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Stress-Strength Reliability Bayesian of Generalized Exponential-Poisson Distriution for Complete Data

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A B S T R A C T
In this paper, a Bayesian analysis is made to estimate the Reliability of two stress-strength model systems. First: the reliability R_1 of one component strengths X under stress Y. Second: reliability R_2 of one component strength under two stresses. Where X and Y are independent Generalized Exponential-poison random variables with (α, λ, θ) and (β, λ, θ), parameters respectively. The analysis is concerned based on complete data samples using gamma prior under eight different loss functions (Quadratic, Squared error, Weighted, Linear exponential, precautionary, De Groot, Entropy, and non-Linear exponential). The estimators are compared by Mean squared error criteria due to a simulation study and find that the best performance of the estimators found in order as (Quadratic, Weighted, Linear exponential and non-Linear exponential.

Keywords: *Bayesian analysis, generalized exponential-poisso distribution, Stress-Strength,reliability, prior, posterior, loss functions.*

1. Introduction

Kus (2007) introduced a two-parameter distribution known as exponential distribution with poisson distribution [3]. In (1998) Adamidis and Loukas introduced with decreasing failure rate distribution [1]. This distribution is known as exponential-geometric distribution**.** Barreto-Souza and Cribari-Neto with decreasing or increasing the failure rate introduce the generalization of the distribution. The function cumulative distribution (cdf) with two parameter Exponential-Poisson(EP) given as:[2]

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$$F(x) = \frac{1 - \exp[-\lambda(1 - \exp(-\theta x))]}{1 - \exp(-\lambda)} \qquad x > 0; \theta, \lambda > 0$$

The Generalized Exponential–Poisson distribution (GEP) for the random variable X with parameters $(\alpha, \lambda, \theta)$ given as:

$$F(x) = \left[\frac{1-\exp(-\lambda(1-\exp(-\theta x)))}{1-\exp(-\lambda)}\right]^{\alpha} = \left[\frac{1-Ax}{k}\right]^{\alpha} \tag{1}$$

Where $Ax = e^{-\lambda(1-e^{-\theta x})} for\ \alpha > 0$ and $k = 1 - e^{-\lambda}$ with α as shape and (θ, λ) as scale parameters. The corresponding Probability density functions (pdf) define as:

$$f(x) = \frac{\alpha\lambda\theta}{(1-e^{-\lambda})^{\alpha}}\left[1 - e^{-\lambda(1-e^{-\theta x})}\right]^{\alpha-1} e^{-\theta x} e^{-\lambda(1-e^{-\theta x})} \tag{2}$$

The random number X has been generated by inverse function method, which is for uniform random U.

$$X = \frac{-1}{\theta} ln\left(1 + \frac{1}{\lambda} ln\left(1 - U^{\frac{1}{\alpha}}(1 - e^{-\lambda})\right)\right) \tag{3}$$

The measure of system performance referred to as the stress strength parameter is the reliability $R = P(Y < X)$, if X is the strength of acomponent subjected to a stress Y.The system fails if and only if the applied stressis greater than its strength :[4]

the paper organized as in section 2 the general expression of $R_1$and R_2 are given for two stress-strength systems of one components having strengths composed under one stress and under two stresses. the Bayesian estimators are found for R_1 and R_2 under eight different loss functions are given in section 3. Finally, in section 4 the performance of the estimators is illustrating by Experimental simulation study.

2. Reliability of the systems for GEP Stress-Strength Models

The purpose of this section is to obtain the reliabilities expression of two different systems for stress- strength models.

2-1 One component system reliability.

The reliability R_1 of a component operating under stress –strength system given by: [5]

$$R_1 = P(Y < X) \tag{4}$$

Let the strength random variable $X \sim GEP(\alpha, \lambda, \theta)$ and the stress random variable $Y \sim GEP(\beta, \lambda, \theta)$ when X and Y are independent but not identical .Therefore the cdf for Y can be written as:

$$G(y) = \left[\frac{1 - exp\left[-\lambda\left(1 - exp(-\theta y)\right)\right]}{1 - exp(-\lambda)}\right]^{\beta} = \left[\frac{1 - Ay}{k}\right]^{\beta}$$

Now the reliability R_1 from eq. (4) can be given as:

$$R_1 = \int_0^\infty G_y(x)\, f(x)\, dx \;\; = \int_0^\infty \left(\frac{1-A_x}{k}\right)^{\beta} \frac{\alpha\lambda\beta}{k^{\alpha}} A_x e^{-\theta x}(1 - A_x)^{\alpha-1}dx$$

$$= \frac{\alpha\lambda\beta}{k^{\beta+\alpha}} \int_0^\infty (1 - A_x)^{\beta+\alpha-1} A_x e^{-\theta x} dx$$

And since $\int_0^\infty f(x)dx = 1$, So from eq. (2) it can be use as:

$$\int_0^\infty (1 - Ax)^{\alpha-1} Ax\, e^{-\theta x} dx = \frac{k^{\alpha}}{\alpha\lambda\theta} \tag{5}$$

Then,

$$R_1 = \frac{\alpha\lambda\theta}{k^{\beta+\alpha}} \int_0^\infty e^{-\theta x}\, A_x(1 - A_x)^{\beta+\alpha-1} dx = \frac{\alpha}{\alpha + \beta} \tag{6}$$

2.2. Two stress-one strength

$$R_2 = P(\max(Y_1, Y_2) < X)$$

When a component having X strength is exposed to two independent Y_1 *and* Y_2 stresses, stress-strength reliability is obtained as follows :[6]

Let $X \sim GEP(\alpha, \lambda, \theta)$ be strength random variable and $Y_1 \sim (\beta_1, \lambda, \theta), Y_2 \sim (\beta_2, \lambda, \theta)$ be stresses random variable where X,Y_1 and Y_2 are independently distribution.

$$R_2 = P(\max(Y_1, Y_2) < X) = \int_{x=0}^{\infty} P(y_1 < x)\, P(y_2 < x) f(x) dx$$

$$R_2 = \int_{x=0}^{\infty} \int_0^\infty \int_0^\infty g(y_1, y_2, x)\, dy_2 dy_1 dx = \int_{x=0}^{\infty} \int_0^\infty \int_0^\infty g(y_1) g(y_2)\, f(x) dy_2 dy_1 dx$$

$$R_2 = \int_{x=0}^{\infty} G_{y_1}(x) G_{y_2}(x) f(x) dx = \int_{x=0}^{\infty} \left(\frac{1-Ax}{k}\right)^{\beta 1} \left(\frac{1-Ax}{k}\right)^{\beta 2} \frac{\alpha\lambda\theta}{k^{\alpha}}\; e^{-\theta x} Ax\; (1 - Ax)^{\alpha-1}\, \mathrm{dx}$$

$$= \int_{x=0}^{\infty} \left(\frac{1 - Ax}{k}\right)^{\beta 1 + \beta 2} \frac{\alpha\lambda\theta}{k^{\alpha}}\; e^{-\theta x} Ax\; (1 - Ax)^{\alpha-1} dx$$

$$= \frac{\alpha\lambda\theta}{k^{\alpha+\beta 1+\beta 2}} \int_{x=0}^{\infty} e^{-\theta x}\, Ax(1 - Ax)^{\alpha+\beta 1+\beta 2} dx$$

$$R_2 = \frac{\alpha}{\alpha+\beta_1+\beta_2} \tag{7}$$

3. Bayes analysis

In this section, the Bayes estimators of reliabilities R_1 and R_2 are given based on complete samples using gamma prior under Quadratic, Squared error, Weighted, Linear exponential precautionary, De Groot, Entropy, and non-Linear exponential loss functions.

3.1. Complete Data

Let $x_1, x_2, \dots, x_n$ be a random sample form GEP distribution with shape parameter α> 0 then the likelihood function of complete data from (2) :[7]

$$L(\alpha|x) = \prod_{i=1}^{n} f(x_i|\alpha) = \prod_{i=1}^{n}\left[\frac{\alpha\lambda\theta}{k^{\alpha}}\, Ax_{(i)}\, e^{-\theta x}\left(1 - Ax_{(i)}\right)^{\alpha-1}\right]$$

$$= \left(\frac{\alpha\lambda\theta}{k^{\alpha}}\right)^{n}\prod_{i=1}^{n} Ax_{(i)}\, e^{-\theta\sum_{i=1}^{n} x_{(i)}}\prod_{i=1}^{n}\left(1 - Ax_{(i)}\right)^{\alpha-1}$$

$$= \alpha^{n}(\lambda\theta)^{n}k^{-\alpha n}\prod_{i=1}^{n} Ax_{(i)}\, e^{-\theta\sum_{i=1}^{n} x_{(i)}}\prod_{i=1}^{n}\left(1 - Ax_{(i)}\right)^{\alpha-1}$$

$$= \alpha^{n}\,(\lambda\theta)^{n}\, e^{-\alpha n lnk} e^{\sum_{i=1}^{n} lnAx_{(i)}}\, e^{-\theta\sum_{i=1}^{n} x_{(i)}}\, e^{\sum_{i=0}^{n} ln\left(1-Ax_{(i)}\right)^{\alpha-1}}$$

$$= \alpha^{n}\,(\lambda\theta)^{n}\, e^{-\alpha n lnk} e^{\sum_{i=1}^{n} lnAx_{(i)}}\, e^{-\theta\sum_{i=1}^{n} x_{(i)}} e^{-\alpha\sum_{i=1}^{n} ln\left(1-Ax_{(i)}\right)^{-1}} e^{\sum_{i=1}^{n} ln\left(1-Ax_{(i)}\right)^{-1}}$$

Now let $W_2 = (\lambda\theta)^{n}\, e^{\left(\sum_{i=1}^{n} lnAx_{(i)} - \theta\sum_{i=1}^{n} x_{(i)} + \sum_{i=1}^{n} ln\left(1-Ax_{(i)}\right)^{-1}\right)}$

And let $\mu x = nlnk + \sum_{i=1}^{n} ln\left(1 - Ax_{(i)}\right)^{-1}$

Then

$$L(\alpha|x) = \alpha^{n}\, W_2\, e^{-\alpha\left(lnk + \sum_{i=1}^{n} ln\left(1-Ax_{(i)}\right)^{-1}\right)} = \alpha^{n}\, W_2\, e^{-\alpha\mu_x} \tag{8}$$

3.2. Bayes procedur

By using Bayes method for finding the posterior function under gamma prior function:

$$\pi(\alpha) = \frac{b^{a}\alpha^{a-1}e^{-\alpha b}}{\Gamma(a)},\quad a > 0, b > 0 \tag{9}$$

The posterior function is:

$$P(\alpha|x) = \frac{L(\alpha|x)\,\pi(\alpha)}{\int_0^{\infty} L(\alpha|x)\,\pi(\alpha)d\alpha}$$

By using (8)and(9) we get:

$P(\alpha|x) = \dfrac{W_2\frac{b^{a}}{\Gamma a}\,\alpha^{n+a-1}e^{-\alpha(\mu_x+b)}}{W_2\frac{b^{a}}{\Gamma a}\int_0^{\infty}\alpha^{n+a-1}e^{-\alpha(\mu_x+b)}d\alpha}$ by using

$$\int_0^\infty x^{\alpha-1}\ e^{-\beta x} dx = \frac{\Gamma\alpha}{\beta^\alpha} \tag{10}$$

$$P(\alpha|x) = \frac{\alpha^{n+a-1}\ e^{-\alpha(\mu_x+b)}}{\Gamma(n+a)/(\mu_x+b)^{n+a}} = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1}\ e^{-\alpha(\mu_x+b)} \tag{11}$$

3.2.1. Squared loss function

The Bayes estimator for α using squared as loss function given as:[8]

$$\hat{\alpha}_s = \mathrm{E}(\alpha \,|\, \mathrm{x}) \tag{12}$$

$$\mathrm{E}(\alpha \,|\, \mathrm{x}) = \int_0^\infty \alpha\, P(\alpha|x)\, d\alpha = \int_0^\infty \alpha \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1}\ e^{-\alpha(\mu_x+b)} d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a}\ e^{-\alpha(\mu_x+b)} d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \frac{\Gamma(n+a+1)}{(\mu_x+b)^{n+a+1}} = \frac{(n+a)}{(\mu_x+b)}$$

Then the estimates will be as:

$$\hat{\alpha}_s = \frac{(n+a)}{(\mu_x+b)} \quad , \quad \hat{\beta}_{is} = \frac{(m_i+a)}{(\mu_{yi}+b)} \quad , i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1s} = \frac{\hat{\alpha}_s}{\hat{\alpha}_s+\hat{\beta}_s} \qquad , \hat{R}_{2s} = \frac{\hat{\alpha}_s}{\hat{\alpha}_s+\hat{\beta}_{1s}+\hat{\beta}_{2s}}$$

3.2.2. Quadratic loss function

The Bayes estimator for α using Quadratic as loss function given as:[9]

$$\hat{\alpha}_Q = \frac{E(\alpha^{-1}|x)}{E(\alpha^{-2}|x)}$$

$$E(\alpha^{-1}|x) = \int_0^\infty \alpha^{-1}\, P(\alpha|x) d\alpha = \int_0^\infty \alpha^{-1} \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1}\ e^{-\alpha(\mu_x+b)} d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a-2}\ e^{-\alpha(\mu_x+b)} d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \frac{\Gamma(n+a-1)}{(\mu_x+b)^{n+a-1}}$$

$$E(\alpha^{-1}|x) = \frac{(\mu_x+b)}{(n+a-1)} \tag{13}$$

$$E(\alpha^{-2}|x) = \int_0^\infty \alpha^{-2}\, P(\alpha|x) d\alpha = \int_0^\infty \alpha^{-2} \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1}\ e^{-\alpha(\mu_x+b)} d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a-3}\ e^{-\alpha(\mu_x+b)} d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \frac{\Gamma(n+a-2)}{(\mu_x+b)^{n+a-2}}$$

$$E(\alpha^{-2}|x) = \frac{(\mu_x+b)^2}{(n+a-1)(n+a-2)}$$

$$\hat{\alpha}_Q = \frac{(\mu_x+b)}{(n+a-1)} / \frac{(\mu_x+b)^2}{(n+a-1)(n+a-2)}$$

Then the estimates will be as:

$$\hat{\alpha}_Q = \frac{(n+a-2)}{(\mu_x+b)} \quad , \hat{\beta}_{iQ} = \frac{(m_i+a-2)}{(\mu_{yi}+b)}, i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1Q} = \frac{\hat{\alpha}_Q}{\hat{\alpha}_Q+\hat{\beta}_Q} \qquad , \hat{R}_{2Q} = \frac{\hat{\alpha}_Q}{\hat{\alpha}_Q+\hat{\beta}_{1Q}+\hat{\beta}_{2Q}}$$

3.2.3 Weighted loss function

The Bayes estimator for α using weighted as loss function given as:[8]

$$\hat{\alpha}_w = \frac{1}{E(\alpha^{-1}|x)} \tag{14}$$

By compensating (12) in (13) we get:

$$\hat{\alpha}_w = \frac{1}{(\mu_x + b)/(n + a - 1)} = \frac{(n + a - 1)}{(\mu_x + b)}$$

Then the estimates will be as:

$$\hat{\alpha}_{w=} \frac{(n + a - 1)}{(\mu_x + b)} \quad , \hat{\beta}_{iw} = \frac{(m_i + a - 1)}{\left(\mu_{yi} + b\right)} \quad , i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1w} = \frac{\hat{\alpha}_w}{\hat{\alpha}_w + \hat{\beta}_w} \qquad , \hat{R}_{2w} = \frac{\hat{\alpha}_w}{\hat{\alpha}_w + \hat{\beta}_{1w} + \hat{\beta}_{2w}}$$

3.2.4 Linear exponential

The Bayes estimator for α using Linear exponential as loss function given as:[10]

$$\hat{\alpha}_L = \frac{-1}{c} \, lnE(e^{-c\alpha}|x)$$

Where $E(e^{-c\alpha}|x) = \int_0^\infty e^{-c\alpha} \, P(\alpha|x) d\alpha$

$$= \int_0^\infty e^{-c\alpha} \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1} \, e^{-\alpha(\mu_x+b)} d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a-1} \, e^{-\alpha(\mu_x+b+c)} d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \frac{\Gamma(n+a)}{(\mu_x+b+c)^{(n+a)}} = \frac{(\mu_x+b)^{n+a}}{(\mu_x+b+c)^{(n+a)}} \tag{15}$$

Then the estimates will be as:

$$\hat{\alpha}_L = \frac{-1}{c}\ln\left(\frac{(\mu_x+b)}{(\mu_x+b+c)}\right)^{n+a} = \frac{-(n+a)}{c} ln\frac{(\mu_x+b)}{(\mu_x+b+c)}$$

$$\hat{\beta}_{iL=}\frac{-(m_i+a)}{c} ln\frac{(\mu_{yi}+b)}{(\mu_{yi}+b+c)} \qquad ,i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1L} = \frac{\hat{\alpha}_L}{\hat{\alpha}_L+\hat{\beta}_L} \qquad ,\hat{R}_{2L} = \frac{\hat{\alpha}_L}{\hat{\alpha}_L+\hat{\beta}_{1L}+\hat{\beta}_{2L}}$$

3.2.5. Non-Linear Exponential loss function

The Bayes estimator for α using Non- Linear exponential as loss function given as;[11]

$$\hat{\alpha}_N = \frac{-1}{c+2}[\, lnE(e^{-c\alpha}|x) - 2\,E(\alpha|x)] \tag{10}$$

By compensating (12) and (15) in (16) we get the estimates will be as:

$$\hat{\alpha}_N = \frac{-1}{c+2}\left[\ln\left(\frac{(\mu_x+b)}{(\mu_x+b+c)}\right)^{n+a} - 2\left(\frac{(n+a)}{(\mu_x+b)}\right)\right]$$

$$\hat{\beta}_{iN} = \frac{-1}{c+2}\left[\ln\left(\frac{(\mu_{yi}+b)}{(\mu_{yi}+b+c)}\right)^{n+a} - 2\left(\frac{(m_i+a)}{(\mu_{yi}+b)}\right)\right] \qquad ,i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1NL} = \frac{\hat{\alpha}_{NL}}{\hat{\alpha}_{NL}+\hat{\beta}_{NL}} \qquad ,\hat{R}_{2NL} = \frac{\hat{\alpha}_{NL}}{\hat{\alpha}_{NL}+\hat{\beta}_{1NL}+\hat{\beta}_{2NL}}$$

3.2.6. Precautionary loss function

The Bayes estimator for α using Precautionary as loss function given as:[12]

$$\hat{\alpha}_p = \sqrt{E(\alpha^2|x)}$$

$$E(\alpha^2|x) = \int_0^\infty \alpha^2\, p(\alpha|x)d\alpha\alpha = \int_0^\infty \alpha^2 \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1}\, e^{-\alpha(\mu_x+b)}d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)}\int_0^\infty \alpha^{n+a+1}\, e^{-\alpha(\mu_x+b)}d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)}\,\frac{\Gamma(n+a+2)}{(\mu_x+b)^{n+a+2}}$$

$$E(\alpha^2|x) = \frac{(n+a+1)(n+a)}{(\mu_x+b)^2} \quad (17)$$

Then the estimates will be as:

$$\hat{\alpha}_p = \sqrt{\frac{(n+a+1)(n+a)}{(\mu_x+b)^2}} \quad ,\hat{\beta}_{ip} = \sqrt{\frac{(m_i+a+1)(m_i+a)}{(\mu_{yi}+b)^2}} \ ,i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1P} = \frac{\hat{\alpha}_P}{\hat{\alpha}_P + \hat{\beta}_P} \qquad ,\hat{R}_{2P} = \frac{\hat{\alpha}_P}{\hat{\alpha}_P + \hat{\beta}_{1P} + \hat{\beta}_{2P}}$$

3.2.7. Entropy loss function

The Bayes estimator for α using Entropy as loss function given as:[13]

$$\hat{\alpha}_E = (E(\alpha^{-t}|x))^{\frac{-1}{t}}$$

$$E(\alpha^{-t}|x) = \int_0^\infty \alpha^{-t} \ P(\alpha|x)d\alpha = \int_0^\infty \alpha^{-t} \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \alpha^{n+a-1} \, e^{-\alpha(\mu_x+b)} d\alpha$$

$$= \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \int_0^\infty \alpha^{n+a-t-1} \, e^{-\alpha(\mu_x+b)} d\alpha = \frac{(\mu_x+b)^{n+a}}{\Gamma(n+a)} \frac{\Gamma(n+a-t)}{(\mu_x+b)^{n+a-t}}$$

$$E(\alpha^{-t}|x) = \frac{\Gamma(n+a-t}{\Gamma(n+a)} \ (\mu_x+b)^t$$

$$\hat{\alpha}_E = \left(\frac{\Gamma(n+a-t)}{\Gamma(n+a)} \ (\mu_x+b)^t\right)^{\frac{-1}{t}} = \sqrt[t]{\left(\frac{\Gamma(n+a-t)}{\Gamma(n+a)} \ (\mu_x+b)^t\right)^{-1}}$$

Then the estimates will be as:

$$\hat{\alpha}_E = \sqrt[t]{\frac{\Gamma(n+a)}{\Gamma(n+a-t)}} \Big/ (\mu_x+b) \quad ,\hat{\beta}_{iE} = \sqrt[t]{\frac{\Gamma(m_i+a)}{\Gamma(m_i+a-t)}} \Big/ (\mu_{yi}+b) \qquad ,i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1E} = \frac{\hat{\alpha}_E}{\hat{\alpha}_E + \hat{\beta}_E} \qquad ,\hat{R}_{2E} = \frac{\hat{\alpha}_E}{\hat{\alpha}_E + \hat{\beta}_{1E} + \hat{\beta}_{2E}}$$

3.2.8. De-Groot loss function

The Bayes estimator for α using De-Groot as loss function given as:[14]

$$\hat{\alpha}_D = \frac{E(\alpha^2|x)}{E(\alpha|x)} \tag{18}$$

By compensating (12) and (17) in (18) we get: $\hat{\alpha}_D = \frac{\frac{(n+a+1)(n+a)}{(\mu_x+b)^2}}{\frac{(n+a)}{(\mu_x+b)}}$

Then the estimates will be as:

$$\hat{\alpha}_D = \frac{(n+a+1)}{(\mu_x+b)} \quad , \qquad \hat{\beta}_{iD} = \frac{(m_i+a+1)}{(\mu_{yi}+b)} \text{ }, i = 1,2$$

And the reliabilities estimation function in eq.(6) and (7) we get:

$$\hat{R}_{1D} = \frac{\hat{\alpha}_D}{\hat{\alpha}_D + \hat{\beta}_D} \qquad , \hat{R}_{2D} = \frac{\hat{\alpha}_D}{\hat{\alpha}_D + \hat{\beta}_{1D} + \hat{\beta}_{2D}}$$

4. Simulation study

In this section, observe the simulation experiments of the Bayes estimators proposed in the previous section (3), Monte Carlo simulation is performed to compare the performance of different estimation of R_1 and R_2. The simulation study has been carried out for four samples size and the values of (c=1,t=2), the parameter values of the shape parameter (α) and the prior distribution a=2 and b=3 the results have been replicated sufficiently (L=1000)time for each experiment, the result presented of some of numerical experiment to compare the performance of the Bayes estimators under gamma prior distribution and eight loss function proposed in the previous sections, we have presented the simulation results using MATLAB (2013)program. A simulation results are conducted to examine and compare the performance of the estimates for shape respecting to the MSE. The estimator has the smallest value of MSE. When we have as well as complete type data sample as shown in the last column in the tables from (3 to 12).

Table 1. The experiments for real R_1 value

Experiment	α	β	θ	Λ	R_1
1	2	1.5	3	2.5	0.57142
2	1.4	1.5	2	2.5	0.48275

3	1.3	1.5	2.5	2	0.46428
4	2.5	1.5	1	1.5	0.62500
5	2.3	1.5	2	1.9	0.60596

Table 2. The experiments for real R_2 value

Experiment	α	β_1	β_2	Θ	λ	R_2
1	1.7	2	1	3	2.5	0.36170
2	1.7	2	2.5	3	1.5	0.27419
3	1.2	2	2.5	3	1.5	0.21052
4	1.5	2	2	1.9	1.5	0.27272
5	1	2	3	1.9	1.7	0.16667

Table 3. R_1 estimators performance for Experiments 1.

n,m	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30	0.00596	0.00468	0.00447	0.00360	0.00558	0.00524	0.00492	0.00475	BD
30,15	0.00373	0.00353	0.00354	0.00399	0.00364	0.00357	0.00331	0.00345	BL
50,15	0.00386	0.00319	0.00315	0.00378	0.00361	0.00341	0.00309	0.00315	BP,BNL
50,30	0.00246	0.00249	0.00250	0.00251	0.00246	0.00247	0.00238	0.00245	BL
15,50	0.00646	0.00441	0.00408	0.00333	0.00584	0.00528	0.00490	0.00456	BD
30,50	0.00303	0.00276	0.00271	0.00245	0.00295	0.00288	0.00280	0.00277	BD
100.15	0.00461	0.00301	0.00281	0.00395	0.00408	0.00363	0.00318	0.00305	BP
100,30	0.00233	0.00210	0.00208	0.00231	0.00225	0.00218	0.00207	0.00209	BL
100,50	0.00143	0.00139	0.00139	0.00147	0.00142	0.00141	0.00136	0.00138	BL
15,100	0.00724	0.00423	0.00385	0.00316	0.00635	0.00556	0.00509	0.00455	BD
30,100	0.00312	0.00248	0.00237	0.00211	0.00294	0.00276	0.00266	0.00253	BD
50,1	0.001	0.00	0.00	0.001	0.001	0.001	0.001	0.00	BD

00	81	166	163	52	76	73	69	167	

Table 4. R_1 estimators performance for Experiments 2.

n.m	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30	0.00 465	0.00 427	0.00 424	0.00 448	0.00 451	0.00 440	0.00 408	0.00 420	BL
30,15	0.00 442	0.00 384	0.00 377	0.00 488	0.00 422	0.00 406	0.00 375	0.00 380	BL
50,15	0.00 498	0.00 366	0.00 349	0.00 485	0.00 455	0.00 418	0.00 378	0.00 369	BP
50,3 0	0.00 270	0.00 253	0.00 251	0.00 295	0.00 265	0.00 261	0.00 250	0.00 252	BL
15,5 0	0.00 485	0.00 373	0.00 360	0.00 346	0.00 448	0.00 416	0.00 375	0.00 373	BD
30,5 0	0.00 256	0.00 246	0.00 245	0.00 249	0.00 253	0.00 250	0.00 239	0.00 244	BL
100. 15	0.00 572	0.00 372	0.00 345	0.00 491	0.00 507	0.00 452	0.00 400	0.00 380	BP
100, 30	0.00 264	0.00 225	0.00 219	0.00 260	0.00 252	0.00 241	0.00 228	0.00 226	BP
100, 50	0.00 164	0.00 155	0.00 153	0.00 170	0.00 161	0.00 159	0.00 154	0.00 155	BP
15,1 00	0.00 524	0.00 361	0.00 342	0.00 325	0.00 469	0.00 424	0.00 375	0.00 365	BD
30,1 00	0.00 248	0.00 216	0.00 212	0.00 208	0.00 238	0.00 229	0.00 217	0.00 216	BD
50,1 00	0.00 174	0.00 168	0.00 167	0.00 168	0.00 172	0.00 170	0.00 166	0.00 167	BL

Table (5): R_1estimators performance for Experiments 3.

n.m	BQ	BS	BP	BD	BE	BW	BL	BNL	BE ST
15,3 0	0.004 62	0.004 33	0.004 32	0.004 68	0.004 50	0.004 41	0.004 11	0.004 25	BL
30,1 5	0.004 77	0.004 05	0.003 95	0.005 30	0.004 51	0.004 34	0.004 01	0.004 03	BP
50,1 5	0.004 92	0.003 58	0.003 41	0.004 80	0.004 49	0.004 12	0.003 73	0.003 62	BP

50,30	0.00295	0.00278	0.00275	0.00319	0.00289	0.00285	0.00274	0.00276	BL
15,50	0.00457	0.00376	0.00369	0.00375	0.00428	0.00404	0.00368	0.00372	BL
30,50	0.00283	0.00275	0.00274	0.00282	0.00280	0.00278	0.00267	0.00272	BL
100.15	0.00553	0.00357	0.00331	0.00474	0.00490	0.00435	0.00385	0.00365	BP
100,30	0.00251	0.00214	0.00209	0.00248	0.00240	0.00229	0.00218	0.00215	BP
100,50	0.00176	0.00169	0.00168	0.00181	0.00174	0.00172	0.00167	0.00168	BL
15,100	0.00480	0.00343	0.00330	0.00322	0.00433	0.00394	0.00349	0.00344	BD
30,100	0.00233	0.00214	0.00213	0.00217	0.00226	0.00220	0.00210	0.00212	BL
50,100	0.00168	0.00164	0.00164	0.00167	0.00166	0.00165	0.00161	0.00163	BL

Table 6. R_1estimators performance for Experiments 4.

n.m	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30	0.00718	0.00548	0.00516	0.00376	0.00669	0.00623	0.00611	0.00567	BD
30,15	0.00350	0.00373	0.00383	0.00353	0.00352	0.00357	0.00343	0.00363	BL
50,15	0.00331	0.00292	0.00293	0.00326	0.00314	0.00301	0.00275	0.00286	BL
50,30	0.00214	0.00221	0.00224	0.00213	0.00215	0.00217	0.00212	0.00218	BL
15,50	0.00828	0.00556	0.00508	0.00388	0.00749	0.00676	0.00654	0.00586	BD
30,50	0.00320	0.00282	0.00274	0.00228	0.00309	0.00299	0.00300	0.00287	BD
100.15	0.00375	0.00271	0.00263	0.00329	0.00338	0.00308	0.00272	0.00270	BP
100,30	0.00187	0.00178	0.00179	0.00185	0.00182	0.00179	0.00171	0.00157	BNL

100,50	0.00146	0.00145	0.00146	0.00147	0.00145	0.00145	0.00142	0.00144	BL
15,100	0.00942	0.00570	0.00505	0.00402	0.00833	0.00732	0.00702	0.00610	BD
30,100	0.00338	0.00256	0.00240	0.00201	0.00315	0.00293	0.00292	0.00267	BD
50,100	0.00189	0.00170	0.00166	0.00150	0.00184	0.00179	0.00180	0.00173	BD

Table 7. R_1estimators performance for Experiments 5**.**

n.m	BQ	BS	BP	BD	BE	BW	BL	BNL	BET
15,30	0.00656	0.00507	0.00480	0.00366	0.00613	0.00473	0.00552	0.00520	BD
30,15	0.00326	0.00324	0.00329	0.00342	0.00322	0.00320	0.00301	0.00316	BL
50,15	0.00354	0.00297	0.00295	0.00347	0.00332	0.00315	0.00284	0.00293	BL
50,30	0.00237	0.00244	0.00247	0.00237	0.00238	0.00240	0.00234	0.00241	BL
15,50	0.00794	0.00534	0.00488	0.00377	0.00718	0.00648	0.00617	0.00559	BD
30,50	0.00301	0.00268	0.00262	0.00255	0.00292	0.00283	0.00280	0.00272	BD
100.15	0.00419	0.00289	0.00275	0.00363	0.00375	0.00338	0.00296	0.00291	BP
100,30	0.00201	0.00185	0.00184	0.00199	0.00194	0.00190	0.00180	0.00183	BL
100,50	0.00145	0.00147	0.00148	0.00146	0.00145	0.00145	0.00143	0.00145	BL
15,100	0.00870	0.00519	0.00458	0.00365	0.00766	0.00671	0.00633	0.00553	BD
30,100	0.00324	0.00252	0.00239	0.00208	0.00303	0.000284	0.00279	0.00260	BD
50,100	0.00185	0.00166	0.00162	0.00146	0.00180	0.00175	0.00174	0.00168	BD

Table 8. R_2estimators performance for Experiments 1.

n.m_1, m_2	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30,30	0.00334	0.00287	0.00281	0.00282	0.00318	0.00305	0.00279	0.00284	BL
30,15,15	0.00352	0.00278	0.00268	0.00405	0.00329	0.00308	0.00278	0.00278	BP
50,15,15	0.00380	0.00244	0.00225	0.00363	0.00336	0.00299	0.00263	0.00250	BP
50,30,30	0.00167	0.00159	0.00158	0.00182	0.00164	0.00162	0.00154	0.00157	BL
15,50,50	0.00357	0.00251	0.00236	0.00214	0.00323	0.00293	0.00264	0.00254	BD
30,50,50	0.00185	0.00175	0.00173	0.00175	0.00181	0.00179	0.00170	0.00173	BL
100.15,15	0.00440	0.00231	0.00202	0.00356	0.00373	0.00316	0.00272	0.00243	BP
100,30,30	0.00177	0.00137	0.00130	0.00173	0.00164	0.00154	0.00144	0.00139	BP
100,50,50	0.00103	0.00095	0.00094	0.00108	0.00100	0.00098	0.00095	0.00095	BP
15,100,100	0.00421	0.00267	0.00246	0.00221	0.00372	0.00329	0.00293	0.00274	BD
30,100,100	0.00189	0.00160	0.00156	0.00150	0.00180	0.00172	0.00162	0.00160	BD
50,100,100	0.00119	0.00113	0.00113	0.00112	0.00117	0.00116	0.00112	0.00113	BD, BL

Table 9. R_2 estimators performance for Experiments 2.

n. m_1, m_2	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30,30	0.00197	0.00186	0.00187	0.00221	0.00191	0.00188	0.00172	0.00181	BL
30,15,15	0.00386	0.00280	0.00262	0.00453	0.00355	0.00326	0.00311	0.00289	BP
50,15,15	0.00454	0.00274	0.00245	0.00438	0.00400	0.00351	0.00332	0.00292	BP
50,30,30	0.00171	0.00149	0.00145	0.00199	0.00165	0.00159	0.00157	0.00152	BP

15,50,50	0.00226	0.00173	0.00168	0.00171	0.00207	0.00192	0.00171	0.00172	BD,BL
30,50,50	0.00136	0.00136	0.00137	0.00151	0.00136	0.00135	0.00130	0.00134	BL
100.15,15	0.00575	0.00310	0.00266	0.00474	0.00495	0.00423	0.00397	0.00336	BP
100,30,30	0.00186	0.00135	0.00125	0.00182	0.00172	0.00158	0.00155	0.00141	BP
100,50,50	0.00091	0.00079	0.00077	0.00098	0.00088	0.00085	0.00084	0.00081	BP
15,100,100	0.00291	0.00186	0.00172	0.00157	0.00257	0.00228	0.00200	0.00190	BD
30,100,100	0.00135	0.00116	0.00114	0.00112	0.00128	0.00123	0.00015	0.00116	BL
50,100,100	0.00078	0.00076	0.00076	0.00079	0.00077	0.00077	0.00074	0.00076	BL

Table 10. R_2 estimatores performance for Experiments 3.

n.m_1, m_2	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30,30	0.00154	0.00179	0.00187	0.00187	0.00159	0.00164	0.00167	0.00175	BQ
30,15,15	0.00378	0.00274	0.00256	0.00441	0.00348	0.00320	0.00319	0.00288	BP
50,15,15	0.00405	0.00250	0.00223	0.00391	0.00358	0.00317	0.00308	0.00267	BP
50,30,30	0.00148	0.00127	0.00123	0.00173	0.00142	0.00137	0.00139	0.00131	BP
15,50,50	0.00142	0.00144	0.00150	0.00182	0.00139	0.00138	0.00133	0.00140	BL
30,50,50	0.00102	0.00111	0.00114	0.00140	0.00104	0.00106	0.00108	0.00110	BQ
100.15,15	0.00442	0.00232	0.00197	0.00362	0.00379	0.00322	0.00307	0.00255	BP
100,30,30	0.00147	0.00104	0.00096	0.00143	0.00134	0.00123	0.00124	0.00110	BP
100,50,50	0.00074	0.00064	0.00061	0.00081	0.00071	0.00069	0.00070	0.00066	BP
15,100,100	0.00152	0.00133	0.00136	0.00151	0.00142	0.00135	0.00124	0.00130	BL
30,100,100	0.00089	0.00093	0.00095	0.00106	0.00089	0.00090	0.00088	0.00091	BL
50,100,100	0.00059	0.00063	0.00065	0.00073	0.00060	0.00061	0.00062	0.00063	BQ

Table 11. R_2 estimators performance for Experiments 4**.**

n. m_1, m_2	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30,30	0.00210	0.00209	0.00212	0.00259	0.00207	0.00206	0.00193	0.00203	BL
30,15,1	0.00362	0.00265	0.00248	0.00425	0.00333	0.00307	0.00290	0.00272	BP

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50,15,15	0.00392	0.00241	0.00217	0.00379	0.00346	0.00305	0.00283	0.00253	BP
50,30,30	0.00161	0.00141	0.00137	0.00187	0.00155	0.00150	0.00147	0.00143	BP
15,50,50	0.00218	0.00178	0.00177	0.00188	0.00203	0.00191	0.00172	0.00176	BL
30,50,50	0.00128	0.00131	0.00133	0.00152	0.00128	0.00129	0.00125	0.00129	BL
100.15,15	0.00488	0.00260	0.00224	0.00400	0.00418	0.00356	0.00326	0.00280	BP
100,30,30	0.00162	0.00119	0.00111	0.00159	0.00150	0.00138	0.00134	0.00123	BP
100,50,50	0.00089	0.00078	0.00076	0.00095	0.00086	0.00083	0.00083	0.00080	BP
15,100,100	0.00246	0.00170	0.00163	0.00161	0.00220	0.00198	0.00174	0.00171	BD
30,100,100	0.00121	0.00109	0.00108	0.00111	0.00116	0.00113	0.00107	0.00108	BL
50,100,100	0.00081	0.00079	0.00079	0.00082	0.00080	0.00079	0.00077	0.00078	BL

Table 10. R_2 estimators performance for Experiments 5.

n. m_1, m_2	BQ	BS	BP	BD	BE	BW	BL	BNL	BEST
15,30,30	0.00125	0.00160	0.00170	0.00246	0.00132	0.00141	0.00156	0.00159	BQ
30,15,15	0.00370	0.00271	0.00253	0.00429	0.00342	0.00315	0.00327	0.00288	BP
50,15,15	0.00409	0.00260	0.00233	0.00397	0.00366	0.00325	0.00327	0.00280	BP
50,30,30	0.00132	0.00112	0.00107	0.00157	0.00127	0.00121	0.00129	0.00117	BP
15,50,50	0.00103	0.00122	0.00130	0.00166	0.00105	0.00109	0.00113	0.00119	BQ
30,50,50	0.00080	0.00091	0.00094	0.00121	0.00083	0.00085	0.00092	0.00091	BQ
100.15,15	0.00429	0.00024	0.00207	0.00358	0.00372	0.00321	0.00316	0.00263	BS
100,30,30	0.00141	0.00102	0.00094	0.00138	0.00130	0.00120	0.00124	0.00109	BP
100,50,50	0.00067	0.00057	0.00055	0.00073	0.00064	0.00062	0.00065	0.00059	BP
15,100,100	0.00097	0.00099	0.00105	0.00123	0.00094	0.00093	0.00091	0.00096	BL
30,100,100	0.00066	0.00073	0.00076	0.00087	0.00067	0.00068	0.00070	0.0072	BQ
50,100,100	0.00045	0.00049	0.00050	0.00057	0.00046	0.00047	0.00048	0.00049	BQ

5-Conclusion

From above results, it observed that in general the best performance in order are as (BL, BP, BD, BQ, BS, BNL) respectively under the different sample sizes and for the different experiments studied of the Bayes method.

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