An Analytical study of Optimal Die Design of Elliptic Tube

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Abstract

In this study, die profile of the elliptic tube extrusion is analysis to predict the optimum die design by using upper bound theory. Developed manner in determining the relative extrusion power by dividing the extrusion die for different sector and calculate the relative deformation power of each sector under different condition which are reduction of area, friction factor and relative die length. The study predicts a new equation for determining the optimum die length with taking in the count the effects of those variables. The surface of the die is generated by an envelope of straight lines drawn from the points on the perimeter at the entry section to corresponding points at exist of the die. An upper bound technique based on the kinematically admissible velocity field is used to determine the forming stress. A complex program built in Visual Fortran V5.0 used to calculate relative extrusion power with variable friction factor (relative optimum extrusion power for values of reduction area are changed from 52.6% at area reduction equals to 0.15 to 82.8% at area reduction equals to 0.4). ANSYS program is used to compare the behavior and values of relative extrusion stress for optimum relative die length at constant reduction of area (R.A.=25%). The results are compared with other papers theoretical and experimental results are found to be in a very good agreement.

Keyword: extrusion, upper bound theory, reduction of area, friction factor, kinematically admissible velocity field

دراسة تحليلية لتصميم القالب الأمثل للأنابيب البيضوية

الخلاصة

الدراسة الحالية بحثت في شكل القالب لبثق الأنابيب البيضوية لإيجاد التصميم الامثل للقالب باستخدام نظرية الحد الأعلى. إسلوب مطور في حساب طاقات التشكيل النسبية من خلال تقسيم قالب البثق إلى عدد من الأجزاء وحساب طاقات التشكيل لكل جزء عند ظروف مختلفة متمثلة بنسبة النقصان بالمساحة ، عامل الاحتكاك وطول القالب النسبي. الدراسة استنتجت معادلة لتحديد طول القالب الأمثل مع الأخذ بنظر الاعتبار تأثير المتغيرات أعلاه. سطح القالب يتكون بواسطة غلاف من الخطوط المستقيمة الممتدة بين نقاط على السطح عند مقطع الدخول ونقاط على سطح الخروج. تقنية الحد الأعلى اعتمدت على حيز السرعة الحركية الممكن المستخدم لحساب إجهاد

التشكيل. تم بناء برنامج معقد باستخدام الـ (Visual Fortran V5.0) لحساب طاقات التشكيل النسبية لقيم معامل احتكاك متغيره (طاقة البثق النسبية المثلى لقيم من نسب انخفاض في المساحة فقد تغيرت من 52.6% عند انخفاض المساحة بمقدار 0.15 الى 82.8% عند انخفاض المساحة بمقدار 0.4). تم استخدام برنامج ANSYS لمقارنة سلوك وقيم اجهاد البثق النسبي عند قيم طول القالب النسبية المثالية عند نسبة تخفيض تساوي 25%. النتائج المستحصلة قورنت مع النتائج النظرية والعملية لبحوث أخرى ووجدت في تطابق جيد جداً.

الكلمات المرشدة: البثق ، نظرية لحد الأعلى ، النقصان بالمساحة ، عامل الاحتكاك، حيز السرعة الحركية الممكن.

Notation:

A_f	final cross section area (mm^2)
A_o	Initial cross section area (mm^2)
AM	Fraction factor
$G(\varphi,z)$	Function determined from the determinant of Jacobian matrix
$F(x,y)$	Shape function of product
L	Die length (mm)
δL	Length of each sector of die (mm)
J, n ,z	Polar coordinate system
R_o	Radius of initial cross section (mm)
V_o , V_f	Velocity at initial and final cross section (mm/sec)
V_x ,V_y, V_z	Velocity components in the Cartesian coordinate system (mm/sec)
$WE_{1,2}$	The energy loss due to the velocity discontinuity at entry shear surface (N.mm/sec)
$WI_{1,2}$	The internal energies of the deformation (N.mm/sec)
$WF_{1,2}$	The energy loss due to the velocity discontinuity at exit shear surface (N.mm/sec)
$Ws_{1,2}$	The energy of friction between metal and die surface (N.mm/sec)
$WT_{1,2}$	Total energy of extrusion process (N.mm/sec)
REF	The Relative Extrusion Power per unit volume ($1/m^3$)
XL	Relative die length (L/R_o)
z, y, x	Cartesian coordinate system
σ_o	Yield stress (N/mm^2)
α_1, α_2, ….α_n	Slop of the each sector in the die (degree)

Introduction

The analysis of the stresses in the metal working has been an important area of plasticity for the past few years. Since the forces and deformations generally are quite complex.

An upper bound analysis provides an overestimation of the required deformation force. It is more accurate because it will always result in an overestimation of the load that press or the machine will be called upon to deliver.

Extrusion die profile plays an important role on material flow, microstructure evaluation, speed of production and left out material in the die. Conical dies, designed using conventional method, suffer from two major drawbacks. First, the formation of dead metal zone and secondly large amount of left out material in the die cavity[1]. Attempts to design streamlined dies using Bezier curve and upper bound theorem [2].

Kim et. al. optimized die profile of axisymmetric extrusion of Metal Matrix Composites MMCs using FEM in order to obtain uniform strain rate profile. J. S. Gunasekera and S Hoshino investigate a new method for obtaining optimal die shape which produces minimal stress in extrusion or drawing of nonaxisymmetric sections from round bar stock [3].

An upper bound solution for extrusion of " triangular" section product through taper die from round sectioned billet has been developed. A simple discontinuouskinematically admissible velocity field with optimization parameter is proposed [4]. Optimized the die profile using Bezier curve to get uniform microstructure in hot extrusion studied by Lee et al.[5].

An elliptical tubes are used in many applications like ,elliptical tube for bridge rail, fitness equipment applications(it often use custom oval and elliptical tubing to give their machines a more high-tech appearance), various storm water(in these systems can be designed and used in conjunction with stormceptor that uses include water quality treatment device for the removal of total suspended),and there are more.

Theoretical analysis

The upper bound solution is constructed on what is known as kinematically admissible velocity field. A velocity field is said to be kinematically admissible if it is consistent with the velocity boundary condition both in rigid as well as plastic zone.

The principle use of analytical study of metal working process is for determining the forces required to produce deformation for a certain geometry prescribed by the process and is the ability to make an accurate prediction of stress, strain, and velocity at every point in the deformed region of the work piece.

Limiting the Kinematically Admissible Velocity Field [6]:

1-Constant volume of deformed metal inside die.

2- The kinematically admissible velocity field limited by streamline drawing between entry and exist surface S_i, S_o.

3- Deformation region limited by entry and exist surface.

4- Incompressible material.

5- Die divided to many sector and determine the total energy of each sector shown in Figure (1).

The maximum shear stress (τ) calculate according Von Misses theory [6]. For achieving a good streaming of metal inside the extrusion die and absence from defects which created from distortion done by dividing the entry and exist cross section to same number of sectors with keeping constant reduction area of each sector limited between entry and exist cross section and as follow shown figure(2) [7]:

$$\frac{Area(ABCD)}{Area(A'B'C'D')} = \frac{A_O}{A_f} \text{...(1)}$$

the drawing streamlines between entry surface (Si) and exist surface (So) represented the die surface. Limit the arbitrary function for die surface in figure (3) by determining the coordinate on each surface (Si, So) as follow:

$$F(n\sin\varphi, n\cos\varphi, 0)$$
$$F^{'}(\mu\sin\psi, \mu\cos\psi, L)$$

then describe the streamline function on three-axis (X,Y,Z) to limit the velocity components on that axis and determined Jacobian matrix of that velocity component which used in calculate strain rate and then deformation energies in extrusion process according upper bound theory .

After limited the coordinate of point at entry and exist then for determining the kinematically admissible velocity field will find the linear functions which describe die surface on three axis (X,Y,Z):-

$$\begin{aligned} x &= n\sin\varphi + \mu\sin\psi \\ y &= n\cos\varphi + \mu\cos\psi \text{ .... (2)} \\ z &= Z \end{aligned}$$

The equation (2) can be written referring to (φ) after finding the relationship between (φ) and (ψ) for describing that equation in general form:-

$$\begin{aligned} x &= n * A(\varphi) + nB(\varphi) * f(z) \\ y &= n * C(\varphi) + nD(\varphi) * f(z) \text{ .... (3)} \\ z &= Z \end{aligned}$$

By limiting the three dimensional displacement derivative and calculate Jacobian matrix then calculate the determine of Jacobian matrix.

The symbol (') represented the derivative of function relative to (φ,z):-

$$det = -n.[A' + Bf'] (C+ Df) -(A+ Bf) (C' + D'f)= - g(\varphi,z)$$

Where is, $f=z/L$

Or,

$$g(\varphi,z)=n.(A' + B'f)(C+ Df)-(A+ Bf) \ *(C' + D'f)\text{........}(4)$$

The calculating velocity component describe as following:

$$Vx = \frac{k.n.B.f'}{g(\varphi,z)}.Vo \quad \text{.......................................} \ (5)$$

$$Vy = \frac{k.n.D.f'}{g(\varphi,z)}.Vo \ \text{.......................................} \ (6)$$

$$Vz = \frac{k}{g(\varphi,z)}.Vo \quad \text{...} \ (7)$$

Where $k=A'C-C'A$

Upper Bound Solution

The total relative powers of circular cross section of extrusion die (WT_1) have been calculated then determine the total relative power of elliptic cross section (WT_2). The final total relative power (WT) will find by subtracting the total relative power of elliptic cross section (WT_2) from that of circular cross section.

The total relative powers of cross section which represent the hole which created by mandrel through extruded metal inside extrusion die. The deformation power in extrusion process classified as follow:

1- plastic deformation power represent deformation power due to increase of strain rate of the extruded metal inside the die and it determine according following equation [8,9,10]:-

$$WI_1 = \frac{2}{\sqrt{3}} * \sigma_o \int_0^L \int_0^{R_o} \int_0^{\varphi} \left[\frac{1}{2} \sum_{i=1}^{3} \sum_{j=3}^{3} \dot{\varepsilon} ij \dot{\varepsilon} ij \right]^{1/2} d\varphi dndz ...(8)$$

2- Deformation power due to the velocity discontinuity at inlet of the die (WE_1) where (z=0) and determine as following equation[8,10]:-

$$WE_1 = \frac{\sigma_o}{\sqrt{3}} \int_0^{R_o}\int_0^{\varphi} \left[Vx^2 + Vy^2\right]^{1/2}_{z=0} / \det /_{Z=0}\, d\varphi dn....(9)$$

3- Deformation power due to the velocity discontinuity at exit of the die (WF_1) where (z=L) and determine as following equation [8,10] :-

$$WF_1 = \frac{\sigma_o}{\sqrt{3}} \int_0^{R_o}\int_0^{\varphi} \left[Vx^2 + Vy^2\right]^{1/2}_{z=L} / \det /_{Z=L}\, d\varphi dn...(10)$$

4- Deformation power due to friction between internal die surface and the extruded metal flow through the die. It calculated according following equation [8,10] :-

$$WS_1 = \frac{AM * \sigma_o}{\sqrt{3}} \int_0^{L}\int_0^{\varphi} \left[Vx^2 + Vy^2 + Vz^2\right]^{1/2} * \frac{1}{\cos\alpha} * \frac{\delta(x,y)}{\delta(\varphi,z)} d\varphi dz.........(\ 11)$$

Where:

$$\frac{\delta(x,y)}{\delta(\varphi,z)} = \left| \begin{matrix} \delta x/\delta\varphi & \delta x/\delta z \\ \delta z/\delta\varphi & \delta z/\delta z \end{matrix} \right|$$

the total relative power extrusion before subtract that power which represented the elliptic hole determine as follow:

$$WT_1 = WI_1 + WE_1 + WF_1 + WS_1 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(12)$$

The total relative power of elliptic hole (WT_2) that will subtracting later from (WT_1) to find final total relative power which required for extrusion elliptic tube. The (WT_2) determine as follow [8, 10]:

1- Plastic deformation power:

$$WI_2 = \frac{2}{\sqrt{3}} * \sigma_o \int_0^{L} \int_0^{F(n,\varphi)} \int_0^{\varphi} \left[\frac{1}{2}\sum_{i=1}^{3}\sum_{j=3}^{3} \varepsilon^{\cdot} ij\, \varepsilon^{\cdot} ij\right]^{1/2} d\varphi dndz....(13)$$

2- The deformation power due to discontinuity at inlet of die (z=0) calculated as follow [8,10]:

$$WE_2 = \frac{\sigma_o}{\sqrt{3}} \int_0^{\varphi}\int_0^{F(n,\varphi)} \left[Vx^2 + Vy^2\right]^{1/2}_{z=0} / \det /_{Z=0}\, d\varphi dn....(14)$$

3- The deformation power due to discontinuity at exit of die (z=L) calculated as follow [8,10]:

$$WF_2 = \frac{\sigma_o}{\sqrt{3}} \int_0^{\varphi} \int_0^{F(n,\varphi)} \left[Vx^2 + Vy^2\right]^{1/2}_{z=L} / \det/_{Z=L}\, dnd\varphi ....(15)$$

4- The deformation power due to friction between the extruded metal and mandrel surface calculated as follow [8,10]:

$$WS_2 = \frac{m * \sigma_o}{\sqrt{3}} \int_0^{L} \int_0^{\varphi} \left[Vx^2 + Vy^2 + Vz^2\right]^{1/2} * \frac{1}{\cos\alpha} * \frac{\delta(x,y)}{\delta(\varphi,z)} d\varphi dz ....(16)$$

Where:

$$\frac{\delta(x,y)}{\delta(n,\varphi)} = \begin{vmatrix} \delta x/\delta\varphi & \delta x/\delta z \\ \delta z/\delta\varphi & \delta Z/\delta z \end{vmatrix}$$

The total relative power which represented the elliptic hole in elliptic tube determine as follow:

$WT_2 = WI_2 + WE_2 + WF_2 + WS_2$…(17)

So, the final total relative power (WT) of elliptic tube will be as following equation:

$WT = WT_1 - WT_2$……..(18)

The **r**elative **e**xtrusion **p**ower (REF) equal:

$$\text{REF} = \frac{WT}{\sigma_0} \qquad ….. (19)$$

Analysis of Finite Elements methods

First, ANSYS program is used to estimate the relative extrusion stress(extrusion stress dividing by yield stress(σ_o))for optimum relative die length for different values of friction(Am=0 ,0.3, 0.5)that obtained from theoretical study (fig.4) and just for reduction of area 25%. In ANSYS program the data of material properties for the die and the mandrel are taken from properties of steel alloy, but the material properties of extruded bar is taken from aluminum alloy(see Fig.(14)).

Result and Discussion

- Figure (4) shown the effect of relative die length (L/R_o) on the total relative deformation power, in the rang (0.8-1.2) the total relative power have high values as a results of increasingin strain rate which increase the plastic deformation power in extrusion process. The minimum total relative power required for extrusion elliptic tube be in the rang (1.2-1.8) as results of increase friction deformation power due to increase friction surface area between the extruded metal and internal surface die. The optimum relative die length will limited for design purpose. The rang upon (1.8) the total energy as a results of high increasing in friction total power due to high increase in surface friction.

- The comparison between figure (4) at reduction of area (25%) with figure (5,6) at reduction area (35%) show that total relative extrusion power increase with approximate rate (18%) due to increase of plastic deformation power as a results of increase the strain rate of extruded metal through the die.

  So that, increasing reduction of area from (25%-35%) will increase the optimum relative die length with approximate rate (1.7%).

- Figure (7) show the effect of die angle (α) on relative extrusion power where increasing die anglefrom (3-6) degree will decrease the relative extrusion power with approximate rate (39.4%) at constant friction factor (AM=0.0) and reduction of area (R.A. =20%). That approximate rate decreases with increase friction factor (AM). The increase of die angle (α) at constant reduction of area which mean reduce the length of extrusion die due to increase in plastic deformation of extruded metal as a results of increasing strain rate so that, decrease in friction power as a results of decreases in surface contact area between extruded metal and die.

the relative extrusion power decrease with approximate ratio (87%) as a results of increasing temperature from (2.7-6) degree at constant R.A.=20% and friction factor AM=0.1.

- Figure (8) show the optimum relative extrusion power increases with increase relative optimum die length and friction factor as a results of increase the friction power between extruded metal and internal surface of extruded die. The approximate ratio of increasing in optimum relative extrusion power show in table (1). Figure (9) show increase the optimum relative extrusion power with approximate ratio (96.25%) as a results of increasing the relative optimum die length due to increase the reduction of area from (0.15%-0.4%) at constant friction factor at (AM=0.2)

Comparison Results

The theoretical values of relative extrusion power for optimum relative die length for Am=0,0.3, 0.5 and for A.R. equals to 25% are compared with relative extrusion stress that obtained from finite elements results (ANSYS results) under the same conditions. From this comparison ,the values of theoretical and numerical extrusion stresses are equals nearly(with error around 5%,6%,10% for Am=0,0.3,0.5 respectively)(see fig.10) and that means the theoretical model is correct.

The results of this study described in relative dimension and relative extrusion power which was give wide information about die design even can apply for extrusion different material and determine the extrusion power required for extrusion process with taking in the count the effects of reduction of area, friction factor and optimum relative die length.

Conclusion

The dividing the extrusion die for number sector give an excellent agreement when compared it with numerical results .

The study predict a new equation to describe the relationship between relative optimum die length with each of friction factor for extrusion elliptic tube in the rang (0.1-0.9) and reduction of area(0.1%-0.4%) in the following equation(see table(1)&table(2):

$$f_B(R.A)= -1.23-0.64*R.A. + 122.7* R.A.^2 + 40.96*R.A.^3$$

$$f_C(R.A)= 182.4-2345.17 *R.A+ 9494.86*R.A.^2 -10304.9 * R.A.^3$$

$$f_D(R.A)= -699.223+ 8129.49 * R.A. - 28608.9 *R.A.^2+ 31509.6* R.A.^3$$

$$f_E(R.A)= 499.99-5826.34*R.A. + 20500.7*R.A.^2-22607.5 *R.A.^3$$

The general function:

$$f(R.A,AM)_{opt.\ die}= f_B(R.A) +f_C(R.A)* AM +f_D(R.A)* AM^2 +f_E(R.A) * AM^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

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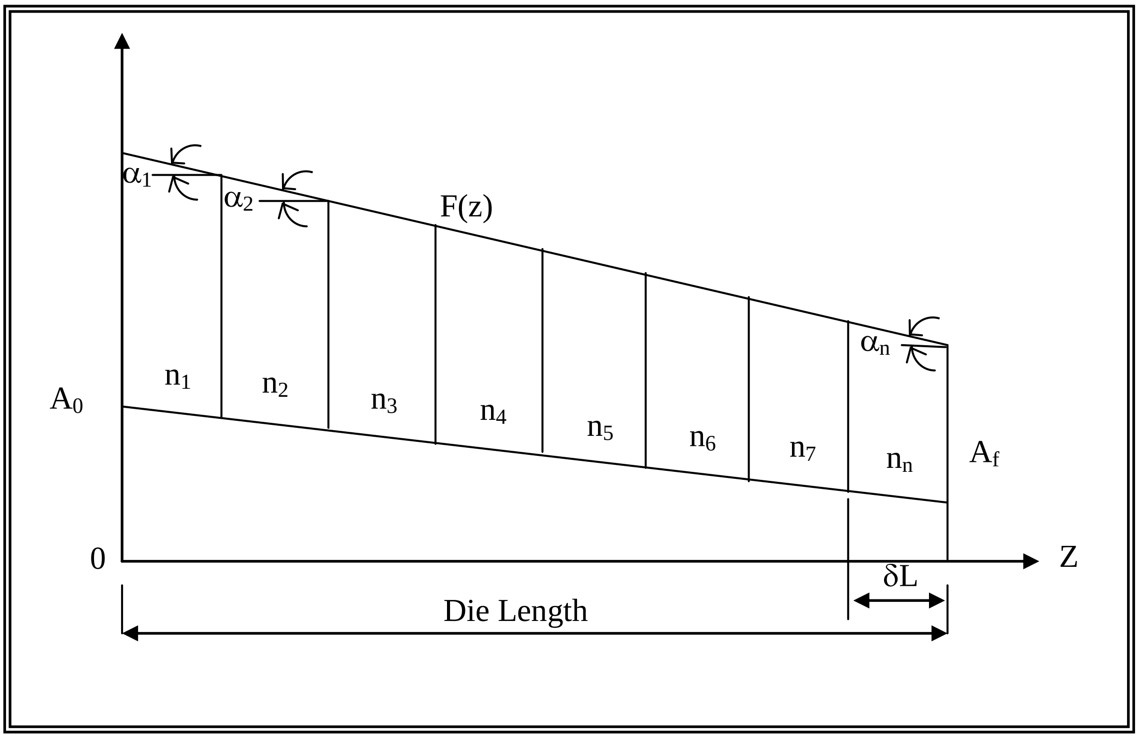


Figure (1) elliptic extrusion die with it sectors and slop angles

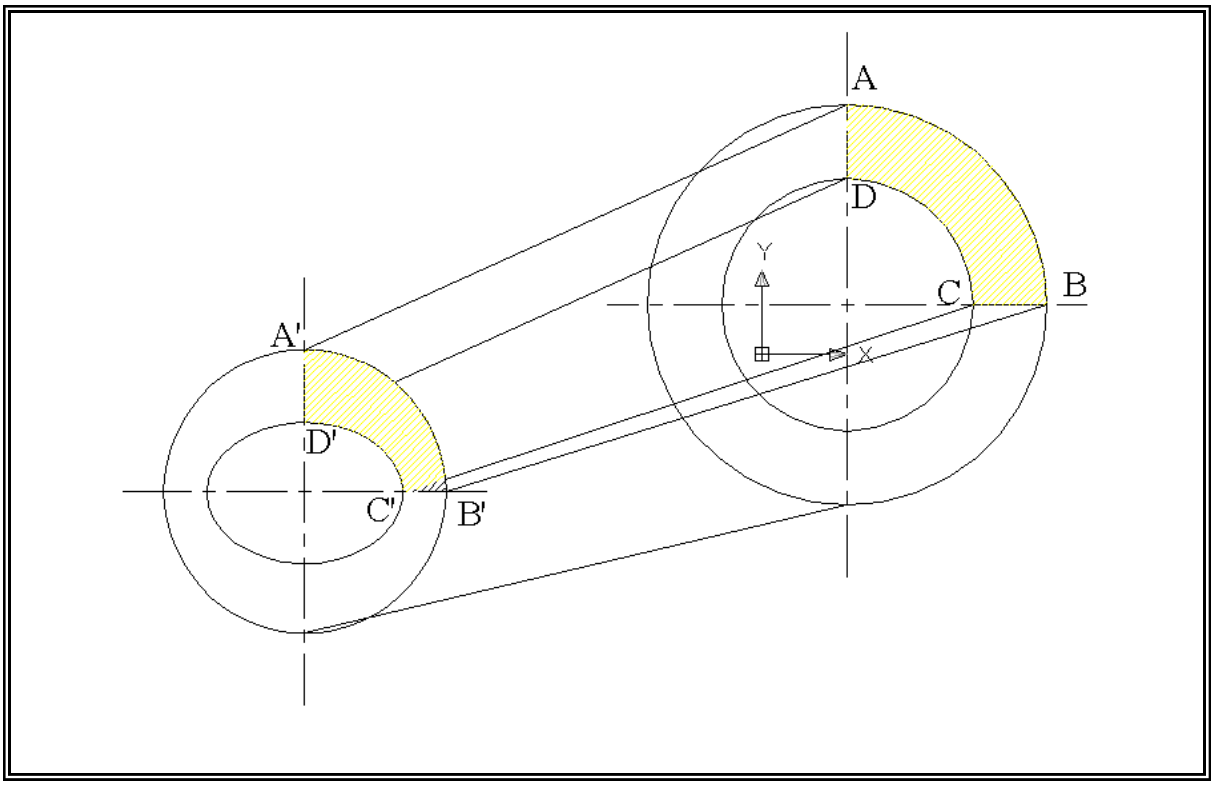


Figure (2) the suggesting extrusion sector confront inlet sector

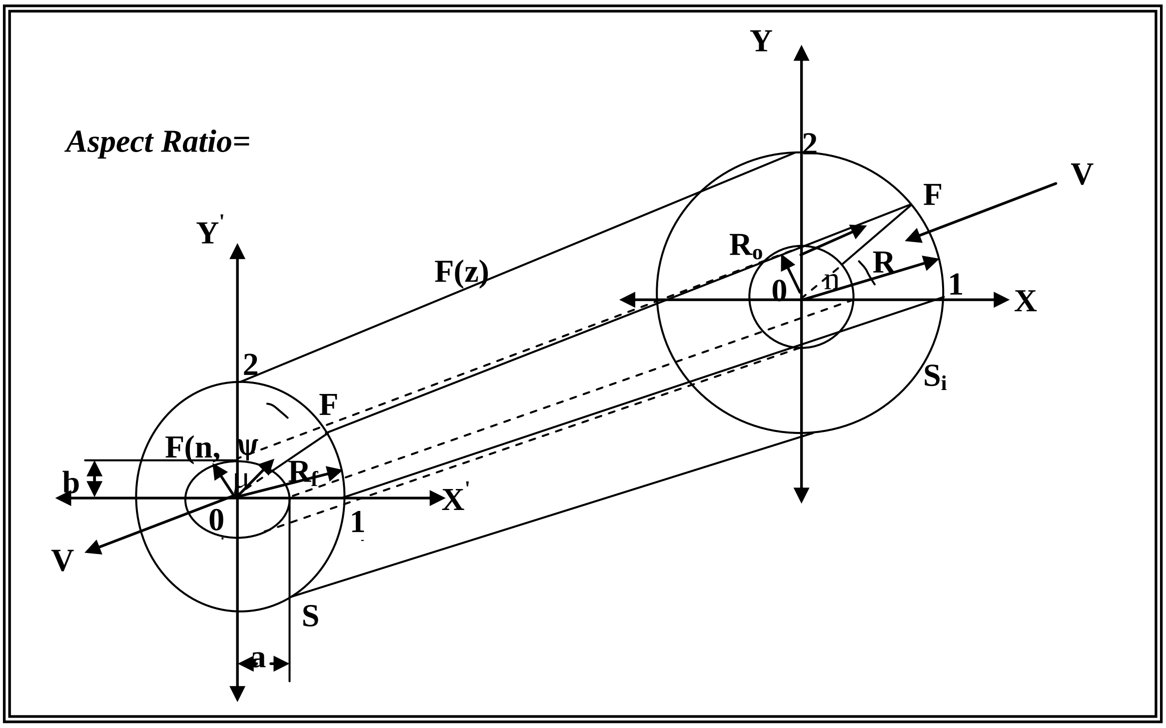


Figure (3) the suggested kinematically admissible velocity field of extrusion elliptic tube

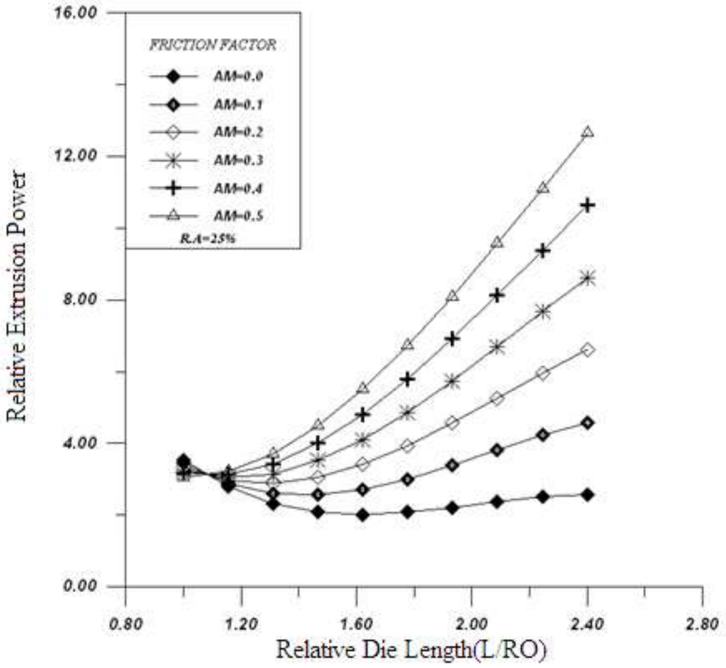


Figure (4) the relationship between relative die length with total relative power at constant reduction of area (R.A.=25%).

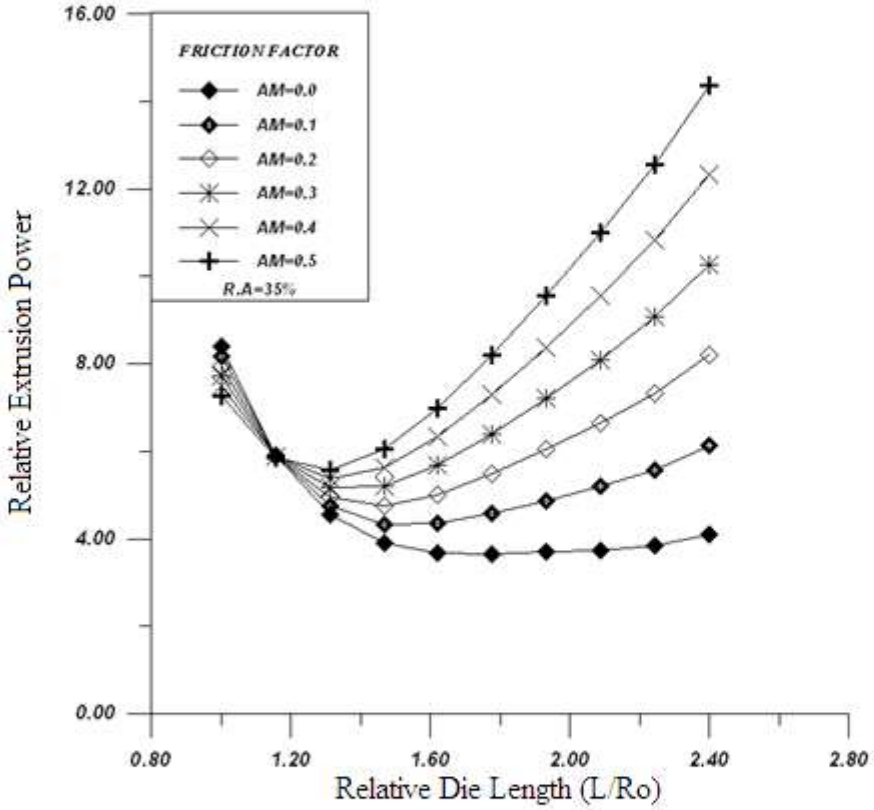
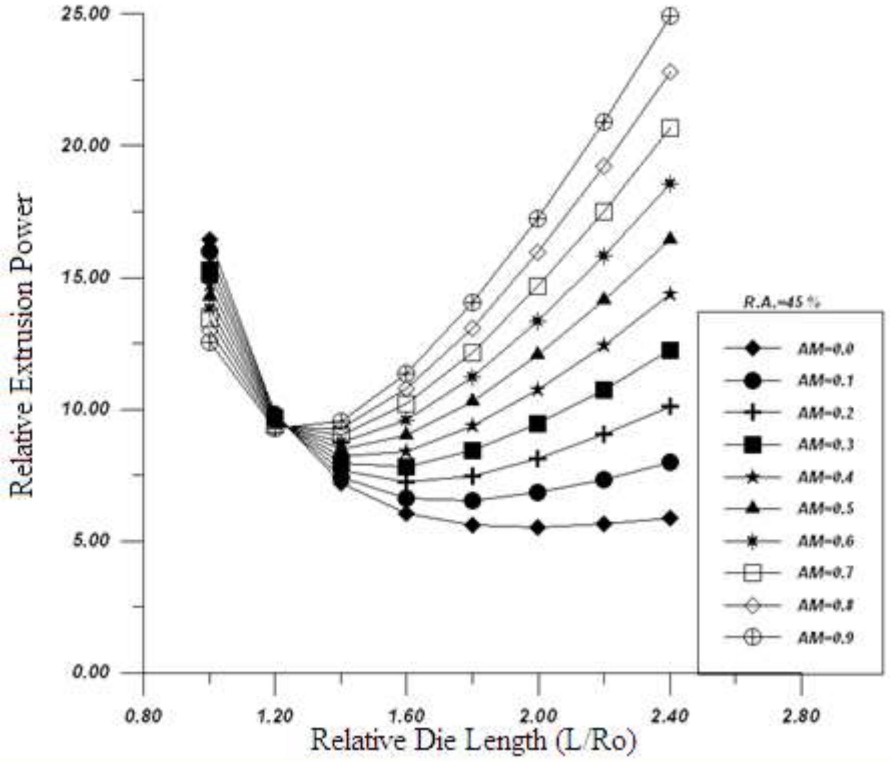


Figure (5) the relationship between relative die length with total relative power at constant reduction of area (R.A.=35%).



Figure(6) the relationship between relative die length with total relative power at constant reduction of area (R.A.=45%).

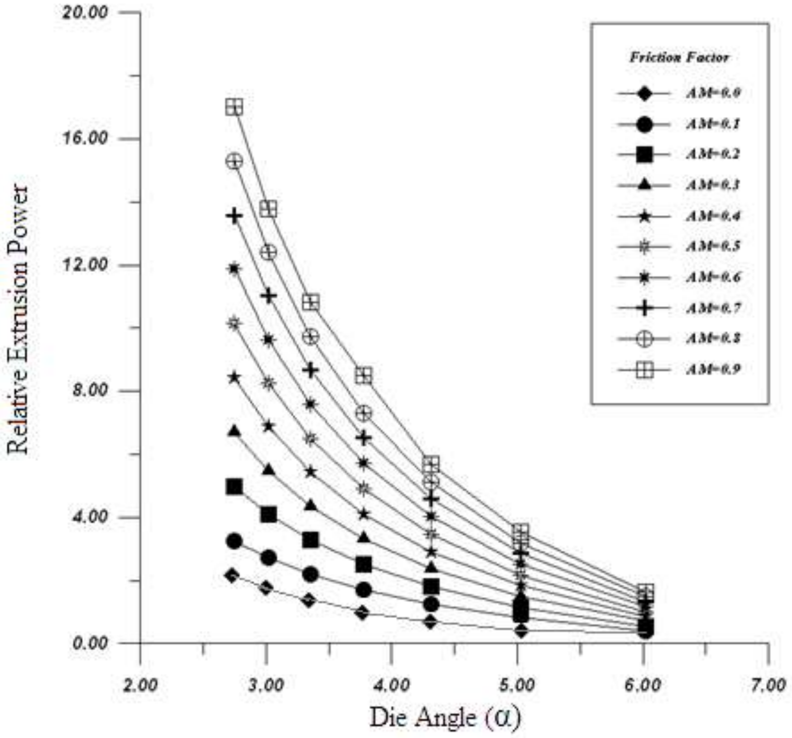
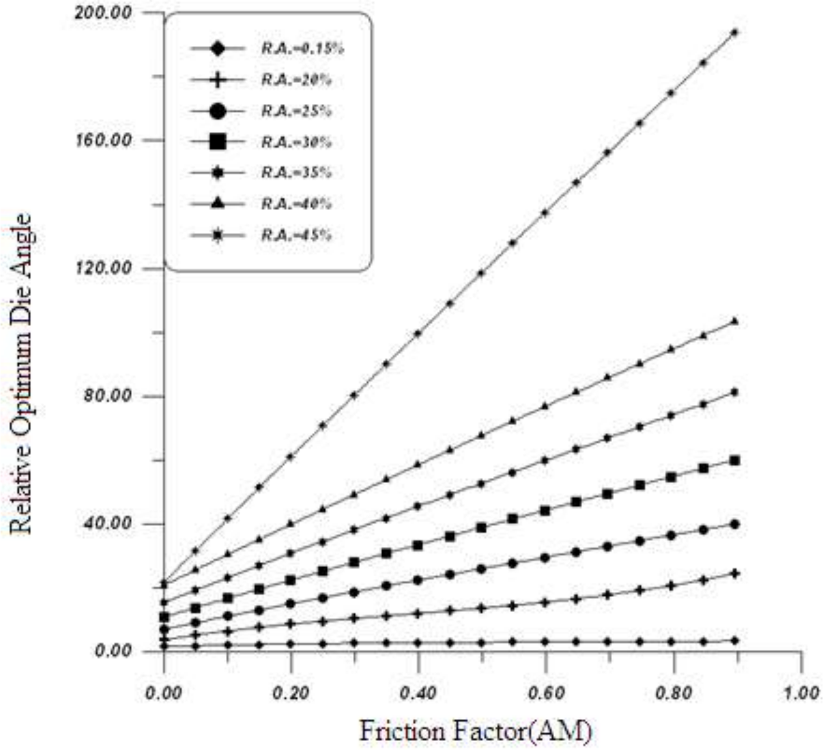


Figure (7) the relationship between slope die angle with relative power at constant reduction of area (R.A.=20%).



Figure(8) the relationship between friction factor and relative optimum die angle power at different reduction of area.

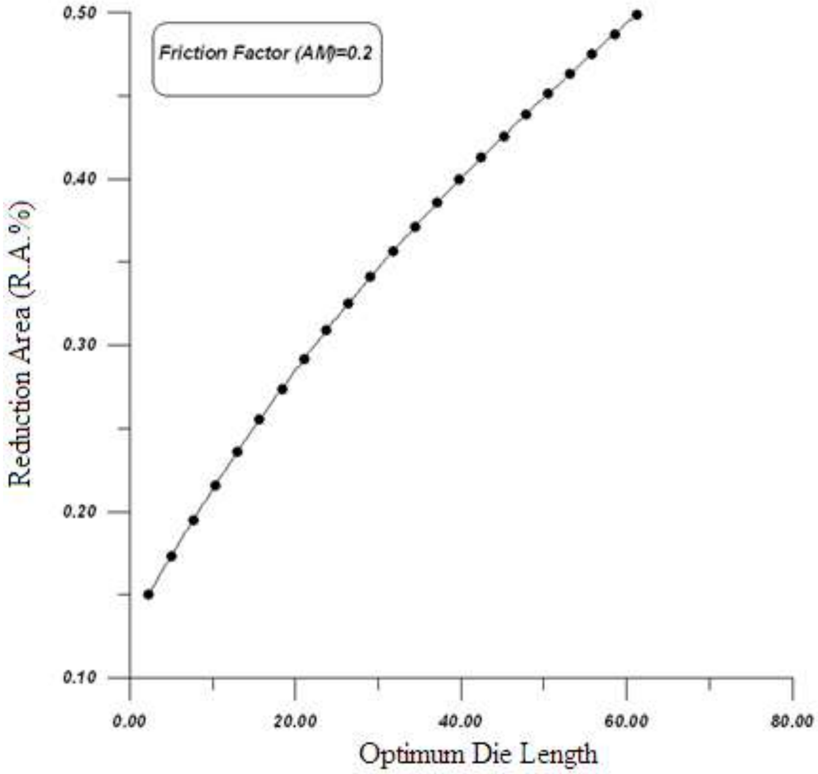


Figure (9) the relationship between optimum die length and reduction of area at (AM=0.2).

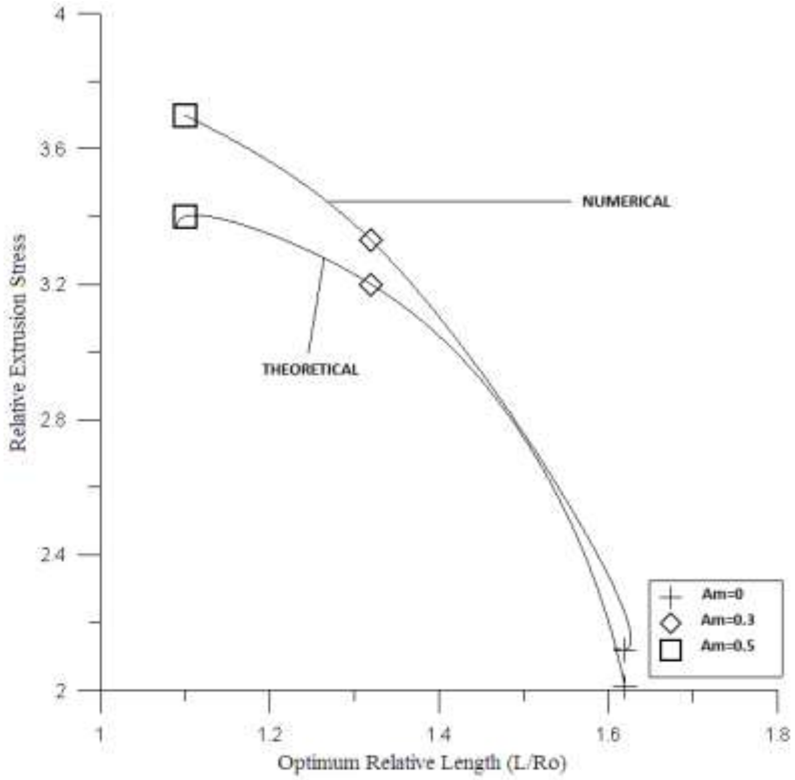


Figure (10) the relationship between relative extrusion stress and optimum relative length(L/Ro) at (R.A.=25%).

Table (1) show the approximate ratio of increasing optimum relative extrusion power with vibration friction factor (0.0-0.9)

Reduction of Area	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**
Approximate ratio of increase relative optimum extrusion power	*52.6%*	*81.5%*	*82.6%*	*82.1%*	*82.2%*	*82.8%*

Table (2) Function of optimum die length for each area reduction

Reduction of area R.A.%	***Function of optimum die length***
0.15	$= 1.54773 + 5.08752* AM - 6.48323*AM^2 + 3.28051* AM^3$
0.2	$= 3.95616 + 22.1515* AM - 4.88491*AM^2 + 1.87126* AM^3$
0.25	$= 6.7852 + 9.38344* AM + 87.8214*AM^2 - 64.6926* AM^3$
0.3	$= 10.8121 + 58.9554* AM - 6.64693*AM^2 + 2.39196* AM^3$
0.35	$= 15.3014 + 78.9967* AM - 10.4913*AM^2 + 4.79408* AM^3$
0.4	$= 20.7631 + 96.505* AM - 4.83105*AM^2$

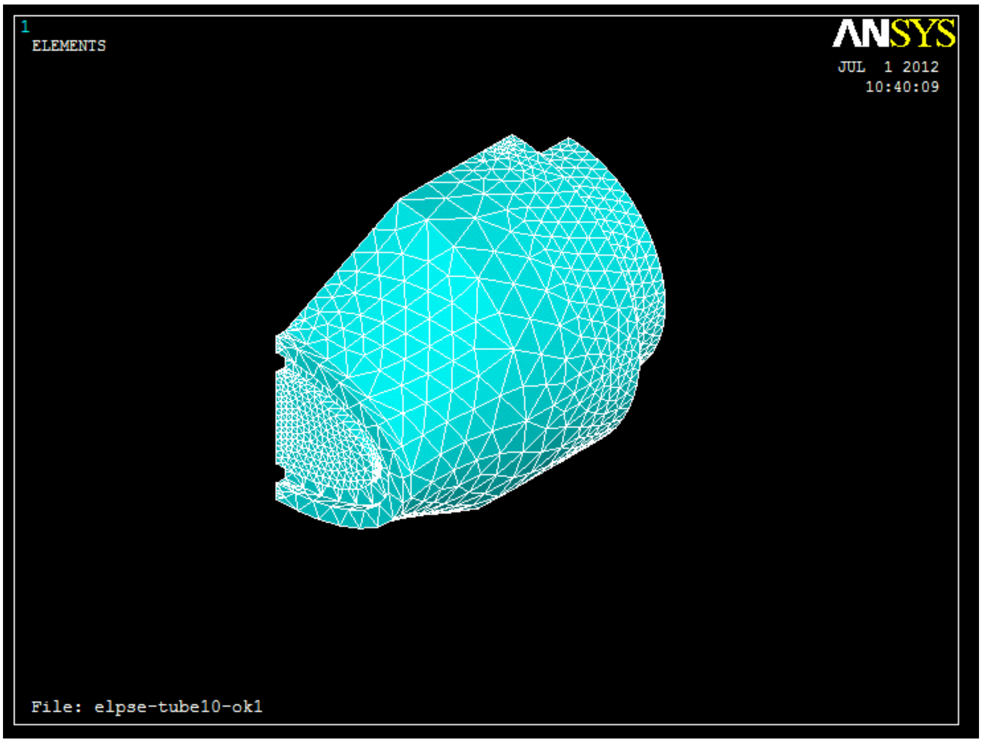


Fig.(11)Elements distribution(type SOLID 185)

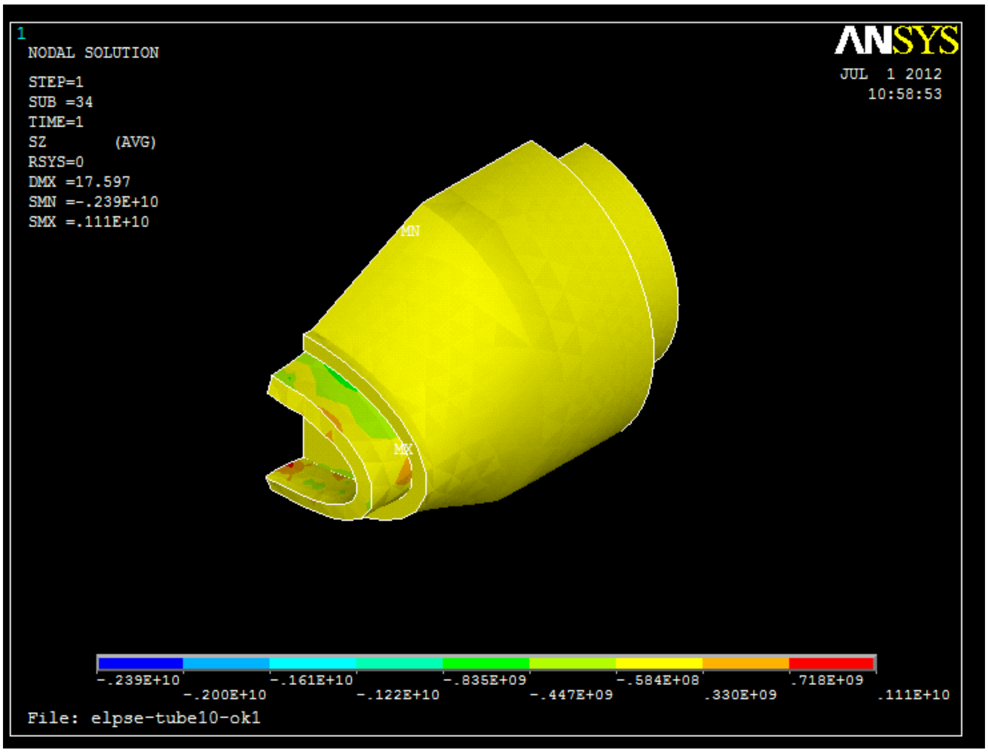
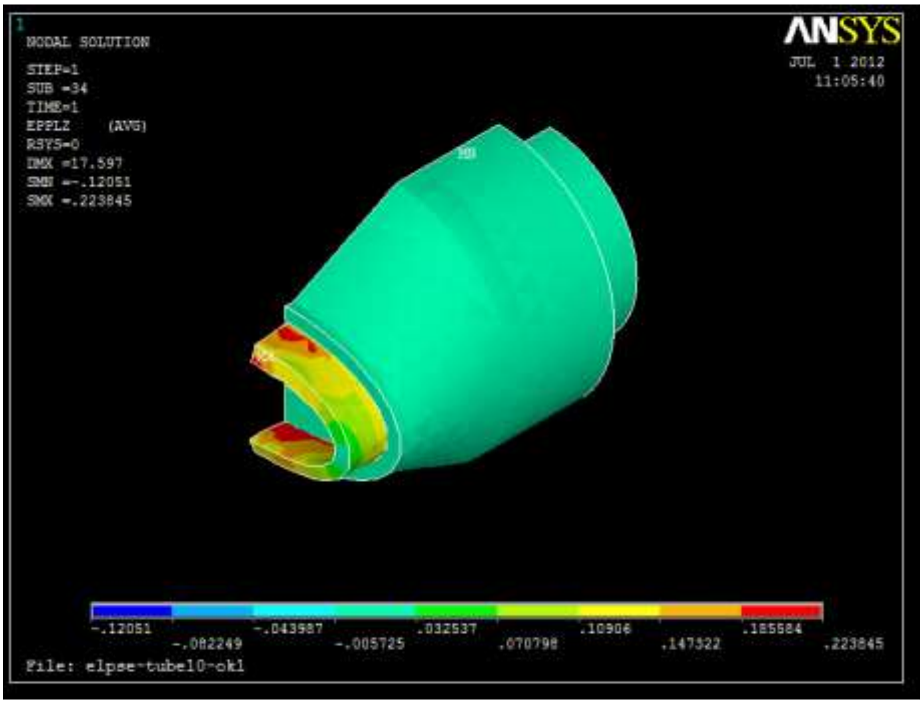


Fig.(12)Stress distribution in direction of forming axis(z-direction)



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1
NODAL SOLUTION

STEP=1
SUB =34
TIME=1
EPPLZ    (AVG)
RSYS=0
DMX =17.597
SMN =-.12051
SMX =.223845
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Fig.(13) plastic strain in direction of forming axis(z-direction)

In ANSYS program the data of material properties and dimensions of the die and the billet are putted in the figure(14):

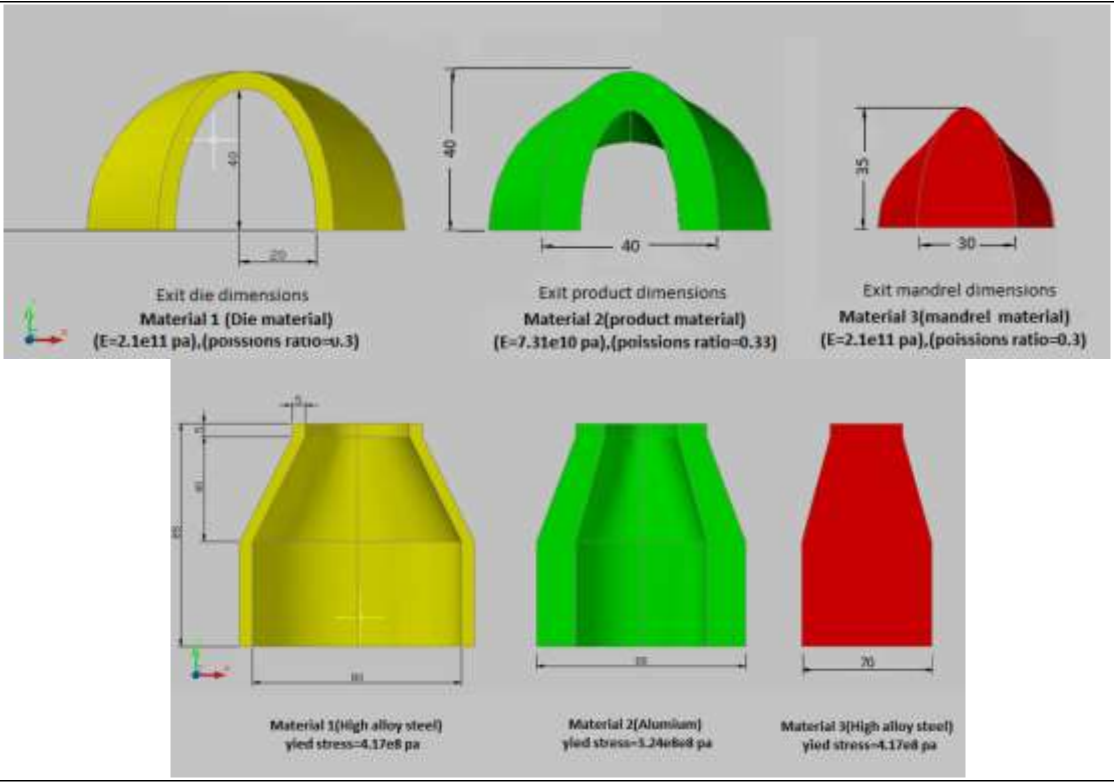

Fig.(14) Dimensions and properties for die, product, and mandrel