EFFECT OF ELASTIC MODULI (E_1 AND E_2) OF A HYBRID COMPOSITE ROD AND ITS CONSTITUENTS ON FREE TORSIONAL NATURAL FREQUENCY

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Received on 04 February 2016 Accepted on 31 May 2016

1. Abstract:

In this work, a hybrid fiber reinforced composite rod fixed from one end, while the other end is left free and subjected to a static torsional load, and left to vibrate freely after load releasing. Fiber volume fraction of 40% is considered to rod of interest, the matrix is composed of 20% short fibers and 40% pure matrix (this type of matrix is called composite matrix, which is well known and widely used in fiber reinforced hybrid composites). Such a ratio is adopted since it gives enough strength and reinforcement and meet the economic requirements as well, as mentioned in Mechanics of Composite Materials by Jones 1999. Four various long and short fibers of the same type and four matrices are taken to construct the rod in order to introduce the different properties to investigate their effects on natural frequency under torsional excitation. The problem is manipulated using software of AnsysV.14. The elastic properties of materials are determined using software of Matlab v.7. The results show that natural frequency is mainly highly affected by matrix longitudinal elastic modulus than that of fibers, and displays a regular pattern of behavior. The fibers show an irregular behavior towards the natural frequency variation. This can be attributed to the interaction effects between three types of constituents, matrix, long and short fibers composing the whole structure of the rod. In addition, there is the effect of anisotropy of material effective elastic properties, which plays an important role in results irregularities.

Key Words: Stacking sequence, Natural frequency, Torsional vibration, Elastic moduli, Constituents, Hybrid composite Rod.

تاثير معاملي المرونة (E_1,E_2) لعمود مصنوع من مادة مركبة هجينة ولمكوناته على التردد الطبيعي الالتوائي الحر

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١. الخلاصة:

تم في هذا العمل محاكاة تثبيت عمود (مصنوع من مادة مركبة هجينية مقواة بالياف) من احدى نهايتيه وتركت النهاية حرة وسلط عليها حملا التوائيا استاتيكيا ثم تركت لتهتز بشكل حر بعد زوال الحمل. اعتمدت نسبة الياف حجمية ٤٠% للعمود موضع الدراسة، ويتكون النسيج الاساس من ٢٠% الياف قصيرة و ٤٠% من نسيج اساس صاف (صرف) (ويسمى هذا النوع من الانسجة الاساسية بالنسيج الاساس المركب وهو نوع معروف وواسع الاستعمال في المواد المركبة الهجينية).وتم اعتماد مثل هذه النسبة لكونها تحقق شرط المتانة والتقوية كما تفي بالعامل الاقتصادي الترشيدي وكما جاء في كتاب (ميكانيك المواد المركبة ل جونز ١٩٩٩). تم أخذ اربعة انواع من الالياف الطويلة والقصيرة من نفس النوع مع اربع انواع من مواد الانسجة الاساسية لبناء العمود وللحصول على مواد مختلفة لدراسة تاثيرها على التردد الطبيعي تحت الاثارة الالتوائية (تحت تاثير الحمل الالتوائي). تم تحليل المسالة بواسطة برنامج (الانسز ١٤). وحساب الخواص المرنة للمواد المستخدمة بواسطة (برنامج الماتلاب ٧). تبين النتائج ان التردد الطبيعي يتاثر كثيرا وبصورة رئيسية بمعامل المرونة للنسيج الاساس اكثر من الليف (الفايبر) كما يظهر نمطا سلوكيا منتظما. حيث ان الالياف تبدي سلوكا غير منتظم تجاه تغير التردد الطبيعي. ويمكن ان يعزى ذلك الى التاثيرات التفاعلية المتبادلة بين النوعين المختلفين من الالياف (الطويلة والقصيرة) مع النسيج الاساس. هذا بالاضافة الى تاثير الخاصية الاتجاهية اللاانتظامية للخواص الميكانيكية الفعالة للمواد والتي تلعب دورا مهما في عدم انتظامية النتائج.

الكلمات المفتاحية: تسلسل التراص، الترد الطبيعي، اهتزازات التوائية، معاملات المرونة، مكونات، عمود مركب هجين.

List of Symbols:

A: Cross Section Area

a_f: The ratio of average fiber length to fiber diameter $=l_f/d_f$

d_f: Fiber diameter

E_{1m}: Longitudinal moduli for a unidirectional discontinuous fiber 0^0 composite matrix combined of resin and discontinuous fiber.

E_{2m}: Transverse moduli for a unidirectional discontinuous fiber 0^0 composite matrix combined of resin and discontinuous fiber.

E_{cm}: Moduli of isotropic composite matrix, combined of resin and random discontinuous fiber.

E_1: Longitudinal modulus for unidirectional continuous fiber 0^0 composite lamina combined of composite matrix and continuous fiber.

E_2: Transverse modulus for unidirectional continuous fiber 0^0 composite lamina combined of composite matrix and continuous fiber.

E_{sf}: Moduli of discontinuous fiber material.

E_f: Moduli of continuous fiber material.

E_m: Moduli of resin material.

f_n :Natural frequency.

G: shear modulus of the shaft.

G_{12m}: Shear modulus for a unidirectional discontinuous fiber 0^0 composite matrix.

G_{cm}: Shear modulus of isotropic composite matrix.

G_{12}: Shear modulus for a unidirectional continuous fiber 0^0 composite lamina.

G_{sf}: Shear modulus of discontinuous fiber material.

G_f: Shear modulus of continuous fiber material.

G_m: Shear of resin material.

G_{23}: Modulus of Rigidity for the material in 2-3 plane.

I: Polar Moment of Inertia.

J, J_p :Polar Second Moment of Area $J = \pi R^4/2$.

L, l: Length of shaft.

l_f: Average fiber length.

n: mode

R: Radius of shaft.

t: Time

α: Angular Acceleration $\alpha = \frac{d^2\theta}{dt^2}$
θ: Angle of Twist
ν_{12m}: The major Poisson's ratio for a unidirectional discontinuous fiber 0^0 composite matrix.
ν_{cm}: Poisson's ratio of isotropic composite matrix.
ν_{12}: The major Poisson's ratio for a unidirectional continuous fiber 0^0 composite lamina.
ν_{sf}: Poisson's ratio for discontinuous fiber material.
ν_f: Poisson's ratio for continuous fiber.
ν_m: Poisson's ratio for resin material.
ρ: Density of material.
$\forall_{sfp}$: Volume fraction of discontinuous fiber, ratio of the volume of discontinuous fiber to the volume of composite lamina.
$\boldsymbol{\omega}_1$: Fundamental frequency of beam.
ω_n :Natural Angular Frequency.
$\forall_{mm}$: Volume fraction of resin matrix, ratio of the volume of resin to the volume of composite matrix.
$\forall_{mp}$: Volume fraction of resin matrix, ratio of the volume of resin to the volume of composite lamina.
$\forall_f$: Volume fraction of continuous fiber, ratio of the volume of continuous fiber to the volume of composite lamina.
$\forall_m$: Volume fraction of matrix, ratio of the volume of composite matrix to the volume of composite lamina.

1. Introduction:

1.1. General Background:

A lamina usually represents a fiber reinforced composite structure, the composite consists of high strength and modulus fibers embedded in or bonded to a matrix. Both the fiber and the matrix retain their distinct properties and together produce properties, which cannot be achieved individually. The fibers are usually the principal load carrying elements in the composite and the purpose of the matrix is essentially to keep the fibers in the desired location. The matrix also serves the purpose of protecting the fiber from heat, corrosion and other environmental effects and damages. The fibers may be glass, carbon, boron, silicon carbide, aluminum oxide etc. Theses fibers may be embedded in the matrix either in a continuous for or discontinuous form (chopped pieces of different length). The matrix may usually be a Polymer, Metal or a Ceramic as a bonding material. To increase the strength of the matrix using the reinforcement by composing or hybridizing it with discontinuous fiber, then reinforcing this composite matrix with continuous long fibers to get the composite lamina combined of bonded material, discontinuous fibers, and continuous fibers as reinforcement phase; of composite matrix, (bonded material and discontinuous fiber), together to made a lamina or plate. Note, the discontinuous fiber can be used in the same type of continuous fiber or different. There are several types of hybrid composites characterized as: (1) interply or tow-by-tow, in which tows of the two or more constituent types of fiber are mixed in a regular or random manner; (2) sandwich hybrids, also known as core-shell, in which one material is sandwiched between two layers of another; (3) interply or laminated, where alternate layers of the two (or more) materials are stacked in a regular manner; (4) intimately mixed hybrids, where the constituent fibers are made to mix as randomly as possible so that no over-concentration of any one type is present in the material; (5) other kinds, such as those reinforced with ribs, pultruded wires, thin veils of fiber or combinations of the above [1]. The hybrid composites extensively used in engineering and industrial applications such as drive shafts on which many researches have been carried out during the last two decades. Hybrid composite shafts made of glass or carbon fibers with epoxy and steel or aluminium have high fundamental bending natural frequency as well as high torque transmission capability [2]. Hybrid materials were defined as mixtures of two or more materials with new properties created by new electron orbitals formed between each material, such as covalent bond between polymer and silanol molecular in inorganic/organic hybrids[1]. Makishima categorized substances into three materials by their chemical-bond modes, i.e., metals, organic materials and their polymers, and ceramics. He also defined hybrid materials as mixtures of two or more materials with newly-formed chemical-bonds. Makishima explained that the difference between hybrids and Nano-hybrids was not so obvious, and that nanocomposites include hybrids and nanohybrids in many cases [5]. Gómez-Romero and Sanchez defined hybrid materials as organic-inorganic hybrid materials or inorganic-biomaterials. They also mentioned that the characteristic scale of hybrid materials was less than 103 nm. They did not provide a strict definition of hybrid materials, and did not mention the formation of new electron orbitals or chemical bonds [6, 7]. In the Makishima classification and the Gomez-Romero & Sanchez definition, mixtures of materials were focused at the viewpoint of the characteristic scale and materials category. Their definitions of "hybrid materials" required an atomic or nanometer-level mixture of materials. Figure-1 shows a classification of materials by different scale levels, as proposed by the Materials Science Society of Japan [6, 5].

1.2. Literature Review:

The vibration of composite materials during the last decades has attracted the attention of the researchers for their wide range and margin of properties tailoring and vast scope of applications. In 1972 Abarcar studied the flexural vibration of cantilever composite beam [8]. In 1979, Teh and Huang presented a finite element approach as a study of vibration of generally orthotropic beams [9]. Kapani and Raciti in 1989 studied the nonlinear vibration of asymmetrically laminated composite beam [10]. Abramovitch studied in 1992 the effect of shear deformation and rotary inertia on vibration of composite beam [11]. A Vibration analysis and finite element modeling for determining shear modulus of pultruded hybrid composites was introduced by Chandrasekhar in 1996 [12]. Yilidirim et. al. made a comparison of in plane natural frequencies of symmetric cross ply laminated beams based on Bernouli-Euller and Temoshinko beam theories in 1999 [13]. Erol & Gürgöze studied the lateral vibration of composite beam during 2004 [14]. Mirtalaie and Hajabasi introduced an analytical approach to investigate the coupled lateral-torsional vibration of laminated composite beam [15] during 2011 along with Hasan and Atlihan who worked numerically and analytically on the study of the effect of the length of delamination and orientation angle on the natural frequencies of symmetric composite beams [16]. Unlike to the works mentioned above, in this work, a cantilever hybrid fiber reinforced composite rod subjected to a static torsional load and left to vibrate freely after load removal is studied to investigate the effect of its elastic moduli on the torsional natural frequencies. This loading application is one of free vibration generation sources or methods. Fiber volume fraction of 40% is considered to rod of interest. The problem is analyzed using ANSYS v.14. The element type adopted is “Beam 188, structural 3-D 2-Node Beam element with DOF: UX, UY, UZ, ROTX, ROTY, ROT as shown in Fig. below:

Beam 188, structural 3-D 2-Node Beam element

The meshing of the structure is mapped meshing as shown in Fig. 12.

2. Mathematical Formulation:

2.1. Hybrid composite lamina (discontinuous random short fiber, resin, continuous fiber):

Fig.5 shows a simple schematic model of a composite lamina consists of a discontinuous random fiber, resin and continuous fiber. The fibers are assumed to be uniformly distributed throughout the composite matrix, combined of discontinuous random short fiber and resin material. A perfect bonding is assumed free of any voids. The fibers and the matrix are both assumed linear and elastic. The elastic properties of such a lamina will be as following [17]. Some of them are shown in table-2 Appendix-1:

$$E_1 = E_f.\forall_f + (1-\forall_f).\mathrm{E_m}\left[\left(\frac{3.(1-\forall_f)+6.a_f.\eta_l.\forall_{sfp}}{8(1-\forall_f)-8\eta_l.\forall_{sfp}}\right)+\left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right] \quad (1)$$

The corresponding major Poisson's ratio is:

$$v_{12} = v_f.\forall_f + v_{cm}(1-\forall_f) \quad (2)$$

Using Eq. 14, and 5 in to 15, results in

$$v_{12} = v_f.\forall_f + \left(\frac{E_{cm}}{2.G_{cm}} - 1\right)(1-\forall_f)$$
(3)

The transverse modulus and minor Poisson's ratio for the loading transverse to the continuous fiber direction as shown in figure (5-b) are:

$$E_2 = \frac{E_f.E_{cm}}{Ef(1-V_f)+E_{cm}.V_f}$$

Or,

$$E_2 = \frac{E_f.E_{cm}\left[\left(\frac{3.(1-\forall_f)+6.a_f.\eta_l.\forall_{sfp}}{8(1-\forall f)-8\eta_l.\forall_{sfp}}\right)+\left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right]}{E_f(1-\forall_f)+E_{cm}.\forall_f\left[\left(\frac{3.(1-\forall f)+6.a_f.\eta_l.\forall sfp}{8(1-\forall f)-8\eta_l.\forall sfp}\right)+\left(\frac{5.(1-\forall_f)+10.\eta_T.\forall_{sfp}}{8(1-\forall_f)-8\eta_T.\forall_{sfp}}\right)\right]} \quad (4)$$

And,

$$v_{21} = \frac{E_2}{E_1}v_{12} \quad (5)$$

Where E_1, E_2 and v_{12} as in Eqs.18, 21 and 22.

For a shear force loading as shown (5-b):

$$G_{12} = \frac{G_f.G_{cm}}{G_f.\forall_m+G_{cm}.\forall_f} = \frac{G_f.G_{cm}}{G_f(1-\forall_f)+G_{cm}.\forall_f} \quad (6)$$

Substitution Eq. 20 in to 26 leads to:

$$G_{12}=\frac{G_f.E_m.\left[\left(\frac{(1-\forall_f)+2.a_f.\eta_l.\forall_{sfp}}{(1-\forall_f)-\eta_l.\forall_{sfp}}\right)+\left(\frac{2.(1-\forall_f)+4.\eta_T.\forall_{sfp}}{(1-\forall_f)-\eta_T.\forall_{sfp}}\right)\right]}{8.G_f.(1-\forall_f)+E_m.\forall_f\left[\left(\frac{(1-\forall_f)+2.af.\eta_l.\forall_{sfp}}{(1-\forall_f)-\eta_l.\forall_{sfp}}\right)+\left(\frac{2.(1-\forall_f)+4.\eta_T.\forall_{sfp}}{(1-\forall_f)-\eta_T.\forall_{sfp}}\right)\right]} \quad (7)$$

There are in all quantities, E_1, E_2, G_{12} and v_{12} to describe the elastic behavior of a hybrid lamina constituted from a composite matrix and continuous reinforcement fibers.

3. Analysis of Torsional Free Vibration of Hybrid Composite Materials-Analytical Approach:

Oscillations can occur in long transmission shafts such as the drill shaft of an oil rig. The theory is similar to that of transverse vibrations and buckling as there can be more than one mode. The derivation uses the wave equation. The work applies only to shafts with a circular cross section.When a long shaft is fixed at one end and free at the other, and a torque T is applied at the free end as shown in Fig.6, then the change in torque magnitude across an element of length δx will be δT, thence the rate of change of the torque transmitted the shaft length as *dT / dx*. Therefore, the net torque on the element is [20]:

$$\delta T=\frac{dT}{dx}\,\delta x \quad (8)$$

From the general torsion equation:

$$T=\frac{G\,J\,\theta}{L} \quad (9)$$

The factor G (shear Modulus) is specified for a cross section lies in a plane perpendicular to the longitudinal axis of the shaft. For the hybrid composite shaft of interest, it is being the factor of G_{23} (transverse shear modulus in x-y plane) since the longitudinal axis is taken as 1 direction along the fiber length as shown in Fig.7, 2-direction along x-axis and 3-direction along y-axis, hence Eq.36 becomes for composites and henceforth:

$$T=\frac{G_{23}\,J\,\theta}{L} \quad (10)$$

However, for a uniform shaft:

$$\frac{\theta}{L}=\frac{d\theta}{dx} \qquad \therefore T=G_{23}J\,\frac{d\theta}{dx} \quad (11)$$

Differentiate with respect to x obtains:

$$\frac{dT}{dx}=G_{23}J\,\frac{d^2\theta}{dx^2} \quad (12)$$

The net torque now is:

$$\delta T=\frac{dT}{dx}\delta x=G_{23}J\frac{d^2\theta}{dx^2}\delta x \quad (13)$$

But the torque on the element must overcome the inertia of the material itself only:

$$\delta T = I\,\alpha = I * \frac{d^2\theta}{dt^2} = G_{23} J \frac{d^2\theta}{dx^2}\delta x \tag{14}$$

It's known that for a solid circular length of shaft:

$$I = \frac{MR^2}{2} = \frac{\rho A\ \delta x\ R^2}{2} = \frac{\rho\pi R^2 \delta x\ R^2}{2} = \rho\delta x J \tag{15}$$

$$\therefore\ \delta T = \rho\,\delta x\, J\,\frac{d^2\theta}{dt^2} = G_{23} J\,\frac{d^2\theta}{dx^2}\,\delta x$$

$$\rho\,\frac{d^2\theta}{dt^2} = G_{23}\,\frac{d^2\theta}{dx^2} \quad \Rightarrow \quad \frac{d^2\theta}{dx^2} = \frac{\rho}{G_{23}}\,\frac{d^2\theta}{dt^2} \tag{16}$$

This is usually expressed as:

$$\frac{d^2\theta}{dx^2} = \frac{1}{c^2}\,\frac{d^2\theta}{dt^2} \tag{17}$$

Where c is the velocity of a wave of propagation of torsional angular deformation.

The standard solution for this 2nd order non-homogeneous differential equation is:

$$\theta = \left[A\,\sin\left(\frac{\omega\,x}{c}\right) + B\,\cos\left(\frac{\omega\,x}{c}\right)\right]\sin\left(\omega\,t\right) \tag{18}$$

A & B are constants determined by the boundary conditions.
Put in the boundary conditions for this shaft. When x = 0, θ = 0 so putting this in Eq.45 results in:

$$0 = [A\sin(0) + B\cos(0)]\sin(\omega t) = [0 + B]\sin(\omega t) \tag{19}$$

It follows that B = 0 and the solution is reduced to:

$$\theta = \left[A\,\sin\left(\frac{\omega\,x}{c}\right)\right]\ \sin\left(\omega\,t\right) \tag{20}$$

Differentiating Eq.47 with respect to x getting:

$$\frac{d\theta}{dx} = \left[\frac{A\ \omega}{c}\cos\ \left(\frac{\omega\,x}{c}\right)\right]\ \sin\omega\,t \tag{21}$$

$$\text{At } x=L \qquad \frac{d\theta}{dx} = 0 \qquad \therefore \left[\frac{A\ \omega}{c}\cos\ \left(\frac{\omega\,L}{c}\right)\right]\ \sin\omega\,t = 0 \tag{22}$$

This can only occur if:

$$\omega = \left(n - \frac{1}{2}\right)\frac{\pi c}{L} \qquad \text{Where n is an integer 1, 2, 3} \tag{23}$$

Thus, the natural frequency of the system can be given as:

$$\omega_n = \left(n - \frac{1}{2}\right)\frac{\pi}{L}\sqrt{\frac{G_{23}}{\rho}} \qquad \text{and} \qquad f_n = \left(n - \frac{1}{2}\right)\frac{1}{2L}\sqrt{\frac{G_{23}}{\rho}} \tag{24}$$

If pure free torsional vibration are to be considered as in the case of the current research, the analysis can be reduced to the following manipulation:

3.1. Equation of Motion for Free Undamped Pure Torsional Vibrations:

Consider now the torsional vibrations of the same bar, shown in Fig.8. This consists of the rotation of each cross-section about the longitudinal axis, which passes through the centroids of the cross-sections. Only the cross-sections having at least two symmetry axes (such as the ellipse seen in Fig.8) will be considered to avoid coupling between twisting and bending displacements. A typical element of length dz, determined by parallel planes located at z and (z+dz), is again chosen. A free body diagram of it is drawn in Fig. 9. A twisting moment M_t is acting on the cross-section taken at the z-plane. This moment is the resultant of the internal shear stresses τ_{zx} and τ_{yz} Fig. 10, which exist on the cross-section and vary as functions of the transverse coordinates y and x (as well as with z and t). The final equation of motion for such a system cab represented in the following form [21]:

$$\sum M_o = J_o\ddot{\theta} + K_t\,\theta = 0 \tag{25}$$

The derivation can be cited in the relevant reference and literatures. The solution of the above equation leads to that for free (undamped) torsional vibration, the angular natural frequency ω_n can be found as:

$$\omega_n = \sqrt{\frac{K_t}{J_o}}\ rad/sec \tag{26}$$

Where K_t is the torsional spring constant of the shaft, J_o is the polar mass moment of inertia for the disk. The torsional spring constant K_t is determined from the relationship between moment M and angular displacement θ of the shaft through the following relationship:

$$M_o = K_t\,\theta \tag{27}$$

Also $M_o = \frac{GJ_p\theta}{l}$ therefore $K_t = \frac{G\,J_p}{l}$ (28)

For a circular shaft J_p is given by $\frac{\pi d^4}{32}$ (mm^4), therefore:

$$K_t = \frac{\pi\,Gd^4}{32l} \tag{29}$$

To find the natural frequency (ω_n or f_n) of the system shown in figure 11, :

$$\omega_n = \sqrt{\frac{\frac{\pi G d^4}{32 l}}{J_o}} \quad \left(\frac{rad}{sec}\right) \qquad \text{or} \qquad f_n = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi G d^4}{32 l}}{J_o}} \quad \left(\frac{cycle}{sec}\right) \tag{30}$$

The factor G (shear Modulus) is specified for a cross section lies in a plane perpendicular to the longitudinal axis of the rod. For composite rod of interest it is being the factor of G_{23} since the longitudinal axis is taken as 1 direction as shown in Fig. (12), thus:

$$\omega_n = \sqrt{\frac{\frac{\pi G_{23} d^4}{32 l}}{J_o}} \quad \left(\frac{rad}{sec}\right) \quad \text{or} \quad f_n = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi G_{23} d^4}{32 l}}{J_o}} \quad \left(\frac{cycle}{sec}\right) \tag{31}$$

Given that the mass moment of inertia for the disk *J*o can be calculated using the formula of:

$$J_o = \frac{1}{2}\, m\, k^2 \tag{32}$$

where k is radius of gyration of the disc about an axis passing through its center and perpendicular to the plane of the disc and k = r (radius for circular discs).

4. Results and Discussion:

Curves of results show graphically in Figs. 13 through 17 that the natural frequency under pure free torsional vibrations for hybrid composite rods have various modes of affecting by the different factors for hybrid composite materials as stated below:

1. With respect to longitudinal modulus of elasticity (Young modulus) E_1, there is a direct proportionality between it and the natural frequency when the variation of this modulus is based on increasing of the fiber volume fraction of short fibers of the composite matrix for all types of hybrid composite samples indicating that regardless to the type of material. The natural frequency under free torsional vibration is directly affected the fiber volume fraction Fig. 13. The lowest set of natural frequencies belong to fiber of the lowest Young modulus.
2. For the transverse modulus, E_2 has less as direct effect as the modulus E_1 even if it is varied according to matrix or fiber type Fig.17. The relation curves have a slope less than that of E_1 curves do at low fiber modulus. However, this is not the case for high moduli as in the case of CT300 and E-glass, due to the contribution of the transverse modulus in the value of shear rigidity of the composite materials and direct proportionality on the natural frequency, as clearly figured out in Fig. 17 referred to above. The reason of dominance of E_1 is that the free torsional vibration occurred about the longitudinal axis, thus the properties of this direction will control on the various responses and characteristics of the rod, the reinforcing long fibers are also laid along it. In addition, there are the discontinuous fibers, randomly distributed in the composite matrix and assumed to behave isotropically strengthening the long ones effect. These factors and other elements are the common constituents of E_1, E_2, G_{12} and G_{23} but E_1 is the most affected factor among them by these constituents so it will highly predominate on the values of natural frequencies.

The matrix has the major and direct-proportional effect on the natural frequency for free torsional vibration more than the fiber does because it is the first part experiences the application of the torque and resists the twisting deformation. Fig.16 clearly demonstrates the uniformity of this effect

through changing values of E_1 of the composite depending on matrix type (Fig.16) and on fiber type as well.
When the effects of longitudinal and transvers moduli on the natural frequency are compared to each other, it is seen that E_1 is more dominant and has a higher slope in their graphs since the long fibers strengthen the torsional rigidity of the rod and subsequently more torsional energy can be stored in as shown in Fig. 18 .

3. Theoretical natural frequencies are calculated using Eqs. (31 & 32) below. To find the natural frequency (ω_n or f_n) of the system shown in figure 11, the following formulas are used:

$$\omega_n = \sqrt{\frac{\frac{\pi G_{23} d^4}{32l}}{J_o}} \quad \left(\frac{rad}{sec}\right) \quad \text{or} \quad f_n = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi G_{23} d^4}{32l}}{J_o}} \quad \left(\frac{cycle}{sec}\right) \tag{31}$$

$$J_o = \frac{1}{2}\, m\, k^2 \tag{32}$$

It can be seen from Figs. 13 through 17 that when fiber type is changed, the whole characteristics of the hybrid material are changed (E_{cm}, E_1, E2 and G_{23}) thus the structural stiffness and the whole vibrational behavior (including its natural frequency of course) of the rod will be affected. Equations 31 and 32 demonstrate this fact clearly.

5. Results Verification:

Table-1 includes the values of the natural frequencies of the rod under consideration obtained from the numerical solution resulting from the ANSYS v-14 package along with those obtained from the analytical solution based the theory referred to above according to Eqs. (31 & 32) for the purpose of results validation. The differences between the analytical and numerical results may be attributed to the collection of pure theoretical assumptions which the analytical relationships based on, while the numerical solution is built on some approximations represented by the discritization of the whole rod domain to a certain number of structural finite elements and determining the responces at specified regions on the rod.

6. Conclusions:

The following conclusions can be drawn out based on the above results:

1. The fiber of the lowest elastic modulus has the lowest set of angular natural frequencies among the others.
2. The fiber volume fraction is directly proportional to the natural frequency under torsional free vibration.
3. The natural frequencies are more susceptible to the longitudinal modulus of elasticity than to the transverse one.

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Table 1: Numerical and analytical values of natural frequency of the rod of interest

Type of composite	f_n (Cycle/sec) Numerical Soution	f_n (Cycle/sec) Analytical solution	Convergence Percentage
Kevlar 49-Epoxy	123	144.260571	85.26%
Kevlar 49-Polyester	157.48	188.016895	83.76%
Kevlar 49-Polypropylene	116.88	138.5419283	84.36%
Kevlar 49-Polyamid	234.51	278.7060848	84.14%
E-Glass –Epoxy	106.02	123.8654923	85.59%
E-Glass -Polyester	135.46	164.0335492	82.58%
E-Glass -Polypropylene	98.495	118.2960421	83.26%
E-Glass -Polyamid	200.23	240.1721691	83.37%
Kevlar29-Epoxy	122.57	144.0380828	85.10%
Kevlar 29-Polyester	156.58	187.551067	83.49%
Kevlar 29-Polypropylene	116.56	138.3781001	84.23%
Kevlar 29-Polyamid	229.93	276.4334901	83.18%
CarbonT300-Epoxy	117.92	137.8583904	85.54%
CarbonT300-Polyester	151.3	181.1654591	83.51%
CarbonT300-Polypropylene	111.04	131.814878	84.24%
CarbonT300-Polyamid	227.73	269.921414	84.37

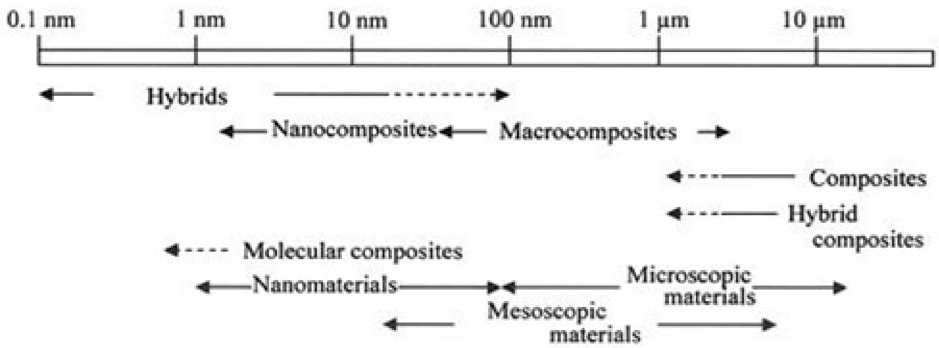

Figure-1. Classification of Materials at Different Scale Levels.

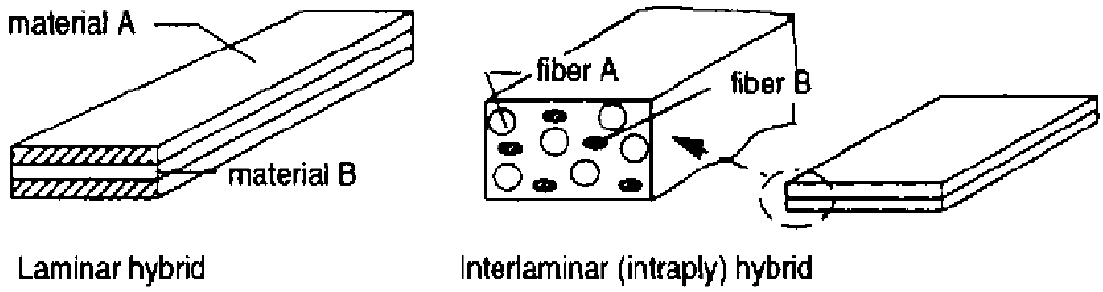

Figure-2: Laminar and Interlaminar (intraply) Hybrid Structures.

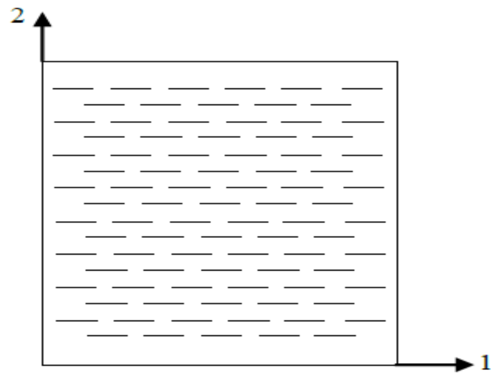

Figure-3: Unidirectional discontinuous Fiber matrix.

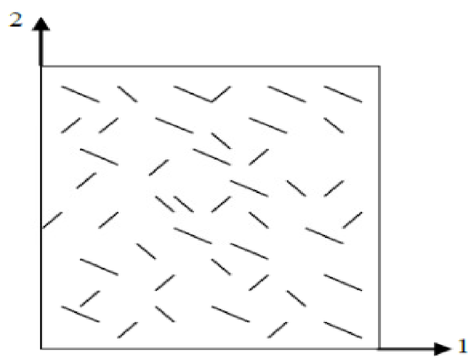

Figure-4: Randomly oriented discontinuous fiber matrix.

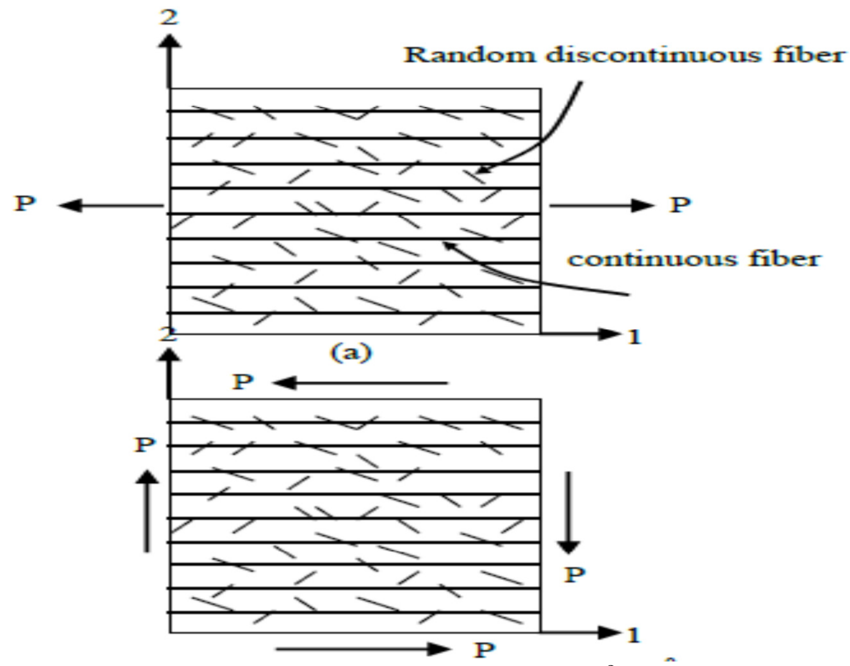

Figure-5: Unidirectional continuous Fiber (0 ^{o}) lamina.

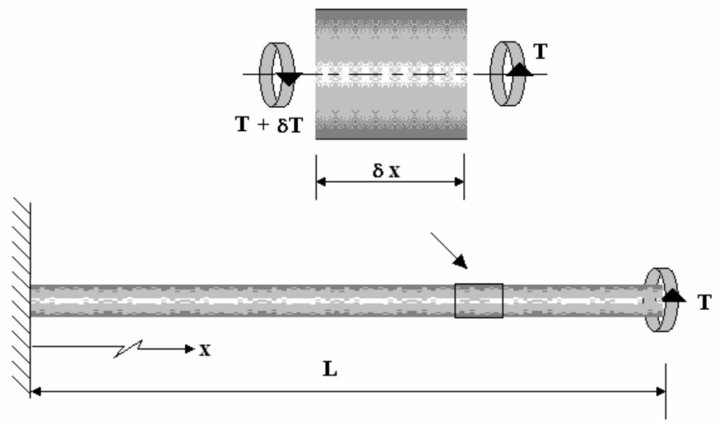

Fig. 6: A cantilever hybrid composite rod subjected to applied torque at the free end.

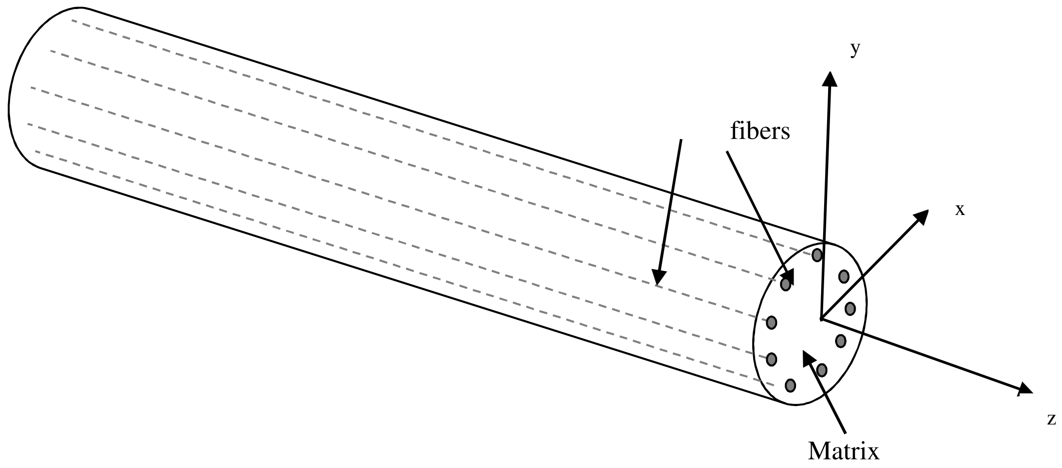

Figure-7: Hybrid fiber reinforced composite beam showing fibers direction and principal elastic properties axes.

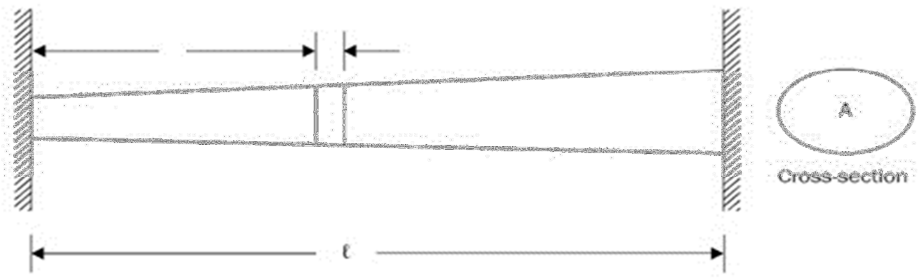

Fig. 8: A bar (or rod) of length ℓ and cross-sectional area A.

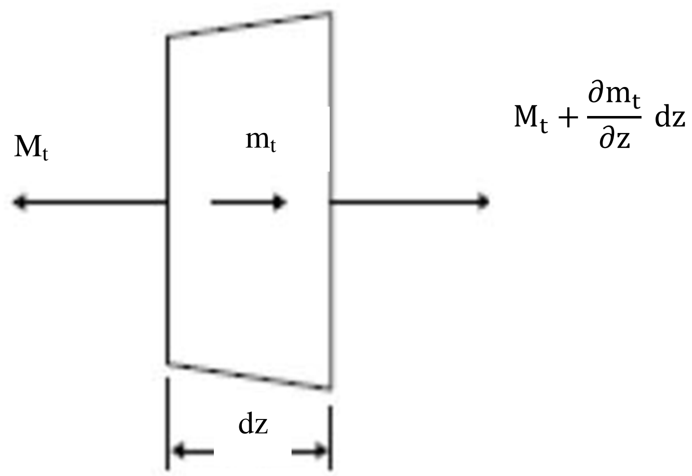

Fig. 9: Free body diagram of a typical element of length dz subjected torsional moments.

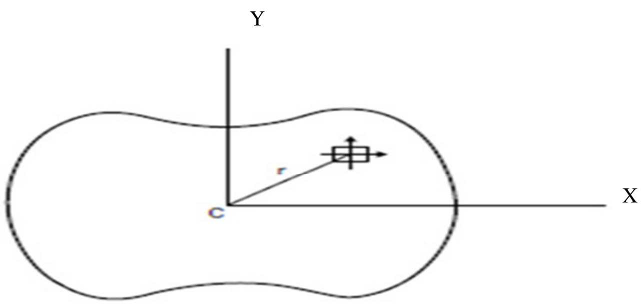

Fig.10: Internal shear stresses τ_{yz} and τ_{xz} that result in a twisting moment M_t.

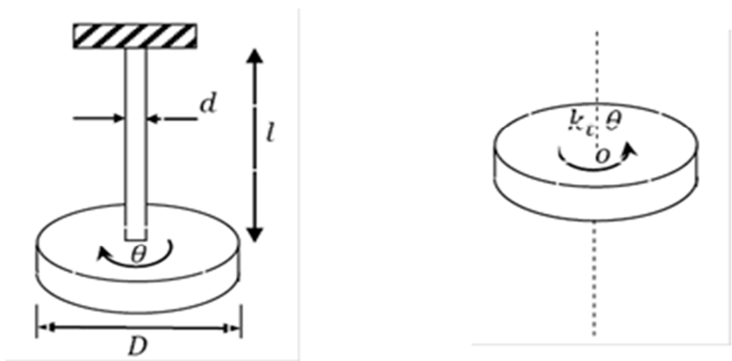

Fig. 11: A torsional undamped excited single degree of freedom.

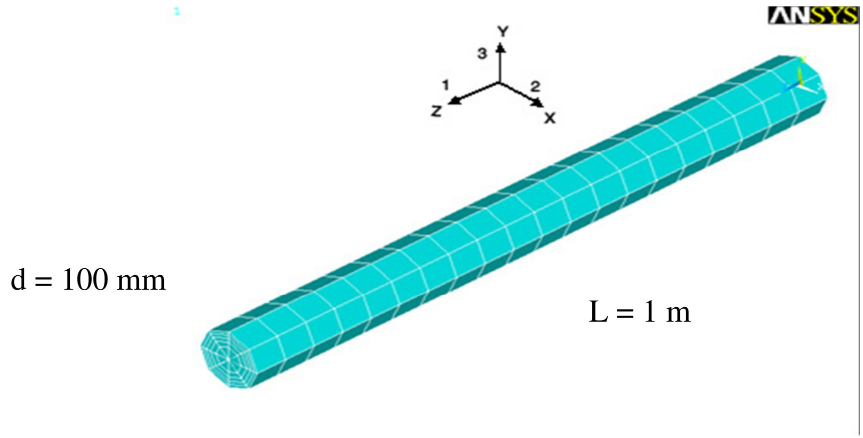

Fig. 12: Hybrid Fiber reinforced composite rod showing its meshing and principal coordinate and elastic properties axes.

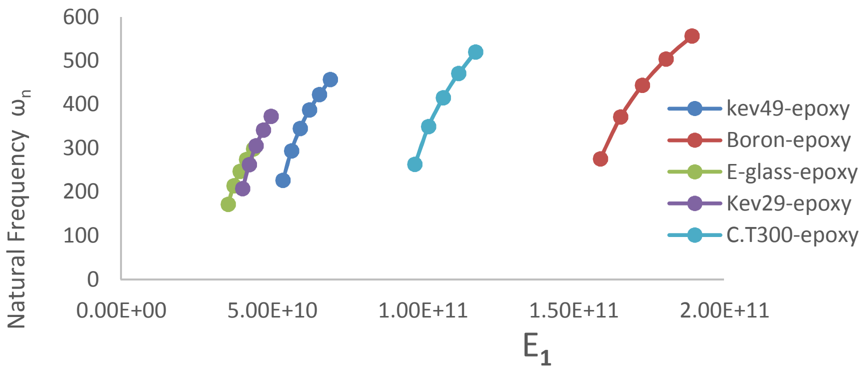

Fig. 13: Effect of Longitudinal Elastic Modulus on the Natural Frequency for epoxy.

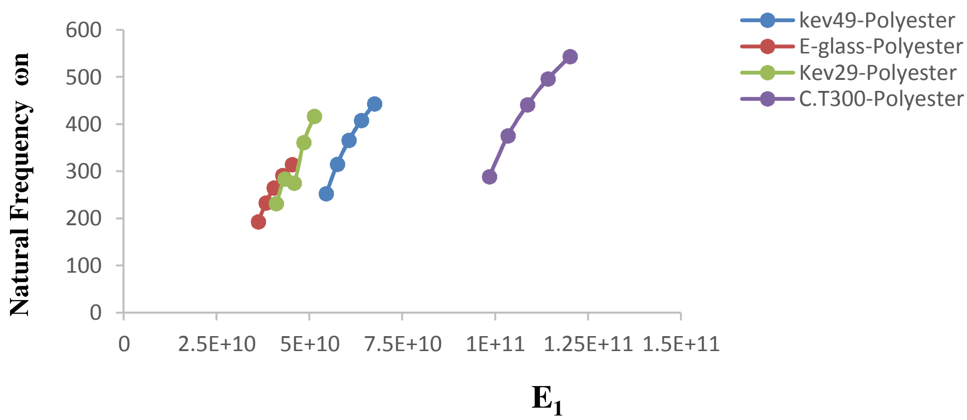

Fig. 14: Effect of Longitudinal Elastic Modulus on the Natural Frequency for polyester.

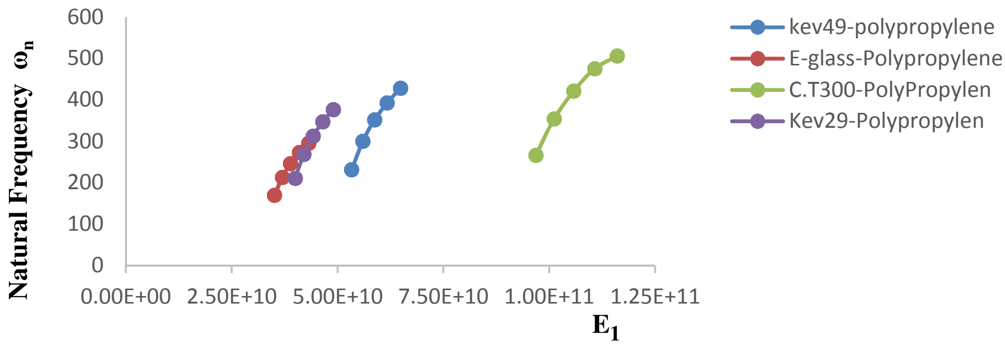

Fig.15: Effect of Longitudinal Elastic Modulus on the Natural Frequency for polypropylene.

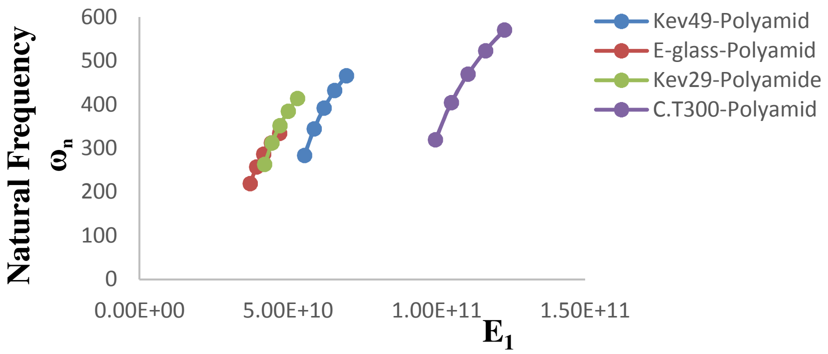

Fig. 16: Effect of Longitudinal Elastic Modulus on the Natural Frequency for polyamide.

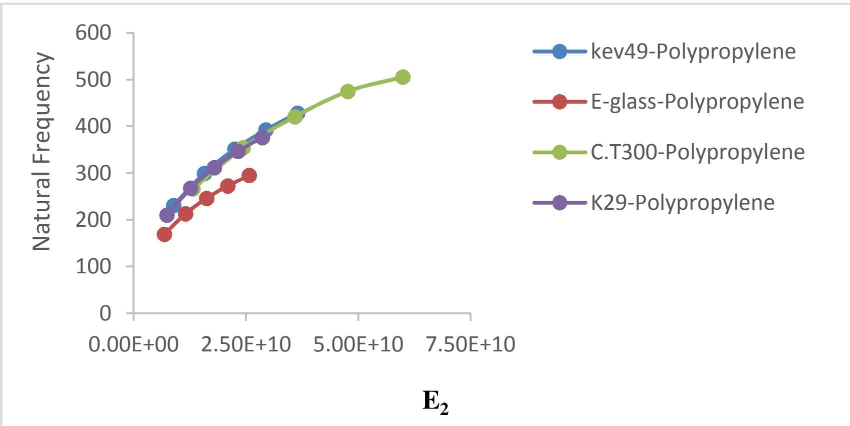

Fig. 17: Effect of Transverse Elastic Modulus on the Natural Frequency for polypropylene.

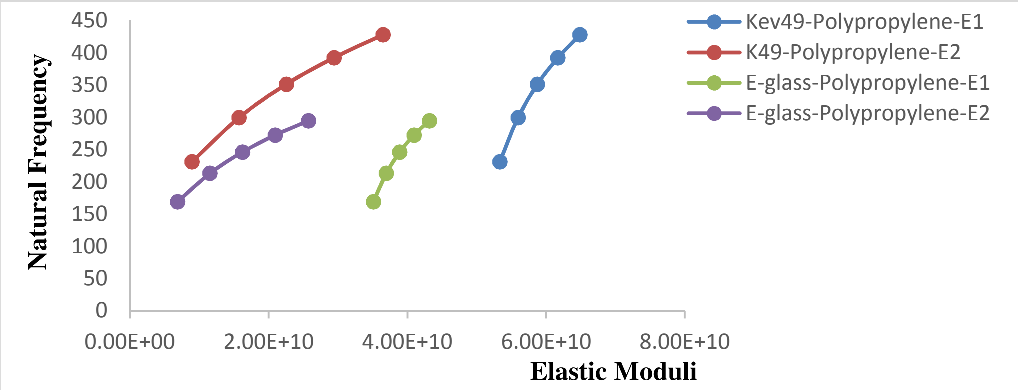

Fig. 18: Comparison of Longitudinal and Transverse Elastic Moduli Effect on the Natural Frequency for polypropylene.

Appendix-1:

Table-2: Properties of hybrid composite materials considered in this work.

Type of hybrid composite	E_1	E_2	G_{12}	ν_{12}	ν_{21}	ν_{23}	G_{23}	ρ Kg/m^3
Kevlar 49-Epoxy	6.24E+10	3.11E+10	4.59E+09	4.12E-01	2.05E-01	5.57E-01	9.98E+09	1371.6
Kevlar 49-Polyester	6.83E+10	4.38E+10	5.24E+09	0.419	2.68E-01	5.27E-01	1.43E+10	1315.2
Kevlar 49-Polypropylene	6.17E+10	2.94E+10	4.48E+09	4.17E-01	1.99E-01	5.73E-01	9.36E+09	1252.8
Kevlar 49-Polyamid	6.57E+10	3.83E+10	5.04E+09	3.91E-01	2.28E-01	4.96E-01	1.28E+10	1317.6
E-Glass –Epoxy	4.15E+10	2.22E+10	7.74E+09	3.51E-01	1.88E-01	4.40E-01	7.71E+09	2114
E-Glass -Polyester	4.28E+10	2.50E+10	8.83E+09	3.39E-01	1.98E-01	4.12E-01	8.85E+09	2096
E-Glass -Polypropylene	4.10E+10	2.09E+10	7.25E+09	3.57E-01	1.82E-01	4.54E-01	7.20E+09	1995.2
E-Glass -Polyamid	4.44E+10	2.82E+10	1.01E+10	3.28E-01	2.08E-01	3.86E-01	1.02E+10	2060
Kevlar29-Epoxy	4.71E+10	2.46E+10	3.28E09	4.06E-01	2.12E-01	5.39E-01	8.00E+09	1371.6
Kevlar 29-Polyester	4.85E+10	2.76E+10	3.43E+09	3.95E-01	2.24E-01	5.06E-01	9.15E+09	1353.6
Kevlar 29-Polypropylene	4.65E+10	2.33E+10	3.20E+09	4.12E-01	2.06E-01	5.56E-01	7.47E+09	1252.8
Kevlar 29-Polyamid	5.01E+10	3.10E+10	3.59E+09	3.83E-01	2.37E-01	4.74E-01	1.05E+10	1317.6
CarbonT300-Epoxy	1.12E+11	5.05E+10	1.25E+10	4.02E-01	1.81E-01	5.50E-01	1.63E+10	1582.8
CarbonT300-Polyester	1.14E+11	5.57E+10	1.35E+10	3.95E-01	1.92E-01	5.26E-01	1.82E+10	1564.8
CarbonT300-Polypropylene	1.11E+11	4.77E+10	1.20E+10	4.05E-01	1.74E-01	5.62E-01	1.53E+10	1464
CarbonT300-Polyamid	1.16E+11	6.08E+10	1.44E+10	3.86E-01	2.02E-01	5.02E-01	2.03E+10	1528.8

  * The hybrids mentioned above are composed of composite matrix with a volume fraction of 60% made of V_m= 36%, $V_{S.F}$= 24%, the short fibers are randomly distributed within the resin matrix. The composite matrix is reinforced by long Fibers with a volume fraction of 40%.
* The set of hybrid materials listed in the table above represents only one of the various sets of volume fractions of constituents considered in this work.