# TRANSIENT STRESS ANALYSIS OF SIMPLY SUPPORT BEAM EXCITED UNDER MOVING LOAD

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**ABSTRACT**

In this paper, an experimental and numerical study of dynamic deflection and dynamic bending stress of beam structure under moving load has been carried out. The moving load is constant in magnitude and travels at a uniform speed. The dynamic analysis of beam-type structure is done by taking three (39.24, 58.86 and 78.48) N concentrated loads; each one of them travels at two uniform speeds(0.2 and 0.25) m/s . The theoretical analysis is based On Euler- Bernoulli theory and Fourier series solution. A finite element model of a beam vibrating under moving load is established by ANSYS software. The comparison between the numerical dynamic stresses of beam via ANSYS software with the experimental results showed that the percentage error of (15%). Effect of the speed and load variables on the dynamic stress and dynamic deflection is investigated. It is concluded that the influence of variable speed on the dynamic deflection and dynamic bending stress is more than the effect of variable load on them. On the other hand, the dynamic bending stress is more sensitive to the variable speed and load than the dynamic deflection. Dynamic bending stress and dynamic deflection due to moving load could become significantly higher than those obtained with the stationary loads, thus they must be considered.

**Key words:** simply supported beam, Euler- Bernoulli theory, Fourier series solution, moving load

**الخلاصة**: يقدم هذا البحث دراسة عمليه وتحليليه عن الاجهاد والانفعال الديناميكي الناتجين عن اهتزاز العارضة قسريا تحت تأثير حمل يتحرك عليها بسرعه منتظمة، وتم ذلك باختيار ثلاثة احمال (٣٩.٢٤، ٥٨.٨٦ و ٧٨.٤٨) نيوتن كل واحد من هذه الأحمال يتحرك بسرعتين (٠.٢ و ٠.٢٥) م/ثا . بالاعتماد على نظريه أويلر- بيرنولي ومبدأ متسلسلة فورير حيث تم انجاز الجانب النظري من هذا البحث. باستخدام طريقة العناصر المحددة تم محاكاة عارضة مسنده بمسند بسيط في طرفيها تهتز قسريا بحمل يتحرك عليها بسرعة منتظمة بمساعدة برنامج ANSYS. بالمقارنة بين النتائج العملية ونتائج برنامج ANSYS تبين ان نسبة الخطأ لا تتجاوز الـ (١٥٪).

بعد ذلك تم أجراء دراسة عمليه عن تأثير تغير السرعة والحمل المنتقل على الاجهاد والانفعال الديناميكي لمنتصف العارضة المسندة بمسند بسيط في طرفيها، وقد تبين ان الاجهاد والانفعال الديناميكي يكون اكثر تأثرا بتغير السرعة عما هو حاصل نتيجة تغير الحمل. من جهة اخرى ان التغير في الاجهاد الديناميكي نتيجة تغير السرعة يكون اكبر بالمقارنة مع التغير الذي يحدث في الانفعال الديناميكي. واخيرا فأن النتائج اوضحت ان الاجهاد والانفعال الديناميكي الناتجين عن اهتزاز العارضة قسريا تحت تأثير حمل يتحرك

بسرعه منتظمة يكون اكبر من الاجهاد والانفعال الناتجين في الحالة الاستاتيكية عند منتصف العارضة. لذلك فان دراسة الاجهادات والانفعالات الديناميكية تحت تأثير الحمل المنتقل لابد من اخذها في عين الاعتبار.

## 1. INTRODUCTION:

In recent years, all branches of transport have experienced great advances, characterized by the increasingly high speed and weight of vehicles and other moving bodies. As a result, corresponding structures have been subjected to vibration and dynamic stress far longer than ever before. The moving load problem has been the subject of numerous research efforts in the last century. The importance of this problem is manifested in numerous applications in the field of transportation. Bridges, guide ways, overhead cranes, cableways, rails, roadways, runways, tunnels, launchers and pipelines are examples of structural elements designed to support moving loads so the literature concerning the forced vibration analysis of structures with moving bodies is sparse.

Many methods are applied to determine dynamic responses under moving load, some yield to exact solution, such as **Fryba** [1] who used Fourier sine (finite) and Laplace-Carson integral transformation to determine the dynamic response of beams due to moving loads exactly and obtained a response in the form of series solutions. **Michaltsos et al** [2] formulated and solved the transverse vibration of beam under moving load problem by using finite Fourier series. He presented the effect of load's speed on the dynamic response of beam. **M.A. Foda and Z. Abdul-Jabbar** [3] used a green function approach to study the dynamic response of a simply supported Bernoulli–Euler beam with finite length subjected to a moving load traversing with constant speed through its span. In addition to the previous methods, some researchers innovate numerical methods that yield to approximate solution which has an excellent agreement with the exact solution. **Hamada** [4] presented a method, based on the double Laplace transformation, to obtain the dynamic response of uniform Euler-Bernoulli vibrated under moving load. **H.P. Lee** [5] presented a numerical integration programs using the fourth order Runge-Kutta method to solve the equation of motion of Euler -Bernoulli beam for investigating the dynamic responses of both a simply supported beam and a fixed-fixed beam vibrating by moving load. **Husain Mehdi et al** [6] investigated the dynamic response of Euler-Bernoulli uniform beam under moving load, The finite element method and numerical time integration method (New mark method) were employed in the dynamic response analysis.

It should be noted, many researchers were applied different type of moving load. **Jing Ji et al** [7] studied the deformation of bridge subjected to vehicles with different velocities. Finite element model of bridge was established by ANSYS software. Through the numerical simulation analysis, dynamic response of the characteristics of the bridge body is acquired when the vehicle can be considered as concentrated load passes through the bridge at different speeds or as harmonic load with different frequents. **M.Mohsen**[8] analyzed the dynamic response of elastic homogeneous isotropic beams with various boundary conditions subjected to a harmonic force travelling with a uniform and variable velocity. **M.Abu-hilal [9]** investigated the dynamic response of elastic homogenous isotropic Euler-Bernoulli beam with general boundary conditions subject to random moving concentrated load. **Y.-H. Lin** [10] presented the theory for dynamic response of the beam-type structures vibrated by uniform partially distributed moving loads.

In this paper, dynamic deflection and dynamic bending stress of copper alloy (c85700) simply supported beam under moving load are presented. The moving load is constant in magnitude and travels at a uniform speed. The dynamic analysis of beam-type structure is done by taking three (39.24, 58.86 and 78.48) N concentrated loads; each one of them travels at two uniform speeds(0.2 and 0.25) m/s. On the basis of Euler- Bernoulli theory and Fourier series solution, the theoretical analysis is presented. A finite element model of a beam vibrating under moving load is established by ANSYS software version (11).

## 2. THEORETICAL PRESENTATION:

Consider a simply supported beam subjected to a concentrated force F (constant magnitude) moves towards the right with a constant speed, as illustrated in **Figure (1).** When the time is equal to 0, F is located in the left supporting place, and when the time is equal to T, F moves to the right supporting place, According to the vibration analysis, The governing differential equation that describes the vibration of beam under moving load is[11] ;

$$\mathrm{EI}\frac{\partial^4 y_{(x,t)}}{\partial x^4} + \rho A\frac{\partial^2 y_{(x,t)}}{\partial t^2} = f_{(x,t)} \tag{1}$$

$y_{(x,t)}$ : beam deflection at point $x$ and time $t$, $x$ an length coordinate with the origin at the left-hand end of the beam, $t$: time coordinate, E young's modulus of elasticity of the beam, I second moment of inertia of the beam cross section, $\rho$ density of the material of beam, $A$ area of cross section of beam and $f_{(x,t)}$ : external force .

The concentrated force is represented by using a Fourier series. For this, the concentrated load *P* effecting at $x = d$ is assumed to be distributed uniformly over an elemental length 2Δx centered at $x = d$, as shown in Fig. (2). Now the distributed force, $f(x)$, can be defined as [12]:

$$f(x) = \begin{cases} 0 & \text{for } 0 < x < d - \Delta x \\ \frac{\mathrm{F}}{2\Delta x} & \text{for } d - \Delta x \le x \le d + \Delta x \\ 0 & \text{for } d + \Delta x < x < l \end{cases} \tag{2}$$

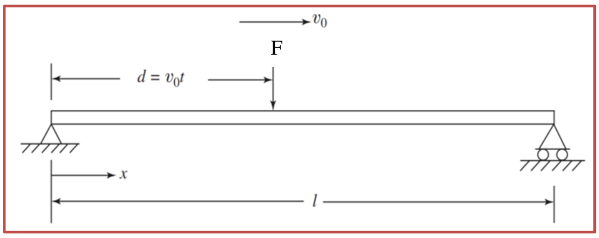


**Figure (1);** simply supported beam subjected to a moving concentrated load.

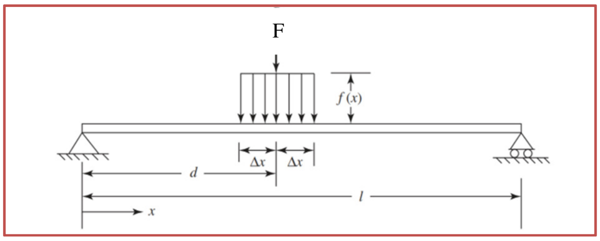


**Figure (2)** Concentrated load assumed to be uniformly distributed over a length 2Δx

From Fourier series analysis, it is known that if a function *f (x)* is defined only over a finite interval (e.g., from $x_0$ to $x_0 + L$), the definition of the function *f (x)* can be extended for all values of *x* and can be considered to be periodic with period *L*. The Fourier series expansion of the extended periodic function converges to the function *f (x)* in the original interval from $x_0$ to $x_0 + L$. As a specific case, if the function *f (x)* is defined over the interval 0 to *l*, its Fourier series expansion in terms of only sine terms is given by [13]

$$f_{(x)} = \sum_{n=1}^{\infty} f_n \sin\left(\frac{n\pi}{l} x\right) \tag{3}$$

Where,

$$f_n = \frac{2}{l}\int_0^l f_{(x)} * \sin\left(\frac{n\pi}{l} x\right) \partial x \tag{4}$$

According to equation (2), equation (4) yields to;

$$f_n = \frac{2}{l}\int_0^{d-\Delta x} (0) * \sin\left(\frac{n\pi}{l} x\right) \partial x + \frac{2}{l}\int_{d-\Delta x}^{d+\Delta x} \frac{p}{2\Delta x} * \sin\left(\frac{n\pi}{l} x\right) \partial x + \frac{2}{l}\int_{d+\Delta x}^{l} (0) * \sin\left(\frac{n\pi}{l} x\right) \partial x \tag{5}$$

Solution the integration and simplifying the equation (5) yields to:

$$f_n = \frac{2p}{l} \sin\left(\frac{n\pi}{l}\, d\right) \tag{6}$$

Substituting equation (6) into (3) yields:

$$f_{(x)} = \frac{2p}{l}\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{l}\, d\right) \sin\left(\frac{n\pi}{l} x\right) \tag{7}$$

Using **(d = u*t) in** Eq. (7), the load distribution will be represented in terms of *x and* t as:

$$f_{(x,t)} = \frac{2p}{l}\sum_{n=1}^{\infty} \sin\left(\frac{n\pi u}{l}\right) t \sin\left(\frac{n\pi}{l}\right) x \tag{8}$$

Now, substituting the equation (8) into equation (1) gives:

$$\mathrm{EI}\frac{\partial^4 y_{(x,t)}}{\partial x^4} + \rho A \frac{\partial^2 y_{(x,t)}}{\partial t^2} = \frac{2p}{l}\sum_{n=1}^{\infty} \sin\left(\frac{n\pi u}{l}\right) t \sin\left(\frac{n\pi}{l}\right) x \tag{9}$$

The above equation represents the governing partial differential equation for force transverse of beam traversed by moving load. To solve this equation;

$$y_{(x,t)} = y_{(x)} * y_{(t)} \tag{10}$$

From the modal analysis of simply supported beam [13], the Eigen function is $y_{(x)} = \sin\left(\frac{n\pi}{l}\right) x$ in above equation so it yields to

$$y_{(x,t)} = \sin\left(\frac{n\pi}{l}\right) x * y_{(t)} \tag{11}$$

Substituting equations (11) into equation (9) gives:

$$\mathrm{EI}y_{(t)}\left(\frac{n\pi}{l}\right)^4\sin\left(\frac{n\pi}{l}\right)x+\rho A\sin\left(\frac{n\pi}{l}\right)x\frac{\partial^2 y_{(t)}}{\partial t^2}=\frac{2p}{l}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi u}{l}\right)t\sin\left(\frac{n\pi}{l}\right)x \tag{12}$$

Multiply equation (12) by $\left\{\sin\left(\frac{n\pi}{l}\right)x\right\}$ and integrating it from 0 to L yield to:

$$\mathrm{EI}y_{(t)}\left(\frac{n\pi}{l}\right)^4+\rho A\frac{\partial^2 y_{(t)}}{\partial t^2}=\frac{2p}{l}\sin\left(\frac{n\pi u}{l}\right)t \tag{13}$$

From the modal analysis of simply supported beam [] , the Eigen value is $\omega_n=(n\pi)^2\sqrt{\frac{EI}{\rho AL^4}}$ in above equation so it yields to

$$\frac{\partial^2 y_{(t)}}{\partial t^2}+\omega_n y_{(t)}=\frac{2p}{l\rho A}\sin\left(\frac{n\pi u}{l}\right)t \tag{14}$$

Equation (14) can be observed as a nonhomogeneous second order ordinary differential equation, so that the solution is

$$y_{(t)}=A\sin\omega_n t+\mathrm{B}\cos\omega_n t+\frac{2p}{l\rho A}*\frac{1}{{\omega_n}^2-\left(\frac{2\pi u}{l}\right)^2}*\sin\left(\frac{2\pi u}{l}\right)t \tag{15}$$

Where, A and B are constants obtained from applying initial conditions for zero displacement and zero velocity gives:

$$y_{(t)}=\frac{2p}{l\rho A}*\frac{1}{{\omega_n}^2-\left(\frac{2\pi u}{l}\right)^2}*\left[\sin\left(\frac{2\pi u}{l}\right)t-\left(\frac{2\pi u}{l\omega_n}\right)\sin\omega_n t\right] \tag{16}$$

Substituted equation (16) in equation (11) yields to:

$$y_{(x,t)}=\frac{2p}{l\rho A}*\frac{1}{{\omega_n}^2-\left(\frac{2\pi u}{l}\right)^2}*\left[\sin\left(\frac{2\pi u}{l}\right)t-\left(\frac{2\pi u}{l\omega_n}\right)\sin\omega_n t\right]\sin\left(\frac{n\pi}{l}\right)x \tag{17}$$

Rearrangement equation (17) and using $\omega_n=(n\pi)^2\sqrt{\frac{EI}{\rho AL^4}}$ yields to:

$$y_{(x,t)}=\frac{2pl^3}{EI\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}*\frac{1}{1-\left(\frac{2\pi u}{l{\omega_n}^2}\right)^2}\sin\left(\frac{n\pi}{l}\right)x\left[\sin\left(\frac{2\pi u}{l}\right)t-\left(\frac{2\pi u}{l\omega_n}\right)\sin\omega_n t\right] \tag{18}$$

The above equation represents the dynamic response of transverse vibration of a uniform Euler - Bernoulli beam traversed under moving load. Simply supported boundary conditions are considered.

## 3. EXPERIMENTAL INVESTIGATIONS:

The experimental work divided in three parts, the first part is interested with free transverse vibration of simply supported beam, the second part is mainly focused on transverse vibration of beam traversed under moving load and the third part investigated the dynamic stress due to moving load.

### 3.1 Free Transverse Vibration of Simply Supported Beam:

The vibration test involves studying the fundamental natural frequency for the beam. The material of beam was brass alloy (c 85700 ), **Table (1)** illustrated the mechanical properties of alloy that used in this test , dimension of beam was (0.01× 0.025×0.84) m**,** boundary conditions taken as a simply supported beam**,** where the used technique depended on reference [14]. **Figure (3)** illustrates the beam sample which is tested to evaluate the fundamental natural frequency. **Figure (3)** consists of the following parts:

1-Rig structure :It is one of the most important devices of vibration Laboratories called universal vibration apparatus (tm16) produced by Technical Teaching Equipment For Engineering Company[15].

2-The supporting conditions in the ends of beam are rolling supported in the left end and pinned at right.

3-Impact hammer of model (086C01-PCB Piezotronics vibration division) is used.

4-The amplifier is used with the model No. (480E09).

5-Digital storage oscilloscope model (ADS 1202CL+) and serial No.01020200300012 is employed.

6-The model of accelerometer (352C68) is also used.

### 3.2 Force Transverse Vibration of simply supported beam By Moving Load Test.

Force transverse vibration of uniform beam by moving load test involves studying the dynamic response due to the moving load. The experimental results are obtained where constant moving load with uniform speed passed through beam span from lift to right. In this test, ,three different moving loads are applied, each one of them travels at two uniform speeds over beam span, see **Table (2)** .The Global Coordinates Center at left end supported. The start point of load was (70 mm) and end point was (720 mm) from Global Coordinates Center.

The rig of this test manufactures to complete the practical requirements illustrated in **Figure (4**). **Figure (4)** consists of the following parts

1. Rig stricture.
2. Beam.
3. Supported part.
4. Carriage for moving load.

The design and implementation of carriage should satisfy the practical requirements of moving load problem, which are:

- Minimum attached area between the carriage and beam to decrease the friction.
- Simple and easily assembling and disassembling with beam.
- Rolling motion should not cause sliding between the beam and carriage during motion.
- There are no adverse effects on the beam surface during motion.
- The possibility of moving with any speed.

- Can carry any load into the elastic reign.
- Carriage designed in minimum size to consider the carriage and the load as concentrated load.

Solidworks software was used to illustrate the carriage design **Figure (5)** depicting the final shape of carriage and the drawing in SOLIDWORKS software.

5. Masses:
6. Dc motor gear box.

The **Table (3)** lists information about the Dc motor gear box. A pulley is attached to the motor and connected with a metallic string which links the pulley with the carriage to transient the motion.

7. Power supply model PS-305D.
8. Measure unit: It consists of minor power supply , data acquisition model 6009, accelerometer model ADXL335, wire and on/off switch
9. Computer pc: where the Lab view program installed and by help of sound and vibration tools in this program the single and double integration operation will be done, which yield to in axel form tables and curves of velocity and amplitude vs time.

### 3.3 Dynamic Stresses By Moving Load

The purpose of the experimental work is to estimate the dynamic bending stress at mid span beam produced from force transverse vibration due to moving load. Through the observation of the **Figure (6)**, the moving load position is obtained according to variables (a and b) instantaneously .the moving load is located at the three prospects as illustrated [16];

1 – Moving load travels before mid-span beam (0≤a≤ L/2),

$$M = \frac{pbx}{l} \tag{19}$$

Where, $x = {}^{l}/_{2}\ and\ b = (l - a)$

Substituting the above variable in equation (19) and rearrangement yields:

$$p = \frac{2M}{(l - a)} \tag{20}$$

Substitute equation (20) in equation (18) gives:

$$M = \frac{EI\pi^4 y_{(x,t)}(l - a)}{4l^3 \sum_{n=1}^{\infty} \frac{1}{n^4} * \frac{1}{1 - \left(\frac{2\pi u}{l\omega_n{}^2}\right)^2} sin\left(\frac{n\pi}{l}\right) x \left[sin\left(\frac{2\pi u}{l}\right) t - \left(\frac{2\pi u}{l\omega_n}\right) sin\, \omega_n t\right]} \tag{21}$$

2 – Moving load travels after mid-span beam (L/2≤a≤L),

$$M = \frac{pbx}{l} - p(x - a) \tag{22}$$

Where, $x = {}^{l}/_{2}\ and\ b = (l - a)$, so substituting those variables in equation and rearrangement gives:

$$p = \frac{2M}{a} \tag{23}$$

Substitute equation (23) into equation (18) gives;

$$M = \frac{EI\pi^4 y_{(x,t)} a}{4l^3 \sum_{n=1}^{\infty} \frac{1}{n^4} * \frac{1}{1-\left(\frac{2\pi u}{l\omega_n{}^2}\right)^2} sin\left(\frac{n\pi}{l}\right) x \left[sin\left(\frac{2\pi u}{l}\right) t - \left(\frac{2\pi u}{l\omega_n}\right) sin\,\omega_n t\right]} \tag{24}$$

The experimental result of previous test ($y_{(x,t)}$) is applied into equation (26) and equation (29) to obtain the dynamic bending moment. The flexure formula is employed to convert the dynamic bending moment to dynamic bending stresses as shown,

$$\sigma_{bending} = -\frac{My}{I} \tag{25}$$

Where,

$y$: The distance from neutral axis to any point in the cross section, in the present work, this point is located at the bottom surface, $y = {}^{h}/_{2} = -0.005m$

$I$ : Second moment of area

$$I = \frac{bh^3}{12}$$

$b_{(width\ of\ beam)} = 0.025m$ and $h_{(high\ of\ beam)}=0.01m$ so I= 2.083 $m^4$

## 4. FINITE ELEMENT MOEL

To simulate the moving load on beam problem, the beam is drawn 2D as rectangle by two corners (0, 0) and (0.79, 0.01). **Figure (7);** show the finite element grid for the beam model. A (PLANE 82) was used to build the finite element model inside the frameworks described above; the element is defined by eight nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The comparison between the numerical and the experimental results showed that the PLANE 82 is suitable to simulate the transverse vibration under moving load for a simply supported beam. The plane stress option with unit thickness was used and scaled to the actual model thickness of 0.01 m. To specify the boundary conditions, all the nodes on right terminal end of the beam were selected and given zero displacement in both directions X and Y, while the lift terminal end of the beam were selected and given zero displacement in directions X only, as illustrated in **Figure (8)** .

### 4.1 Modal Analysis

Having obtained the finite element model, a modal analysis has been conducted on the beam. The global stiffness matrix [K] and global mass matrix [M] was obtained by assembling the element stiffness and mass matrices, respectively. The natural frequencies of the beam were obtained by solving the Eigen value problem given by the following equation:

$$[K]\{U\} = \omega_n^2 [M] \{U\} \quad (26)$$

Where, $\omega_n$ is the natural frequency of the system and {U} is the corresponding normalized Eigen vector (mode shape). The Eigen values and Eigen vectors were obtained using Block Lanczos method. The first five natural frequencies and corresponding normalized Eigen vectors were calculated using this technique. The first five natural frequencies of the beam obtained for the selected model are given in **Table (4).**

**4.2 Load distribution on beam:**

In order to conduct a static stress analysis, the loads have to be evaluated. The load on the mid span of the beam finite element model produces the largest bending stress. For node26, the maximum bending stresses are (-18.57 MPa, -27.9 MPa and -37.181 MPa) for (39.24 N, 58.86 N and 78.48 N), respectively. The magnitude of load at any point of contact on beam surface as the load moves from left end to the right end of beam depends on the moving speed. **Figure (10)** shows the magnitude of loads at various points along the path of contact. The normal load P acting on the beam of the proposed model is taken as (39.24 N, 58.86 N and 78.48 N), each one moves in two different speeds as ( 0.2 m/s and 0.25 m/s) respectively. Nodes starting from the extreme left end are (node 85) to the final point of contact and (node 53) of the model.

**4.3 Transient analysis**

After computing the natural frequencies and the mode shapes the dynamic response is obtained using the modal superposition technique [17]. The method is computationally efficient, particularly for a large sized problem.

**Time steps:**

The time of contact T of beam depends on speed of the moving load on beam contact surface. The time taken for any moving load can be divided into required number of intervals in the present work, eight intervals will be considered. One time step ΔT can be calculated by considering the number of modes, which are expected to contribute to the dynamic response. So, the first frequency of the model is (24.008 cycle/sec) which is taken from the finite element modal analysis results in Table (4);

$$\Delta T = \frac{1}{10 \times f_1} \quad (27)$$

The total time obtained are (3.25 sec) and (2.6 sec), respectively for the two indicated speeds. The total number of time steps for each speed can be evaluated by

$$\mathrm{TNTS} = \frac{total\ time}{\Delta T} \quad (28)$$

Where, TNTS is the total number of time steps. So, the total number of time steps for the model is 780 and 624, respectively for the two mentioned speeds which mean that there are about 97.5 and78 time step between each two nodes when speed is 0.2m/s and 0.25m/s, respectively, as shown in figure (3.8). At any time, two nodes are considered to calculate the load vector for that time step. The actual load at a point is distributed in inverse proportion to its distance from either node, to the two nodes under consideration. The Initial conditions for displacements and velocities for the beam in the proposed model are taken as zero for all degrees of freedom. The first mode is selected for the mode superposition technique. At each time interval, the acting load is calculated and fed into the mode superposition part in ANSYS software, and the corresponding deformation and stress are thus obtained. The dynamic displacement and dynamic stress in the Y direction in the mid span portion of the selected beam model (node 26 is mid-span of beam), are plotted in condition of the three indicated loads. This dynamic analysis is carried out for the following three speeds of moving load namely 0.2 m/s and 0.25 m/s respectively.

## 5. RESULTS AND DISCUSSION

The obtained results from the numerical and experimental work are discussion in this section. The results are divided into three parts, as mentioned in the previous section, first is the evaluation of the fundamental natural frequency of simply supported beam, obtain the dynamic response (amplitude) of beam under moving load, get the dynamic bending stresses of beam due to moving loads.

### 5.1 Free Transverse Vibration of Beam.

The results of this part include the calculation of the first mode of the natural frequency of beam type-structure. Through the analysis of the accelerometer signal with sigview software, the natural frequency of the beam was evaluated. This software is used to transform the signal obtained from time domain into frequency domain by using **FFT** function. The experimental signal is acquired from the oscilloscope and drawn via Excel Microsoft office, as shown in **Figure (9)** while **Figure (10)** illustrates the **FFT** function. The comparison between numerical natural frequency of beam via ANSYS software and experimental result showed that the percentage error between them is (4.09%). The natural frequency that evaluated numerically is (24.008Hz) and its value that computed experimentally is (23.026 Hz).

### 5.2 Force Transverse Vibration of Beam under Moving Load

The experimental and numerical results were obtained when a constant moving load with a uniform speed passed through beam span from left to right. In this part, there are three different moving loads, each one of them travels at two uniform speeds over beam span, see **Table (2).**

When the concentrated moving load (39.34 N) traveled with following speeds (0.2 and 0.25 m/s), the behavior of dynamic response is shown in **Figures (11), (13)** respectively**.** These Figures display the amplitude in meter unit in y- axis, while time of carriage motion represent in the x-axis. To validate the experimental result, finite element modeling by ANSYS software is achieved, as shown in **Figures (12) and (14).**

It's clear from these figures that the dynamic deflection due to moving load could become significantly higher than those obtained with the stationary loads because the obtained deflection is result from two component, due to static and dynamic of moving load . **Table (5)** illustrates the maximum dynamic deflection increase with increasing the speed of moving load, the percentage of increase of dynamic deflection according to static deflection also included in the **Table (5)** where, the maximum dynamic deflection for other cases shown in same table.

### 5.3 Dynamic bending stresses of beam due to moving load:

When the concentrated moving load (39.34 N) traveled with following speeds (0.2 and 0.25 m/s), the behavior of dynamic bending stresses is shown in **Figures (15) and (17),** respectively. These figures display the relationship between the dynamic stresses in y-axis, and time of carriage motion in x-axis.

To validate the experimental results, finite element modeling by ANSYS software is achieved as shown in **Figures (16) and (18).** It's clear from these figures that the dynamic stresses due to moving load could become significantly higher than those obtained with the stationary loads because the obtained bending stresses is result from two component, due to static and dynamic of moving load. **Table (6)** illustrates the dynamic bending stress increase with increasing the speed of moving load, the percentage of increase of dynamic bending stress according to static bending stresses also included in the **Table (6)**. As well the maximum dynamic bending stress for other cases shown in same table.

When the moving load reached to mid-span of beam, the beam vibrated freely via two components, weight of beam and the moving load at mid-span. The vibration of beam appears as a result of these two components. Dunkerley's method [18] was used to estimate the natural frequency of beam.

$$\frac{1}{\omega_n^2}=\frac{1}{\omega_{11}^2}+\frac{1}{\omega_{22}^2} \qquad \text{Dunkerley formula}$$

$\omega_n$: circular nature frequency of beam
$\omega_{11}$: experimental value of circular nature frequency due to beam's weight
$\omega_{22}$: circular nature frequency due to concentrated load

$$\omega_{22}=\sqrt{\frac{k}{M}} \quad (\frac{rad}{s}) \tag{29}$$

K is the stiffness of beam and can be calculated for mid-span of simply supported beam

$$k=\frac{48EI}{l^3} \tag{30}$$

By substitute equation (30) into (29):

$$\omega_{22}=\sqrt{\frac{48EI}{l^3 M_{moving\ load}}} \tag{31}$$

Where $M_{moving\ load}$ is mass of moving load

Substituting equation (31) in Dunkerley formula yields:

$$\frac{1}{\omega_n^2}=\frac{1}{\omega_{11}^2}+\frac{l^3 M_{moving\ load}}{48EI} \tag{32}$$

According to Dunckerley's principle, the natural frequency of the beam depends on the stiffness of beam. The stiffness of beam varies with position of moving load, so it can conclude that the natural frequency of beam had a value at each position of moving load. The dynamic response signal was analyzed with **FFT** function by using sigview program; the purpose of this analysis is to get the natural frequency of beam under moving load. As previously mentioned, there are two frequencies, constant due to uniform speed of moving load and variable natural frequency due to position of moving load along the beam span (Dunkerly's principle). The value of the ratio force frequency to the variable

nature frequency is varies with position of moving load along the beam span. Whenever this ratio is close to one, the concern of resonant is increasing.
The reason expected for the sudden drop in dynamic bending stress behavior at mid-span is the ratio of force frequency to variable natural frequency in mid-span had the highest value compared with another position of moving load along the beam span, as well as the static bending moment had a maximum value at mid-span beam according to the principle of strength of material [16].

## 6. CONCLUSIONS:

The main conclusions of this thesis are :

1. The natural frequency increases with the decrease of the load at mid-span. In this work, for the indicated moving loads the highest value occurred at moving load (39.24 kg) according to Dunkerley's principle .
2. The force frequency increases with increasing the speed of moving load and the highest value of force frequency occurred at 0.25 m/s .
3. A ratio of the force frequency to the natural frequency in mid-span had the highest value compared with another position of moving load along the beam span. When the load (78.48 kg) traveled with speed (0.25 m/s), this ratio had the greatest value that is (0.61).
4. The dynamic deflection and dynamic stresses due to moving load could become significantly higher than that obtained with the stationary loads.
5. The dynamic deflection and dynamic stresses increase with increase the speed of moving load and the magnitude of moving load.
6. During the process, the variation of speed and load effects on the dynamic deflection and dynamic bending stress. The influence of variable speed on the dynamic deflection and dynamic bending stress more than the effect of variable load on them. On the other hand, the dynamic bending stresses is more sensitive to the variable speed and load than the dynamic deflection.

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**Table (1):** mechanical properties of copper alloy (c85700)

| Mechanical properties | Value |
|---|---|
| Young's modules | 87 GPa |
| Passion ration | 0.35 |
| Density | 7904.76 kg/m$^3$ |
| Second moment of inertia | 2.083*10$^{-9}$ m$^4$ |

**Table (2):** Moving loads with uniform speeds.

| Masses(kg) | Load(N) | Speed(m/s) |
|---|---|---|
| 4 | 39.24 | 0.20 |
| | 39.24 | 0.25 |
| 6 | 58.86 | 0.20 |
| | 58.86 | 0.25 |
| 8 | 78.48 | 0.20 |
| | 78.48 | 0.25 |

**Table (3) :** Characteristics of Dc motor gear box

| | |
|---|---|
| Model: gmx-8pvo17d | Power rating : 27(W) |
| Current rating : 1(A) | Speed rating : 143(r.p.m) |
| Voltage rating : 19(V) | Efficiency :99% |

**Table (4):**The first five natural frequencies

| Mode | Natural frequencies(cycle/s) |
|---|---|
| 1 | 24.088 |
| 2 | 96.160 |
| 3 | 215.63 |
| 4 | 381.33 |
| 5 | 590.21 |

**Table (5):** maximum dynamic deflection and the rate of increase of the deflection

| load | maximum static deflection (m) | speed (m/s) | maximum dynamic deflection (m) | | The rate of increase % | |
|---|---|---|---|---|---|---|
| | | | experimentally | numerically | experimentally | numerically |
| 39.24 | -0.0022 | 0.2 | -0.002520416 | -0.0027 | 14.5643527 | 22.72 |
| | | 0.25 | -0.002878441 | -0.0035 | 30.83824176 | 59.090 |
| 58.86 | -0.0033 | 0.2 | -0.003482576 | -0.0041 | 5.532598664 | 24.24 |
| | | 0.25 | -0.003556374 | -0.005 | 7.768923564 | 51.51 |
| 78.48 | -0.0044 | 0.2 | -0.00460624 | -0.0056 | 4.687273403 | 27.27 |
| | | 0.25 | -0.0048 | -0.007 | 9.090909091 | 59.09 |

**Table (6):** maximum dynamic bending stress and the rate of increase of the stress

| load | maximum static bending stresses(Mpa) | speed (m/s) | Maximum dynamic bending stresses(Mpa) | | The rate of increase % | |
|---|---|---|---|---|---|---|
| | | | experimentally | numerically | experimentally | numerically |
| 39.24 | -18.57 | 0.2 | -27.3856 | -28 | 47.4722671 | 50.78082929 |
| | | 0.25 | -34.0552 | -30.708 | 83.38826064 | 65.3634895 |
| 58.86 | -27.9 | 0.2 | -37.8756 | -42 | 35.75483871 | 50.53763441 |
| | | 0.25 | -45.8542 | -46 | 64.35197133 | 64.87455197 |
| 78.48 | -37.181 | 0.2 | -49.713 | -56 | 33.70538716 | 50.6145612 |
| | | 0.25 | -54.0084 | -60.91 | 45.25806191 | 63.82023076 |

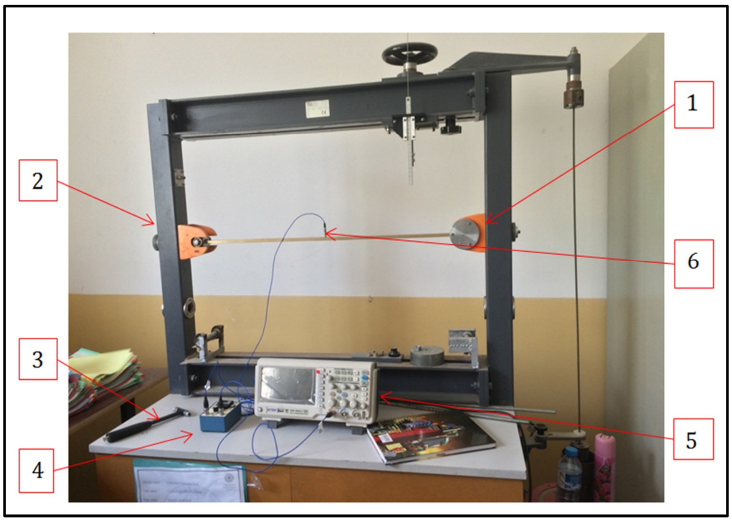



**Figure (3)**: Illustration of free vibration rig

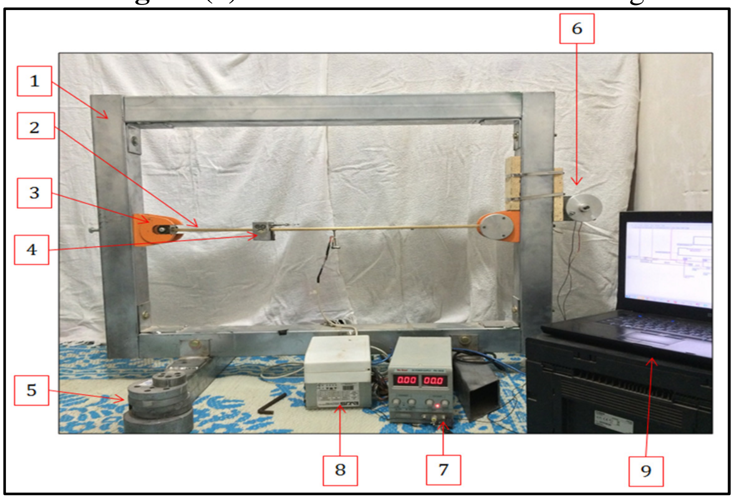
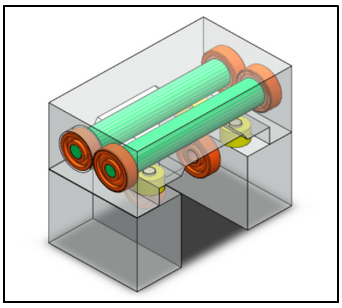
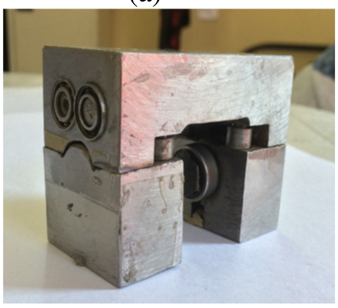



**Figure (4):** Rig of force vibration by moving load



(a)



(b)

**Figure (5)**: (a) drawing in SOLDWORKS and (b) final shape of carriage

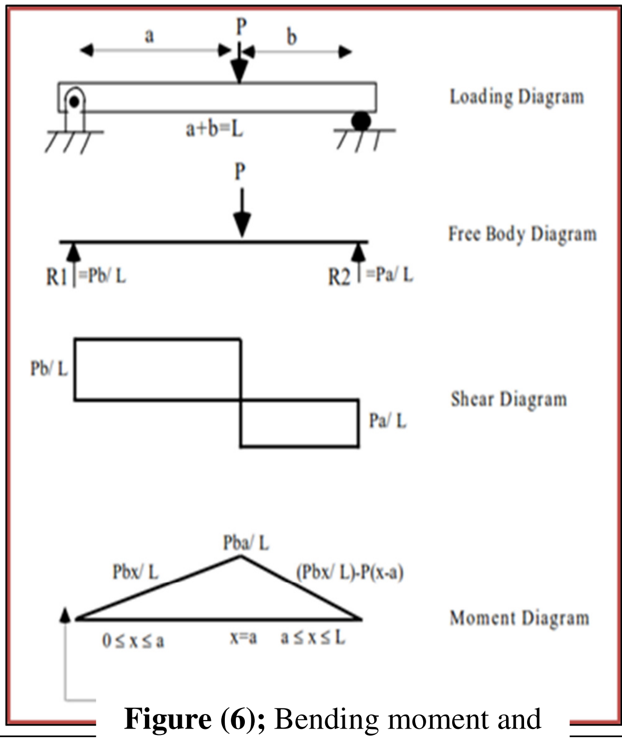



**Figure (6);** Bending moment and shear force diagram

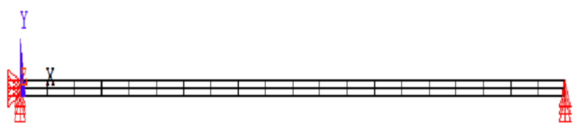



**Figure (7):** Finite element mash and boundary conditions for beam model

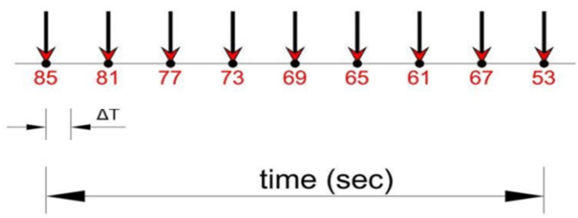



**Figure (8):** Load on various nodes of the selected model

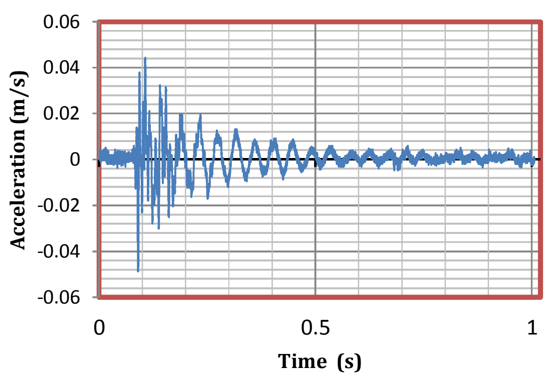



**Figure (9):** Experimental response in time domain.

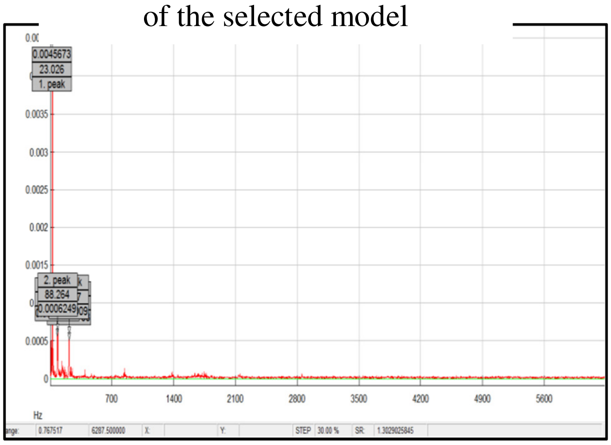



**Figure (10):** Analysis of the experimental signal by sigview software.

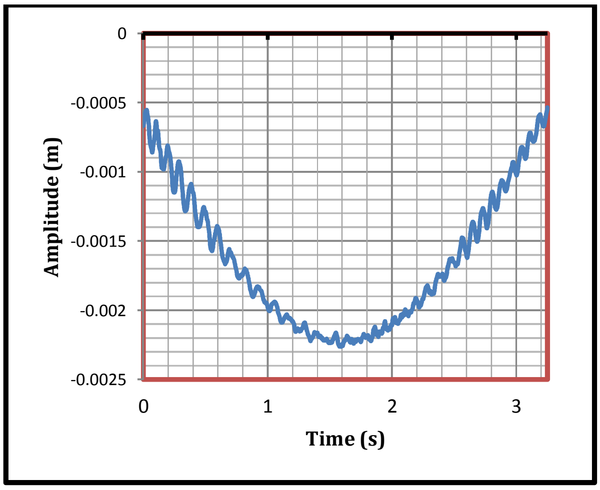



**Figure (11):** Experimental dynamic response of mid-span when load 39.24N travel at 0.2 m/s

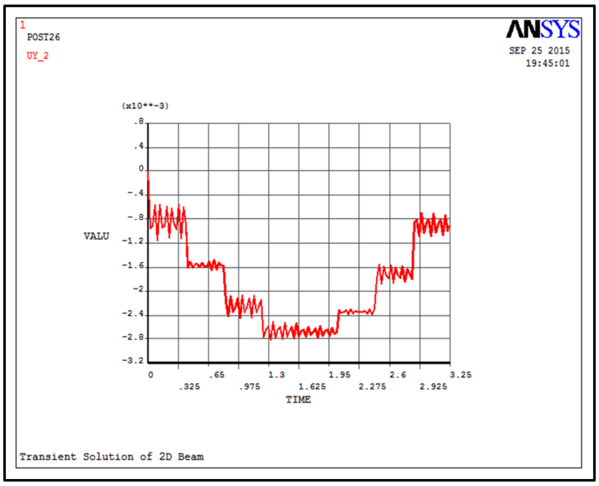



**Figure (12):** Numerical dynamic response of mid-span when load 39.24 N travels at 0.2 m/s

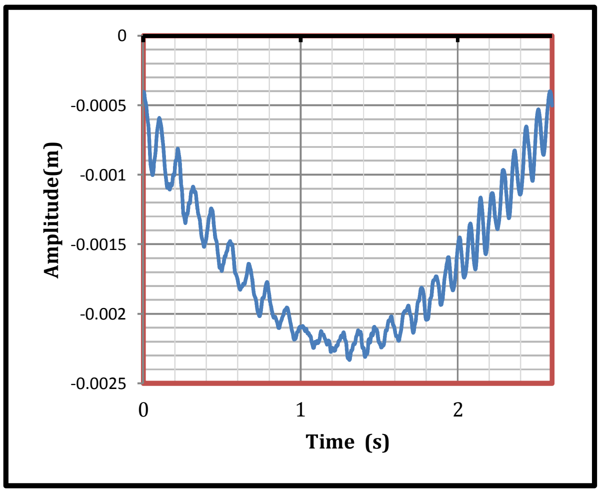



**Figure (13):** Experimental dynamic response of mid-span when load 39.24 N travels at 0.25 m/s

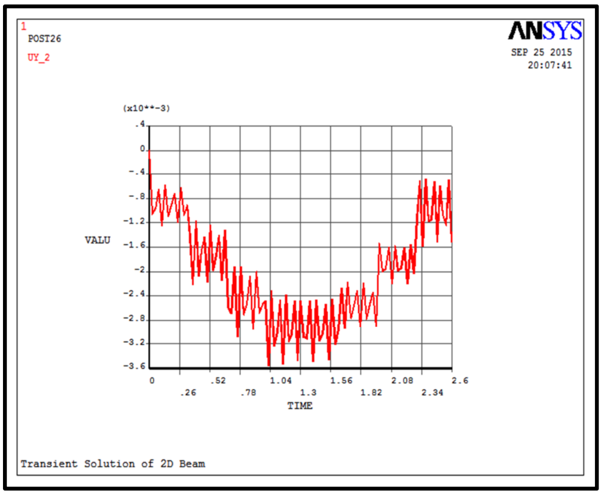



**Figure (14):** Numerical dynamic response of mid-span when load 39.24 N travels at 0.25 m/s

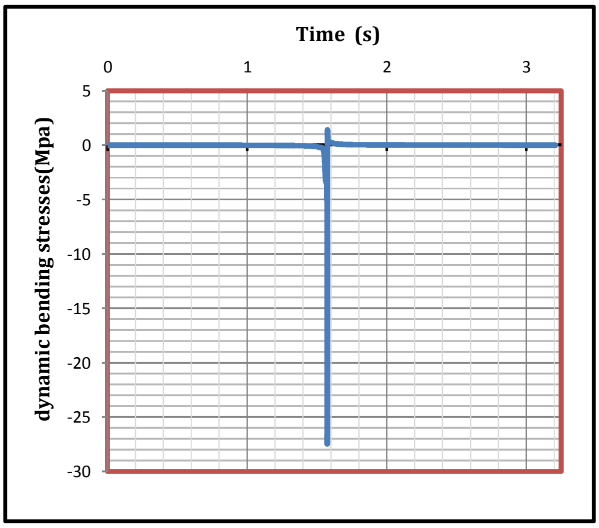



**Figure (15):** Experimental dynamic bending stresses of mid-span when load 39.24 N travels at 0.2 m/s

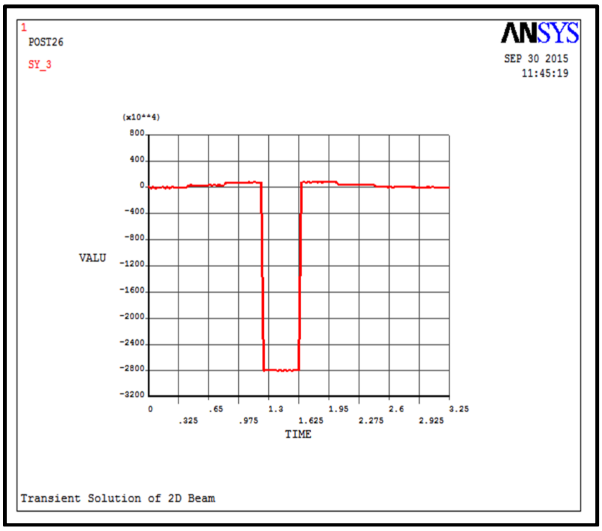



**Figure (16):** Numerical dynamic bending stresses of mid-span when load 39.24 N travels at 0.2 m/s

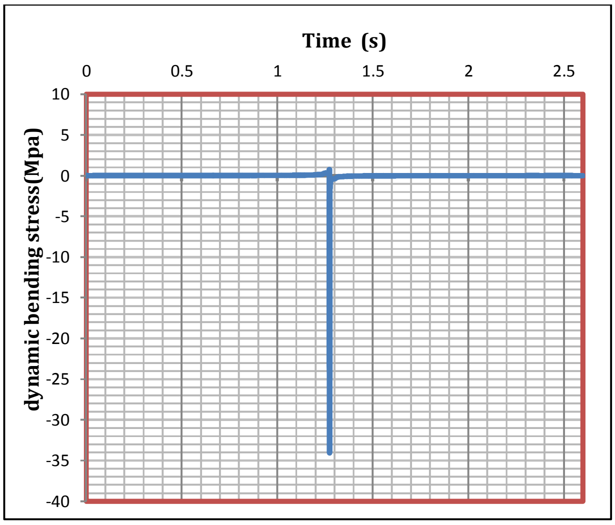



**Figure (17):** Experimental dynamic bending stresses of mid-span when load 39.24 N travels at 0.25 m/

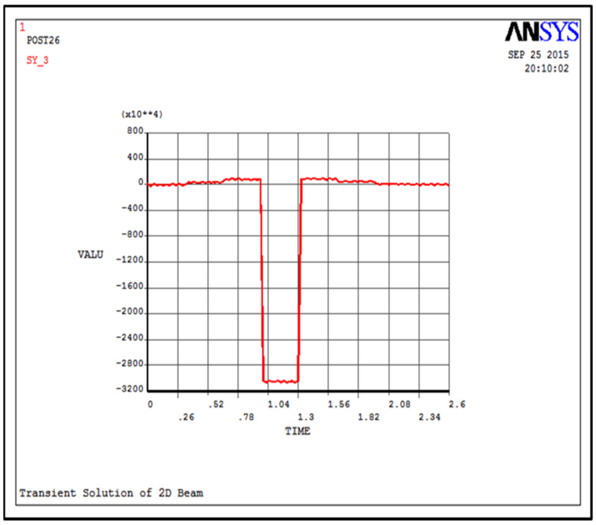



**Figure (18):** Numerical dynamic bending stresses of mid-span when load 39.24 N travels at 0.25 m/s