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Pseudo-Sc-injective Modules

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ABSTRACT
Let R be associative ring with unit element ,and X be unitary right. R-module .in this paper, we introduce and study the concept, pseudo-Sc-X-1injective module which is the generalization1 of pseudo injective module. and we give an example: of pseudo-Sc-X-injective module which is not pseudo-X-injective module. Many, properties of this concept are introduced and also, we are consider some of their characterizations. We also, study some properties related to Co-⋖opfian and ⋖opfion modules.

Keywords : *injective module ,pseudo-injective1 module; closed-pseudo injective module. pseudo-Sc-injective module , small-closed sub module .*

1. Introduction

Through this introduction, *we will* mention some ,concepts related to; our concept as well as some well-known concepts, that we. need to complete this work. "Let X and Y be ,two R-module,Y is called , (pseudo-)X-injectvie if for every sub. module D of X any R-homomorphism1 (R-monomorphism) h: D → Y can .be extended to; an R-homomorphism α: X → Y. An 'R-module Y is called ; injective if it is X-injective for each R-module' X". "An R-module X is said to be :-quasi-injective (pseudo--injective), if it is (pseudo)X-injective". see [7] ,[9], [5] ."A sub module D of an ,R-module X is, said to, be small in X (briefly $D \leq_s X$), if D + A = X for every sub module A of X , then A = X .Dually, a nonzero sub. module ⋖ of ,X is, called essential (briefly $⋖ \leq_e X$),if ⋖ ∩ A ≠ 0 for each nonzero sub module A of X [8], if this is case, then we say that X is essential extension of D. A sub module D of X is

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called small-essential and denoted by $D \leq_{se} X$,if $D \cap A = 0$ with, $A \leq_s X$ implies $A = 0$". "A sub module D of an R-module X is, called: closed in X ,if it has no proper essential extension in X". [6] .in [2] , "A sub .module D of an R-module X is¸ said to be small--closed (simply s-closed) if D has no proper. small-essential extension in X ,i.e . if $D \leq_{se} A \leq X$ then $D = A$". Cleary; every s-closed sub module in R-module X is closed in M but. the converse not true in general, [2] ."it, is well-know that ,every directsummand is closed by [6] ,but in case s-closed sub module there is no relationship with direct summands, [2] .M.S.Abbas and F.∢.Mohammed in [2] " are presented the concept of small-closed injective (shortly Scl-injective)" modules. "Let; X_1 and X_2 be an R-modules .X is –called Scl-X_1-injective if for every homomorphism $\alpha: D \rightarrow X_2$, where D is a s-closed sub module of X_1 can be extended to a homomorphism $\beta: X_1 \rightarrow X_2$. V.Kumar ,A.J. Gupta, B.M.Pandeya and ..M.K.Patel in [14] , was introduced the notion of closed ,pseudo-X-injective ."Let X and Y .be two R—modules.Then ,Y is called "closed pseudo1-X-injective" if for, every closed submodule D of X ,any monomorphism from D to Y can be extended to a homomorphism from X to Y ". So we have the following implications :

injective module → quasi-injective module, → pseudo-injective module → closed-pseudo injective module .

The1 **following**1 **symbols :** $D \leq X$, $D \leq_c X$; $D \leq_{sc} X$ ¸ are ,denotes to that D is sub module , closed sub module ;small-closed sub module respectively.

2. Pseudo-small closed-injective 2.

Definition 2.1 :- A right R--module Yis "called pseudo-small closed-X-injective".(shortly, pseudo-Sc-X-injective) ,if for every small closed sub module D of X and ,any monomorphism from D to Y can be extended¸ to a homomorphism from X to Y . if X, is pseudo-Sc-X-injective then it is called pseudo-Sc-injective module .

Examples and1**Remarks 2.2 :-**

(1) Every c-pseudo injective, is pseudo-Sc-injective :but the converse is not. true .

(2) Every pseudo, injective is pseudo-Sc-injective module but the converse is not true for example : let E be a field and $R = \begin{pmatrix} E & E \\ 0 & E \end{pmatrix}$, $X_R = \begin{pmatrix} E & E \\ 0 & 0 \end{pmatrix}$,$Y_R = \begin{pmatrix} 0 & 0 \\ 0 & E \end{pmatrix}$,where X and Y are right modules. Then . Y is Sc-pseudo-X-injective modules .Fact Y is c-pseudo – injective .in [14] ,but Y is not pseudo-X-injective .

(3) Every injective module is Sc-pseudo injective *but* the converse. is not true. For example. : Z is Sc-pseudo-- injective module but not Z--injective module.

So, we obtain from .above the following implications for R,-modules :

injective → pseudo injective, → c-pseudo-injective ˛→ Sc-pseudo injective .

Now, we discuss some properties of pseudo-Sc-injective modules.

Proposition 2.3 :- if ,Y is pseudo-Sc-X-injective module then* Y is pseudo-Sc-W-injective. for any small closed sub module W of X.

Proof :- Assume that B $\leq_{sc}$ W , where W $\leq_{sc}$ X by [2 ;proposition (2.11)] then B $\leq_{sc}$ X ., and α : B → Y is a monomorphism . As Y is pseudo-Sc-M-injective ; therefore α can: be extended to., a homomorphism ά :X → Y. The restriction $ά|_A$ is a homomorphism from W to Y , which1 extends α. ≺ence Y is pseudo-W-injective .

Definition 2.4 :- Let X and Y be modules. A monomorphism f :Y → X is small-closed in case im f $\leq_{sc}$ X .

Proposition 2.5 :- if 'Y is pseudo-Sc-X-1injective module then1 any Sc-monomorphis α :Y→X splits.

Proof :- Let α:Y→X be Sc-monomorphism .(i.e) α(Y) $\leq_{sc}$ X ,and $α^{-1}$:α(Y) → Y be, the inverse of α .As Y is pseudo-Sc-M-injective module,then.. there exists a homomorphism ά : X → Y that extends $α^{-1}$ .Set u = ά α . Then. u is clearly an identity map on Y . Thus by [8 ;corollary (3.4.11)], α splits .

Proposition 2.6 :- Every directsummand of pseudo—Sc-injective" module is also pseudo-Sc-injective .

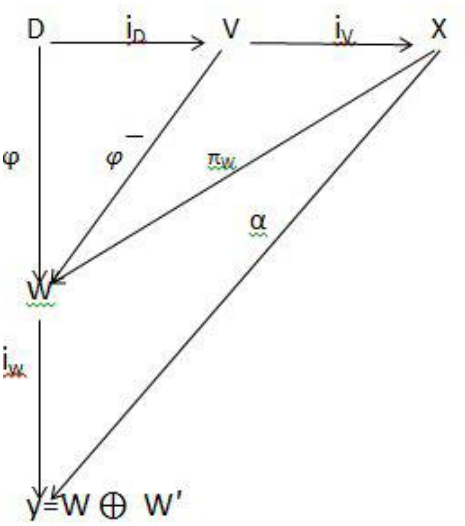
Proof :- Let X be pseudo-Sc --injective˛ module and Y be a 1direct .summand of X . Let B $\leq_{sc}$Y , i: B → Y and i_2: Y→ X be inclusions ; and let α :B → Y be- a monomorphism , since X is pseudo-Sc-injective , therefore exists β : X→X such that $\beta \circ i_2 \circ i_1 = i \circ \alpha \rightarrow p \circ \beta \circ i_2 \circ i_1 = p \circ i \circ \alpha$, where i: Y → X , p :X → Y are the inclusion and projection maps respectively . Take $\lambda = p \circ \beta \circ i_2$ and $p \circ i = i_Y$. Therefore $\lambda \circ i_1 = i \circ \alpha \rightarrow \lambda \circ i_1 = \alpha$.

Proposition 2.7 :- Let ,X ,and Y be right R--module .if Y is' pseudo-Sc-X-injective module ,W is a direct˛ summand of Y and V is a s-closed, sub module of X then . **(i)** W is a pseudo-Sc-V-injective module .

(ii) W is a pseudo-Sc-X-injective module.

(iii) Y is a pseudo-Sc-V-injective module.

Proof :- **(i)** Let $D \leq_{sc} V$, and φ: D → W ,be a -monomorphism . Since W is a direct summand of X ; there- exists a sub module W′ of Y . Such that¸ Y = W⊕ W′ . Let i_D: D → V be an inclusion map , i_V : V → X be an inclusion map; and i_W :W → Y = W ⊕ W′ be an injection map .Consider the following diagram :-



Since, Y is pseudọ-Sc-X-.injective module and $i_W \circ \varphi$ is a monomorphism .. there, exists α :X → Y a homomorphism 1such1 that1 $\alpha \circ i_V \circ i_D = i_W \circ \varphi$. Choose $\bar{\varphi} = \pi_W \circ \alpha \circ i_V$, where π_W :X → W be a projection map. Clearly; $\bar{\varphi}$:V → W be a homomorphism and $\bar{\varphi} \circ i_D = \pi_W \circ \alpha \circ i_V \circ i_D = \pi_W \circ i_W \circ \varphi = \varphi$. ⊰ence W is pseudo-Sc-V-injective module.

(ii) proof by proposition (2.6) . proof by-proposition1 (2.3). **iii)(**

Corollary 2.8 :- Let X and Y be -a right R-modules . *Then* Y is pseudo—Sc-X-injective module if and, only if Y is pseudo-Sc-B-injective module for, every s-closed sub module B of X .

Proof :- Suppose that Y is pseudo-X-injective module .By proposition (2.7(iii)); we have Y is pseudo-Sc-B-injective module ′for every s-closed· sub module B of X .

Conversely; since X is s-closed sub module of X andby assumption we .have Y is pseudo-Sc-X-injective module .

Now, we- need the following. Lemma **Lemma 2.9 :-** [2 ;corollary (2.7)]

Let D and 'Q be sub modules of an R-module X .if Q is an s-closed" sub module in X, then Q /D is an s-closed sub module in X/ D .

Lemma 2.10 :- [2 ;Remark 2.2(6)] Now, we need the.. following Lemma

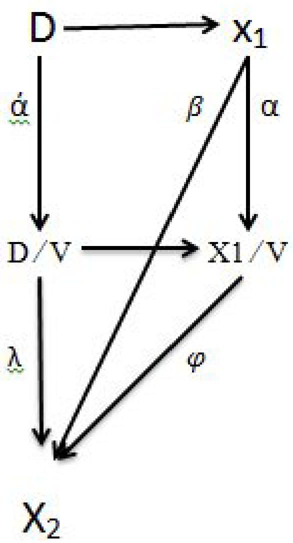
Let $D \leq Q$ be sub1 moduleof X with D is s-closed in X . Then $Q \leq_{se} X$ if and only if $Q/D \leq_{se} X/D$.

Lemma 2.11 :- if $D \leq_{sc} X$,then the s-closed sub module of X/D ,are of the form $\prec/D$ where $\prec \leq_{sc} X$ and $D \leq \prec$.

Proof :- Suppose that $D \leq_{sc} X$,and we prove that $\prec \leq_{sc} X$ .By lemma (2.9) above $\prec/D \leq_{sc} X/D$,for every $\prec \leq_{sc} X$ such that $D \leq \prec$. if $Y \leq X$ is such that $\prec \leq_{se} Y$,then by above lemma (2.10) $\prec/D \leq_{se} Y/D$. Because $\prec/D \leq_{sc} X/D$ We can conclude $\prec = Y$ and that $\prec \leq_{sc} X$.

Proposition- 2.12 :- Let "$X_1$and X_2 be .R-module . if X_2 is pseudo-Sc-X_1-injective module then X_2 is pseudo-Sc-X_1/Y-injective for every s-closed sub module Y of X_1 .

Proof :- Let $D/V \leq_{sc} X_1/V$. Consider $\lambda : D/V \rightarrow X_2$ is a monomorphism and by lemma (2.11) above we have $D \leq_{sc} X_1$. Consider the following ,diagram :-



Let $\alpha: X_1 \rightarrow X_1/V$ and $\acute{\alpha}: D \rightarrow D/V$ be the canonical epi .As X_2 is Sc-X_1-injective ,there exists $\beta : X_1 \rightarrow X_2$ that extends $\lambda\, \acute{\alpha}$,since $V \leq \ker \beta$, the existence of a homomorphism φ: $X_1/V \rightarrow X_2$ such· that $\varphi \circ \alpha = \beta$ is garunteed . For every $a \in D$, $\varphi(a + V) = \varphi \circ \alpha(a) = \beta(a) = \lambda \circ \acute{\alpha}(a) = \lambda\,(a + V)$. Therefore φ extends λ and X_2 is pseudo-Sc-X_1/Y-injective .

The R-module X_1 and X_2 are. relatively (mutually) pseudo-Sc-injective if X_i is pseudo-Sc-X_j-injective for all distinct $i,j \in i$,where i is the index set.

The1 following1 result1 is1 generalization 1of 1 [5 ; Theorem1 (2.2)].

Proposition, 2.13 :- if $X_1 \oplus X_2$ is pseudo-Sc-injective^ modules, then X_1 and X_2 are mutually Sc-injective .

Proof :- Suppose that $X_1 \oplus X_2$ be pseudo-Sc-injective module. To show that X_1 is pseudo-Sc-X_2-injective, let $D \leq_{sc} X_2$ and λ:D → X_1 be a homomorphism Define ψ:D → $X_1 \oplus X_2$ by ψ(a) = (λ(a),a) , ∀ a ∈ D. it is clear that ψ is R---monomorphism . Since Y is isomorphic. To" a direct summand of X ⊕ Y then 1by proposition (2.3) .We have $X_1 \oplus X_2$ is pseudo-Sc-X-injective ·thus, there exists an R-homomorphism h:X_2 → $X_1 \oplus X_2$ such 1that'1 φ = h ∘ i . *where iD* → *X*2 is the inclusion1 map, let, T_1:$X_1 \oplus X_2$ → X_1 be the natural projection .Now, $T_1 \circ \psi = T_1 \circ h \circ i$; hence $\lambda = T_1 \circ h \circ i$ ..Then $T_1 \circ h$ is a homomorphism ,extending λ ..Therefore X_1 is Sc-X_2-injective1 As· 1same way can prove: that X_2 is pseudo-Sc-X_1--1injective .

Corollary 2.14 :- if1 ⊕i ∈i X_i is a pseudo-Sc—injective, then X_i is, a pseudo-Sc-X_j-injective for: all distinct $i, j \in I$.

Corollary 2.15 :- Y is. quasi-injective R-module if and only., if Y^2 is1 pseudo-Sc –Y-injective .

Proof :- ⇒ it, *is* clear .

⇒ if Y^2 is. pseudo-Sc-Y-injective , .thus by proposition (2.13), Y. is Y-injective, .this means Y is quasi.-injective .

Proposition 2.16 :- Let W be s-closed, sub module of R-module X .if W- is pseudo-Sc-X-injective ; *then* W is a direct summand. of X .

Proof :- 1Since W is pseudo-Sc-X-.injective R-module ∃ an :R-homomorphism h: X → W. That extends the identity1i:W → W. ≺ence .by [81 ;corollary (3.4.10)], X =W ⊕ ker h .So·, that W is .a direct summand, , of X.

Proposition 2.17 :- if .,X is pseudo-Sc-injective *and* $Y \leq_{sc} X$,then any map h:Y → X can be extended to X, provided that ker $h \leq_{se} Y$.

Proof :- Let X be pseudo-Sc-injective ·module and $Y \leq_{sc} X$ .Let h:Y → X be given map *with* ker $h \leq_{se} Y$ .Consider a map g = (i_Y – h):Y → X. Cleary ker g = 0 ,and hence g has an, extension q to X ;because X is pseudo-Sc-injective .Then i_X – q is extension of h to X.

3. CSC-modules and some related modules in terms pseudo-Sc-injective modules

Definition 3.1 :- An ~R-module X is said *to* be complete Small-closed modul (briefly **CSC** module), if each sub module of X is a Small-closed .

Examples and Remarks 3.2 :-

(1) Z_4 as Z-module is CSC module .

(2) Z_6 as Z-module is not CSC module .

(3) From (1) it is clear Z as Z-module is not semi simple module and (2) is semi simple module .This means there is no relationship between semi simple and CSC and because there is no relationship between direct summand and small-closed for example by (2) .

Proposition 3.3 :- Let .X . be a CSC module. .Then, . the following statements. Are equivalent :

(i) Y is pseudo-X-injective.

(ii) Y is pseudo-Sc-X--injective .

Proof -:- **i** → **ii** it is clear

ii →**i** let $D \leq X$,and β: D → Y be a monomorphism ,since X is CSC module ,then $D \leq_{sc} X$, and by pseudo-Sc-X.injectivity of Y ther exists h: X →Y such that $h \circ i = \beta$. Therefore1 Y *is* pseudo-X-injective .

Recall that a nonzero R-module.. X is a hollow if every proper sub module of X is small [8].

in.. case of hollow modules ,the concept of closed and s-closed are equivalent [2]. So it is easy to get the proof of the following" proposition .

Proposition 3.4 :- Let X be a hollow R-module. .Then the, following statements′are equivalent :

(i) Y is c-pseudo-X-injective .

(ii) Y is pseudo-Sc-X-injective .

Theorem1 **3.5 :-** Let X be a hollow and CSC, *then* thẹ following; statements arẹ equivalent :

(i) Y is pseudo-X-injective .

(ii) Y is c-pseudo-X-1injective .

(iii) Y is pseudo-Sc-X-injective .

Proof :- i → ii → iii ; it is clear .

ii →1 **i** let ,D ≤ X and β:D → Y bea monomorphism . .Now ,by CSC and according to [2] ,every s-closed R-module is closed .We $get\ D\ \leq c\ X$ and since "Y is closed-pseudo-X-injectivity ,the n there exists h: X → Y such 1 that h ∘ i = β .Therefore Y is pseudo-X-injective .

iii →**ii** by proposition (3.4).

iii → **i** by proposition (3.3).

Recall that an R-module X is multiplication if each sub module of X has the form i_X for some ideal i of R. [9].

Proposition 2.20 :- Every s-closed sub module of multiplication s-closed pseudo injective R-module is s-closed pseud injective .

Proof :- ..Let W be a s-closed sub module ofa s-closed sub module ≺ of X and let h:W → ≺ be an R-monomorphism1 .Since" ≺ $\leq_{sc}$ X . it follows that by [2 ;proposition (2.11)] , W is also a s-closed sub module of X Since X is pseudo-Sc-injective , then there exist an R-homomorphism φ: X →X that ~extends h .Since X is multiplication module , we have ≺ = X i for some i of R .Thus $\varphi|_{\prec}$ = φ(≺) = φ(X i) = φ(X) i ≤ X i = ≺ . This show that ≺1 is 1pseudo-Sc-injective .

Proposition 2.21 :- Let X_1 and ,,X_2 be R-modules and X= $X_1 \oplus X_2$ .Then is scl-X-injective if and only if ,fore every sub module Y of X such that $Y \cap X_2 = 0$ and $T_1(Y)$ is a s-closed sub module of X_1 ¸ there a sub module Y′ of X such that Y ≤ Y′ and X= $Y' \oplus X_2$, where T_1 the natural projection of X in to X_1 .

Proof:- Similar to proving [2 ;Theorem (3.5)]

Some, general properties of pseudo-Sc-injectivity are given in the following results

Proposition1 **2.22 :-** Let X and Y_i(i ∈ 1i) be R-modules. Then1 $\prod i \in_i Y_i$ is pseudo-Sc-X-1injective if and $only\ if$ Y_i is pseudo-Sc-X-injective,for every i ∈ i .

Proof:- Follows ,from the definition and injections and projections associated with the directproduct .

The following corollary is immediately., fromproposition1 (2.22).

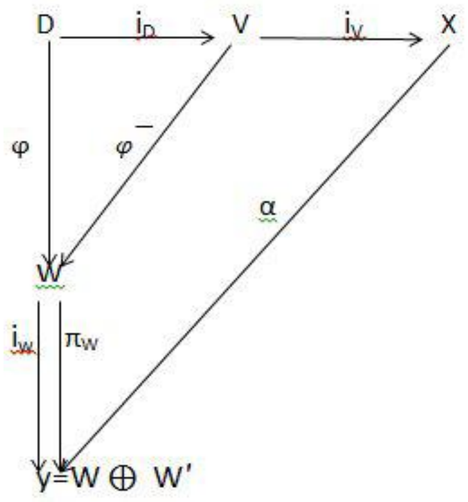
Corollary 2.23 :- Let X and Y_i be R-1modules *where* $i \in I$ and i is finite1 index set , if $\oplus_{i=1}^{n} Y_i$ is pseudo-Sc-X-injective , ∀1 i ∈ i then Yi is pseudo-Sc-X-injective . in particular every direct~summand of pseudo-Sc-injective R-module is pseudo-Sc-injective .

Proposition 2.24 :- Let X be a right R-1moduleand B $\leq_{sc}$ X . if B is, pseudo-Sc-X-injective module then¸ B is a direct summand- of X .

Proof :- The proof is routine.

Proposition 2.25 :- Let X and Y be right R-.modules .if Y is pseudo-Sc-X-injective module; W is adirect summand of Y and V is a direct summand of X then W is pseud-Sc-V-injective module .

Proof :- *Let D* $\leq_{sc}$ V and φ:D → W be a monomorphism ¸since W is *a* direct summand of Y; and V is a direc summand of X . ∃ sub module W′ of Y and V′ of X. such that Y = W ⊕ W′ , X, = V ⊕ V′ . Let i_W :W → Y be an injective map¸ i_V : V → X an injective map and i_D :D → V an inclusion map. Consider the following diagram



Since V is 1a 1direct1 summand 1of X, V $\leq_{sc}$ X and V is not an s-essential 1in X. Then D $\leq_{sc}$ X .But Y is pseudo-Sc-X-injective module · So $i_W \circ \varphi$ can be extended to α :X → Y a homomorphism ;such that $\alpha \circ i_V \circ i_D = i_W \circ \varphi$ .Choose $\overline{\varphi} = \pi_W$:Y → W be an projection map .We have $\overline{\varphi}$ is an extension of φ .Therefore W is pseudo-Sc-V-injective module .

By varadarajan.K.[13]

A right R-1module X is called Co-∢opfian (∢opfian) if every injective (surjective) endomorphism h: X → X is an automorphism .

According to [10]

A right R-module X is^called directly finite if it *is* not isomorphic toaproper direct summand of X.

Lemma 2.26 :- in [10 ;proposition (1.25)]

An right R-1module X is directlyfinite if and only if $h \circ g = i$ implies that $g \circ h = i$ for all $h,g \in S = End_R(X)$ where i is an identity map from X to X.

Proposition 2.27 :- A pseudo-Sc-injective module X is a directly,finite if and.. only. if it isCo-⊀opfian .

Proof :- Let h be an injective endomorphismof X an $i_X : X \rightarrow X$ be an identity homomorphism.Since X is pseudo-Sc-injective. modulẹ there exists a· homomorphism $g:X \rightarrow X$ such that $g \circ h = i_X$. By lemma (2.26) we have $h \circ g = i_X$ which implies. that h is an automorphism .⊀ence X is Co--⊀opfian .

Conversely ; assume that X is ,Co-⊀opfian ,let $h,g \in S = End_R(X)$ such that $h \circ g = i$ .Then.. g is an injective homomorphism and g^{-1} exists .Thus, $h = g \circ g^{-1} = i \circ g^{-1} = g^{-1}$ .Sọ $g \circ h = g \circ g^{-1} = T$ .By lemma (2.26), we have X is directly finite .

Since. Every, indecomposable modul. is directly* finite then by.. proposition (2.27), we obtain the following corollary .

Corollary 2.28 :- if X is anindecomposable pseudo-Sc-injective R-module^ then X is *a* Co-⊀opfian .

in [13]; was proved. that every ⊀opfian R-module is directly .,finite Thus the following,, result follows from proposition1 (2.27) .

Corollary 2.29 :- if X is *a* pseudo-Sc-injective *and* ⊀opfian R-module. Then X is a Co-⊀opfian .

Corollary 2.30 :- Let X be Sc-injectiveand ⊀opfian module ,then it is a Co-⊀opfian

Corollary 2.31 :- An Sc-injective R-module X is ,a directly finite if and only if it is Co-⊀opfian .

in , [4] , anR-module X is direct-injective ,if given any direct`summand D of X, an injection map $j_D : D \rightarrow X$ and every R-monomorphism $\alpha: D \rightarrow X$ *there is* an R-endomorphism β of X *such that* $\beta\ \alpha = j_D$.

Proposition 2.32 :- Every pseudo-Sc-injective cSc module is~direct-1injective .

in [11], recal that a., right R-module X is called divisible, iff or each x ∈X and"for each r ∈ R which is not left zero-divisor..,*there* exist x′∈ X such that x = x′r .

in1 [4], was proved that every direct-injective R-module is divisible.Thus we have the^following corollary which follows *from* proposition1 (2.32).

Corollary 2.33 :- Every pseudo-Sc-injective cSc module is, divisible .

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