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New Formula for Conjugate Gradient Method to Unconstrained Optimization

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A B S T R A C T
This study presents a new conjugate gradient formula (CGM), which is obtained by fulfilling two conditions, firstly the conjugate property and secondly the descent property, in order to address the issue of unconstrained optimization. The gradient, which is the initial derivative of the objective function, is a continuous Lipchitz work and the objective function is differentiable. Using a new variable that serves as the gradient for the objective coefficient function, to create a new course of action based on the old variable and to validate the effectiveness of the new CDM, the table with total iterations and job evaluation was used to display the results of our new methodology using the MATLAB application

Key Words**:** conjugate gradient method, objective function, descent direction, conjugate property, unconstrained optimization.

1. Introduction

several changed CDM to CGM based about the final gradient and orientation (which only depends on the goal function's first derivative) because to the method's simplicity. In [1], a significant unconstrained optimization difficulty was overcome by changing the CGM to always point downward. The authors of [2], who also established the global convergence and descent property, proposed the modified hybrid CGM. New nonlinear CGM was suggested by the authors in [3]. using incomplete line search to address a general optimization problem. The steepest descent approach is paired with CGM in order to tackle an unconstrained optimization problem. A modified PRP CGM was suggested by the authors of [4]. The other members of [5] proposed a novel conjugate descent-based approach. A novel nonlinear hybrid was presented by the author of [6]. a strategy to guarantee that each iteration has the descent attribute Last but not least, the authors created the CGM in [7] as a specific example of for resolving a set of linear equations is one of the more general iterations. In this study, we propose a new CDM for a sol unconstrained optimization problem that guarantees the descent and conjugate properties for al work shown a deviation form the standard topological approach by using the quasi- feasible directions. We'll present a brand-new formula for the direction that minimizes the function

$$min_{x \in R^n} f: R^n \longrightarrow R \tag{1}$$

so that for the unrestricted optimization. Mahmood presents a novel conjugate direction in [9] that addresses the issue of unrestricted optimization

1. Algorithm of_ CGM

One of the most important features of this method is that all directions are conjugate, meaning that $d_{j+1}^T G d_{\mathcal{K}} = o$

1. Let $\varkappa_0 \in \mathcal{R}^n$,is a starting point and $\epsilon > o$, $j = o$.
2. Compute $\nabla f(\varkappa_j) = \mathcal{H}_j$.
3. $d_{\mathcal{K}} = -\mathcal{H}_{\mathcal{K}}$
4. If $\|\mathcal{H}_j\| < \epsilon$, We finish Otherwise, continue to step 5
5. Determined $m_j > 0$such that $f(\varkappa\varkappa_j + m_j d_j) < f(\varkappa_j)$ and set $\varkappa_{j+1} = \varkappa_j + m_j d_j$
6. Determined the following direction utilizing some a conjugate formula, such as

$$d_{j+1} = -\mathcal{H}_j + \frac{\mathcal{H}_{j+1}^T \mathcal{H}_{j+1}}{\mathcal{H}_j^T \mathcal{H}_j} d_j \quad \text{(Fletcher-Reeves Formula)}$$

$j = j + 1$ proceed to 4

2. New CG Method with _PRP _ parameter

Let $f(\varkappa) = \frac{1}{2}\varkappa^T G \varkappa + b\varkappa + \mathcal{C}$ be differentiable objective function and $\varkappa$, b $\in \mathcal{R}^n$ and $\mathcal{C} \in R$ where G is positive- definite and symmetric matrix of type G keep in mind that ,according to the CGM [8] property $\mathcal{H}(\varkappa) = G(\varkappa)$ and that $\mathcal{H}_j^T \mathcal{H}_{\mathcal{K}} = 0$, $\mathcal{K} = 0,1,\ldots\ldots, j-1$

We suggest a new direction as:

$$d_{j+1} = \theta_j d_j - \mathcal{B}_j^{\mathcal{PRP}} \mathcal{H}_{j+1} \tag{2}$$

$$d_{j+1} = \frac{\mathcal{B}_j^{\mathcal{PRP}} \mathcal{H}_{j+1}^T G d_j}{d_j^T G d_j} d_j - \mathcal{B}_j^{\mathcal{PRP}} \mathcal{H}_{j+1}$$

$$d_{j+1} = \frac{\frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\mathcal{H}_{j+1}^{T}Gd_{j}}{d_{j}^{T}Gd_{j}}\ d_{j} - \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\ \mathcal{H}_{j+1}]$$

$$d_{j+1} = \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\ [\frac{\mathcal{H}_{j+1}^{T}Gd_{j}}{d_{j}^{T}Gd_{j}}\ d_{j} - \mathcal{H}_{j+1} \tag{3}$$

Finding the formula for θ_{j} such that the direction specified in Eq.(4), is a descending direction is the issue $\mathcal{H}_{j+1}^{T}d_{j+1} < 0$, and the conjugate direction is

$$d_{j+1}^{T}Gd_{j} = 0 \tag{4}$$

We have the following equation to determine θ_{j} using the conjugate definition.

When we substitute Eq. (3), for Eq. (4), We arrive to the following formula:

$$d_{j+1}^{T}Gd_{k} = [\theta_{j}\ d_{j} - \mathcal{B}_{j}^{\mathcal{PRP}}\mathcal{H}_{j+1}]^{T}Gd_{j} = 0 \quad .k = 0,1,2,\ldots,j \tag{5}$$

$$d_{j+1}^{T}Gd_{k} = \theta_{j}d_{j}^{T}Gd_{j} - \mathcal{B}_{j}^{\mathcal{PRP}}\mathcal{H}_{j+1}^{T}Gd_{j} = 0$$

$\theta_{k} = \frac{\mathcal{B}_{j}^{\mathcal{PRP}}\mathcal{H}_{j+1}^{T}Gd_{j}}{d_{j}^{T}Gd_{j}}$. There for, we get the following equation from Eq (5),

$$d_{j+1} = \theta_{j}d_{j} - \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\mathcal{H}_{j+1} \tag{6}$$

The new CDM based on the CGM is represented in Eq. (6),

Additionally, direction of the Eq. (6), is a descending direction. we will prove that as follows:

By definition of the descent property

$d_{j+1}^{T}\mathcal{H}_{j+1}$ must be negative as follows

$$d_{j+1}^{T}\mathcal{H}_{j+1} = [\ \theta_{j}d_{j} - \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\ \mathcal{H}_{j+1}]^{T}\mathcal{H}_{j+1}$$

$$= \left[\theta_{j}d_{j}^{T}\mathcal{H}_{j+1} - \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\mathcal{H}_{j+1}^{T}\mathcal{H}_{j+1}\right]$$

$$= [\theta_{j}d_{j}^{T}\mathcal{H}_{j+1} - \frac{\mathcal{H}_{j+1}^{T}[\mathcal{H}_{j+1} - \mathcal{H}_{j}]}{\mathcal{H}_{j}^{T}\mathcal{H}_{j}}\left\|\mathcal{H}_{j+1}^{T}\right\|^{2}]$$

The first part is equal to zero by the principal theorem of CDM [8], and

$$d_{j+1}^T\mathcal{H}_{j+1} = -\frac{\mathcal{H}_{j+1}^T[\mathcal{H}_{j+1}-\mathcal{H}_j]}{\mathcal{H}_j^T\mathcal{H}_j}\left\|\mathcal{H}_{j+1}\right\|^2$$

$$= -\frac{\mathcal{H}_{j+1}^T\mathcal{H}_{j+1}-\mathcal{H}_{j+1}^T\mathcal{H}_j}{\mathcal{H}_j^T\mathcal{H}_j}\left\|\mathcal{H}_{j+1}\right\|^2$$

$$= -\frac{\left\|\mathcal{H}_{j+1}\right\|^2-0}{\mathcal{H}_j^T\mathcal{H}_j}\left\|\mathcal{H}_{j+1}\right\|^2 \quad \text{by the principal theorem of CDM}$$

$$d_{j+1}^T h_{j+1} = -\frac{\left\|\mathcal{H}_{j+1}\right\|^2}{\mathcal{H}_j^T\mathcal{H}_j}\left\|\mathcal{H}_{j+1}\right\|^2 < 0$$

$$d_{j+1}^T h_{j+1} = -\frac{\left\|\mathcal{H}_{j+1}\right\|^4}{\mathcal{H}_j^T\mathcal{H}_j} < 0 \tag{7}$$

Since $\left\|\mathcal{H}_{j+1}\right\|^4$ positive value then d_{j+1} in Eq.(7) is descent direction

3. New CG Algorithm

Now we will show the new algorithm of CG as follows:

1. put $\varkappa_0 \in \mathcal{R}^n, eps > o, j = o$
2. Determined $\nabla f(\varkappa_j) = \mathcal{H}_j$
3. Put $d_j = -\mathcal{H}_j$
4. $If \left\|\mathcal{H}_j\right\| < \epsilon$ We finish Otherwise, continue to step 5
5. Compute $m_j > 0$ such that $f(\varkappa_j + m_j d_j) < f(\varkappa_j)$, and set $\varkappa_{j+1} = \varkappa_j + m_j d_j$

6. Determine the next direction by $d_{j+1} \quad \frac{\mathcal{H}_{j+1}^T[\mathcal{H}_{j+1}-\mathcal{H}_j]}{\mathcal{H}_j^T\mathcal{H}_j}[\frac{\mathcal{H}_{j+1}^T G d_j}{d_j^T G d_j} d_j - \mathcal{H}_{j+1}]$

5. Numerical Result

Many standard function are tested in this part by using the new approach. We write the outcomes in a table containing number of iteration and the function evaluation. We represent the test list function by using MATLAB - Lenovo core i3 as follows:

$1 -$ Least square equation for two dimension

$f(\varkappa) = (1-\varkappa_1)^2 + (1-\varkappa_2)^2.$

$2 -$ Rosen brock's function

$f(\varkappa) = (1-\varkappa_1)^2 + (\varkappa_2-\varkappa_1)^2$

3 − Rosen broc'k cliff function,

$$f(\varkappa) = 10^{-4}(\varkappa_1 - 3)^2 - (\varkappa_1 - \varkappa_2) + e^{20(x_1 - x_2)}.$$

4- Generalized Edeger function

$$f(\varkappa) = \sum_{i=1}^{\frac{n}{2}} [(\varkappa_{2i-1} - 2)^4 + (\varkappa_{2i-1} - 2)^2 \varkappa_{2i}^2 + (\varkappa_{2i} + 1)^2].$$

5- Extended Himmelblau function

$$f(\varkappa) = \sum_{i=1}^{\frac{n}{2}} (\varkappa_{2i-1}^2 + \varkappa_{2i} - 11)^2 + (\varkappa_{2i-1} + \varkappa_{2i}^2 - 7)^2.$$

6- Rosen rock's function, [34]

$$f(\varkappa) = \sum_{i=1}^{\frac{n}{2}} [100(\varkappa_i - \varkappa_i^3)^2 + (1 - \varkappa_i)^2].$$

7- Watson function,

$$F(\varkappa) = \sum_{i=1}^{j} f_i^2(\varkappa)$$

$$f_i(\varkappa) = \sum_{j=2}^{3} (j-1)\varkappa_j t_j^{j-2} - \left(\sum_{j=1}^{3} \varkappa_j t_j^{j-1}\right)^2 - 1. \text{ and } t_j = \frac{i}{29}.$$

8- Trigonometric function,

$$100(\varkappa_2 - Sin\varkappa_1)^2 + 0.25\varkappa_1^2$$

9- Rosenbrock function

$$f(\varkappa) = 100(\varkappa_2 - \varkappa_1^2)^2 + (1 - \varkappa_1)^2$$

10- Extended Rosen brock function

$$f(\varkappa) = \sum_{i=1}^{n-1} [100(\varkappa_{i+1} - \varkappa_i^2)^2 + (1 - \varkappa_i)^2]$$

11- Powell singular function

$$f(\varkappa) = (\varkappa_1 + 10\varkappa_2)^2 + 5(\varkappa_3 - \varkappa_4)^2 + (\varkappa_2 - 2x_3)^4 10(\varkappa_1 - \varkappa_4)^4$$

12- Cube function

$$f(\varkappa) = 100(x_2 - x_1^3)^2 + (1 - x_1)^2$$

13- Trigonometric function,

$$f(\varkappa) = \sum_{i=1}^{n} \left[n + i(1 - cosx_i) - sinx_i - \sum_{j=1}^{n} cosx_j \right]^2$$

Table 1. Numerical Results

f.no	Starting point	Dim	CGM Iter	CGM f.min	MCGM Iter	MCGM f.min	The best
1	[0;0]	2	2	8.7909e-017	2	8.2008e-016	MCGM
2	[−1;0;1;2;3;4;5;6]	8	3	4.8714	8	1.0000	MCGM
3	[2;2;2…]	10	15	4. 0753	2	0.1999	MCGM
4	[0;0;0]	3	3	1.2007	4	0.7817	MCGM
5	[0;0]	2	6	3.9917e-008	2	1.7728e-010	MCGM
6	[0;0;0;0;0;0]	6	13	8.0949e-009	2	5.3821e-010	MCGM
7	[0;-1;-2;-3;-4]	5	22	16.9496	3	2.6721e-013	MCGM
8	[1;1;1..]	11	19	1.8498	3	0.2837	MCGM
9	[0;0;0;0]	4	28	4.7959	3	0.8485	MCGM
10	[0;0]	2	14	1.1099	3	0.8485	MCGM
11	[-0.1;-0.2;0.3;0]	4	41	2.2182	79	0.4652	MCGM
12	[0;0;0]	3	2	2.6812	74	0.8441	MCGM
13	[-0.5;-0.5]	2	5	8.9936e-011	4	8.1673e-007	MCGM

6. CONCLUSION

A novel CGM was suggested, and a new gradient coefficient was derived, to calculate the conjugation and descending direction. The new derived formula, the results of which we gave through the preceding table, has been proposed and will be applied within one of the solutions or formulae in order to address the unlimited optimization problem. The problem with unconstrained optimization was successfully solved by the novel technique. to ascertain the descending directions and conjugation.

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