



## A STUDY OF THE EFFECT OF PARAMETERS ON THE STRESS IN HELICAL SPRING WIRE

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*Received on 21 June 2017*

*Accepted on 23 January 2017*

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**Abstract:** *The stress of the helical spring changes with several parameter are studied using (solidworks2014) program. The stress effect is by coil diameter (D) and the shape of section of the helical spring wire (circular ,rectangular and square) for the same diameter, where the stress on the helical spring wire increases by increasing the coil diameter (D), also the stress on the helical spring wire affect by changing the shape of the spring wire section .*

### 1. INTRODUCTION

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A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :[1]

1. To cushion, absorb or control energy due to either shock or vibration as in car springs , railway buffers, air-craft landing gears, shock absorbers and vibration dampers
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc. [2]

### 1.2. HELICAL SPRING

The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig. (a) and tension helical spring as shown in Fig. (b).[3]

### 1.3. THE AIM OF RESEARCH

- 1- Study the effect of changing the shape of the spring section ( circular , rectangular , square ) on the stresses generated in the spring wire and calculate the maximum shear stress.

2-study the effect of changing the coil diameter for the helical spring on the stresses generated in the spring wire and calculate the maximum shear .

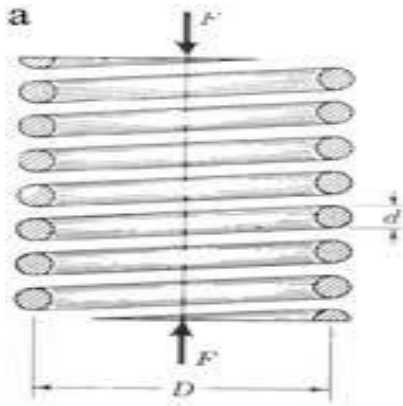


Fig.(a)

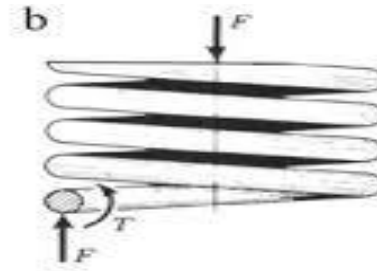


Fig.(b)

## 2. THEORETICAL DERIVATION OF SHEAR STRESS IN THE HELICAL SPRING WIRE

### 2.1. FOR CIRCULAR SPRING SECTION :

we have found out the direction of the internal torsion  $T$  and internal shear force  $F$  at the section due to the external load  $F$  acting at the center of the coil. The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the Fig(c). The broken arrows show the shear stresses ( $\tau_T$ ) arising due to the torsion  $T$  and solid arrows show the shear stresses ( $\tau_F$ ) due to the force  $F$ . It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress ( $\tau_T + \tau_F$ ) always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crack, is always initiated from the inner radius of the spring.[4]

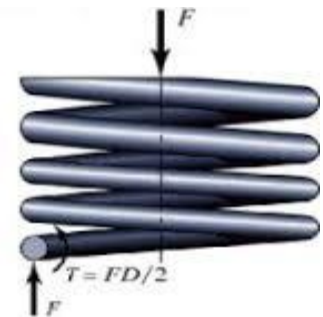


Figure (c)

The radius of the spring is given by  $D/2$ . Note that  $D$  is the mean diameter of the spring. The torque  $T$  acting on the spring is :

$$T = (F \cdot D) / 2 \dots \dots \dots (1)$$

If  $d$  is the diameter of the coil wire and polar moment of inertia,  $I_P = \pi d^4 / 32$  the shear stress in the spring wire due to torsion is

$$\tau_T = TR / I_P = ((F \cdot D) / 2 \cdot d / 2) / ((\pi d^4) / 32) = 8FD / (\pi d^3) \dots \dots \dots (2)$$

Average shear stress in the spring wire due to force  $F$  is

$$\tau_F = F / (\pi d^2 / 4) = 4F / (\pi d^2) \dots \dots \dots (3)$$



Therefore, maximum shear stress the spring wire is

$$\tau_T + \tau_F = 8FD/(\pi d^3) + 4F/(\pi d^2) \dots\dots\dots(4)$$

or 
$$\tau_{max} = 8FD/(\pi d^3) [ 1 + (1/(2D/d)) ] \dots\dots\dots(5)$$

or 
$$\tau_{max} = 8FD/(\pi d^3) [ 1 + (1/2C) ]$$
 , where  $C = D/d$  is called the spring index.

Finally 
$$\tau_{max} = ( k_s 8FD/(\pi d^3) )$$
 where,  $k_s = 1 + (1/2C)$

The above equation gives maximum shear stress occurring in a spring.  $k_s$  is the shear stress correction factor. [5]

## 2.2. FOR RECTANGULAR SPRING SECTION :

If  $a$  ,  $b$  is the length and width respectively of the spring wire section , the polar moment of inertia

$$IP = ((b a^3)/12)$$

The torque  $T$  acting on the spring is :

$$T = F D/2 \dots\dots\dots(1)$$

The shear stress in the spring wire due to torsion is

$$\tau_T = (T \cdot R)/IP = T \cdot (a/2) / ((b a^3)/12) = 6T/(b a^2) \dots\dots\dots(2)$$

$$\tau_T = (6F \cdot (D/2) ) / (b a^2) = (3F \cdot D) / (b a^2) \dots\dots\dots(3)$$

Average shear stress in the spring wire due to force  $F$  is

$$\tau_F = F/A = F/(b \cdot a) \dots\dots\dots(4)$$

Therefore, maximum shear stress the spring wire is

$$\tau_{max} = \tau_T + \tau_F = (3F \cdot D) / (b a^2) + F / (b \cdot a)$$

$$\tau_{max} = (3F \cdot D) / (b a^2) [ 1 + ( 1 / ( 3 D/a) ) ]$$

Where  $D/a = C$

$$\tau_{max} = (3F \cdot D) / (b a^2) [ 1 + (3 / C) ] \dots\dots\dots(5)$$



### 2.3. FOR SQUARE SPRING SECTION :

If  $a$  is the length of the spring wire section , the polar moment of inertia ,  $IP = a^4/12$  And  $D = (a/2)$  , the torque  $T$  acting on the spring is :

$$T = (F \cdot D)/2 \dots\dots\dots(1)$$

The shear stress in the spring wire due to torsion is

$$\tau_T = (T \cdot R)/IP = ( (T \cdot a)/2 ) / ( a^4/12 ) = 6T/a^3$$

$$\tau_T = (6F D/2)/a^3 = 3FD/a^3 \dots\dots\dots(2)$$

Average shear stress in the spring wire due to force  $F$  is

$$\tau_F = F/A = F/a^2 \dots\dots\dots(3)$$

Therefore , maximum shear stress the spring wire is

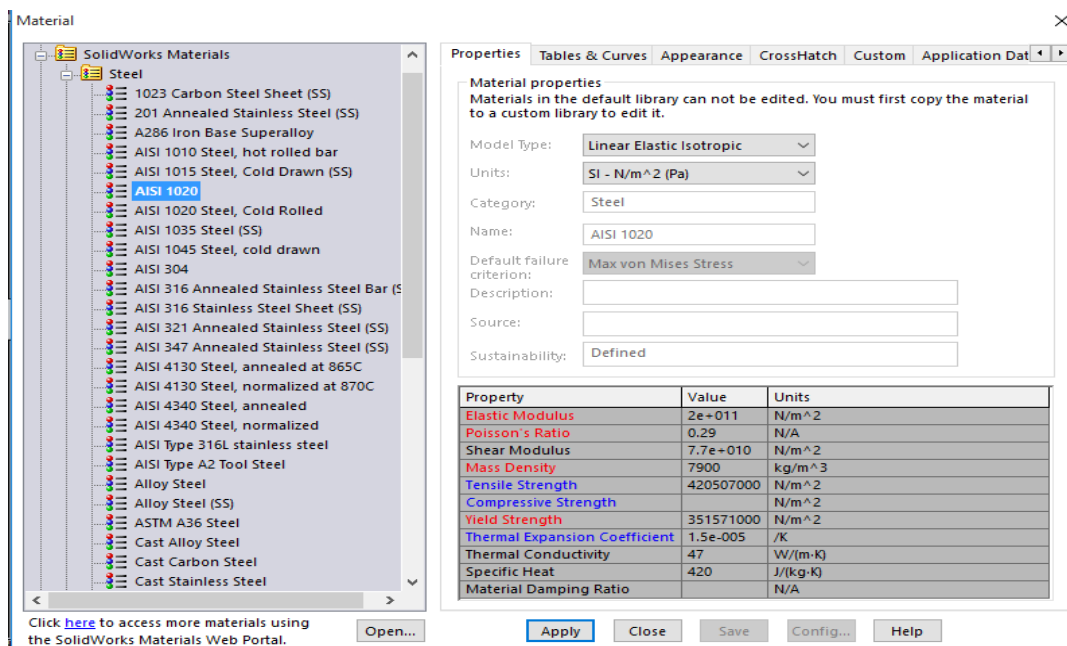
$$\tau_{max} = \tau_T + \tau_F = 3FD/a^3 + F/a^2$$

$$\tau_{max} = 3FD/a^3 [ 1 + ( 1 / ( 3 \cdot ( D/a ) ) ] \dots\dots\dots(4)$$

Where  $D/a = C$

$$\tau_{max} = 3FD/a^3 [ 1 + 1/3C ] \dots\dots\dots(5)$$

All quantities in above equations are now known. Steel AISI 1020 material database.





## 2.4. THEORETICAL SOLUTION :

### 2.4.1. CIRCULAR CROSS SECTION

$$D= 30 \text{ mm} \quad d = 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = \tau_{\max} = 8FD/(\pi d^3) [ 1+(1/2C) ]$$

$$\tau_{\max} = [ ( 8*100*0.03) / \pi (0.008)^3 ] * [ 1+ ( 1 / ( 2* (0.03/0.008) ) ) ] = 16910212 .7 \text{ N/m}^2$$

$$D= 40 \text{ mm} \quad d = 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = [ ( 8*100*0.04) / \pi (0.008)^3 ] * [ 1+ ( 1 / ( 2* (0.04/0.008) ) ) ] = 19894367.89 \text{ N/m}^2$$

$$D= 50 \text{ mm} \quad d= 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = [ ( 8*100*0.05) / \pi (0.008)^3 ] * [ 1+ ( 1 / ( 2* (0.05/0.008) ) ) ] = 26857396 .65 \text{ N/m}^2$$

### 2.4.2. SQUARE CROSS SECTION

$$D= 30 \text{ mm} \quad a = b = 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = 3FD/a^3 [ 1 + ( 1 / ( 3* ( D/a) ) ) ]$$

$$\tau_{\max} = [ ( 3*100*0.03) / ( 0.008)^3 ] * [ 1+ ( 1 / ( 3* (0.03/0.008) ) ) ] = 19140625 \text{ N/m}^2$$

$$D= 40 \text{ mm} \quad a = b = 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = 3FD/a^3 * [ 1+ (1/(3D/a) ) ]$$

$$\tau_{\max} = [ ( 3*100*0.04) / ( 0.008)^3 ] * [ 1+ ( 1 / ( 3* (0.04/0.008) ) ) ] = 25000000 \text{ N/m}$$

$$D= 50 \text{ mm} \quad a = b = 8 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = 3FD/a^3 [ 1 + ( 1 / ( 3* ( D/a) ) ) ]$$

$$\tau_{\max} = [ ( 3*100*0.05) / ( 0.008)^3 ] * [ 1+ ( 1 / ( 3* (0.05/0.008) ) ) ] = 30859375 \text{ N/m}^2$$

### 2.4.3. RECTANGULAR CROSS SECTION

$$D= 30 \text{ mm} \quad a =8 \text{ mm} \quad b = 6 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = (3F*D)/(ba^2) [ 1 + ( 1 / ( 3 D/a) ) ]$$

$$\tau_{\max} = [ ( 3*100*0.03) / (0.006)(0.008)^2 ] * [ 1+ ( 1 / ( 3*(0.03/0.008) ) ) ] = 25520833 \text{ N/m}^2$$

$$D= 40 \text{ mm} \quad a =8 \text{ mm} \quad b = 6 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = [ ( 3*100*0.04) / (0.006)(0.008)^2 ] * [ 1+ ( 1 / ( 3*(0.04/0.008) ) ) ] = 33333333 \text{ N/m}^2$$

$$D= 50 \text{ mm} \quad a =8 \text{ mm} \quad b = 6 \text{ mm} \quad F= 100 \text{ KN}$$

$$\tau_{\max} = [ ( 3 \cdot 100 \cdot 0.05 ) / ( 0.006 ) ( 0.008 ) ^ 2 ] * [ 1 + ( 1 / ( 3 \cdot ( 0.05 / 0.008 ) ) ) ] = 41145833 \text{ N/m}^2$$

### 3. ANALYSIS SETUP

A three dimensional helix was created and meshed in solid works 2014 . under these condition , static study advisor and one fixed end and one free end and constant load. The available design parameters of the helical to be the wire diameter (d) and the coil diameter (D) the helix diameter measured from the wire centerline . pitch of spring (p) is 12 mm and the number of revolutions (N) is 6 . for the three type of spring section (circular, rectangular, square) The helical spring parameters were taken from an existing prototype with parameters in tables (1-a) ,(1-b) and (1-c) respectively.

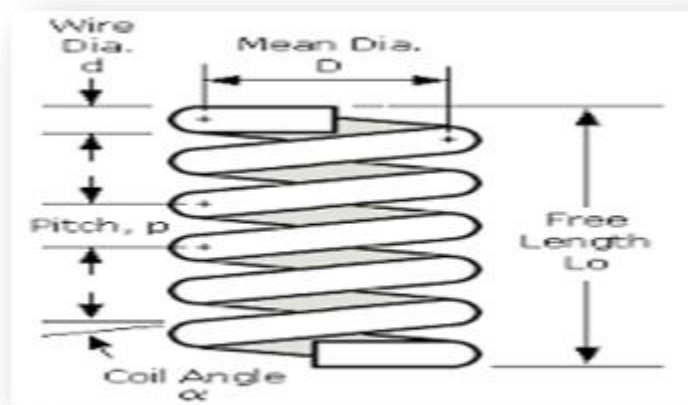


FIGURE (1) : HELICAL SPRING PARAMETER .

D	d	N	$\tau_{\max}$
30	8	6	80914616
40	8	6	114889872
50	8	6	128394864

Table (1-a) circular parameter

D	a*a	N	$\tau_{\max}$
30	8*8	6	95181256
40	8*8	6	114110088
50	8*8	6	142575808

Table (1-b) square parameter

D	a*b	N	$\tau_{\max}$
30	8*6	6	51650756
40	8*6	6	75681192
50	8*6	6	93188664

Table(1-c) rectangular parameter

Using the boundary conditions of one fixed end and one free end and constant load as show in figure (2-a) and (2-b) .

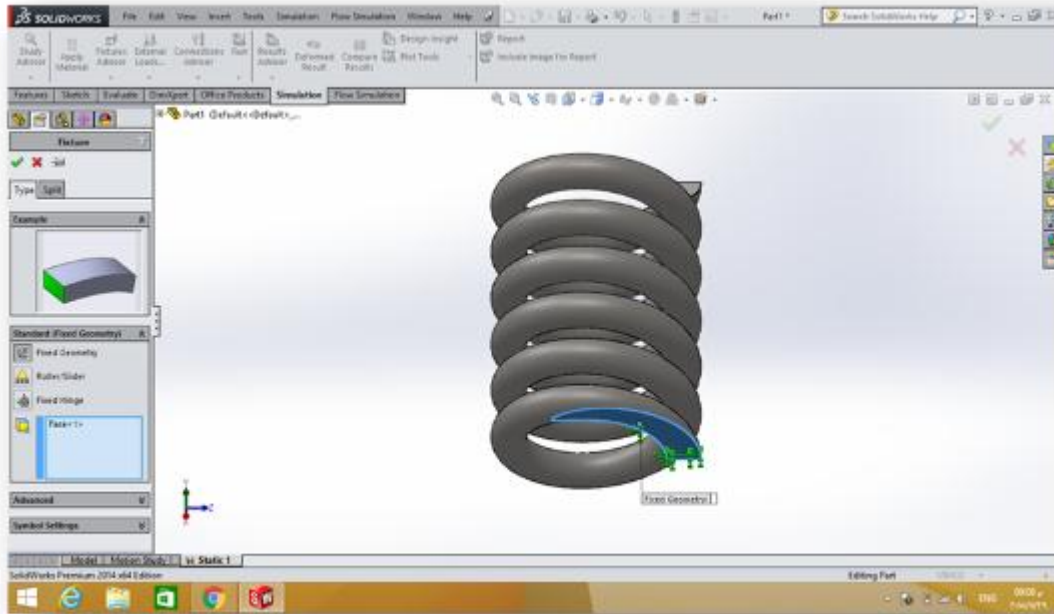


Figure ( 2-a )

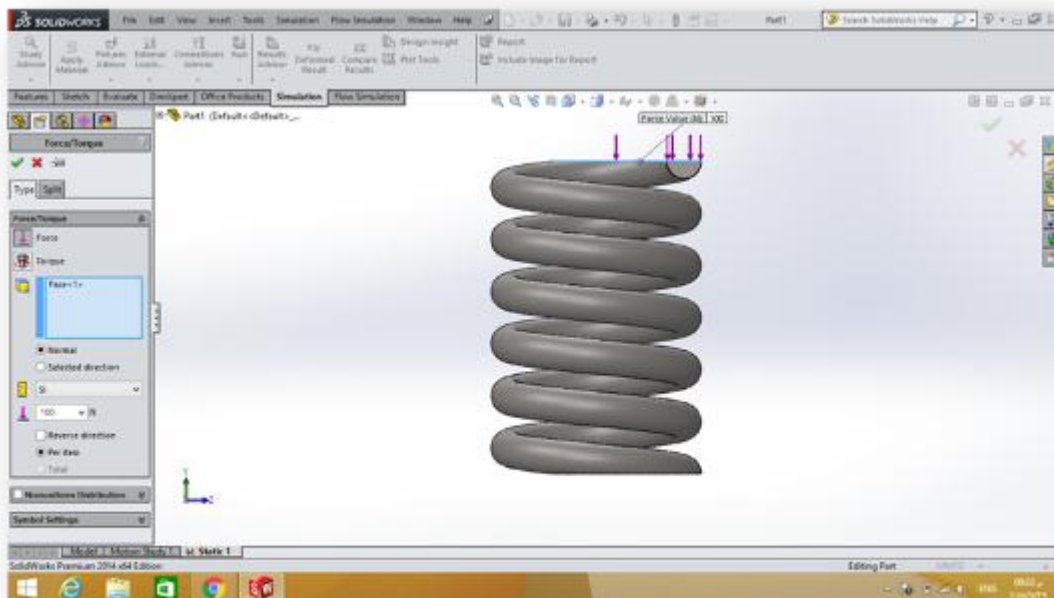
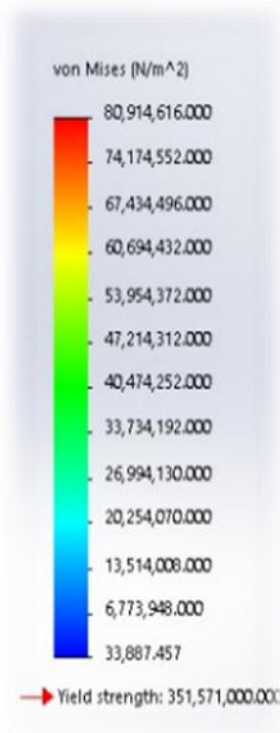


Figure ( 2-b )

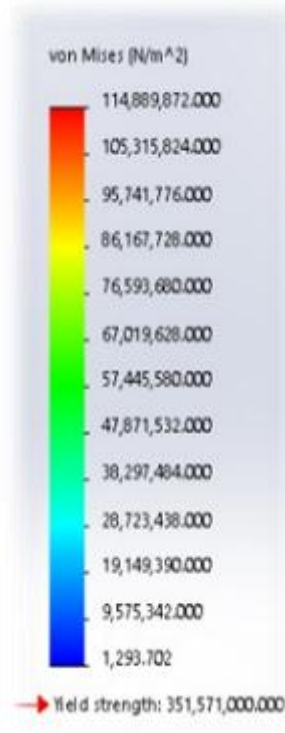
#### 4. ANALYSES MODELS

##### 4.1. FOR CIRCULAR SPRING SECTION :

For D = 30 mm



For D = 40 mm



For D = 50 mm

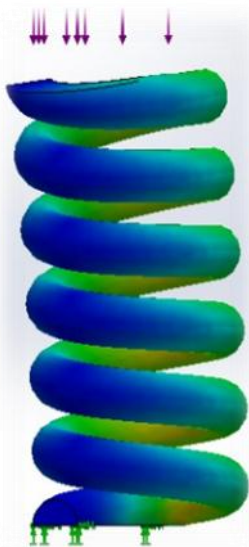
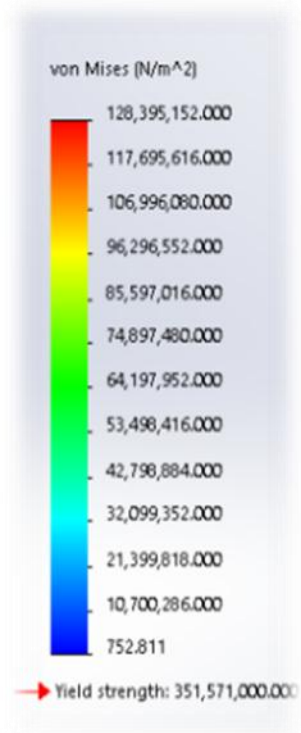


Figure (3-a)



Figure (3-b)

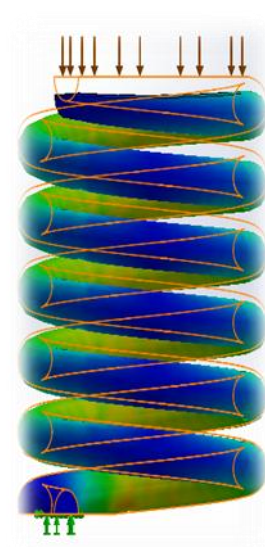
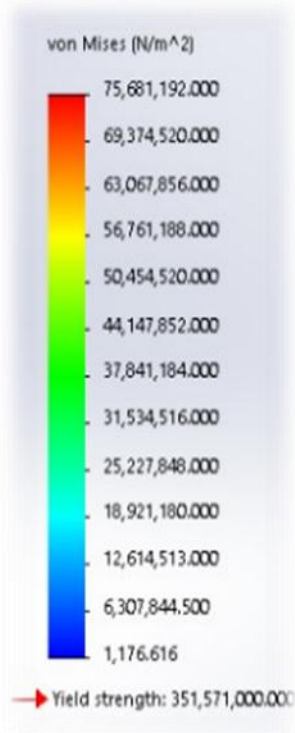


Figure (3-c)

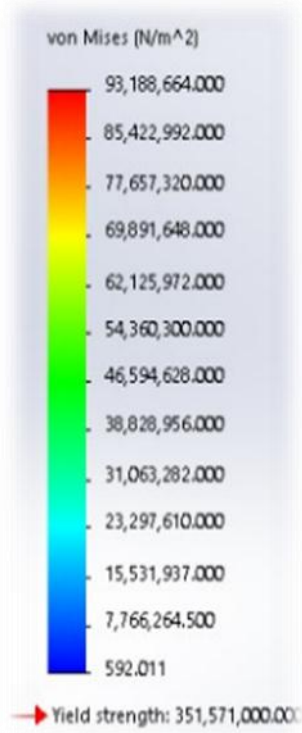


4.2. FOR SQUARE SPRING SECTION :

FOR D = 30 mm



FOR D = 40 mm



FOR D = 50 mm

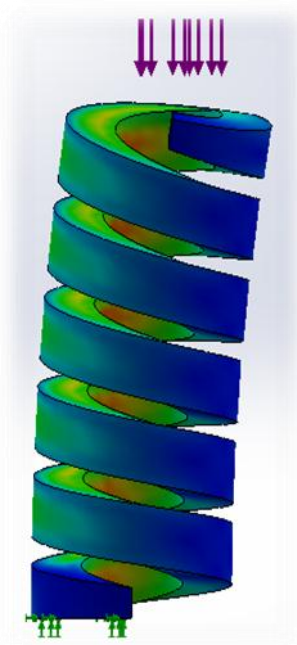
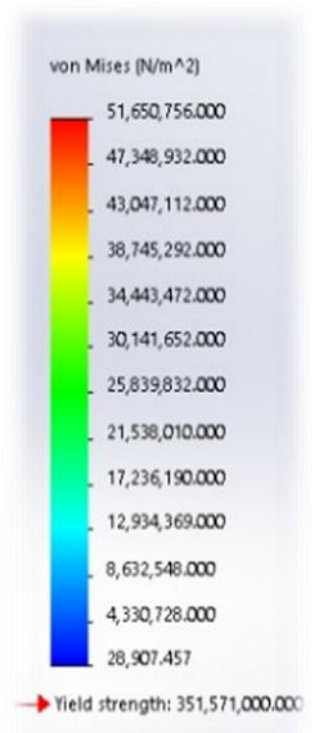


Figure (4-a)

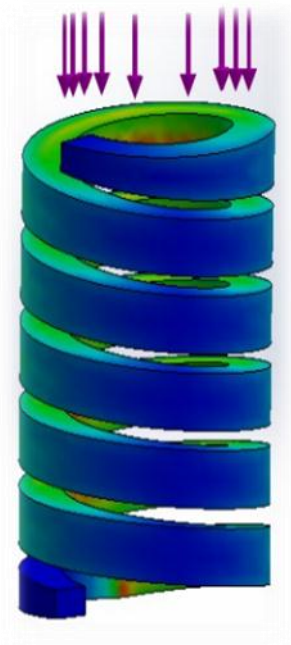


Figure (4-b)

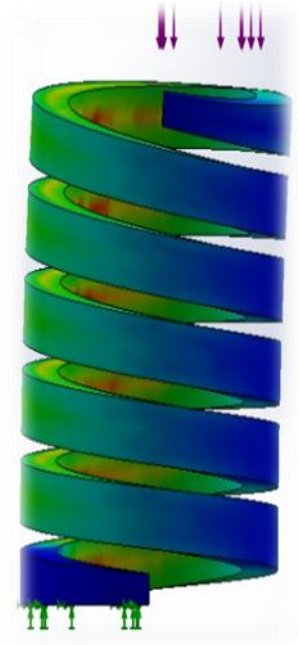


Figure (4-c)

4.3. FOR RECTANGULAR SPRING SECTION :

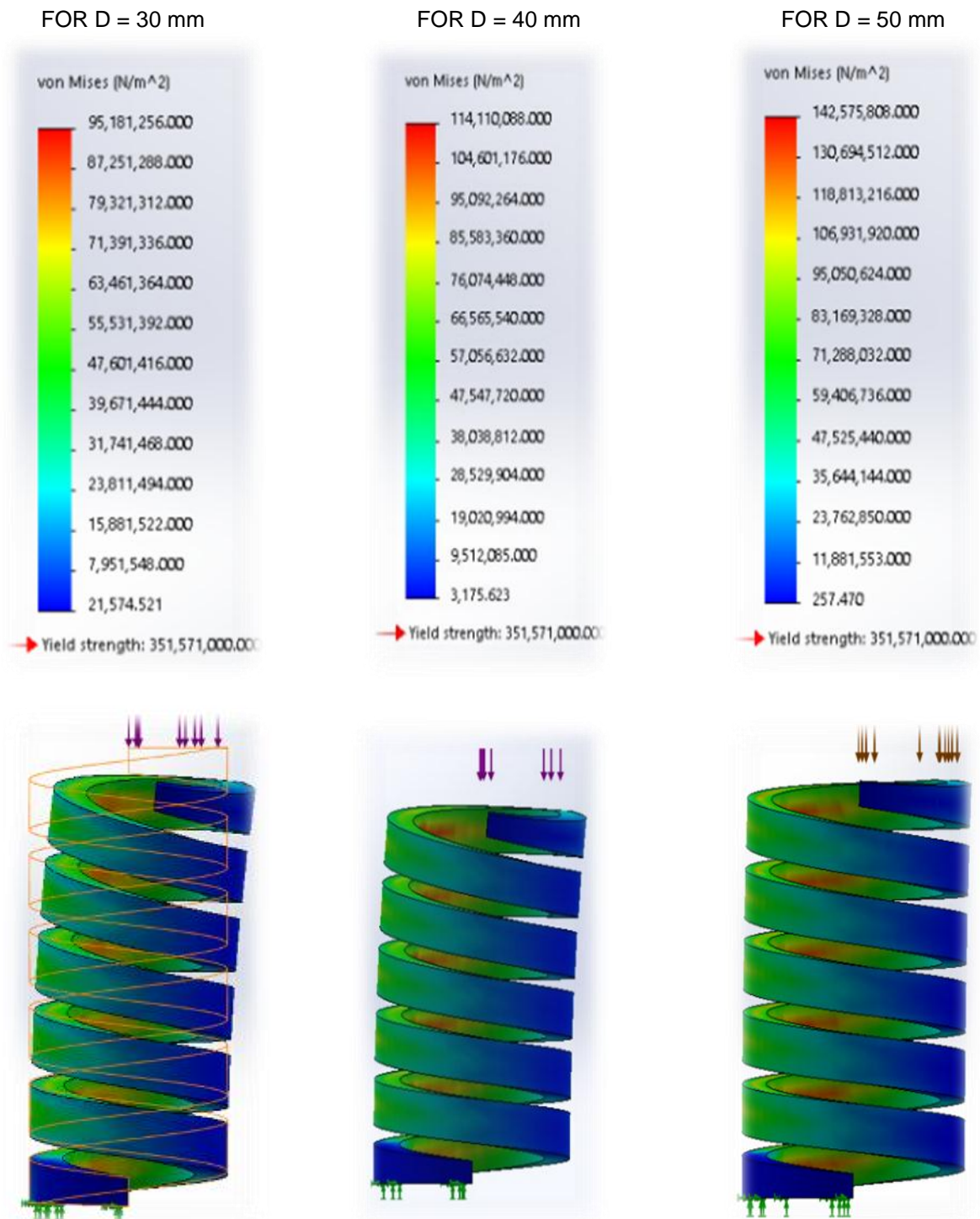


Figure (5-a)

Figure (5-b)

Figure (5-c)



## 5. DISCUSSION :

According to results of the modeling analysis we notice in figure (6-a) that in which we calculate the maximum stresses on the spring of circular cross section , where the stress is increase by increasing the coil diameter (D) , the lowest stresses are appearing in this section since it is suitable for the most application . in figure (6-b) that in which we calculate the maximum stresses on the spring of square cross section , where the stress is greater than that in circular , in figure (6-c) that in which we calculate the maximum stresses on the spring of rectangular cross section , where the stress is highest of the three sections because of the edges and corners that rises the stresses of the rectangular cross section as compared with the circular and rectangular cross section .

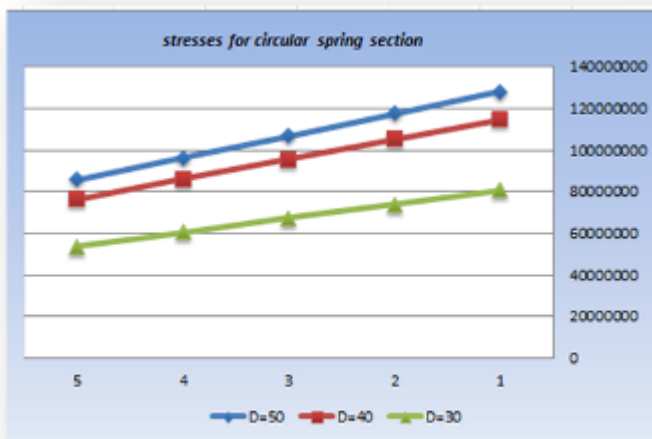
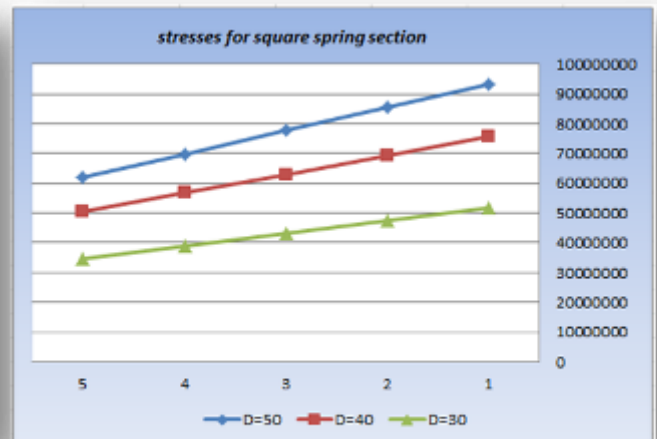
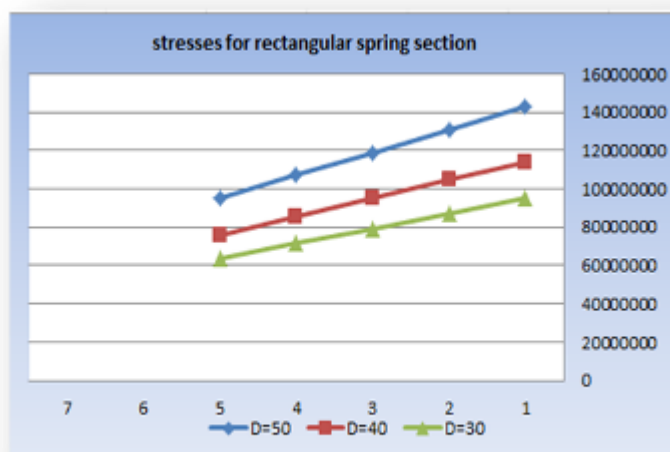


Figure (6-a) von-mises stresses of circular section for five node



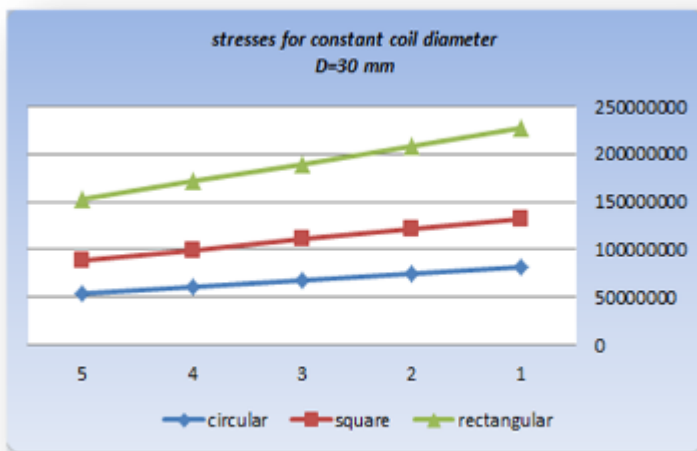
Figure(6-b) von-mises stresses of section of square for five nodes



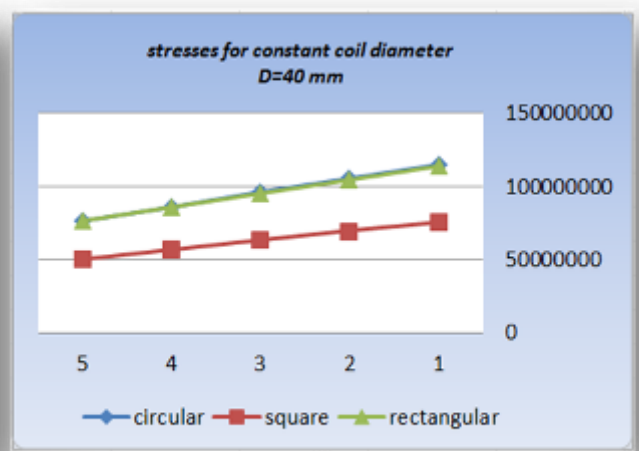
Figure(6-c) von-mises stresses of rectangular for five nodes vs. three diameter



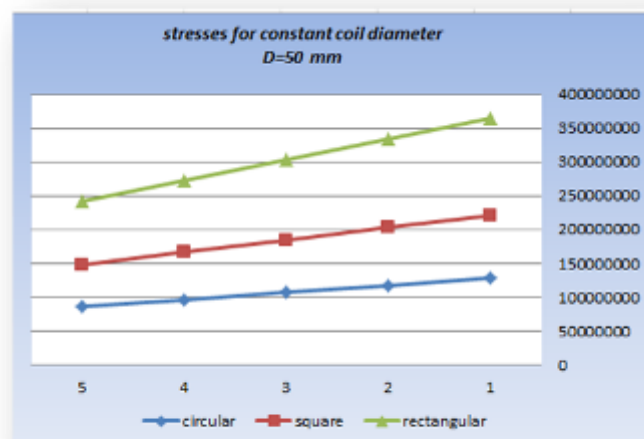
In figure (7-a) that in which we calculate the maximum stresses on the spring for it is three types for the same coil diameter ( $D=30$ ), where the highest stress is appear in the rectangular section and the lowest stress in the circular section, This is due to the shape of the spring section where the circular section has not edges and angles, and the area of the rectangular spring less than the square spring section. In figure (7-b) that in which we calculate the maximum stresses on the spring for it is three types for the same coil diameter ( $D=40$ ), where the stress is increases by increasing the coil diameter. In figure (7-c) that in which we calculate the maximum stresses on the spring for it is three types for the same coil diameter ( $D=50$ ), where the greatest stress is appearing with the greater coil diameter.



Figure(7-a) von-mises stresses of constant coil diameter  $D=30$  for three spring section



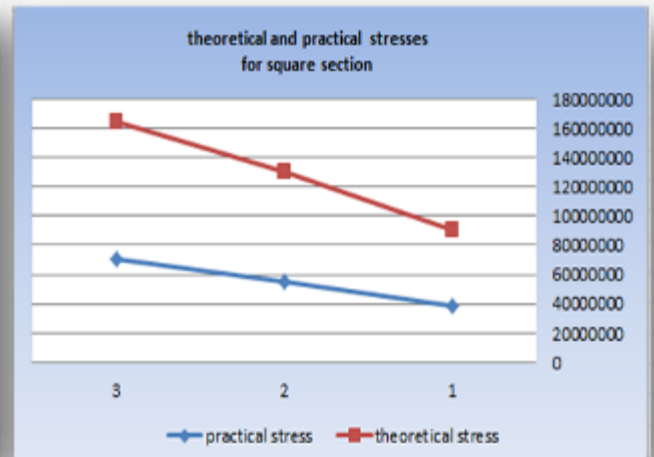
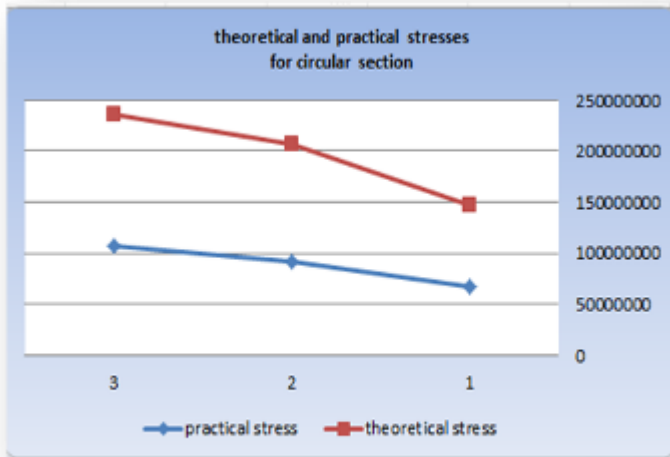
Figure(7-b) von-mises stresses of constant coil diameter  $D=40$  for three spring section



Figure(7-c) von-mises stresses of constsnt coil diameter  $D=50$  for three spring section



In figure (8) we comparing the theoretical results with the results of the program with little differences .  
Finally the circular section is the favorite for the spring design under the same condition of the square and rectangular spring section .



Figure(8-a) comparing between theoretical and practical stresses for circular section

Figure(8-b) comparing between theoretical and practical stresses for square section

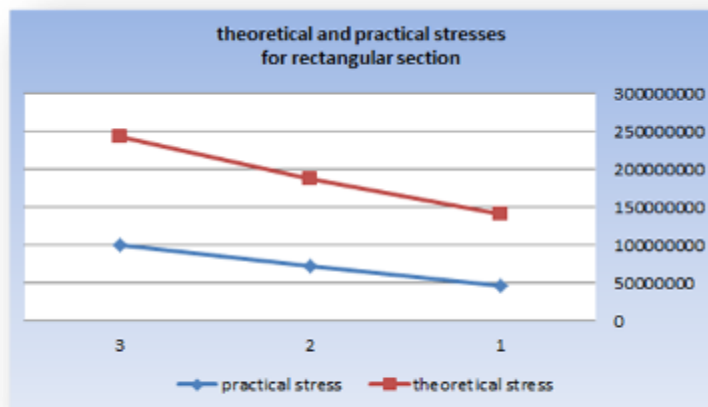


Figure (8-a) comparing between theoretical and practical stresses for rectangular section



## 6. CONCLUSION:

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- 1- From the result we notice that the stresses are rising by increasing the coil diameter (D) for the same spring section .
- 2-The lower stress is being in the circular spring section as compared with the square and rectangular spring section.
- 3- There are an similarity between the analysis by the program result and the theoretical result.

## REFERENCES

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1. **Yildirim, V.**, 2002, *Expressions for predicting fundamental natural frequencies of non cylindrical helical spring* ,pp 259-370
2. **Pomeranz, S.**, 2000, *Using a Computer Algebra System to Teach the Finite Element Method*, *Inte, J., Engng*, pp 362-368.
3. **Mohamed, T., & Said, A., & Mohamed, H.**, 2008, *A Finite Element For Dynamic Analysis Of A Cylindrical Isotropic Helical Spring*, *Transations of the ASME* vol.123 118-124.
4. **Azzaz, S.A. ,& Kaoua, & Dahmoun, D.**, 2004, *A Twin Helical Spring Numerical Modeling Under Tension Loading*, *ASME J Eng Ind*, vol 95, pp 1139 – 1148.
5. **Budynas, & Richard, G., & Charles, R., & Mischke, & Joseph, E., Shigley**, 2003, *Mechanical Engineering Design*, McGraw-Hill Professional.