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PARAMETRIC STUDY ON A PREDICTION THE DRAG COEFFICIENT FOR A SINGLE GROWING BUBBLE IN UNIFORMLY SUPERHEATED PURE LIQUIDS

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ABSTRACT
A relation for predicting the drag coefficient for the process of bubble growing in a convective surrounding (uniformly superheated pure liquids) was developed. The non-dimensional parameters governing the motion and heat transfer are identified and a theoretical study of the effects of the variation of these parameters is made. The drag coefficient decreases with time for all accelerations, as well as with augmentation of the bubble acceleration at each instant of time, independently of the internal vapor parameters for bubble. The theoretical results obtained were agreeable with the previous theoretical data available with error ± 8.80 % % for non-dimensional time τ=1 and within ± 5.4 % for non-dimensional time τ=10.

KEYWORD: Bubble Growth, Drag Coefficient, Non-Dimensional Parameters.

دراسة المعاملات على التنبأ بمعامل الاعاقة لنمو فقاعة داخل سوائل منتظمة نقية محمصة

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الكلية التقنية النجف

الموجز
تم تطوير علاقة للتنبأ بمعامل الاعاقة في اجراء نمو فقاعة في بيئة متعلقة بحمل حراري (سوائل منتظمة نقية محمصة). تم التحقق من المعاملات غير البعدية السائدة او المسيطرة على الحركة وانتقال الحرارة ودراسة تأثير تغيير هذه العوامل على نمو وتكوين الفقاعة. معامل السحب ينخفض مع الزمن لمختلف التعجيلات ، بالاضافة الى ذلك فان تعزيز تعجيل الفقاعة لكل زمن لحظي يكون بشكل مستقل لمعاملات البخار الداخلية للفقاعة. النَتائِج النظرية كَانتْ متوافقة مع البيانات السابقةِ النظريةِ المتوفرةِ بنسبة خطأ % 8.80 ± للمعامل τ=1 و % 5.4 ± للمعامل τ=10.

SYMBOLS

Cd	Coefficient of drag
Cp	Specific heat at constant pressure, kJ/kg K
Fr	Froude number
g	Gravitational acceleration (m/s^2)
$\dot{G}$	Acceleration ratio
h	Heat transfer coefficient, W/m^2 K
h_{fg}	Latent heat of vaporization of the bubble (J/kg)

Ja	Jacob number, $\rho_3\, Cp_3\, \Delta T_{3-2} / \rho_1\, \lambda$
k	Thermal conductivity (w/m. k)
Nu	Nuselt number, *h(2R)/k*
Pe	Peclet number, *U(2R)/α*
R	Polar coordinate, m
T	Absolute temperature, K
T	Time, s
U_b	Bubble velocity, m/s
u_r	Radial component of liquid velocity, m/s
u_θ	tangential component of fluid velocity, m/s

Greek

α	Thermal diffusivity, m^2/s
θ	Polar coordinates
φ	Velocity potential
ρ	Liquid density (distilled water), kg/m^3
τ	Dimensionless time

Subscripts

l	Liquid phase
v	Vapor phase

INTRODUCTION

Bubble growth is the basis of the process of desalination of sea or brackish water by freezing. The other important industrial applications are the design and development of an efficient type of heat exchanger without internal piping.

Boiling is one of the most efficient ways to achieve high heat fluxes at a reasonable wall superheat. The high heat transfer coefficients that can be achieved are of utmost interest for many applications in the field of refrigeration, cooling of electronic devices as well as power generation (**Christian K. and Peter S., 2010**).

Fully quantitative predictive models are still under development, since they require information about heat transfer mechanisms derived from theoretical models and from detailed experiments on bubble dynamics (**Hetsroni G. et al, 2006**).

In formulating the problem of the flow past a bubble, a difficulty arises in connection with the modeling of the mechanical properties of the phase interface. The classic assumption is the continuity of the shear stress on the interface. Then the viscous drag on a gas bubble of constant radius at small Reynolds numbers can be calculated using the Hadamard-Rybchinsky formula. However, it has proved that the Stokes formula for a rigid sphere gives a better description of experiments with rising gas bubbles. This formula was obtained on the assumption that the fluid velocity on the bubble surface is equal to the velocity of the bubble center of mass, i.e. that the surface withstands any shear stress.

A numerical approach to the drag coefficients of spherical particles can be made up to a Reynolds number range of several hundreds, but even this range is not wide enough to cover the necessary Reynolds number range needed for practical use. Consequently, correlations of the drag coefficients applicable for a wide range of Reynolds numbers based on sufficiently reliable experimental data are needed (**Koichi Asano, 2006**).

(**Lapple and Shepherd, 1940**) first proposed their famous correlation for the drag coefficients of a spherical particle based on a wide range of experimental data. Many correlations have been reported, which are well summarized by (**Clift et al., 1978**) among which is the useful correlation by (**Brauer and Sucker, 1978**).

For an evaluation of the drag force of a growing vapor bubble at rectilinear accelerated ascension, (**Askovic, 2002**) developed a simple relation for predicting the drag coefficient by applying the inviscid approximation. He investigated analytically that drag coefficient decreases with time for all accelerations.
Bubble growth rates were extensively investigated in the last few decades. Generally, the work has been initially divided into the following two main regions : growth rates controlled by inertia forces, applicable in the range of a relatively low pressure and high Jakob numbers, e.g. Rayleigh and growth rates for heat diffusion controlled growth, e.g. (**Plesset and Zwick, 1954**). (**Birkhoff et al., 1954**). (**Lien and Griffith, 1969**) experimentally investigated bubble growth in uniformly superheated water covering the low pressure range. They concluded that the bubble growth at very low pressure is controlled solely by inertia forces and that, as the pressure increases heat diffusion becomes a predominant factor, which at the upper part of their pressure range completely controlled the bubble growth. They also found that the interface resistance at the vapor-liquid interface has no significant factor on bubble growth.
Knowledge of the heat and mass transfer associated with a moving bubble or droplet is of importance to a variety of industrial processes. (**Boussinesque, 1905**) has been the first to obtain a solution for the heat transfer rate from a fluid sphere of uniform and constant surface temperature, moving at a constant speed in another fluid of infinite extent. (**Ruckenstein, 1959**) studied the heat transfer between a vapor bubble in motion and the liquid from which the bubble was generated. Amongst the relatively small number of papers on deforming bubble in movement, the most often is used an impulsively started motion in a quiescent liquid initially at rest. Generally speaking, the viscous effect is small when the Reynolds number exceeds two or three hundred. It may be of interest to note that if the hydrodynamic boundary layers are developing simultaneously with the thermal boundary layer, the in viscid approximation is even better (**K. Stewartson, 1960**). (**Dominique Legendre et al., 1998**) studied the heat transfer rate and the hydrodynamic forces experienced by a single vapor bubble of variable radius moving in a superheated or subcooled liquid by means of numerical simulation. They solved for that purpose the full Navier–Stokes equations and the temperature equation are in a frame of reference where the bubble surface is steady. They determined the time evolution of the bubble radius by solving the energy balance at the bubble surface. (**Kendoush, 2004**) derived n analytical equation for prediction the drag coefficient of growing bubble in terms of Reynolds, Peclet, Jakob numbers and dimensionless time.
As no exact relation for drag coefficient for the case of an evaporating and moving bubble in superheated liquid, the relation for the drag coefficient of drag, applicable to motion of bubbles in superheated liquid, found in this work. In spite of a large number of theoretical and experimental investigations, a generalized theoretical analysis of bubble growing in a convective surrounding (uniformly superheated pure liquids) is found to be lacking in the literature. (**Even Askovic, 2002**) investigated that the drag coefficient decreases with time for all acceleration but he not study the non-dimensional parameters effect on the drag coefficient. A study of the various parameters governing the bubble growth process is essential to understand the phenomenon bubble growth in a convective surrounding. The present investigation deals with a parametric study of single bubble growth. It also aims to predict the bubble growth process in any uniformly superheated pure liquids.

THEORETICAL ANALYSIS

For the problem of rectilinear and accelerated movement of a sphere in an in viscid stationary fluid (**Koichi Asano, 2006, Askovic, 2002, Battya, et al., 1984**) the velocity potential ϕ, and the normal v_r and tangential v_θ velocity components of the in viscid fluid flowing past a spherical particle of variable radius r are as follows

$$\varphi = -\frac{U_b}{2}\frac{R^3}{r^2}\cos\theta - \dot{R}\frac{R^2}{r} \qquad (1)$$

$$v_r = U_b \frac{R^3}{r^3} \cos\theta + \dot{R} \frac{R^2}{r^2} \tag{2}$$

$$v_\theta = \frac{U_b}{2} \frac{R^3}{r^3} \sin\theta \tag{3}$$

Where r is the radius-vector, θ is the angle between the radius-vector, U_b is the bubble velocity, $\dot{R}$ = dR/dt , is the bubble expansion rate and where R is the bubble radius as shown in **Figure 1**;
To calculate the pressure distribution on the bubble surface, using the Bernoulli equation for the external unsteady flow (**Askovic, 2002**):

$$\left[p + \frac{1}{2}\rho_l \left(v_r^2 + v_\theta^2\right) + \rho_l \frac{\partial\varphi}{\partial t}\right]_{r=R} = \left[p + \frac{1}{2}\rho_l \left(v_r^2 + v_\theta^2\right) + \rho_l \frac{\partial\varphi}{\partial t}\right]_{r\to\infty} \tag{4}$$

Where

$$(p)_{r=R} = p_\infty - \frac{1}{2}\rho_l \left(v_r^2 + v_\theta^2\right)_{r=R} - \rho_l \left[\frac{\partial\varphi}{\partial t}\right]_{r=R} \tag{5}$$

Differentiate eq. (1) respect to time t and substitute with equations (2) and (3) in eq. (5) yields:

$$(p)_{r=R} = p_\infty + \frac{1}{2}\rho_l U_b{}^2 \left(1 - \frac{9}{4} sin^2\theta\right) + \rho_l \left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) + \frac{1}{2}\rho_l \left(R\dot{U}_b + 3\dot{R}U_b\right) cos\theta \tag{6}$$

The total drag force D of a growing vapor bubble at an accelerated ascension can be expressed as the sum of the three forces contributed by friction, pressure and evaporation. It can be written as:

$$D = 8\mu_f \pi R\dot{R} - 2\pi R^2 \int_0^\pi (p)_{r=R} \cos\theta \sin\theta \, d\theta + \frac{4}{3}\pi\rho_l gR^3 \tag{7}$$

Where the first term is the friction drag force(**Kendoush, 2005**) and the last term represents the buoyancy force experienced by the bubble. Substitute eq. (6) in eq. (7), after integration, yields:

$$D = 8\mu_f \pi R\dot{R} - \frac{2}{3}\pi\rho_l R^3 \dot{U}_b - 2\pi\rho_l R^2 \dot{R} U_b + \frac{4}{3}\pi\rho_l gR^3 \tag{8}$$

It is customary to introduce the drag coefficient C_d, defined by:

$$C_d = \frac{D}{\frac{1}{2}\pi\rho_l U_b{}^2 R^2} \tag{9}$$

Substitute eq. (8) in eq. (9) , we obtain:

$$C_d = \frac{8\mu_f \pi R\dot{R}}{\frac{1}{2}\pi\rho_l U_b{}^2 R^2} - \frac{8}{3}\frac{g}{\dot{U_b}^2}\frac{R}{t^2} - \frac{4}{3\dot{U}_b}\frac{R}{t^2} - \frac{4}{U_b}\frac{\dot{R}}{t} \tag{10}$$

A combination of energy and mass balance results in the following relation for the bubble growth rate (**Mokhtarzadeh M.R. and A.A. EL-Shirbini, 1985**):

$$\frac{dR}{dt} = \dot{R} = \frac{(\rho_l - \rho_{v)}}{\rho_l \rho_v h_{fg}} h \left(T_c - T_d\right) \tag{11}$$

eq. (10) can be transferred in the dimensionless form as:

$$C_d = \frac{32}{Re}\left[1 + \sqrt{\frac{Ja}{2F_r \tau\, P_e}}\right] + G\left[\dot{G}\ \frac{4/3}{F_r\, \tau^2} - \frac{2/3}{F_r\, \tau^2} - \frac{4(1-M)}{\tau\, P_e}\ Ja\ Nu\right] \tag{12}$$

Where

$$Ja = \frac{\rho\ \Delta T\ Cp}{\rho_v\, h_{fg}} = \frac{St}{M\ N} \tag{13}$$

Where

$$St = \frac{C_p \Delta T}{h_{fg}} \tag{14}$$

$$M = \frac{\rho_v}{\rho_l} \tag{15}$$

$$N = \frac{\rho_l}{\rho} \tag{16}$$

$$Nu = \frac{2hR}{k} \tag{17}$$

$$F_r = \frac{{U_b}^2}{2Rg} \tag{18}$$

$$Pr = \frac{\mu\ C_p}{k} \tag{19}$$

$$Re = \frac{2RU_b\ \rho}{\mu} \tag{20}$$

$$\tau = \frac{g\ t}{U_b} \tag{21}$$

$$\dot{G} = \frac{g}{U_b} \tag{22}$$

(**Sideman and Taitel, 1964**) have obtained experimentally the instantaneous heat transfer coefficients for three different systems with various drop diameters and at several temperature differences and percentages of evaporation. The present analysis is carried out using their experimental values and the following correlation is obtained:

$$Nu = 0.64\ Pe^{0.5} Ja^{-0.35} \tag{23}$$

This relation is used in this work for the calculation of the instantaneous Nusselt number.
The temperature of the uniformly superheated pure liquid is assumed to be constant throughout the column for raising the bubble. The effect of the hydrostatic head is not considered for smaller bubble or high temperature differences. The properties of the fluids at the appropriate temperatures are obtained from the published literature and transferred to non-dimensional parameters (**Battya et al., 1984**) and they printed on the figures.
Substitute eq. (23) in eq. (12) and rearrangement yields:

$$C_d = \frac{32}{Re}\left[1 + \sqrt{\frac{Ja}{2F_r \tau\, P_e}}\right] + \dot{G}\left[\dot{G}\ \frac{4/3}{F_r\, \tau^2} - \frac{2/3}{F_r\, \tau^2} - \frac{2.56(1-M)}{\tau\, Pe^{0.5}}\ Ja^{0.65}\right] \tag{24}$$

RESULTS AND DISCUSSION

In this study, presenting the effect of non-dimensional parameters on the drag coefficient, the equation (10) which is the similar to the equation obtained by (**Askovic, 2002**)but with addition of the shear force effect which is obtained by (**Kendoush, 2005**) and we got on equation (24) with non-dimensional parameters. Equation (24) depends on parameters J_a, F_r, P_e, τ and$\dot{G}$. The available experimental data taken in this work is from experimental results of (**Sideman and Taitel, 1964**) and (**Battya, 1984**), when single pentane drop-bubble of 3.6, 3.62 and 3.2 mm diameter evaporate along a column of distilled water at ΔT= 1.6, 3.8 and 8 K, respectively. **Figures 2-4** show the variation of the drag coefficient C_d with non-dimensional time τ for some arbitrarily assigned values of the acceleration parameter: $\dot{G}$ =4, 5, and 6 and varies non-dimensional parameters J_a, F_r, P_e,. It is clearly shown that when the acceleration parameter $\dot{G}$ increases the drag coefficient increases at low dimensionless time due to decreases in bubble acceleration$\dot{U}_b$. **Figures 2-4** predict the drag coefficient for the process of bubble growing in a convective surrounding (uniformly superheated pure liquids). **Figure 5** shows the effect of Prandtl number. A higher Prandtl number decreases the bubble growth rate. A higher Prandtl number decreases the drag coefficient. Figure 6 indicates the important of system Jakob number. Therefore, a higher Jacob number increases the bubble growth rate and reduces the Nusselt number. A higher bubble growth rate is accompanied by a higher acceleration of the bubble. Therefore, a higher Jakob number increases the drag coefficient. **Figure 7** shows the time dependent drag coefficient versus Reynolds number comparing with the theoretical results were given by Kendoush, for time constant τ =1,and 10. The figure shows that the drag coefficient values obtained with τ =1 is bigger than that obtained when τ =10, and the present model results are bigger than that obtained by Kendoush for all Reynolds values tested , but a good agreement is achieved generally.

CONCLUSIONS

Besides the hypothesis of the non-viscous external flow, all the results presented in **Figures 2-4** seem to be acceptable, due to the buoyancy effect, the drag coefficient decreases with time for all values of the acceleration non-dimensional parameters. The bubble growth rate, Nusselt number and the time and height required to predict the drag coefficient depend mainly on the acceleration parameter, dimensionless time, Froude, Peclet and Jakob numbers. The parametric studies presented here help in the prediction of the bubble growing in a convective surrounding (uniformly superheated pure liquids). A good agreement is achieved between the present model results of drag coefficient and Kendoush's results for acceleration $\dot{G} = 16$ .the error between present work and Kendoush's results is within ± 8.80 % for τ=1 and within ± 5.4 % for τ=10 The present results need more validity, by developing an experimental work.

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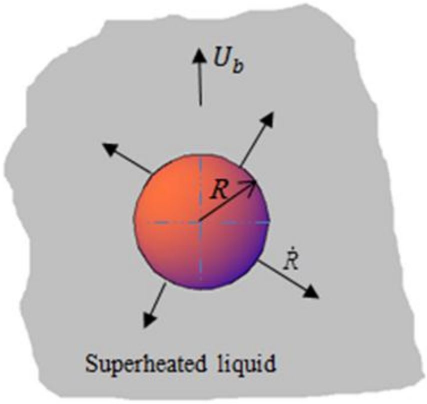

Figure 1 Growth of single spherical bubble in superheated liquid

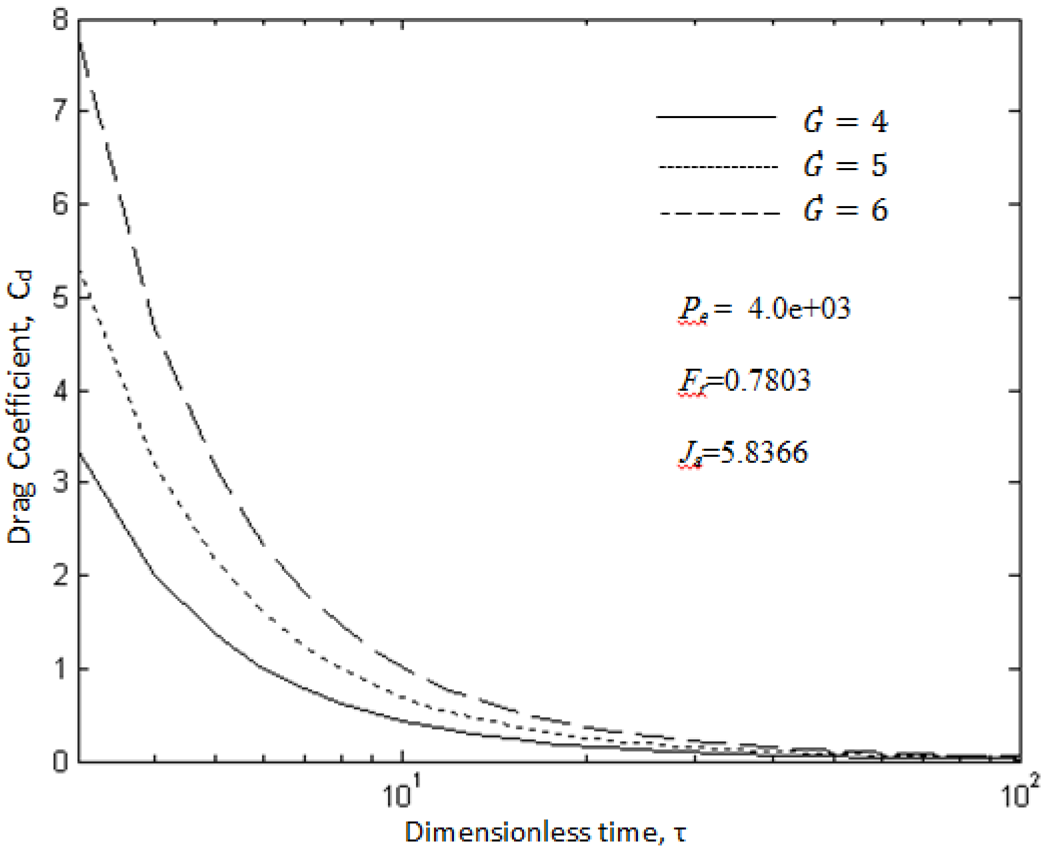

Figure 2 Drag coefficient C_d vs. Dimensionless time τ for various parametric studies P_e, F_r and J_a

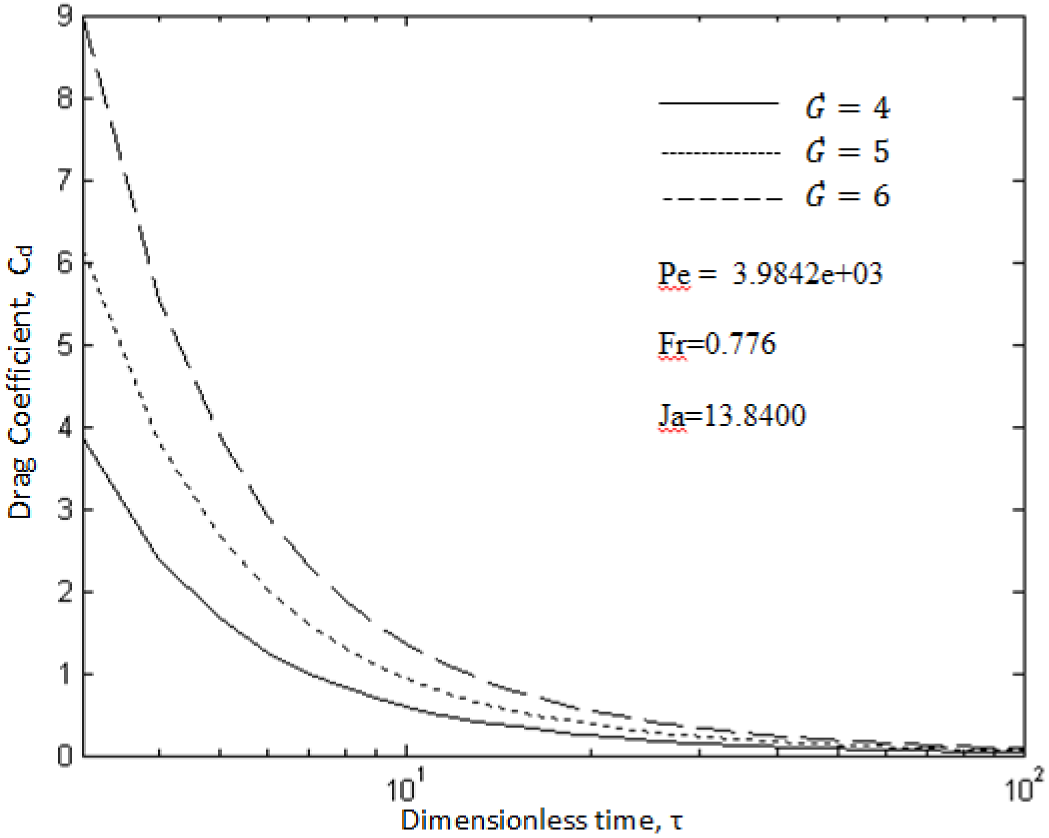

Figure 3 Drag coefficient C_d vs. Dimensionless time τ for various parametric studies P_e, F_r and J_a

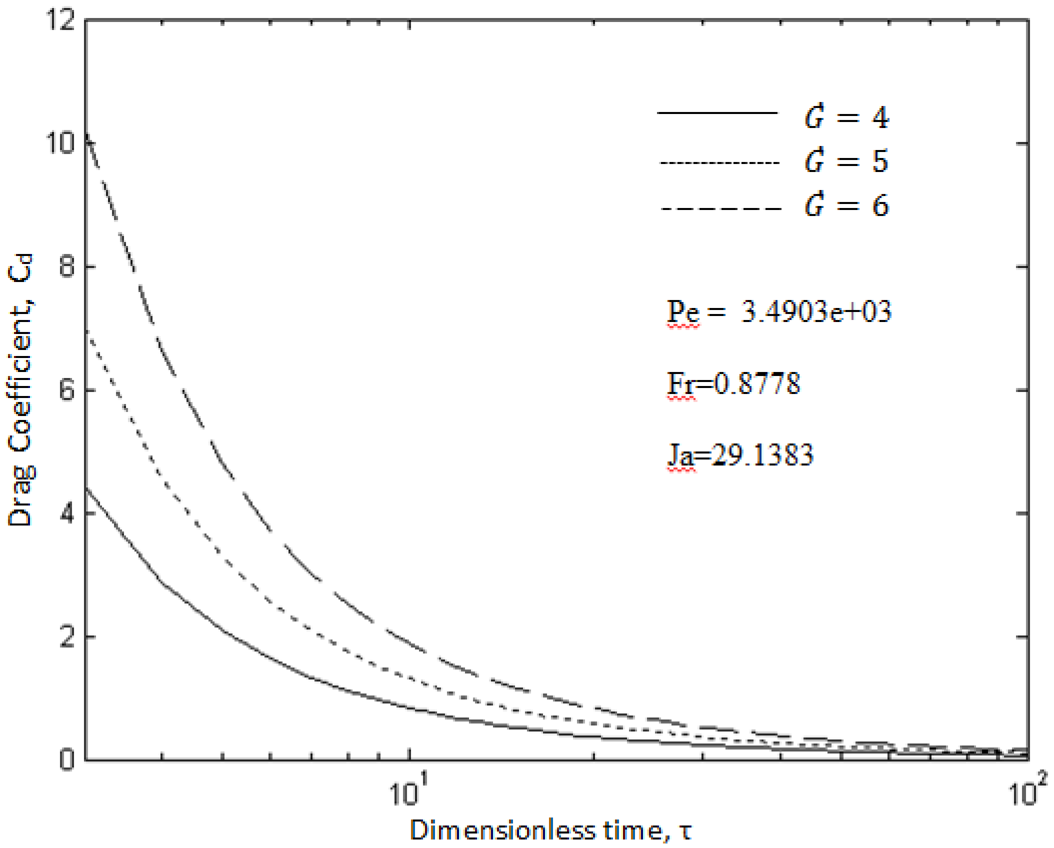

Figure 4 Drag coefficient C_d vs. Dimensionless time τ for various parametric studies P_e, F_r and J_a

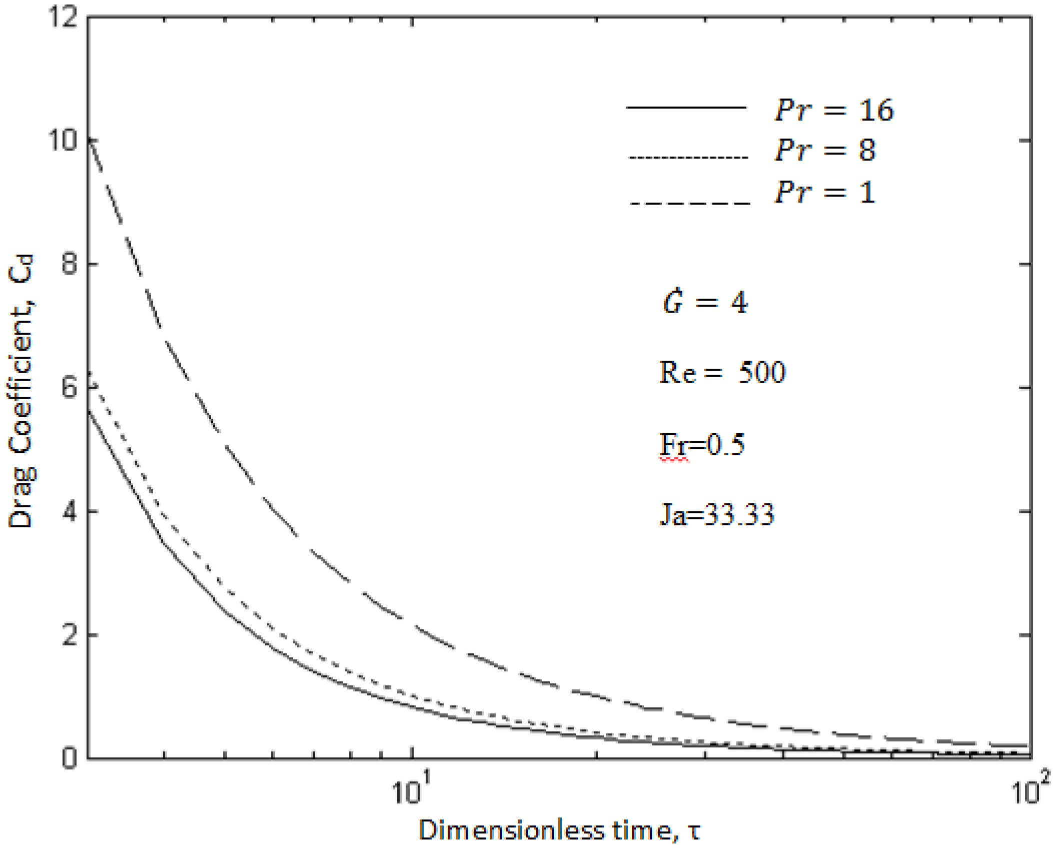

Figure 5 Drag coefficient C_d vs. Dimensionless time τ for various parametric studies R_e, F_r and J_a with different P_r

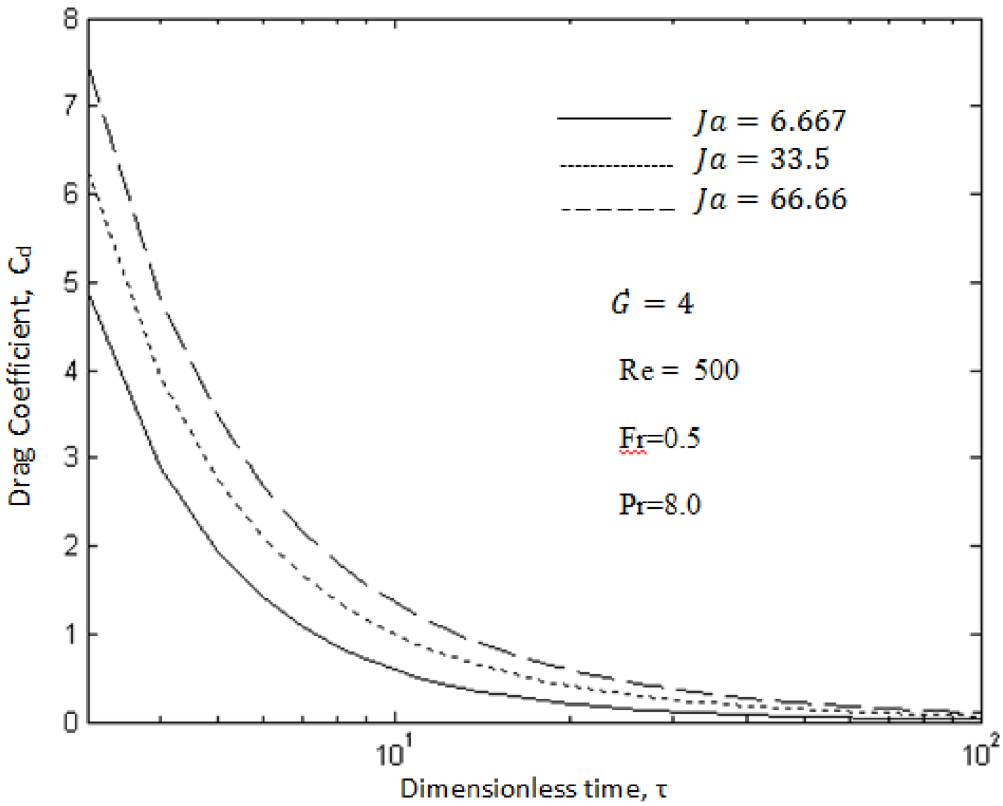

Figure 6 Drag coefficient C_d vs. Dimensionless time τ for various parametric studies P_e and F_r with different J_a

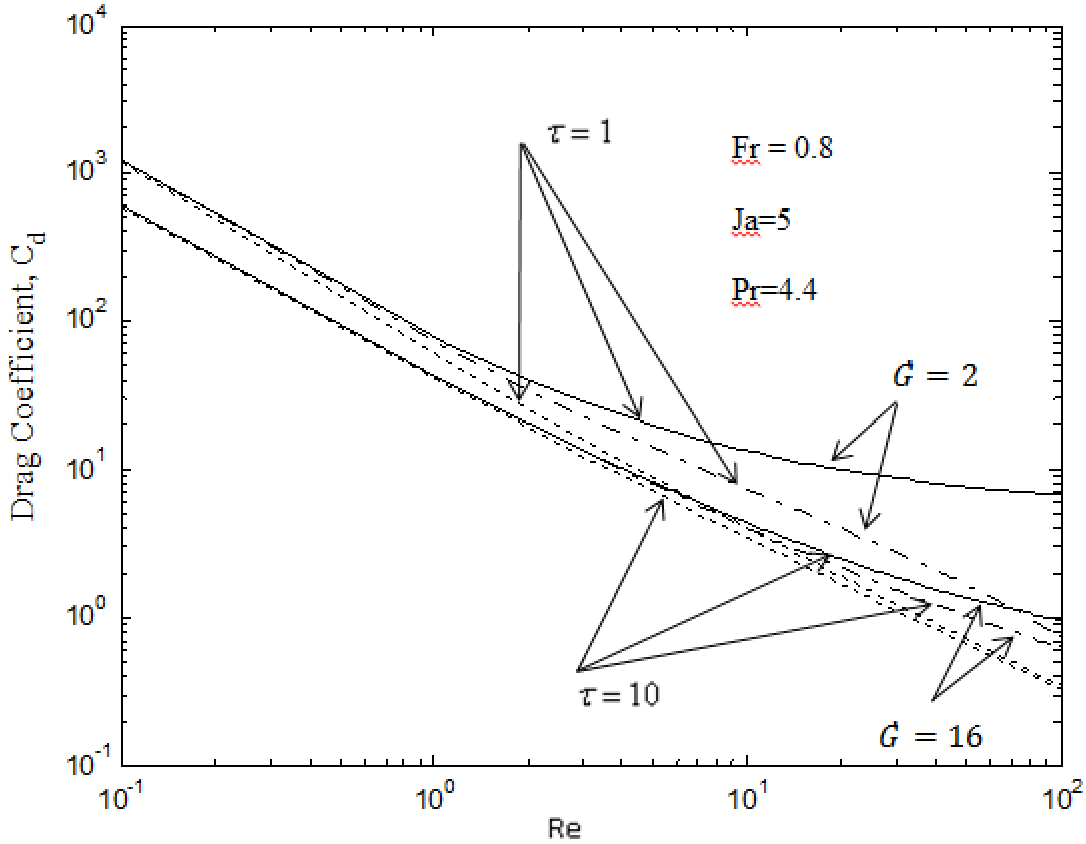

Figure 7 Comparison between the present solution of drag coefficient C_d vs. Reynolds number for various non-dimensional time τ =1 and 10 and various acceleration G (2, 16) and comparison the present work with analytical solution of (**Kendoush, 2005**)