

Discontinuous Speed Control of PMDC Motors with Chattering Attenuation

Bashar F. Midhat

Control and Systems Engineering Department, University of Technology, Baghdad, Iraq
60170@uotechnology.edu.iq

Abstract— In most applications, electric drives are actuated using on/off devices due to their low cost and also due to the relatively high power consumption of the electric drives which make applying linear power amplifiers very costly. In this paper, the operation of PMDC motors under discontinuous control action is analyzed. In addition, to reduce chattering, boundary layer solution has been addressed. Both suggested control techniques have been applied to a PMDC motor model in a software simulation using MATLAB. The results show better performance of boundary layer technique due to the reduced chattering.

Index Terms— PMDC Motor, Discontinuous Control, Boundary Layer.

I. INTRODUCTION

DC motors are widely used in many applications to provide rotational mechanical motion. Due to their direct linear relation between the armature voltage and angular velocity, the DC motors are considered to be easier to control and the actuating circuits are less expensive [1].

Due to easy design, PMDC motors are commonly controlled by PID controller. To adjust PID parameters, it is required to do some experiment or to determine the best way mathematical model of the system. However, conventional PID does not work attractively in applications of nonlinear, complex and of which system model cannot be defined precisely [2,3]. A speed control of separately excited DC motor using various PI, PID and Ziegler Nichols methods was performed in [4]. Classical methods such as Ziegler Nichols which are used to adjust parameters of PID controller can help for determining of these parameters just during design stage. However, they cannot be provided to online adjusting of controller parameters [5,6]. Because PID controller with constant coefficient cannot show a good performance, until now, various modified PIDs such as Adaptive and self-tuning have been developed to cope with these effects [7]. In addition, traditional PID controllers cannot provide steady state error of time varying systems to desired value [8]. Therefore, iterative learning PID control (IL-PID) is proposed to eliminate deficiencies of traditional PID methods [9,10]. A control of a permanent magnet DC motor (PMDC) is realized by using PI and fuzzy logic control in a simulation study [11]. A speed controller for a PMDC motor via self-tuning PID control method and also use fuzzy logic for a self-tuning is designed in [12]. Velagic et al. in [13] carried out speed control of a PMDC motor by fuzzy logic controller, Comparisons are made between PID and fuzzy control under disturbing Effect in both simulation and real time application. Kumar et al. in [14], Ziegler Nichols, modified Ziegler Nichols methods and PI controller which are designed via Particle Swarm Optimization (PSO) are used for speed control of DC motor.

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

An LQR method to optimally tune the PID gains was presented by Yu et al, [15]. In this method the response of the system is near optimal but it requires mathematical calculations and solving equations. Lin et al, [16] have applied GA based PID control for brushless DC motor. Genetic algorithm is originally motivated by the mechanism of natural selection and evolutionary genetics.

Methodologies such as nonlinear control [17], optimal control [18], variable structure control [19], adaptive control [20] and particle swarm optimization strategy [21] have been widely proposed for linear brushless permanent magnet DC motor.

In most of the previous works mentioned above, practical implementation of proposed controllers may have some cost. In addition, for those controllers to operate properly, an additional estimation of performance variables other than the output measurement are required, which adds an extra cost and complexity for the implementation.

The main contribution of this work is to develop a low-cost control algorithm that is able to control the speed of PMDC motors depending only on velocity measurement. For this reason, in this work, a discontinuous controller is proposed for the problem of speed control of PMDC motors using only output measurement. In addition, the problem of chattering is discussed and a solution of boundary layer is proposed in order to reduce chattering and improve performance.

This paper is organized as follows: the mathematical model of PMDC motor is introduced and a model reduction is discussed in section II. In section III, the application of discontinuous control for the problem of speed control of PMDC motors is discussed and a boundary layer solution is introduced for the chattering problem in section IV. Numerical simulations are performed in section V and conclusions are presented in section VI.

II. REDUCED MODEL OF PMDC MOTOR

In this section, a reduced mathematical model for the PMDC motor is introduced since it plays a role in designing an adequate control law. In general, the PMDC motor consists of two coupled electrical and mechanical systems defined by Eq's (1) and (2).

$$\frac{di_a}{dt} = \frac{1}{L_a} v_a - \frac{R_a}{L_a} i_a - \frac{K_b}{L_a} w \quad (1)$$

$$\dot{w} = \frac{K_t}{J} i_a - \frac{B}{J} w + \frac{1}{J} d(t) \quad (2)$$

Where the above model variables are defined in Table I below.

Symbol	Description	Units
w	Armature Angular Velocity	rad/sec
i_a	Armature Coil Current	Amp
J	Armature Plus Load Moment of Inertia	kgm^2
L_a	Armature coil Inductance	H
R_a	Armature Coil Resistance	Ohm
b	Armature Plus Load	Nms/rad
k_t	Motor Torque Constant	Nm/Amp
k_b	Back emf Constant	$volt \cdot sec/rad$

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

As seen from Eq's. (1) and (2) above, the PMDCM model constitutes of two distinguished time constants: the electrical time constant (L_a/R_a) and the mechanical time constant (J/b).

In many practical applications, its considered that the electrical time constant (L_a/R_a) is much smaller than the mechanical time constant (J/b), i.e., the time response of the motor electrical circuit is much faster than the mechanical dynamics. For this reason, the electrical circuit of the PMDC motor can be written as:

$$v_a = i_a R_a + k_b w \quad (3)$$

From the above equation, the armature current can be written as

$$i_a = \frac{v_a}{R_a} - \frac{k_b}{R_a} w \quad (4)$$

Substitute for i_a from Eq. (4) above into Eq. (2) results in

$$\dot{w} = \frac{k_t}{J} \left(\frac{1}{R_a} v_a - \frac{k_b}{R_a} w \right) - \frac{b}{J} w - \frac{1}{J} d \quad (5)$$

Arrange the above equation results in

$$\dot{w} = - \left(\frac{k_t k_b + b R_a}{J R_a} \right) w + \frac{k_t}{J R_a} v_a - \frac{1}{J} d \quad (6)$$

The equation above represents the reduced PMDC motor dynamics. It can be written as

$$\dot{w} = a w + b u + h d \quad (7)$$

Where

$$u = v_a \quad , \quad a = - \left(\frac{k_t k_b + b R_a}{J R_a} \right) \quad , \quad b = \frac{k_t}{J R_a} \quad , \quad h = - \frac{1}{J}$$

III. DISCONTINUOUS CONTROL ANALYSIS

A. Discontinuous Control Design

In this work, the main objective of the controller is to enforce the PMDC motor to run at a desired angular velocity w_d even in presence of external disturbance torque.

The first step is to define the tracking error e as

$$e = w - w_d \quad (8)$$

where w_d is the desired angular velocity (rad/sec).

Differentiate Eq. (8) above

$$\dot{e} = \dot{w} - \dot{w}_d \quad (9)$$

Substitute for \dot{w} from Eq. (7) into Eq. (9) above results in

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

$$\dot{e} = aw + bu + hd - \dot{w}_d \quad (10)$$

From Eq. (8) we get

$$w = e + w_d \quad (11)$$

Substitute in Eq. (10)

$$\dot{e} = a(e + w_d) + bu + hd - \dot{w}_d \quad (12)$$

Eq. (12) above represents the error dynamics of the PMDC motor and the main aim of the controller is to make the tracking error e reach zero which results in $e = 0 \rightarrow w = w_d$.

In this work, the control action is defined by discontinuous control law in Eq. (13) below

$$u = u_o \text{sign}(e) = \begin{cases} u_o & e > 0 (w > w_d) \\ -u_o & e < 0 (w < w_d) \end{cases} \quad (13)$$

To find the stability criteria of Eq. (12), the Lyapunov stability criterion can be applied as follows: First, Define a positive definite Lyapunov function candidate as

$$V = \frac{1}{2}e^2 \quad \forall e \neq 0 \quad (14)$$

According to the Lyapunov stability criterion, the system of Eq. (12) achieves an asymptotic stability if and only if the time derivative of the Lyapunov is positive definite, i.e.

$$\dot{V} = e\dot{e} < 0 \quad \forall e \neq 0 \quad (15)$$

Substitute for \dot{e} from Eq. (12) into Eq. (15) above results in

$$\dot{V} = e[a(e + w_d) + bu + hd - \dot{w}_d] < 0 \quad (16)$$

Substitute for u from Eq. (13) into Eq. (16) above

$$e[a(e + w_d) + bu_o \text{sign}(e) + hd - \dot{w}_d] < 0 \quad (17)$$

Rearranging and using the fact that $e \cdot \text{sign}(e) = |e|$, then Eq. (17) above can be written as

$$ae^2 + e[aw_d + hd - \dot{w}_d] + bu_o|e| < 0 \quad (18)$$

Divide by $|e|$

$$a|e| + \text{sign}(e)[aw_d + hd - \dot{w}_d] + bu_o < 0 \quad (19)$$

Which results in

$$u_o < -\frac{1}{b} \text{sup}(aw_d + hd - \dot{w}_d) \quad (20)$$

The result of the inequality of Eq. (20) above gives sufficient condition for the proposed control law to stabilize the dynamical system of Eq. (12).

A Block diagram for the control system along with the proposed control algorithm is described in Fig. 1 below.

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

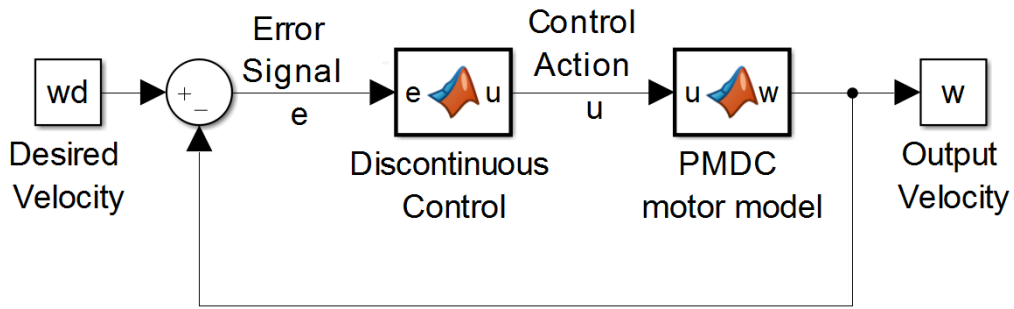


FIG. 1. BLOCK DIAGRAM OF PROPOSED CONTROL.

B. Chattering Reduction Using Boundary Layer Method

From the discussion in the above section, the control action u was introduced as a discontinuous function of the error e (Eq. (13), Fig.2.a). The resulting control signal will change its value between its min and max values (u_o and $-u_o$) in order to keep the error as small as possible since the signum function is undefined at $e=0$, that phenomenon is known as chattering phenomenon (Fig. 2.a). A drawback of chattering is energy losses and/or components damage due to the fast switching of the control action.

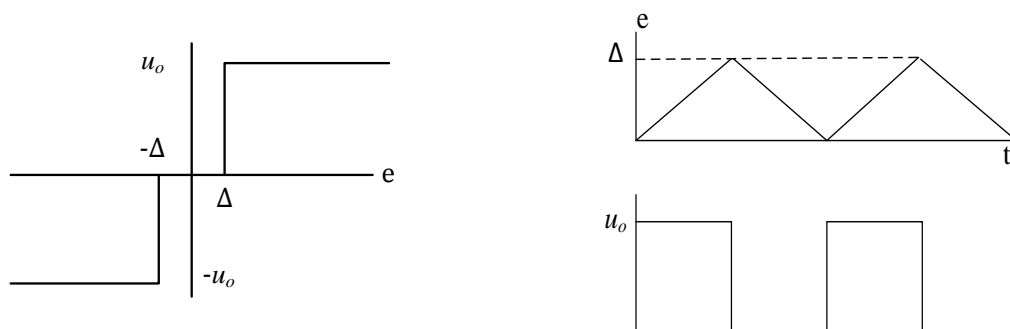
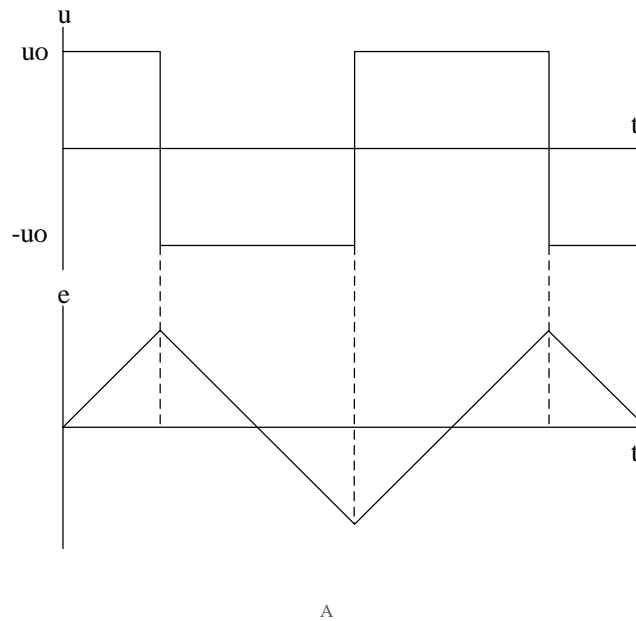


FIG. 2 (A). EXPLANATION OF CHATTERING PHENOMENA.(B). EXPLANATION OF BOUNDARY LAYER SOLUTION.

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

To find a suitable evaluation for the boundary layer value, the following assumption is considered: assuming that the tracking error e rises or decays, then the rate of change of the error, \dot{e} , must not be greater than $\frac{\Delta}{T}$, where T is the system sampling time (sec). Fig. 3 below shows an illustration for this assumption

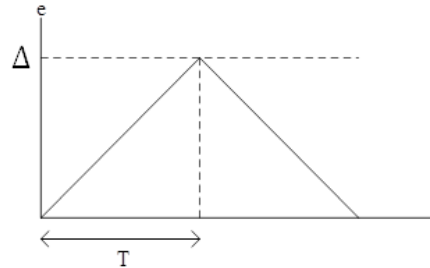


FIG. 3. GEOMETRICAL REPRESENTATION OF BOUNDARY LAYER.

A mathematical representation for this problem can be formulated as

$$\dot{e} < \frac{\Delta}{T}$$

Which leads to

$$\Delta > T \dot{e} \quad (22)$$

Eq. (22) above shows that the larger the sampling time and the rate of change of error, the larger boundary layer required to prevent chattering.

In order to apply the above condition for

$$\Delta > T \cdot \sup(\dot{e}) = T \cdot \sup[a(e + w_d) + bu + hd - \dot{w}_d] \quad (23)$$

IV. SIMULATIONS RESULTS

In this section, the proposed control method is applied to the PMDC motor in software simulation using MATLAB. In the following simulations, the digital sampling time was considered to be 1ms. The motor parameters were considered for general purpose industrial PMDC motor from Moog Components shown in Table II below:

TABLE II. PMDC MOTOR PARAMETER VALUES [5]

Parameter	Value	Units
J	6.63×10^{-3}	kgm^2
L_a	0.012	H
R_a	0.7	Ohm
b	0.37×10^{-3}	Nms/rad
k_t	0.1413	Nm/Amp
k_b	0.1412	$volt \cdot sec/rad$

The motor was considered to be supplied by a 24 volt power supply.

Two performance indices are considered in this work, error and control action. For the error, the performance index is defined as the integral of the error square as in Eq. (24) below

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

$$J_e = \int_0^t e^2 dt \quad (24)$$

In the same manner, the performance index of the control action is defined as

$$J_u = \int_0^t u^2 dt \quad (25)$$

Also, the total performance index J_T of the system can be defined as the sum of both the error and the control action performance index

$$J_T = J_e + J_u$$

The simulation results of both discontinuous and boundary layer methods were plotted together to show the difference between the two schemes.

A. Constant desired velocity with no external disturbance load

In the first simulation, the motor is desired to run at a constant desired angular velocity. The desired angular velocity is assumed to be $v_d = 100 \text{ rad/sec}$. In order to determine the value of the boundary layer Δ , the values in Table II were applied to Eq. (22) and the boundary layer value was found to be $\Delta = 0.45 \text{ rad/sec}$ assuming that the sampling time for the simulation was taken as 0.001 sec and the maximum disturbance value is 1 Nm.

According to Eq. (23), the minimum control action that stabilizes the system was found to be 19.27 volts using the parameters given above which is below the 24 volts supply voltage which means that the controller with supplied voltage should be able to drive the motor at the desired velocity even in presence of the external load torque.

Fig. 's 4-10 show the simulation results. Fig. 4 below shows time response of the motor angular velocity. It shows how the motor speed starts from zero value to the desired speed. Fig. 5 below shows the external disturbance load that exerts on the motor. It can be seen how the motor keeps running on the desired angular velocity even when disturbance load is applied to the motor.

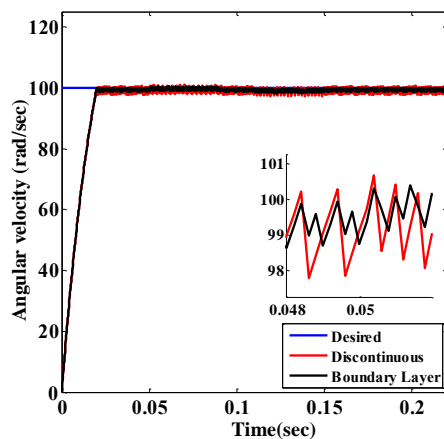


FIG. 4. ANGULAR VELOCITY.

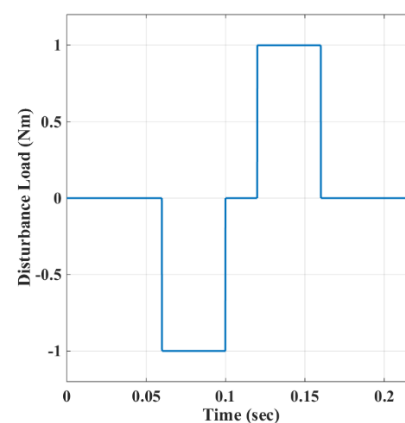


FIG. 5. DISTURBANCE LOAD TORQUE.

Fig. 6 shows the control action of both discontinuous and boundary layer controllers. It can be seen how discontinuous control action chatters between its min and max value to keep the error value as close to zero while in the boundary layer control the control action it only changes its value between zero and its max value. For demonstration purpose, the discontinuous control action is filtered using a low pass filter to show its equivalent DC value and plotted in Fig. 7. The equivalent control action

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

represents the required continuous control action which if replaced instead of the discontinuous one will give the same results. Also, Fig. 8 shows armature current. Fig. 9 shows the power consumption of the motor for both discontinuous and boundary layer control actions which shows that the boundary layer results in a notable reduction in power consumption which is very important aspect.

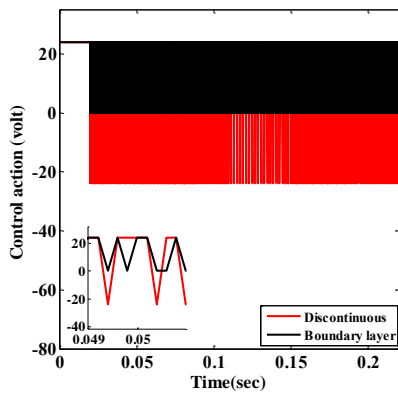


FIG. 6. CONTROL ACTION.

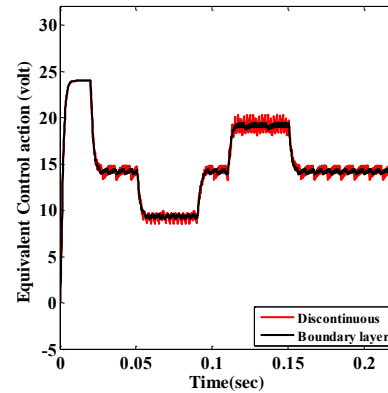


FIG. 7. EQUIVALENT CONTROL ACTION.

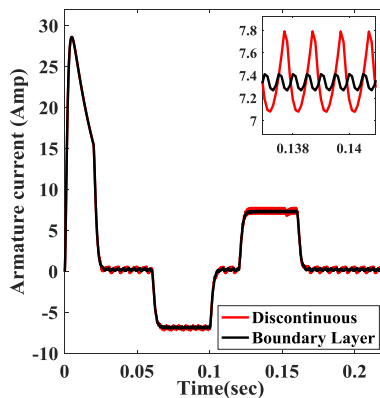


FIG. 8. ARMATURE CURRENT.

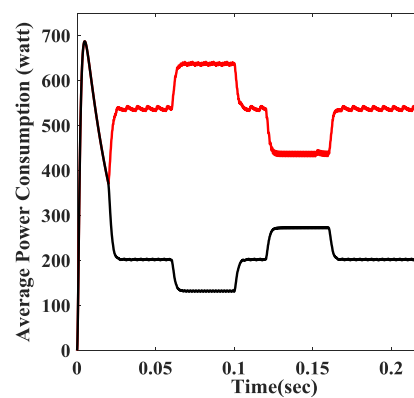


FIG. 9. POWER CONSUMPTION.

Fig. 10 shows the performance indices of both discontinuous and boundary layer controllers. It can be seen how that the error and control action performance indices in boundary layer control is notably less than the discontinuous control action which definitely means less tracking error and also less energy consumption.

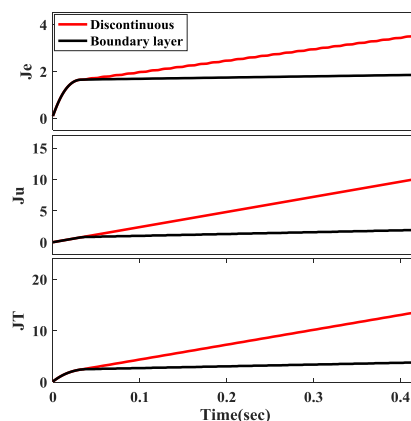


FIG. 10. PERFORMANCE INDICES.

It can be seen that, despite similar output response for both controllers, Fig. 's 9, 10 reveal the real difference between them. Fig. 9 shows the power consumption of motor for each one of the two controllers. It can be seen that in the case of the boundary layer controller the power consumption of

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

the motor has an average value of about 200 watts which is notably less than when the sole discontinuous control action is applied where the power consumption has an average value of 550 watts. Also, *Fig. 10* shows the performance index of each controller where it can be seen that the performance index for the boundary layer is about half of that in case of discontinuous control action which means a less energy spent during the operation of the motor.

In addition, *Fig. 4* shows how, due to the discontinuous control, the angular velocity oscillates between ± 3 rad/sec (3%) of the desired angular velocity while in the case of boundary layer it oscillates only about ± 1 rad/sec (1%) of the desired velocity which indicates an improvement in performance. Also, *Fig. 8* shows that the armature current oscillates between (7 – 7.8) Amp. While in the case of boundary layer it only oscillates between (7.3 – 7.4) Amp. Which indicates less chattering in armature current.

B. Variable desired velocity with external disturbance load

In the second set of simulations, the motor is tasked to run at a variable desired angular velocity. In these simulations, the desired angular velocity is assumed to be in the form $v_d = 100\sin(10t)$. In this case, the control action required to drive the motor was found to be 22.42 volts which is still below the supply voltage and the controller should be able to drive the motor at the desired velocity. *Fig. 's 11-17* show the simulation results.

As in the previous simulation, *Fig. 11* shows the motor angular velocity. *Fig. 's 13, 15* show the control action and armature current respectively. *Fig. 's 16, 17* show the power consumption and performance indices respectively of both discontinuous and boundary layer controllers.

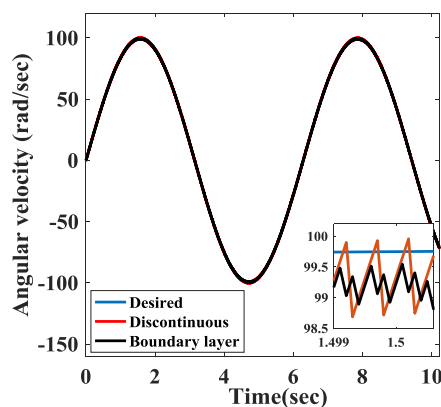


FIG. 11. ANGULAR VELOCITY.

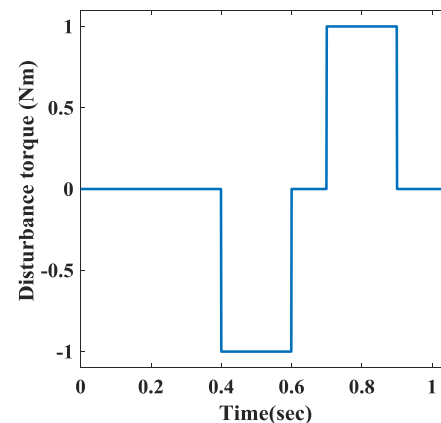


FIG. 12. DISTURBANCE LOAD TORQUE.

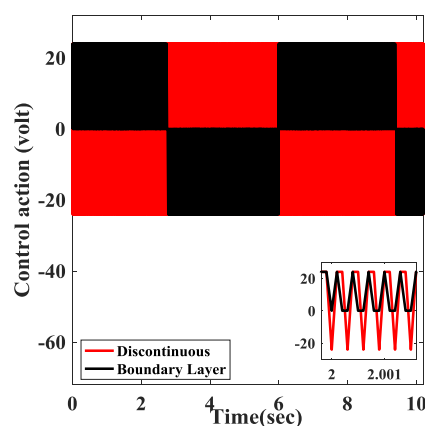


FIG. 13. CONTROL ACTION.

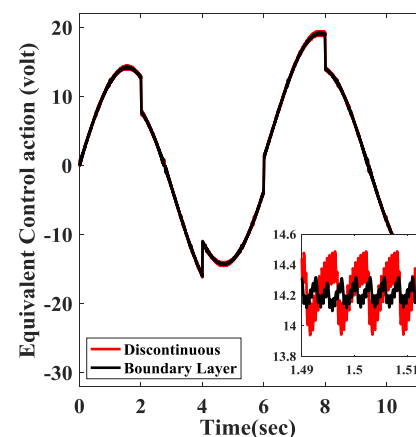


FIG. 14. EQUIVALENT CONTROL ACTION.

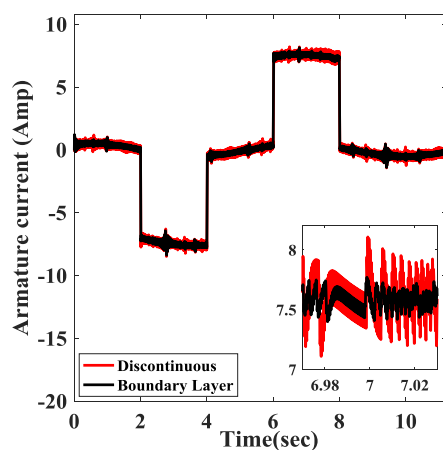
DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

FIG. 15. ARMATURE CURRENT.

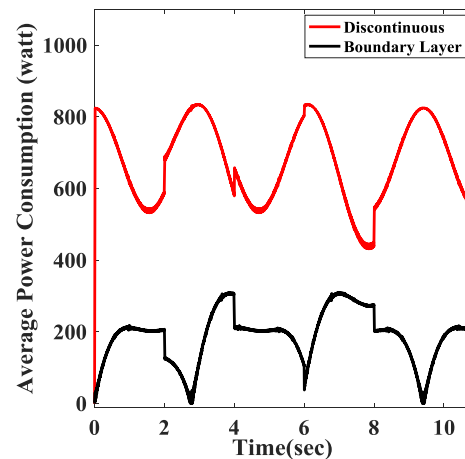


FIG. 16. POWER CONSUMPTION.

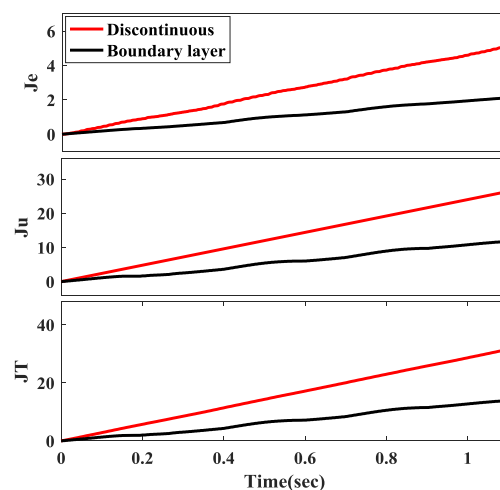


FIG. 17. PERFORMANCE INDICES.

From Fig. 's 16, 17 above, it can be seen that how the boundary layer method consumes a considerably less power than the discontinuous control action. Also, the performance index is much less in the case of boundary layer method than the one in the discontinuous control action which indicates a less energy spent to drive the motor. A final note can be stated about the increasing performance index value which is due to the varying desired velocity which enforces the control action to reverse its polarity in order to keep tracking with the desired one.

V. CONCLUSIONS

In this work, a discontinuous control is proposed for the problem of speed control of PMDC motors. The aim of this paper is to introduce an inexpensive control methodology which reduces the control circuit complexity and cost. Also, a boundary layer solution is proposed for reduction of the chattering problem which presents in applying the discontinuous control action. The results show that the proposed discontinuous control law introduced in Eq. (13) works as desired in tracking the desired angular velocity in presence of external disturbances. Also, the proposed control method proved to be able to reject the external disturbances exerted on the motor. In addition, the boundary layer technique succeeds

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

in reducing the chattering in the control action which results in less oscillations in output signal and hence an improved performance. An important indicator for the proposed control method are the performance indices shown in *Fig 10, 17*. The performance indices show the boundary layer has notably less performance index values which is a desired aspect in evaluation of control systems since it means less error and control action is required to stabilize the system, hence, a less energy consumption and loss are presented. Finally, the most important aspect can be seen from the results is that the power consumption when applying the boundary layer solution is less than when applying the discontinuous control action which is a very important aspect in real life meaning less energy consumption and more saving.

REFERENCES

- [1] M. Sarwer, Md. Abdur Rafiq and B.C. Ghosh: Sliding Mode Speed Controller of a D.C Motor Drive, Journal of Electrical Engineering, The Institution of Engineers, Bangladesh, Vol.31, No.1, 2014, pp. 45-49.
- [2] D. Yulin, "The analysis and implement of PLC-based PI control for the permanent magnet DC motor", Proceedings of Second International Conference on Communication Systems, Networks and Applications, 2020, pp. 95-104.
- [3] G. Huang, "PC-based PID speed control in DC motor", Proceedings of International Conference on Audio, Language and Image Processing, 2008, pp. 400-407.
- [4] U. Kumar, D. Dohera,, "Separately excited DC motor speed control of using various tuning conventional controllers", International Research Journal of Engineering and Technology, Vol.2, No.8, 2015.
- [5] K. Krishnan, G. Karpagam. "Comparison of PID Controller Tuning Techniques for a FOPDT System." International Journal of Current Engineering and Technology ,2013, pp. 2667-2670.
- [6] S. Skogestad, "Simple analytic rules for model reduction and PID controller tuning." Journal of process control, Vol. 13, No. 4, pp.291-309, 2013.
- [7] P. Ananthababu, B. Reddy, "Control of PMDC motor using fuzzy PI controller", Proceedings of International Conference on Control, Automation, Communication and Energy Conservation, pp.1-4, 2019.
- [8] E. Gowthaman, C. Balaji, "Self-tuned PID based speed control of PMDC drive", Proceedings of International Multi-conference on Automation, Computing, Communication, Control and Compressed Sensing, pp. 686-692, 2013.
- [9] K. Nouri, R. Dhaouadi, N. Braiek, "Adaptive control of a nonlinear DC motor drive using recurrent neural networks", Applied Soft Computing, Vol. 8, No. 1, pp.371-382, 2018.
- [10] M. K., Khan, "Design and Application of Second Order Sliding Mode Control Algorithms", Doctor of Philosophy, University of Leicester, pp. 11-26, 2013.
- [11] S. Moussavi, Z. Alasvandi, M. Javadi, , "Speed control of permanent magnet DC motor by using combination of adaptive controller and fuzzy controller", International Journal of Computer Applications, Vol.52, No.20, 2022.
- [12] J. Velagic, A. Galijasevic, "Design of fuzzy logic control of permanent magnet DC motor under real constraints and disturbances", Proceedings of International Conference on Control Applications & Intelligent Control, pp. 461-466, 2019.
- [13] R. Kanojiya, P. Meshram, "Optimal tuning of PI controller for speed control of DC motor drive using particle swarm optimization", Proceedings of International Conference on Advances in Power Conversion and Energy Technologies, pp. 1-6, 2019.
- [14] P. Ananthababu, B. Reddy, "Control of PMDC motor using fuzzy PI controller", Proceedings of International Conference on Control, Automation, Communication and Energy Conservation, pp.1-4, 2019.
- [15] P. M. Pelezewski, and U. H. Kunz, "The Optimal Control of a Constrained drive System with brushless dc motor", IEEE Trans. Ind. Electronics Vol. 37, pp 342-348, 2010.
- [16] P. Sowjanya, S.Tarakalyani: PI and Sliding Mode Control For Permanent Magnet Brushless DC Motor, International Journal of Innovative Technology and Research, Vol.1, No.5, 2018, pp. 497-502.
- [17] N. Hemati, J. S. Thorp, M. C. Leu, ,"Robust nonlinear control of Brushless dc motors for direct-drive robotic applications", IEEE Trans.Ind. Electron., Vol. 37, pp. 460-468, 2019.
- [18] P. M. Pelezewski, U. H. Kunz, "The Optimal Control of a Constrained drive System with brushless dc motor. IEEE Trans. Ind. Electronics Vol. 37, pp.342-348.
- [19] F. J. Lin, , K. K. Shyu, Y. S. Lin, "Variable structure adaptive control for PM synchronous servo motor drive," IEE Proc. IEE B: Elect. Power Applications, Vol. 146, pp. 173-185, 2010.
- [20] Cerruto E., Consoli A., Raciti A. and Testa A. , " A Robust Adaptive Controller for PM motors drives in a Robotic Applications", IEEE. Trans. Power Electronics Vol. 10 pp62-71, 2015.
- [21] Nasri, M., Neezamabadi-Pour, H. Malihemaghfoori, "A PSO Based Optimization of PID Controller for a linear BLDC Motor. Proc. of World academy of Science, Engineering and Technology Vol. 20, 2017.

DOI: <https://doi.org/10.33103/uot.ijccce.23.2.5>

- [22] Yaghoub heidari, Abolfazl Ranjbar Noee, Heydar Ali Shayanfar, Soheil Salehi: Robust Control of DC Motor Using Fuzzy Sliding Mode Control with Fractional PID Compensator, *The Journal of Mathematics and Computer Science*, Vol. 10 No. 4, 2015, pp. 238-246.
- [23] R S. Vaez-Zadeh and M. Zamanian: Permanent Magnet DC Motor Sliding Mode Control System, *IJE Transactions A: Basics*, Vol. 16, No. 4, pp.367-375, 2018.
- [24] Emre Hasan Dursun, Akif Durdu: Speed Control of a DC Motor with Variable Load Using Sliding Mode Control, *International Journal of Computer and Electrical Engineering*, Vol. 8, No. 3, pp. 219-226.
- [25] Mohamad Reza Dastranj, Fatemeh Koozneshin, Esmail Sameni and Mehdi Yousefi Tabari: Robust Control of DC Motor Using Fuzzy Sliding Mode Control And Particle Swarm Optimization “PSO” Algorithm, *Australian Journal of Basic and Applied Sciences*, Vol. 5, No. 11, 2011, pp. 424-430.