THEORETICAL COMPARISON BETWEEN POLARIZATION MODE DISPERSION AND BIREFRINGENCE VECTORS IN SINGLE MODE FIBERS

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Abstract

The polarization mode dispersion, $\vec{\tau}$, and birefringence, $\vec{\beta}$, vectors have the same meaning in a certain cases, but in general they are different. In this paper, we introduce a new theoretical comparison between these two vectors through the propagation in optical fibers. The parameters; the angle between $\vec{\tau}$ and $\vec{\beta}$, and the value of $\vec{\beta}$ depend on the adopted form of the vector $\vec{\beta}$. In addition, the value of $\vec{\tau}$ depend on the number of orders that is used to for account the vector $\vec{\tau}$. We introduce also a new formula of the dynamical polarization mode dispersion equation which contains the circular birefringence and the higher orders of the vector $\vec{\tau}$.

الخلاصة

يتماثل المعنى لمتجهي ثنائي الانكسار $\vec{\beta}$ وتشتت نمط الاستقطاب $\vec{\tau}$ في مجموعة محددة من الحالات ولكنهما مختلفان في الصورة العامة. نقدم في هذا البحث دراسة مقارنة جديدة بين هذين المتجهين خلال الانتشار في الألياف البصرية. ان كل من: الزاوية بين المتجهين، قيمة ثنائي الانكسار، وقيمة التأخير الزمني (قيمة تشتت نمط الاستقطاب) هي معاملات تعتمد بالأساس على صيغة متجه ثنائي الانكسار المتبناة بالإضافة إلى عدد الرتب التي تدخل في تكوين متجه تشتت نمط الاستقطاب. قدم البحث أيضا صيغة جديدة للمعادلة الحركية لتشتت نمط الاستقطاب يدخل في تركيبها كل من ثنائي الانكسار الدائري والرتب العليا لمتجه تشتت نمط الاستقطاب.

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1. Introduction

As the bit rate and distance of optical fiber transmission systems continue to increase, the understanding of polarization mode dispersion (PMD) and its system impairments and mitigation are becoming ever more important [1]. In an ideal circularly symmetric fiber, the two orthogonally polarized modes have the same group delay [2]. In reality, fibers have some amount of birefringence due to imperfections in the manufacturing process and/or mechanical stress on the fiber after manufacture. PMD has its origins in this optical birefringence and the random variation of the birefringent axes orientation along the fiber length [3]. PMD causes different delays for different polarizations; when the difference in these delays approaches a significant fraction of the bit period, pulse distortion and system penalties occur. Environmental changes, including temperature and stress, cause the fiber PMD to vary stochastically in time, making PMD particularly difficult to manage [4].

The PMD may induce unacceptable levels of signal degradation in high optical communication systems. The signal degradation takes the form of pulse broadening due to the differential transmission time of two pulses polarized along orthogonal states of polarization (SOP). This kind of PMD is commonly known as first order PMD [5]. A series of PMD compensation methods have been proposed in order to overcome the problem [6]. Under first order PMD, a pulse at the input of a fiber can decompose into two pulses with the orthogonal SOP. Both pulses will arrive at the output of the fiber undistorted and polarized along different SOP, the output SOP being orthogonal. The differential transmission time between these pulses is referred to a differential group delay (DGD) and the input (output) SOP which allow the transmission (reception) of undistorted pulses are known as the principal input (output) states of polarization (PSP's) [1,7]. Both of the PSP's and the DGD are assumed to be frequency independent when only first order PMD is being considered [4].

The higher order PMD effects account for the frequency dependent of the DGD and PSP's. The frequency dependence of the DGD introduces an effective chromatic dispersion of opposite sign on the signals polarized along the output PSP's [8]. Higher order PMD effects have been studied in the literature, but the distortion of specific input pulse induced by higher order PMD is still to be clarified [5].

There is frequent confusion between the terms "axes of birefringence" and "principal states of polarization". The former refers to a local orientation of the fast and slow axes in the fiber, based upon the physical geometry of the fiber, which is described by the birefringence

vector $\vec{\beta}$. The latter refers to the two states of polarization of (a monochromatic) input light pulse which pass through a birefringent medium, including a concatenation of randomly oriented birefringent elements, without spreading [4,8]. The two corresponding states of polarization of such a pulse as it exits the medium are referred to as the output PSP's, which are, in general, different from those of input PSP's [7]. For a constant birefringence medium, the axes of birefringence and the PSP's are the same, but for a complicated medium having local birefringence, which changes along its length, the input and output PSP's in general do not correspond to the axis of birefringence anywhere along the fiber [2].

In this paper, we are introduced a novel theory to account for the relation between the PMD vector, $\vec{\tau}$, and the birefringence vector, $\vec{\beta}$. The theory includes the effects of these two vectors on the dynamical PMD equation.

2. Theory

The principal states model [9], states that; for a length of fiber, there exists for every frequency a special pair of polarization states, called the principal states of polarization (PSP's). A PSP is defined as that input SOP for which the output SOP is independent of frequency to first order, i.e. over a small frequency range. In the absence of polarization dependent loss (PDL), the PSP's are orthogonal. For each pair of input PSP's, there is a corresponding pair of orthogonal PSP's at the fiber output. The input and output PSP's are related by the fiber's transmission matrix (Jones matrix), just as any input polarization is related to a polarization at the fiber output [10,11]. Using the principal states model, PMD can be characterized by the vector [9]

$$\vec{\tau} = \Delta\tau \ \hat{\mathrm{p}} \tag{1}$$

a vector in the three dimensional Stokes space, where the magnitude, Δτ, is the differential group delay (DGD). The unit vector, $\hat{p}$, points in the direction of the slower PSP, whereas the vector $-\hat{p}$ indicates the orthogonal faster PSP. The latter is 180^{o} from $\hat{p}$ in Stokes space.

In the optical fibers, the birefringence vector $\vec{B}$ is defined in two forms as [3,11]

$$\vec{B}_L = \begin{bmatrix} \Delta\beta\cos 2\alpha \\ \Delta\beta\sin 2\alpha \\ 0 \end{bmatrix} \tag{2 a}$$

$$\vec{B}_{NL} = \begin{bmatrix} \Delta\beta\cos 2\alpha \\ \Delta\beta\sin 2\alpha \\ \zeta\, T \end{bmatrix} \tag{2 b}$$

where α is the angle of birefringence in Jones space, $\Delta\beta$ is the magnitude of linear birefringence, i.e. $\Delta\beta = |\vec{B}_L|$, ζ is the photo-elastic coefficient of glass; it is about 0.14-0.16 [5] according to the dopants rate, and T is the twist rate in (rad/m). The angle α is not constant along the fiber; also, $\Delta\beta$ and T. This means that each position of fiber has a birefringence vector differs from another position at random, depending on the values of α , $\Delta\beta$, and T.

Supposing that $\Delta\beta = |\vec{B}_L|$; it is $\vec{\beta}_L = \Delta\beta\hat{r}$: where $\hat{r}$ represents a unit vector in Stokes space. The vector $\hat{r}$ represents a rotation axis of polarization vector, which differs from one section to another randomly. As a consequence, the PMD vector, $\vec{\tau}$, can be defined as a function of $\hat{r}$ and ϕ [4]

$$\vec{\tau} = \phi_w \hat{r} + \hat{r}_w \sin\phi + \hat{r}_w \times \hat{r}(\cos\phi - 1) \tag{3}$$

where $\phi = \Delta\beta\Delta z$ represents the rotation angle of the polarization state vector $\hat{S}$ around the birefringence vector $\vec{\beta}$, and ϕ_w and $\hat{r}_w$ represent their first derivatives of frequency. Eq.(3) obtains that the angle and direction of rotation control the resultant vector $\vec{\tau}$.

Substituting Eq.(2 a) into (3), we can obtain the following

$$\vec{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \varepsilon\, \Delta z \cos(2\alpha) \\ \varepsilon\, \Delta z \sin(2\alpha) \\ 0 \end{bmatrix} + 2\frac{d\alpha}{dw} \begin{bmatrix} -\sin(\phi)\sin(2\alpha) \\ \sin(\phi)\cos(2\alpha) \\ 1-\cos(\phi) \end{bmatrix} \tag{4}$$

where $\varepsilon = {d\Delta\beta}/{dw}$ represents PMD parameter, and Δz is the fiber segment length. Eq.(4) represents the PMD vector considering the linear intrinsic birefringence. On the other hand, $\vec{\tau}$ is a function of w, which it may be written as a Taylor series around the central frequency w_o as follows

$$\vec{\tau}(w) = \vec{\tau}(w_o) + \Delta w \frac{d\vec{\tau}}{dw}\Big|_{w=w_o} + \frac{\Delta w^2}{2}\frac{d^2\vec{\tau}}{dw^2}\Big|_{w=w_0} + \ldots\ldots\ldots\ldots\ldots\ldots \tag{5}$$

By comparing Eqs.(4) and (5), then the first term on the right hand side of Eq.(4) will represent the first order of PMD vector, while the second term indicates all higher order of PMD vector. Accounting that the higher order depends on the value of $d\alpha/dw$. For a very small variations of

α with frequency, the second term on the right hand side of Eq.(4) may be neglected. Elsewhere, the higher order effects must be included through the determination of PMD vector.

Neglecting the higher order effects makes the PMD vector as follows

$$\vec{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \varepsilon\,\Delta z \cos(2\alpha) \\ \varepsilon\,\Delta z \sin(2\alpha) \\ 0 \end{bmatrix} = \varepsilon\,\Delta z\,\hat{r} = \frac{\varepsilon}{\Delta\beta}\Delta z\,\vec{\beta} = \text{const}.\vec{\beta} \tag{6}$$

This means, $\vec{\tau}$ coincides the rotation vector $\hat{r}$ (the birefringence axes coincide the PSP's). In other words, $\vec{\tau}$ is coincides the birefringence vector $\vec{\beta}$ if the intrinsic birefringence is linear and the higher order PMD effects are neglected. Elsewhere, the two vectors are never coincided.

Using Eq.(6), the DGD of the fiber segment can be obtained as

$$DGD = \Delta\tau = |\vec{\tau}| = \varepsilon\,\Delta z \tag{7}$$

The value of DGD represents the delay time between the two components of polarization in a single segment of the optical fiber. Since the DGD's of the fiber segments are random, such that the mean DGD can be calculated as

$$<\Delta\tau> = \frac{1}{N}\sum_{i=1}^{N}\Delta\tau_i \tag{8}$$

For the case of wide frequency band, the higher order effects of the PMD must be included in account. The DGD of this case can be obtained using Eq.(4) as follows

$$DGD = \Delta\tau_e = \sqrt{(\varepsilon\,\Delta z)^2 + 8(1-\cos\phi)^2\,\alpha_w^2} \tag{9}$$

Looking to Eq.(9), one can notice that the DGD is related to the change of α with respect to frequency, and $\Delta\tau_e > \Delta\tau$. This means that the higher order effects increase the DGD. Using Eqs.(3) and (4) one can obtain the angle between the two vectors $\vec{\tau}$ and $\vec{\beta}$ as

$$\psi = \cos^{-1}\left(\frac{\Delta\tau}{\Delta\tau_e}\right) = \cos^{-1}\left(\frac{\varepsilon\Delta z}{\sqrt{(\varepsilon\Delta z)^2 + 8(1-\cos\phi)\alpha_w^2}}\right) = \cos^{-1}\left(\frac{\Delta\tau \text{ without higher order PMD}}{\Delta\tau \text{ with higher order PMD}}\right) \geq 0 \tag{10}$$

Eq.(10) means that the two vectors in the same direction if the higher order PMD is neglected, i.e. $\Delta\tau_e = \Delta\tau$.

For the nonlinear intrinsic birefringence, $\vec{\tau}$ can be calculated using Eqs.(2 b) and (3) as follows

$$\vec{\tau} = \begin{bmatrix} (a_1 + a_3\sin\phi)\cos(2\alpha) + a_6(\cos\phi - 1)\sin(2\alpha) \\ (a_1 + a_3\sin\phi)\sin(2\alpha) - a_6(\cos\phi - 1)\cos(2\alpha) \\ a_2 + a_5\sin\phi \end{bmatrix} + \frac{d\alpha}{dw}\begin{bmatrix} -a_4\sin\phi\sin(2\alpha) + a_7(\cos\phi - 1)\cos(2\alpha) \\ a_4\sin\phi\cos(2\alpha) + a_7(\cos\phi - 1)\sin(2\alpha) \\ a_8(\cos\phi - 1) \end{bmatrix} \qquad (11)$$

where the parameters a_1 into a_8 are defined as

$$a_1 = \frac{\Delta\beta\varepsilon\Delta z}{\Delta\beta_{NL}} \qquad a_2 = -\frac{\Delta\beta\varepsilon\,\zeta\,T}{\Delta\beta_{NL}^2} \qquad a_3 = a_5 = -\frac{\zeta\,T\Delta\beta\varepsilon}{K}$$
$$a_4 = \frac{2\Delta\beta}{\Delta\beta_{NL}} \qquad a_6 = \frac{2\zeta\,T\Delta\beta}{\Delta\beta_{NL}} - \frac{(\zeta\,T)^3\varepsilon}{K} \qquad a_7 = \frac{\zeta\,T\,\Delta\beta^2\varepsilon}{\Delta\beta_{NL}^2} \qquad (12)$$
$$a_8 = -\frac{2b_L}{\Delta\beta_{NL}^2} \qquad K = (\Delta\beta^2 + \zeta^2 T^2)^{3/2}$$

and $\Delta\beta_{NL} = \sqrt{\Delta\beta^2 + (\zeta\,T)^2}$ is the nonlinear birefringence amount.

Eq.(11) represents a new formula of the PMD vector, it gives us the idea about the amount of difficulties to compensate the noise that arise due to PMD when the pulse propagates through optical fibers. Many scientific researches [2,6,7] of PMD compensation have been proposed, which deal only with the first order of PMD. This means that the compensation depend upon the first term on the right hand side of Eq.(11) and assuming that the birefringence vector $\vec{\beta}$ is linear.

3. Dynamical PMD Equation

The output Jones vector $|s>$ is related to the input one $|t>$ as follows [3]

$$|s> = e^{-i\varphi_o}U\,|t> \qquad (13)$$

where U is the Jones matrix and φ_o is the common phase. By differentiation of Eq.(13) with respect to frequency and eliminate $|t>$ yields

$$\frac{d}{dw}|s> = (-i\tau_o + U_w U^\dagger)\,|s> \qquad (14)$$

where $\tau_o = d\varphi_o / dw$ is the common group delay for all polarizations. Multiplying Eq.(14) from the left by $<s|\vec{\sigma}$ yields

$$< s|\vec{\sigma}\frac{d}{dw}|s> = -i\tau_o\hat{S} + < s|\vec{\sigma}U_w U^\dagger|s> \tag{15}$$

where $\vec{\sigma}$ **is a Pauli vector in three dimension and** $\hat{S} = < s|\vec{\sigma}|s>$ **is the Stokes vector of the polarization. Now, one can obtain the transpose of the complex conjugate of Eq.(15) and multiply it from the right by** $\vec{\sigma}|s>$ **to obtain**

$$\frac{d< s|}{dw}\vec{\sigma}|s> = i\tau_o\hat{S} + < s|UU_w^\dagger\vec{\sigma}|s> \tag{16}$$

Differentiation the definition $\hat{S} = < s|\vec{\sigma}|s>$ **with respect to frequency, using Eqs.(15) and (16) and with help of the identity**

$$U_w U^\dagger = \frac{1}{2i}\vec{\tau}\cdot\vec{\sigma} \tag{17}$$

we can obtain

$$\frac{d\hat{S}}{dw} = \vec{\tau}\times\hat{S} \tag{18}$$

By differentiation Eq.(13) with respect to z **and using a similar manner of the one used to drive Eqs.(14)-(18), we get**

$$\frac{d\hat{S}}{dz} = \vec{\beta}\times\hat{S} \tag{19}$$

The vectors $\vec{\tau}$ **and** $\vec{\beta}$ **in Eqs.(18) and (19) contain the effects of higher orders of the vector** $\vec{\tau}$ **and the circular birefringence of the vector** $\vec{\beta}$**. Accordingly, we can rewrite them as follows**

$$\frac{d\hat{S}}{dw} = (\vec{\tau}_L + \vec{\tau}_h)\times\hat{S} \tag{20}$$

By differentiation of Eqs.(20) and (21) with respect to z **and** w**, respectively, yields**

$$\frac{d\hat{S}}{dz} = (\vec{\beta}_L + \vec{\beta}_h)\times\hat{S} \tag{21}$$

$$\frac{\partial^2\hat{S}}{\partial w\partial z} = \frac{\partial(\vec{\tau}_L + \vec{\tau}_h)}{\partial z}\times\hat{S} + (\vec{\tau}_L + \vec{\tau}_h)\times\frac{\partial\hat{S}}{\partial z} \tag{22}$$

$$\frac{\partial^2\hat{S}}{\partial z\partial w} = \frac{\partial(\vec{\beta}_L + \vec{\beta}_h)}{\partial w}\times\hat{S} + (\vec{\beta}_L + \vec{\beta}_h)\times\frac{\partial\hat{S}}{\partial w} \tag{23}$$

Now, Eqs.(22) and (23) are combined by eliminating $\partial^2 \hat{S} / \partial z \partial w$, using Eqs.(20) and (21), and simplified the results by the identity $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})$ yields

$$\frac{\partial \vec{\tau}_L}{\partial z} + \frac{\partial \vec{\tau}_h}{\partial z} = \frac{\partial \vec{\beta}_L}{\partial w} + \frac{\partial \vec{\beta}_h}{\partial w} + [(\vec{\beta}_L + \vec{\beta}_h)\times(\vec{\tau}_L + \vec{\tau}_h)]\times \hat{S} \tag{24}$$

Eq.(24) represents a new form for the dynamical PMD equation, where $\vec{\beta}_h$ is the circular birefringence and $\vec{\tau}_h$ is the higher orders PMD vector. If we ignore the effects of higher order of $\vec{\tau}$ and assuming that the birefringence vector $\vec{\beta}$ is linear, then Eq.(24) will be the same as the form that obtained by Ref. [4].

The evolution of the PMD vector with fiber length is described by Eq.(24) that relating the PMD vector to the microscopic birefringence. Here z is the direction along the fiber. $\vec{\beta}$ is a three-dimensional, local birefringence vector of the fiber, pointing in the direction of the birefringence axes with a magnitude $\Delta\beta_{NL}$. This equation is the basis for the statistical theory of PMD. Its solution is beyond of this work.

4. Discussion and Conclusions

We can find out the vector $\vec{\tau}$ from $\vec{\beta}$ using Eq.(4). The vector $\vec{\tau}$ is linear only if $\vec{\beta}$ is linear and ignoring the higher orders of the vector $\vec{\tau}$, otherwise they are different. When we change the distance this implies to rotating $\hat{S}$ around $\vec{\beta}$ by an angle ϕ. On the other hand, the change of frequency causes to rotate $\hat{S}$ around $\vec{\tau}$ by an angle θ. Fig.(1-a) illustrates the relation among the three vectors $\hat{S}$, $\vec{\beta}$, and $\vec{\tau}$ where the polarization vector $\hat{S}$ is rotating a round $\vec{\beta}$ and $\vec{\tau}$ by changing the distance and frequency, respectively. When we add the higher orders of $\vec{\tau}$, this means that the vector $\vec{\tau}$ is nonlinear which does not coincided with the vector $\vec{\beta}$ as illustrated in Fig.(1-b). The general case considers the birefringence vector is nonlinear and assuming all orders of $\vec{\tau}$ as illustrates in Fig.(1-c), which shows that each vector rotates in Stokes space.

The final sense of the two vectors $\vec{\beta}$, and $\vec{\tau}$ can be imagine into two cases. Firstly, if the distance is fixed and the frequency is changed, then $\vec{\beta}$ remains fixed and $\vec{\tau}$ takes many forms

$\vec{\tau}_i$ ($i = 1, 2, \ldots\ldots\ldots, N$, **where N is the number of individual frequencies) depending on the frequency. Fig.(2-a) illustrates the individual vectors** $\vec{\tau}_i$ **which represent the PMD with each frequency and the resultant vector** $\vec{\tau}_{tot}$**. This means that the final angle between** $\vec{\beta}$ **and** $\vec{\tau}$ **will change depending on the frequency, see Fig.(2-b). Secondly, when we fixed the frequency and changing the distance, then** $\vec{\tau}$ **remain fixed and** $\vec{\beta}$ **takes many forms** $\vec{\beta}_i$ **(**$i = 1, 2, \ldots\ldots\ldots, N$**, where N is the number of fiber segments) depending one the birefringence value and the length of optical fiber. This case is similar to the previous case with replacing** $\vec{\beta}/\vec{\tau}$ **instead of** $\vec{\tau}\ /\vec{\beta}$ **.**

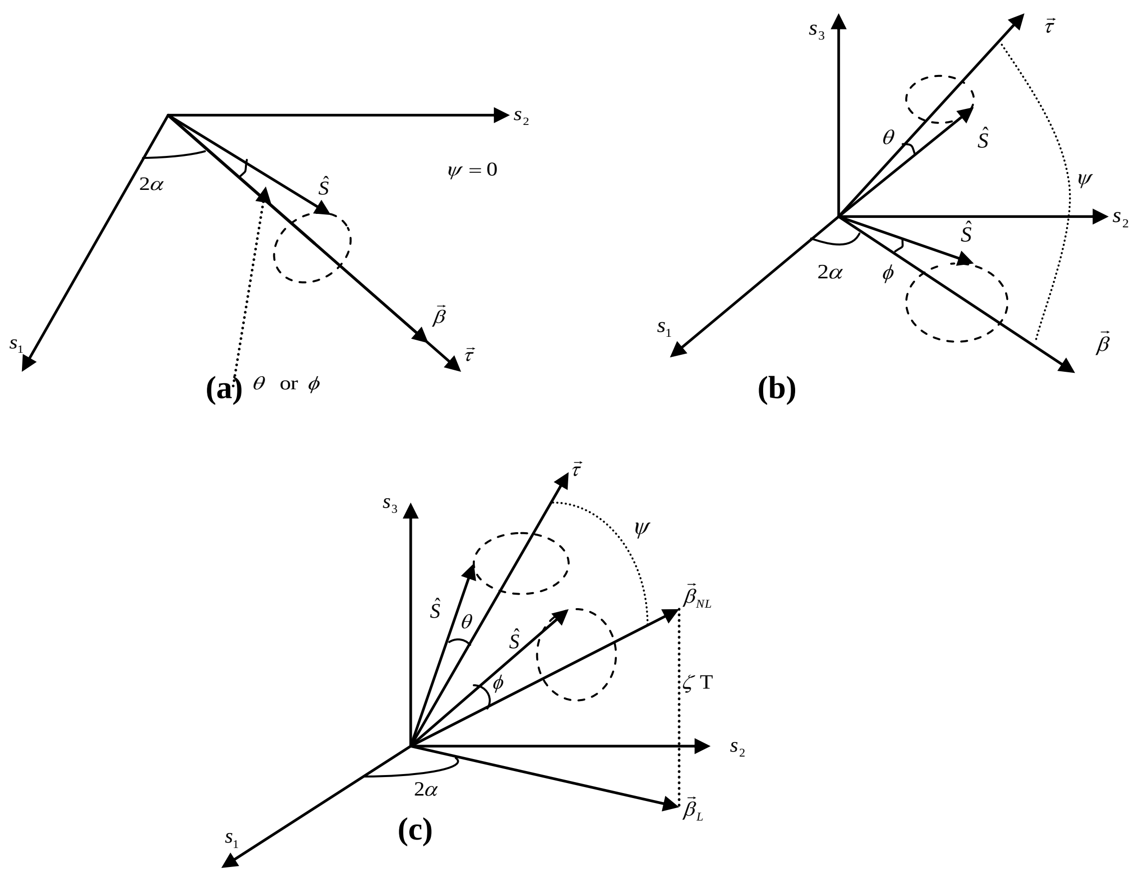
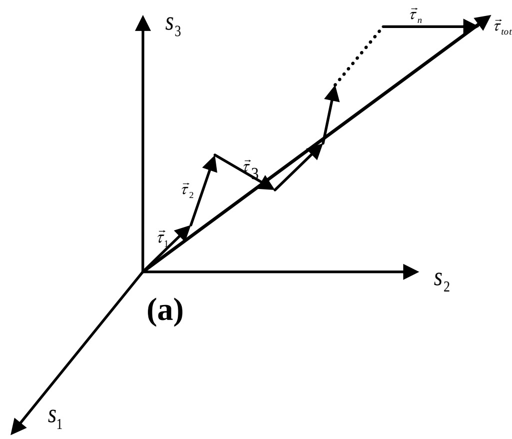


Fig.(1): Rotation of SOP around $\vec{\beta}$ **and** $\vec{\tau}$ **. a) the two vectors** $\vec{\beta}$ **and** $\vec{\tau}$ **are linear b)** $\vec{\beta}$ **is linear and** $\vec{\tau}$ **is nonlinear, c) the two vectors** $\vec{\beta}$ **and** $\vec{\tau}$ **are nonlinear.**



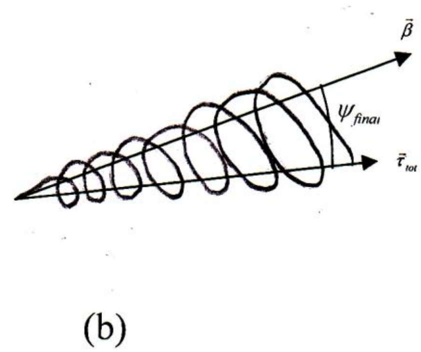


Fig.(2): a) The vector $\vec{\tau}_{tot}$ represents the resultant vector of many different vectors $\vec{\tau}_i$ each one for different frequency , b) the final situation of the vectors $\vec{\beta}$ and $\vec{\tau}$.

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