogonal$

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)MK_i(\alpha)G_i(y) = 0$

By lemma (1.1) we get $D_n(x)K_n(\Gamma)G_n(y) = 0\ \ and\ G_n(y)K_n(\Gamma)D_n(x) = 0$

Hence $D_n(x)K_n(\Gamma)G_n(y) + G_n(y)K_n(\Gamma)D_n(x) = 0$

Also $D_n(x)K_n(\Gamma)G_n(y) = 0$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} d_i(D_i(x)K_i(\alpha)G_i(y)) = 0$

$\sum_{i=1}^{n} d_i\left(D_i\left(\sigma^{n-i}(x)\right)\right)K_i(\alpha)d_i\left(G_i\left(\tau^{n-i}(y)\right)\right) = 0$

$\sum_{i=1}^{n} d_i\left(D_i\left(\sigma^{n-i}(x)\right)\right)K_i(\alpha)G_i\left(d_i\left(\tau^{n-i}(y)\right)\right) = 0$

Replace $D_i(\sigma^{n-i}(x))$ by $\sigma^{n-i}(x)$ and $d_i(y)$ by $\tau^{n-i}(y)$

$\sum_{i=1}^{n} d_i(\sigma^{n-i}(x))K_i(\alpha)G_i(\tau^{n-i}(y)) = 0$

$d_n(x)K_n(\Gamma)G_n(y) = 0$

And

$G_n(x)K_n(\Gamma)D_n(y) = 0$

$\sum_{i=1}^{n} G_i(x)K_i(\alpha)D_i(y) = 0$

$\sum_{i=1}^{n} g_i(G_i(x)K_i(\alpha)D_i(y)) = 0$

$\sum_{i=1}^{n} g_i\left(G_i\left(\sigma^{n-i}(x)\right)\right)K_i(\alpha)g_i\left(D_i\left(\tau^{n-i}(y)\right)\right) = 0$

$\sum_{i=1}^{n} g_i\left(G_i\left(\sigma^{n-i}(x)\right)\right)K_i(\alpha)D_i\left(g_i\left(\tau^{n-i}(y)\right)\right) = 0$

Replace $G_i(\sigma^{n-i}(x))$ by $\tau^{n-i}(y)$ and $g_i(\tau^{n-i}(y))$ by $\sigma^{n-i}(x)$

$\sum_{i=1}^{n} g_i(\tau^{n-i}(y))K_i(\alpha)D_i(\sigma^{n-i}(x)) = 0$

$g_n(y)K_n(\Gamma)D_n(x) = 0$

$d_n(x)K_n(\Gamma)G_n(y) + g_n(y)K_n(\Gamma)D_n(x) = 0$

Theorem 2.5

Let M be 2-torsion free semiprime Γ-ring, $D = (D_n)_{i\in N}$ and $G = (G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ respectively then $D_n\ and\ G_n\ are\ orthogonal$ if and only if for all x,y ∈M

$D_n(x)K_n(\Gamma)G_n(y) = d_n(x)K_n(\Gamma)G_n(y) = 0$

Where $D_n\ and\ G_n\ are\ commutative$

Proof

Suppose $D_n(x)K_n(\Gamma)G_n(y) = 0$

Replace x by xαy

$\sum_{i=1}^{n} D_i(x\alpha y)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} D_i\big(\sigma^{n-i}(x)\big)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)G_i(y) = 0$

By lemma (1.1) we get

$\sum_{i=1}^{n} G_i(y)K_i(\alpha)d_i(\tau^{n-i}(y))K_i(\alpha)D_i(\sigma^{n-i}(x)) = 0$

Hence D_n *and* G_n *are orthogonal*

Conversely:-

Suppose D_n *and* G_n *are orthogonal*

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$

$D_n(x)K_n(\Gamma)G_n(y) = 0$ By lemma (1.1) we get

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} d_i(D_i(x)K_i(\alpha)G_i(y)) = 0$

$\sum_{i=1}^{n} d_i\left(D_i\big(\sigma^{n-i}(x)\big)\right)K_i(\alpha)d_i\left(G_i\big(\tau^{n-i}(y)\big)\right) = 0$

$\sum_{i=1}^{n} d_i\left(D_i\big(\sigma^{n-i}(x)\big)\right)K_i(\alpha)G_i\left(d_i\big(\tau^{n-i}(y)\big)\right) = 0$

Replace $D_i(\sigma^{n-i}(x))$ by $\sigma^{n-i}(x)$ and $d_i(\tau^{n-i}(y))$ by $\tau^{n-i}(y)$

$\sum_{i=1}^{n} d_i(\sigma^{n-i}(x))K_i(\alpha)G_i(\tau^{n-i}(y)) = 0$

$d_n(x)K_n(\Gamma)G_n(y) = 0$

Theorem 2.6

Let M be 2-torsion free semiprime Γ-ring, $D = (D_n)_{i\in N}$ and $G = (G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ respectively then D_n *and* G_n *are orthogonal* if and only if for all x,y ∈M

$D_n(x)K_n(\Gamma)G_n(y) = 0$ *and* $d_nG_n = d_ng_n = 0$

Proof

Suppose D_n *and* G_n *are orthogonal*

$D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(y) = 0$

$D_n(x)K_n(\Gamma)G_n(y) = 0$ By lemma (1.1) we get

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y) = 0$

$\sum_{i=1}^{n} d_i(D_i(x)K_i(\alpha)G_i(y)) = 0$

$\sum_{i=1}^{n} d_i\left(D_i\left(\sigma^{n-i}(x)\right)\right)K_i(\alpha)d_i\left(G_i\left(\tau^{n-i}(y)\right)\right)=0$

Replace $D_i(\sigma^{n-i}(x))$ by $G_i(\sigma^{n-i}(y))$

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(y)\right)\right)K_i(\alpha)d_i\left(G_i\left(\sigma^{n-i}(y)\right)\right)=0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(y)\right)\right)K_i(\alpha)mK_i(\alpha)d_i\left(G_i\left(\sigma^{n-i}(y)\right)\right)=0$

Since M is semiprime

$\sum_{i=1}^{n} d_i\left(G_i\left(\sigma^{n-i}(y)\right)\right)=0$

$d_nG_n=0$

And by ([8] theorem 3 (i))

$d_ng_n=0$

Conversely:-

$D_n(x)K_n(\Gamma)G_n(y)=0$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)G_i(y)=0$

Replace $K_i(\alpha)$ by $K_i(\alpha)mK_i(\alpha)$

$\sum_{i=1}^{n} D_i(x)K_i(\alpha)mK_i(\alpha)G_i(y)=0$

By lemma (1.1) we get

$\sum_{i=1}^{n} G_i(x)K_i(\alpha)mK_i(\alpha)D_i(y)=0$

Hence $D_n\ and\ G_n\ are\ orthogonal$

Theorem 2.7

Let M be 2-torsion free semiprime Γ-ring, $D=(D_n)_{i\in N}$ and $G=(G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d=(d_i)_{i\in N}$ and $g=(g_i)_{i\in N}$ respectively then if $D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(y)=G_n(y)K_n(\Gamma)MK_n(\Gamma)G_n(x)$

Then $(D_n-G_n)\ and\ (D_n+G_n)\ are\ orthogonal$

Proof

$$(D_n+G_n)(x)K_n(\Gamma)MK_n(\Gamma)(D_n-G_n)(x)+(D_n-G_n)(x)K_n(\Gamma)MK_n(\Gamma)(D_n+G_n)(x)$$

$(D_n(x)+G_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x)-G_n(x))$
$+(D_n(x)-G_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x)+G_n(x)$

$=(D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))-(D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$
$+(G_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$

$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))+(D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$
$+(D_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$

$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$

=0

By lemma (1.1) we get

$(D_n+G_n)K_n(\Gamma)MK_n(\Gamma)(D_n-G_n)(x)=0$

And $(D_n-G_n)K_n(\Gamma)MK_n(\Gamma)(D_n+G_n)(x)=0$

Hence (D_n-G_n) *and* (D_n+G_n) *are orthogonal*

Corollary 2.8

Let M be 2-torsion free semiprime Γ-ring, $D=(D_n)_{i\in N}$ and $G=(G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d=(d_i)_{i\in N}$ and $g=(g_i)_{i\in N}$ respectively then if $D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(y)=g_n(y)K_n(\Gamma)MK_n(\Gamma)g_n(x)$

Then (D_n-g_n) *and* (D_n+g_n) *are orthogonal*

Proof

$(D_n+g_n)(x)K_n(\Gamma)MK_n(\Gamma)(D_n-g_n)(x)+(D_n-g_n)(x)K_n(\Gamma)MK_n(\Gamma)(D_n+g_n)(x)$

$=(D_n(x)+g_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x)-g_n(x))$
$+(D_n(x)-g_n(x))K_n(\Gamma)MK_n(\Gamma)(D_n(x)+g_n(x)$

$=(D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))-(D_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))$
$+(g_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$

$-(g_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))+(D_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))$
$+(D_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))$

$-(g_n(x)K_n(\Gamma)MK_n(\Gamma)D_n(x))-(g_n(x)K_n(\Gamma)MK_n(\Gamma)g_n(x))$

=0

By lemma (1.1) we get

$(D_n+g_n)K_n(\Gamma)MK_n(\Gamma)(D_n-g_n)(x)=0$

And $(D_n - g_n)K_n(\Gamma)MK_n(\Gamma)(D_n + g_n)(x) = 0$

Hence $(D_n - g_n)\ and\ (D_n + g_n)\ are\ orthogonal$

Corollary 2.9

Let M be 2-torsion free semiprime Γ-ring, $D = (D_n)_{i\in N}$ and $G = (G_n)_{i\in N}$ generalized higher (σ,τ)-K-derivation with associated higher (σ,τ)-K-derivation $d = (d_i)_{i\in N}$ and $g = (g_i)_{i\in N}$ respectively then if $d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(y) = G_n(y)K_n(\Gamma)MK_n(\Gamma)G_n(x)$

Then $(d_n - G_n)\ and\ (d_n + G_n)\ are\ orthogonal$

Proof

$(d_n + G_n)(x)K_n(\Gamma)MK_n(\Gamma)(d_n - G_n)(x) + (d_n - G_n)(x)K_n(\Gamma)MK_n(\Gamma)(d_n + G_n)(x)$

$(d_n(x) + G_n(x))K_n(\Gamma)MK_n(\Gamma)(d_n(x) - G_n(x))$ $+(d_n(x) - G_n(x))K_n(\Gamma)MK_n(\Gamma)(d_n(x) + G_n(x)$=

$$= (d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x)) - (d_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x)) + (G_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x))$$

$$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x)) + (d_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x)) + (d_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$$

$$-(G_n(x)K_n(\Gamma)MK_n(\Gamma)d_n(x)) - (G_n(x)K_n(\Gamma)MK_n(\Gamma)G_n(x))$$

=0

By lemma (1.1) we get

$(d_n + G_n)K_n(\Gamma)MK_n(\Gamma)(d_n - G_n)(x) = 0$

And $(d_n - G_n)K_n(\Gamma)MK_n(\Gamma)(d_n + G_n)(x) = 0$

Hence $(d_n - G_n)\ and\ (d_n + G_n)\ are\ orthogonal$

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