FREE VIBRATION ANALYSIS OF SQUARED SIMPLY SUPPORTED PLATES CONTAINING VARIOUS CRACK CONFIGURATIONS

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ABSTRACT

This work investigates a free vibration analysis of plates containing a single crack as the crack parameters (i.e. length , orientation and location) is varied. The influences of these parameters on the natural frequencies and the corresponding mode shapes are examined for several squared simply supported plates including internal crack or edge crack or corner crack. Vibration analysis for these plates is carried out using finite element method through ANSYS package version 11 . The validation results are presented and compared with the most recent numerical results found in literature. It is found from present study that the length of the crack as well as its orientation and location are shown to have significant effects on the natural frequency and mode shape of the plates. Some new cases are also discussed in detail. The study is particularly useful in the understanding and offering a better insight into the free vibration of the plates with various crack configurations.

KEYWRDS: Free Vibration, Cracked Plate, Finite Element Method (ANSYS)

تحليل الاهتزاز الحر لصفائح مربعة اسناد بسيط تحتوي أشكال شق مختلفة

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الموجز

هذا العمل يبحث الاهتزاز الحر لصفائح تحتوي على شق مفرد عند تغير محددات الشق (الطول ، الموقع و الميلان).تم دراسة تأثير هذه المحددات على الترددات الطبيعية و أشكال النمط المرادفة لها لصفائح مربعة مسندة اسناد بسيط متعددة تحتوي على شق داخلي ، شق جانبي أو شق زاوية. تحليل الاهتزاز لهذه الصفائح نفذت بأستخدام طريقة العناصر المحددة من خلال الحقيبة البرمجية الانسس. قدمت النتائج المصدقة و قورنت مع أحدث النتائج العددية المتوفرة.و قد وجد من الدراسة الحالية بأن طول الشق و كذلك ميلانه و موقعه لهم تأثيرات واضحة على التردد الطبيعي و شكل النمط للصفائح. بعض الحالات الجديدة ايضاً نوقشت بتفصيل. أن الدراسة مفيدة جداً في فهم و عرض أفضل إلى الاهتزاز الحر للصفائح التي تحتوي على أشكال شق مختلفة.

1. INTRODUCTION

Engineering structures may have structural defects such as cracks during manufacturing or due to service loadings. The presence of a crack in a structural member causes a local flexibility affecting its static and dynamic characteristics such as static deflections, natural frequencies and mode shapes.

Much research work had been done by investigators to study the effects of cracks on the dynamic characteristics of structural elements such as shafts, beams and plates. A comprehensive review on the literature through the period (1971-1992) of the vibration of cracked structures was made by (**Dimarogonas,1996**). In this literature, different modeling techniques of structures with cracks are presented and summarized into three categories, namely: equivalent reduced section, local flexibility from fracture mechanics, and cracked continuous bar or beam. Therefore, the reader can be referred to this review paper on the vibrations of cracked plates studied in that period. Since then, research and publication on this subject has been at an increasing rate. It is well known that exact analytical solutions exist for plates without crack. So numerical solutions or experimental methods can be constructed to consider the vibrations of cracked plates with various crack configurations and arbitrary boundary conditions. Both Rayleigh-Ritz or Ritz method and the finite element method have been often used.

Many published research is available about the vibrations of cracked plates based on the Rayleigh-Ritz method. (**Lee and Lim,1993**) studied the vibration of center cracked rectangular plates with simply support conditions taking into account shear deformation and rotary inertia. (**Liew et al.,1994**) employed the decomposition method to determine the natural frequencies of a plate having an edge or central crack. They assumed the cracked plate domain to be an assemblage of small sub domains with the appropriate functions formed and led to a governing eigenvalue equation. (**Ramamurti and Neogy, 1998**) have applied the generalized Rayleigh-Ritz method to determine the natural frequency of cracked cantilevered plates. (**Khadem and Razaee, 2000**) introduced a modified comparisons functions to analyze a simply supported rectangular plate with a crack having an arbitrary length, depth and location parallel to one side of the plate. Those functions are derived using the Rayleigh–Ritz method. The elastic behavior of the plate at crack location is considered as a line spring with a varying stiffness along the crack. Recently, (**Huang et al., 2008, 2009, 2011**) proposed a set of regular polynomial admissible functions based on Ritz method to describe the stress singularity behaviors around a crack tip. They employed the proposed approach on simply supported and cantilevered plates with single V-notch (**Huang et al., 2008**) , side crack (**Huang et al., 2009**) as well as central crack and side crack (**Huang et al., 2011**).

The finite element method is applied to analyze vibration problems of cracked plates . (**Krawczuk , 1993**) and (**Krawczuk and Ostachowicz, 1994**) calculated the flexibility matrix of a plate with the crack as a sum of the non cracked plate and an additional flexibility matrix caused by the crack. (**Yang and Chen, 1996**) developed an assumed hybrid-stress finite clement model incorporating with two types of multilayer hybrid-stress elements (MLTUP and MLTPH) to study the free vibration of patched cracked laminates. The MLTUP element is used to model the region of an unpatched cracked panel, while the MLTPH element is used to model the region of a cracked panel adhesive patch. (**Ma et al., a-b 2001**) used the commercial finite element ABAQUS Package and selected eight-node two-dimensional shell elements to determine natural frequencies of cantilevered thin plates with horizontal or vertical side cracks to verify the correctness of their experimental results. They utilized an optical method based on the amplitude fluctuation electronic speckle pattern interferometry in their experimental work. Recently, (**Bachene et al., 2009**) applied the extended finite element method to analyze the free vibrations of plates containing central or edge cracks with different boundary conditions.

Further, the finite element results have been used in the area of non-destructive damage evaluation for damage identification in plates (**Cornwell et al., 1999, Anne et al., 2002, Chang and Chen, 2004, Bijaya and Wei-Xin, 2006 and Sandesh and Shankar, 2009**). These methods are based on the fact that local damages usually cause decrease in the plate stiffness, which produces the change in vibration characteristics (such as natural frequencies, mode shapes and curvature mode shapes) of the structure. Damage is determined through the comparison between the undamaged and the

damaged state of the structure. The most common dynamic parameters used in damage detection are natural frequencies and mode shapes.
In summary, to the author's best knowledge, the major concern in the previous studies has been carried out on plates with very specific cracks (i.e. central crack or edge crack) with varying crack length or crack orientation. Few studies have been performed to evaluate the influence of crack location on the natural frequency for edge cracked plates (**Huang et al., 2009, 2011**)and internally cracked plate (**Khadem and Razaee, 2000**). So, the purpose of the present paper is to investigate the effects of crack parameters (i.e. length , location and orientation) on the natural frequency of the plates. In this regard, a set of eigenvalues vibration analysis is conducted for a various cracked plates including edge crack or internal crack or corner crack . The computations are carried out using ANSYS Software version 11, a commercial finite element package.

2- THEORETICAL ANALYSIS

The dynamics of plates, which are continuous elastic systems, can be modeled mathematically by partial differential equations based on Newton's laws or by integral equations based on the considerations of virtual work. It is well known that the natural vibrations of plates are functions of the material properties and the plate geometry only, and are inherent properties of the elastic plate, independent of any load. Thus, the equation of motion for the transverse vibration of a plate as (**Chakraverty, 2009**):

$$D\nabla^4 \mathrm{w}(x,y,t) + \rho h \frac{\partial^2 \mathrm{w}}{\partial t^2}(x,y,t) = o \qquad (1)$$

Where
w(x,y,t) is the transverse displacement of the plate and t is the time.

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ is the plate's flexural rigidity.

E , ν and ρ are the Young's modulus , Poisson's ratio and density of the plate material respectively and h is the plate thickness.

$\nabla^4 \mathrm{w} = \dfrac{\partial^4 \mathrm{w}}{\partial x^4} + 2\dfrac{\partial^4 \mathrm{w}}{\partial x^2 \partial y^2} + \dfrac{\partial^4 \mathrm{w}}{\partial y^4}$ is the bi-harmonic operator.

Deflection w must satisfy the boundary conditions at the plate side (these conditions practically do not differ from those in the case of static equilibrium) and the following initial conditions:
when

$t = 0:\ \mathrm{w} = \mathrm{w}_o(x,y),\ \dfrac{\partial \mathrm{w}}{\partial t} = \mathrm{v}_o(x,y)$ where w_o and v_o are the initial deflection and initial velocity respectively for point (x,y).

Equation (1) is the governing, fourth-order homogeneous partial differential equation of the undamped, free, linear vibrations of plates. A complete solution of the problem of a freely vibrating plate is reduced to determining the deflections at any point for any moment of time. However, the most important part of the problem of free flexural vibrations of plates is to determine the natural frequencies and the mode shapes of the vibration (deflection surfaces in two dimensions) associated with each natural frequency. For such a problem, equation (1) is an eigenvalue problem. The natural frequencies are the eigenvalues and associated shape functions are the eigenfunctions. A solution of equation (1) can be obtained by applying the classical analytical and approximate methods discussed in reference [20]. In the case of a simply supported square plate (**Figure 1**) , the natural frequency may be expressed as :

$$\omega_n = \frac{k}{a^2}\sqrt{\frac{D}{\rho h}} \tag{2}$$

where
ω_n : is the natural frequency
k : is the frequency parameter
a : is the plate side
For example, **Table (1)** gives the first five frequency parameters for a simply supported square plate Chakraverty,2009.
In the cases of cracked plates firstly the eigenvalue vibration analysis is performed then the relevant values of frequency parameter for cracked plate ($k_{cracked}$) are determined in each case. For such cases, the following equation can be considered in order to determine the frequency parameter:

$$k_{cracked} = \omega_n a^2 \sqrt{\frac{\rho h}{D}} \tag{3}$$

3. FINITE ELEMENT MODELING OF CRACKED PLATES
3.1 Model Description
There are many crack parameters involved in modeling and analyzing of the problem under study. These parameters include crack length, location and orientation. Due to such a variety of contributing parameters, three different models are established for analysis purpose. The models cover internally, edge and corner-cracked plates as shown in **Figure 2**. The crack was presumed to be through thickness since thin plate is used and having no friction between their edges and no propagation was allowed.
The considered cracked plates have all their sides simply supported. It are characterized by the following dimensionless geometric parameters: plate's aspect ratio equal to (1), relative plate's thickness (h/a = 1/100), relative crack's length (c/a = 0.1 - 0.5) and orientation ($\Theta = 0^o - 90^o$) as well as relative crack location (x/a = 0.1, 0.3 , 0.4 , 0.5), as indicated in **Table 2**. The plate material considered is supposed to be linear elastic and isotropic with Young's modulus : E=200GN/m^2 , Possion's ratio : $\nu = 0.3$ and density , $\rho = 7860$ kg/m^3.
The commercial finite element package ANSYS version11 is utilized for the modeling and vibration analysis of cracked plates. The "shell93" element of ANSYS element library was used for meshing procedure. This element is suitable for analysis thin-walled structures. It is a eight-node element with six degree of freedom at each node: translations in the nodal x, y, and z directions and rotations about the x, y, and z-axes. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. The adopted finite element mesh is displayed in **Figure 3** where the element’s refinement around the crack’s tips has been carefully considered for an accurate evaluation. The "Block Lanczos" mode extraction method was used to calculate the natural frequency of the cracked plates then the frequency parameters can be computed from equation (3).

3.2 Model Verification
In order to illustrate the accuracy and applicability of the proposed finite element model, two case studies have been selected. Simply supported edge and central cracked square plates with different relative crack length (a/b=0.1 – 0.5) for crack orientation ($\Theta=0^o$, 90^o) which are reported by (**Huang et al., 2009**) and (**Bachene et al., 2009**) respectively, as shown in **Figure 4**. Thus, the quality of the final pattern and density of the finite element mesh have been accepted after several convergence tests in which the mesh density and element shapes have been varied with that case studies. **Figure 4** indicates the effect of the crack relative lengths on the first five frequency parameters (k). It is clearly observed there is a very good convergence between the results of the present study and those obtained in reference (**Huang et al., 2009**) and (**Bachene et al., 2009**).

4- RESULTS AND DISCUSSIONS

It is interesting to observe how the frequency parameters change with crack length, orientation and location. Consequently, **Figures 5** and **6** indicate the values of the first five frequency parameter (k) versus crack orientation (ϴ) for different relative crack length (c/a) and crack location (x/a) of internally cracked plate and edge cracked plate respectively. Also, variations in values of the first five frequency parameter of a plate with corner crack versus relative crack length and crack orientation have been plotted in **Figure 7**. Further, the reduction in frequency parameters due to cracks at different lengths, orientations and locations were computed by the following equation:

$$\mathrm{Re}\,duction\ \% = \frac{k_{uncracked} - k_{cracked}}{k_{uncracked}}\ \% \qquad (4)$$

In general, at the initial investigation of these figures, it is clearly observed that when the length of the crack increases the value of the frequency parameter decreases for all the five modes and for the three types of cracked plates. Although the amount of frequency parameter drop is dependent on the case (i.e. cracked plate) and mode of interest. This phenomenon can be expected as a result of the associated stiffness reduction (**Huang et al., 2009**) and (**Bachene et al., 2009**). Besides, it is seen that small crack (i.e. c/a=0.1) produce a low influence on frequency parameters in various orientations and locations as compared with intact plate, e.g. which are highest reduced only by about 1.564% at (ϴ=0^o and x/a=0.5), 0.217% at (ϴ=45^o and x/a=0.3) and 0.255% at (ϴ=45^o) for internally, edge and corner cracked plates respectively in fifth mode.
Because of the different behavior of the three consideration cases in various lengths, orientations and locations, each case are discussed separately.

4.1 Internally Cracked Plate

In **Figure 5**, the frequency parameters of the first mode are slightly sensitive to the crack orientation than the other modes, thus the variation in highest drop occurs between 10.44% and 11.4% at x/a=0.5. While, in second mode the change in frequency parameters with crack orientation would have very little as the crack moves towards the middle of the plate, with maximum reductions of (15.56-10.79)% , (13.75-12.57)% and (13.26-12.88)% for crack locations at x/a=0.3 , x/a=0.4 and x/a=0.5 respectively. Also, it is noticed that the presence of the crack of any length in different orientations and locations has less influence on the frequency parameters of the third mode with respect to other modes. Thus, the maximum drop occurs at the crack location x/a=0.3 of crack length c/a=0.5 and crack orientation ϴ=45^o by about 3.54%. Moreover, from the observation of this figure, it can be deduced that the behavior in fourth mode is opposite to the behavior in fifth mode, especially for crack length c/a=0.4 and 0.5 at x/a=0.3 and 0.5. Changing the crack orientation from ϴ=0^o to 45^o and as the crack approaches to the plate centre, firstly the frequency parameters reduces then increases at crack location x/a=0.3, while it is always decreased at x/a=0.4 and 0.5 in fourth mode. But, in fifth mode the frequency parameters firstly increases then reduces at x/a=0.4. An interesting phenomenon is presented in **Figure 5**. The largest drop of the frequency parameters is noticed when crack orientation ϴ=45^o of the first and third modes by about 11.4% at x/a=0.5, 3.54% at x/a=0.3 respectively, while in forth mode it is observed when crack orientation ϴ=15^o by about 12.62% at x/a=0.3. Further, the highest reduction in frequency parameters always occurs when crack orientation ϴ=0^o in second and fifth modes, by about 15.56% and 22.49% respectively at x/a=0.3.

4.2 Edge Cracked Plate

Figure 6 shows that in first mode, the frequency parameters are firstly decreased then increased depending on crack orientation, although the opposite happens for crack location x/a=0.5 with maximum reductions of 7.32% at ($\Theta=45^o$ and x/a=0.1), 6.027% at ($\Theta=75^o$ and x/a=0.3) and 5.73% at ($\Theta=90^o$ and x/a=0.5). In second mode, decreasing or increasing in the frequency parameters significantly influence by crack orientation and crack location especially for relative crack length c/a=0.4 and 0.5, thus the largest drop occurs at ($\Theta=90^o$ and x/a=0.1) by about 12.44%. It can be seen that in third mode when the crack is located at x/a=0.3, the frequency parameters are more affected than other locations with maximum reduction of 5.056% at $\Theta=45^o$. Further, in fourth mode the frequency parameters extensively decreases as the crack approaches the plate centre by about 18.37% at ($\Theta=90^o$ and x/a=0.5) and it is less affected by crack orientation at x/a=0.1. Interestingly, the frequency parameters of the fifth mode are slightly sensitive to the crack orientation for relative crack length c/a = 0.1- 0.4, while the influence of crack orientation is very clear for relative crack length c/a=0.5 at crack location x/a=0.1 and 0.5 with greatest reduction of 22.97% and 18.32% respectively at $\Theta=45^o$. Although, when the crack is located at x/a=0.3, the crack orientation are shown to have significant effects on the frequency parameters with maximum drop of 20.93% at $\Theta=90^o$. An important finding have been revealed in **Figure 6** that for the cracks at the location x/a=0.3 an intermediate behavior is observed.

4.3 Corner Cracked Plate

It is obvious from **Figure 7** that the increasing of crack orientation from $\Theta=15^o$ - 45^o generally reduces the frequency parameters of the first, second and forth mode by about 9.94% at c/a=0.5, more than 9.24% at (0.1<c/a<0.5) and more than 7.84% at (0.1<c/a<0.4) respectively. While, for relative crack length c/a=0.5 in the second and fourth modes the highest drop occurs at $\Theta=15^o$ by about 15.47% and 14.34% respectively. In third mode, the frequency parameters are almost unchanged with relative crack length and crack orientation, however there is a very little reduction occurs at c/a=0.5 and $\Theta=15^o$ by about 1.08% . Further, it can be observed for fifth mode that the effect of the crack orientation starts to appear significantly at c/a>0.3. Accordingly, with growing crack orientation the frequency parameters firstly increase then decrease by about 18.82% at $\Theta=45^o$. The above mentioned findings of the reduction in frequency parameters (i.e. decreasing or increasing) for three cases can be explained by deformed mode shapes. So, **Figures 8 and 11** show the first five vibration mode shapes for uncracked plate as well as only for cracked plates characterized by c/a=0.5 , x/a=0.3,0.4,0.5 and $\Theta=0^o$,45^o of internally cracked plate, c/a=0.5 , x/a=0.1,0.3,0.5 and $\Theta=45^o$, 90^o of edge cracked plate and c/a=0.5, $\Theta=15^o$,45^o of corner cracked plate respectively. Since similar results (i.e. similar behavior) were obtained for cracked plates having another crack parameters for brevity. It is seen from **figures 9 and 11** that how the cracks opens and splits the plate depending upon the mode of interest as well as crack parameters with respect to the separator region between convexity surface and concavity surface of the mode shape.
The splitting phenomenon of the plate explains the loss in rigidity of the plate, inducing a drop in the mode frequencies, (**Bachene et al., 2009**). Furthermore, it is evident that the deformed mode shapes in first mode have one region either convexity or concavity. While in second and third modes it have two regions one convexity and the other concavity. Also, in fourth and fifth modes that the deformed mode shapes changes its shape and have more than two regions. Consequently, one region is under tension and the other is under compression. This may be the reason for varying the reduction in frequency parameters by depending upon these region with respect to crack parameters and supported edges.

5. CONCLUSIONS

In the present paper, the free vibration analysis of variously cracked square thin plates has been considered. Finite element method has been preformed through ANSYS Package in order to determine the frequency parameter. In particular, the effects of the crack length, crack orientation and crack location on the natural frequencies and the corresponding mode shapes have been investigated. On the basis of the achieved results the following conclusions can be stated :

1- The numerical simulations show that if the crack length increases, a frequency reduction takes place.
2- The change in frequencies due to the presence of a crack is a function of the crack parameters and it also depends upon the mode shapes of the plate.
3- It is shown from the computed results that the crack orientation has less effect on frequencies for the internal crack plate than the edge crack and corner crack in the first mode.
4- The frequencies of the third modes are less sensitive to the crack parameters than the other modes for the three cases of cracked plates.
5- By depending on the fundamental mode (i.e. first mode), internally cracked plate with crack orientation ($\Theta=45^{o}$) can be considered to be the most dangerous one. Since it has the highest reduction in frequency parameter.

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Table 1 Values of First Five Frequency Parameters of a Simply Supported Square Plate

Mode No.	Frequency Parameters
First	19.739
Second	49.348
Third	49.349
Fourth	79.4
Fifth	100.17

Table 2 The Values of Crack Parameters in The Finite Element Model.

Case study	c/a	Ө	x/a
Internal crack	0.1, 0.2, 0.3, 0.4, 0.5	0, 15, 30, 45	0.3,0.4, 0.5
Edge crack	0.1, 0.2, 0.3, 0.4, 0.5	15, 30, 45, 60, 75, 90	0.1, 0.3, 0.5
Corner crack	0.1, 0.2, 0.3, 0.4, 0.5	15, 30, 45	corner

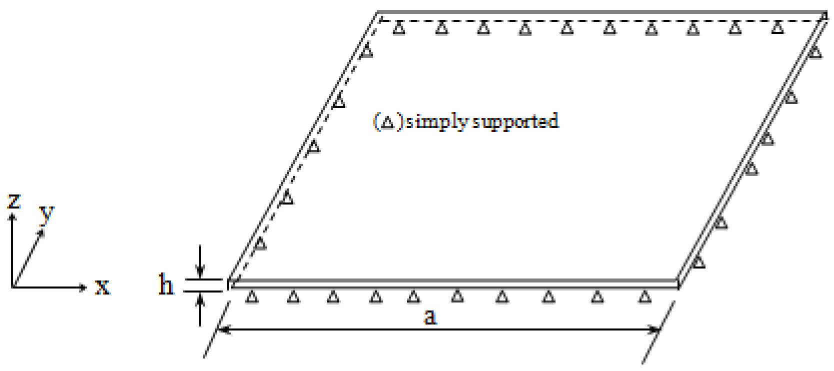


Figure 1 geometry of simply supported square plate.

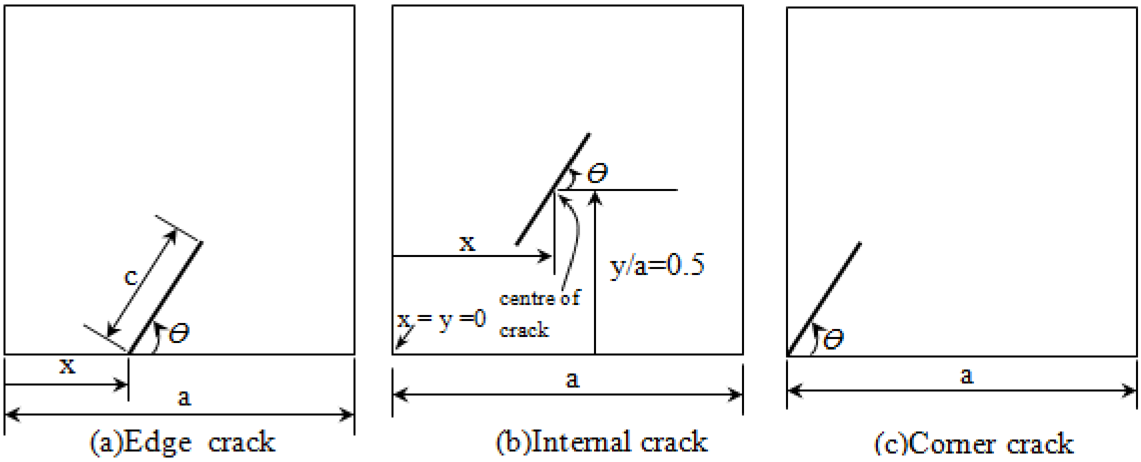


Figure 2 cracked square plate with crack type details.

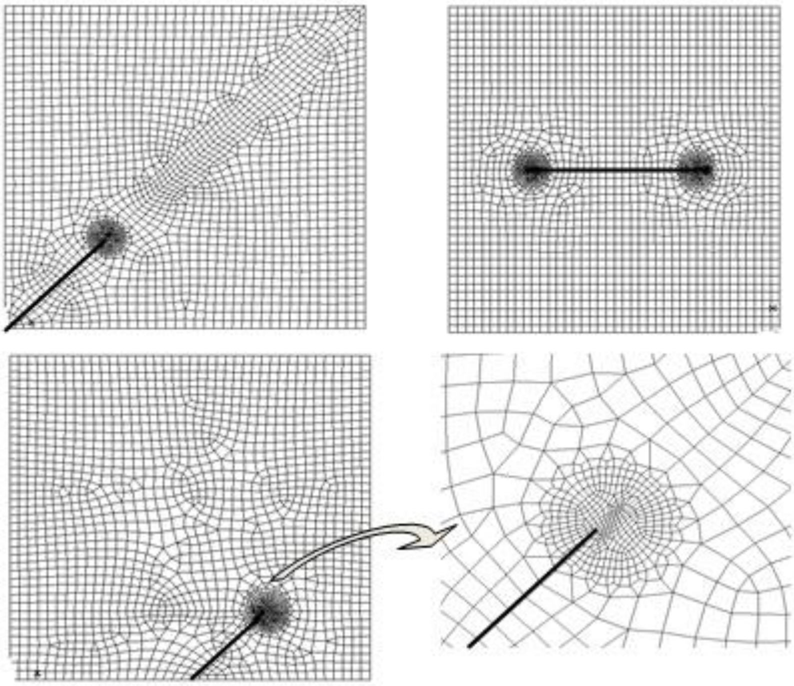
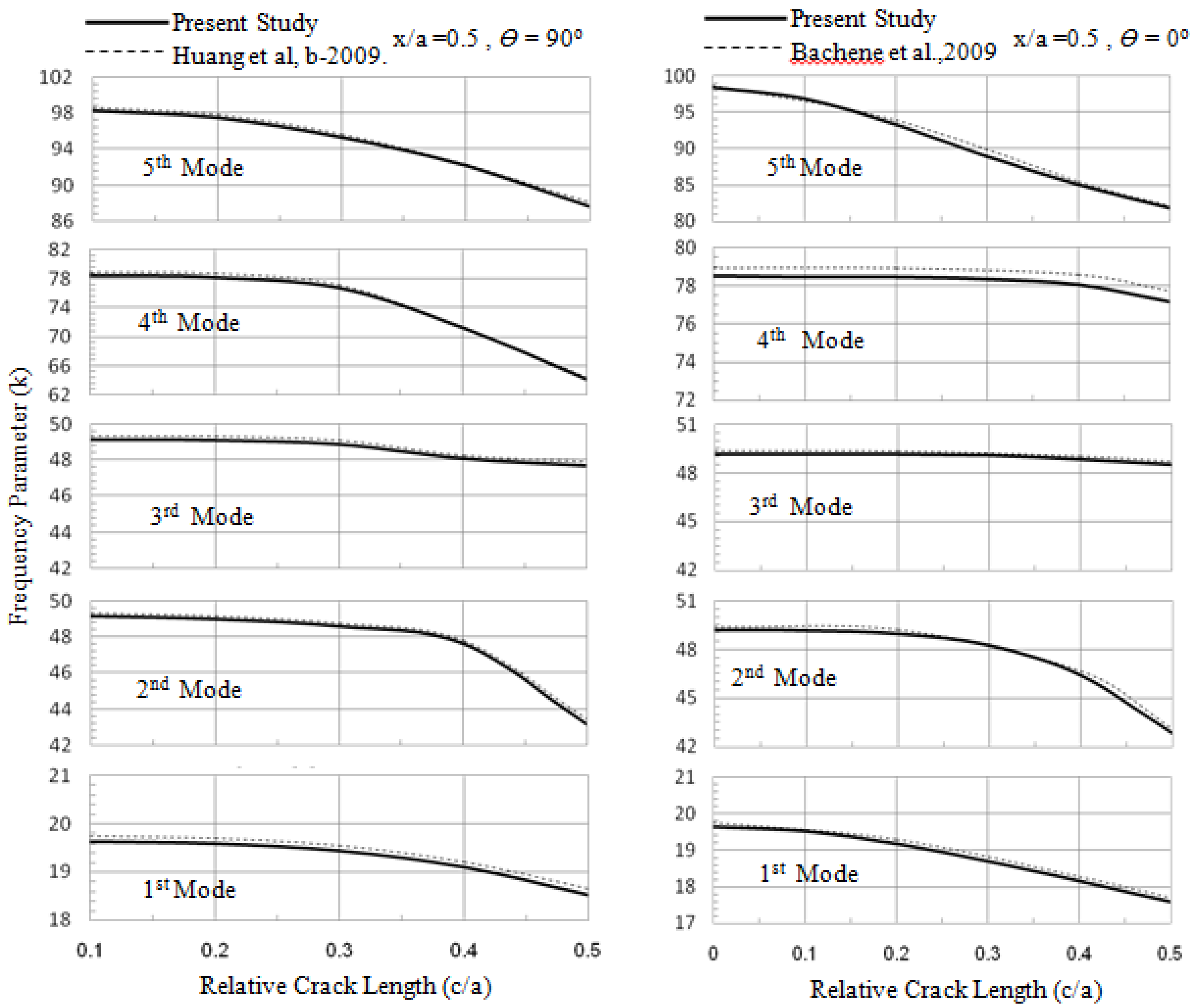


Figure 3 Samples of the finite element model for different type of cracked plate with crack-tip mesh refinement detail.



(a)case study(1): edge cracked plate (b)case study(2): central cracked plate

Figure 4 verification case studies.

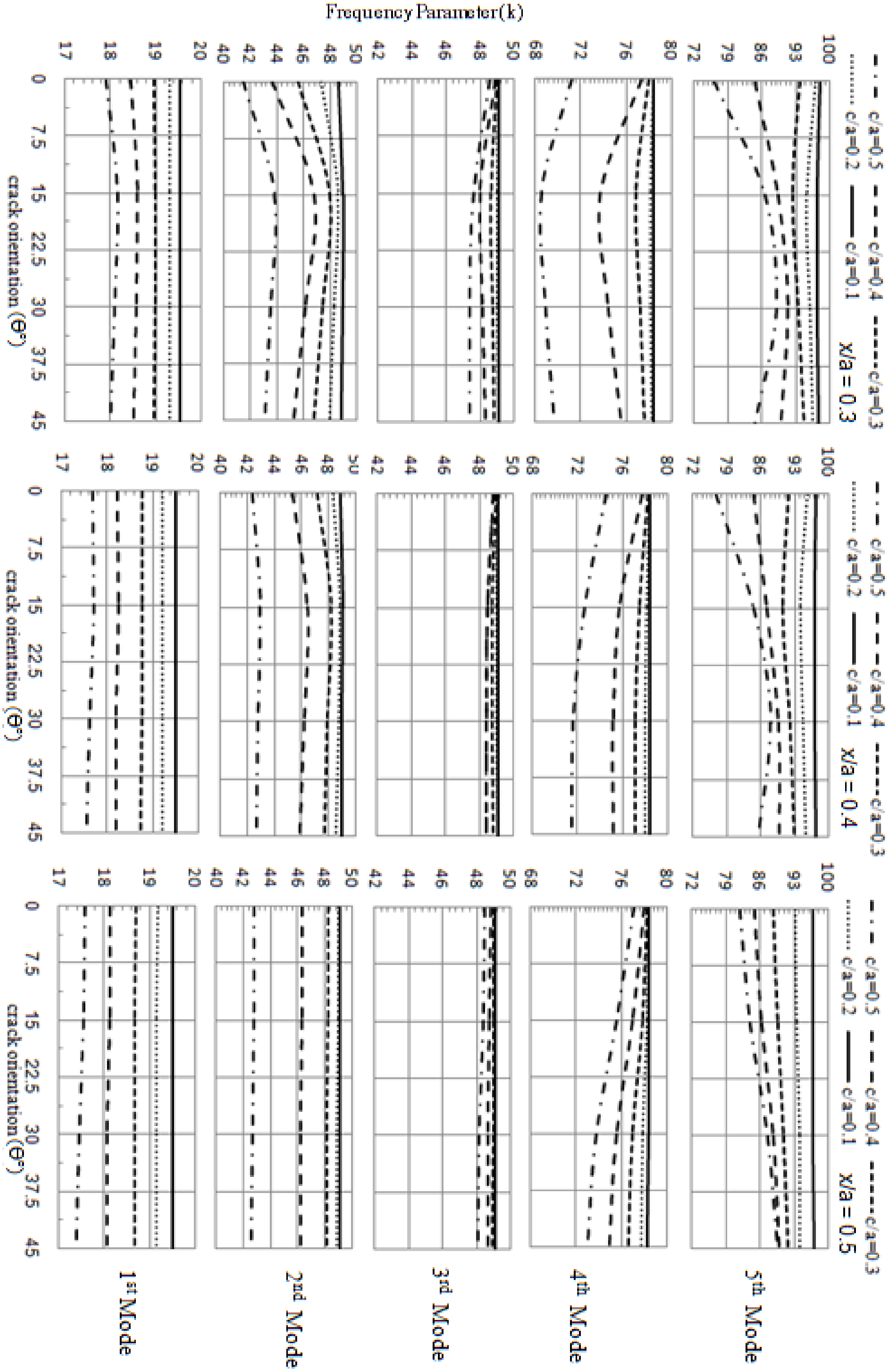


Figure (5) variation of first five frequency parameters to the crack orientation for different relative crack length and crack location in internally cracked plate

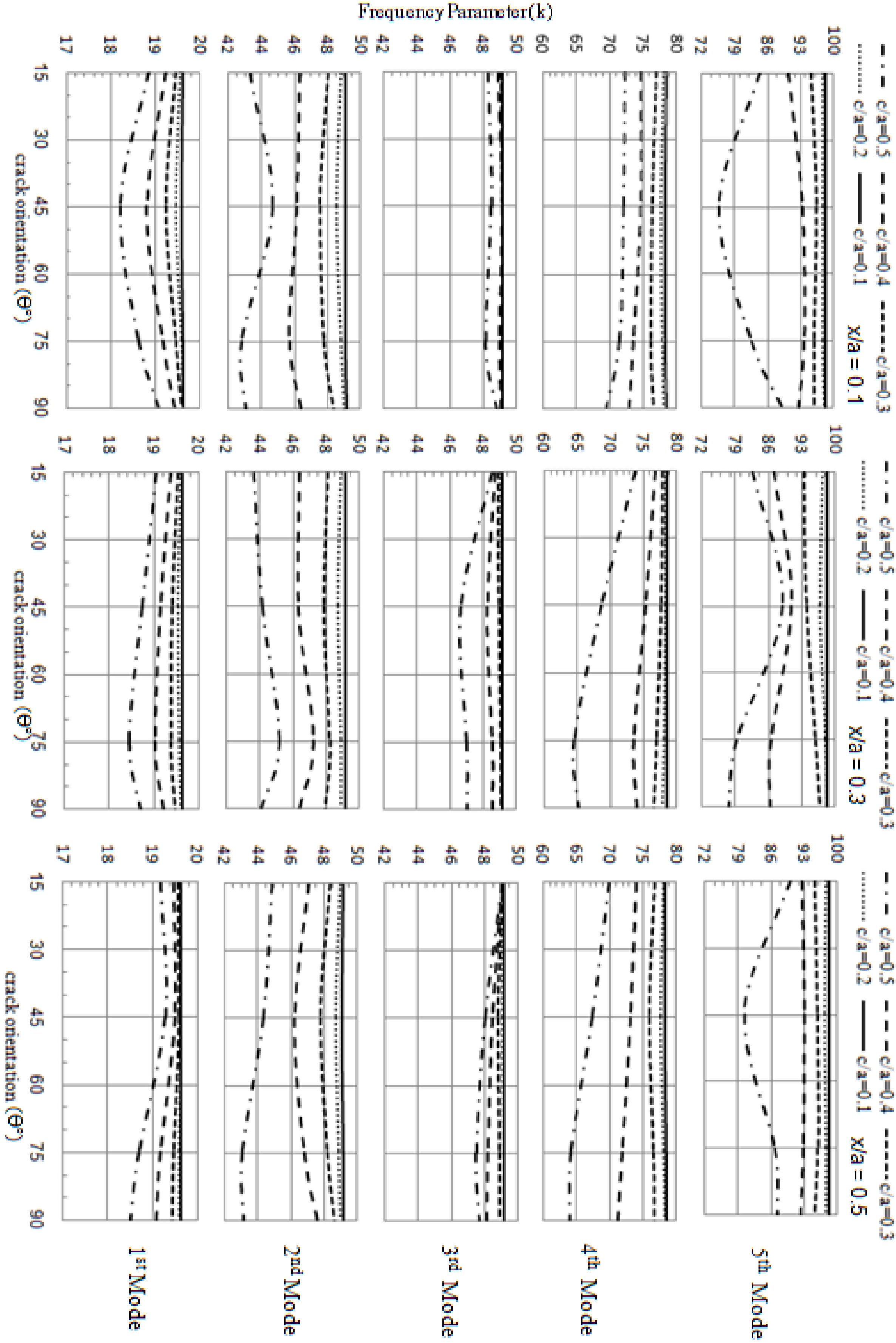


Figure (6) variation of first five frequency parameters to the crack orientation for different relative crack length and crack location in edge cracked plate

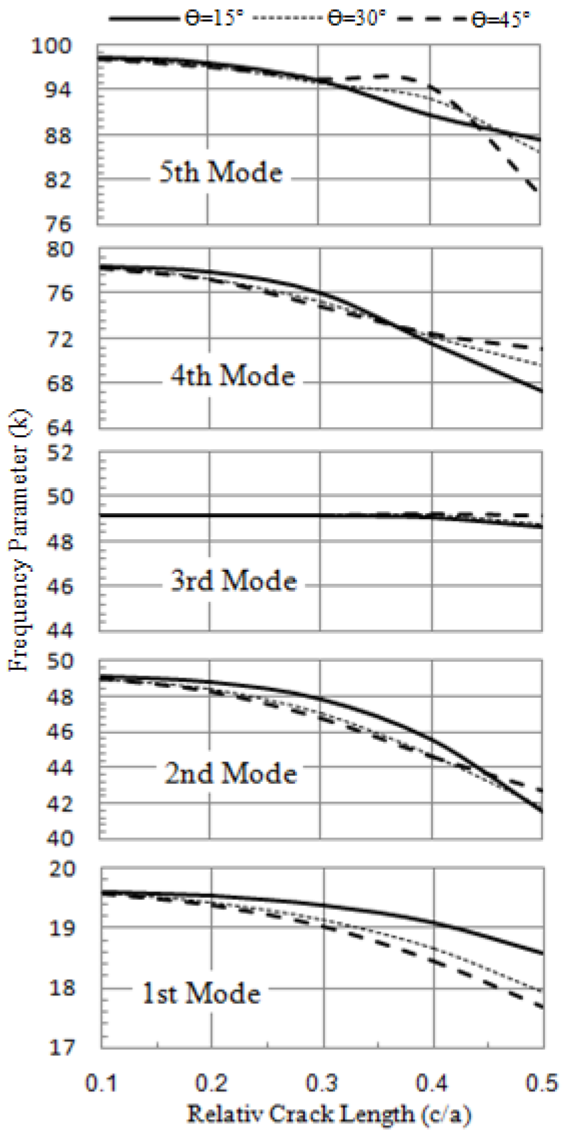


Figure 7 variation of first five frequency parameters to the relative crack length for different crack orientation in corner cracked plate.

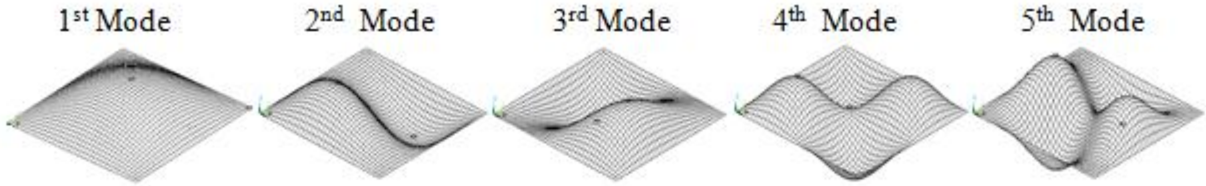


Figure 8 first five deformed mode shapes for simply supported uncracked plate.

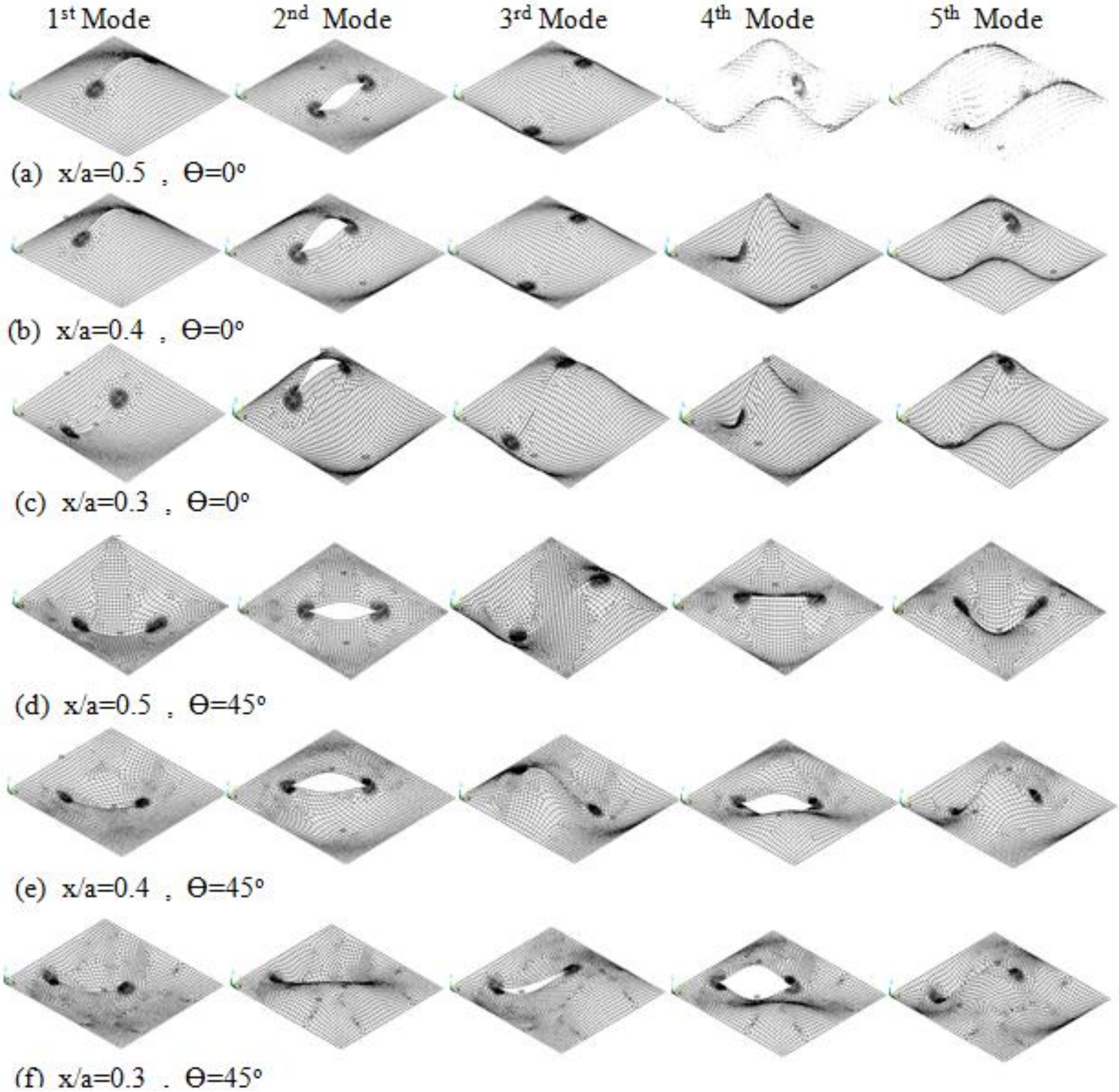


Figure 9 first five deformed mode shapes for simply supported internally cracked plate with relative crack length c/a=0.5

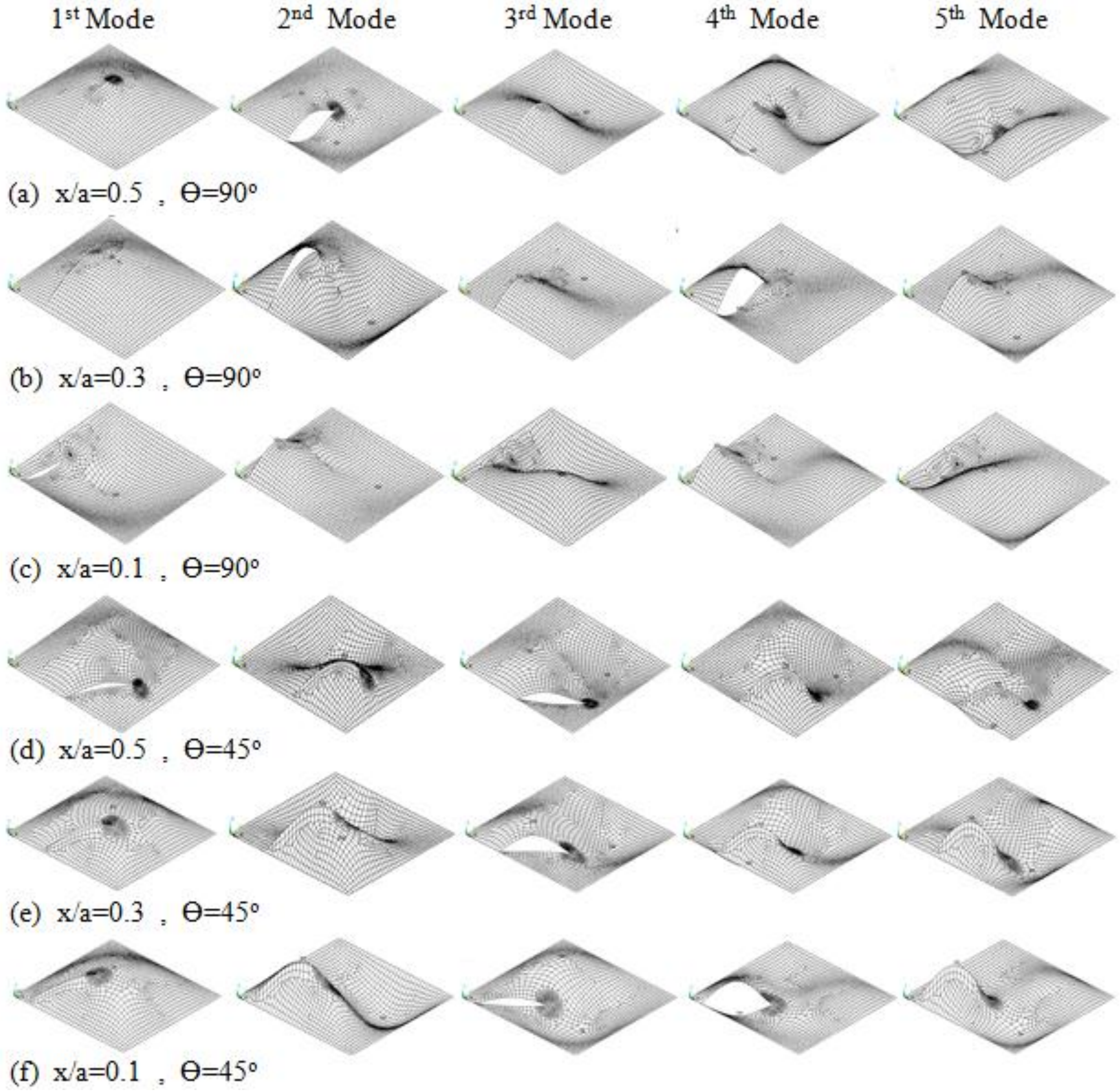


Figure 10 first five deformed mode shapes for simply supported edge cracked plate with relative crack length c/a=0.5.

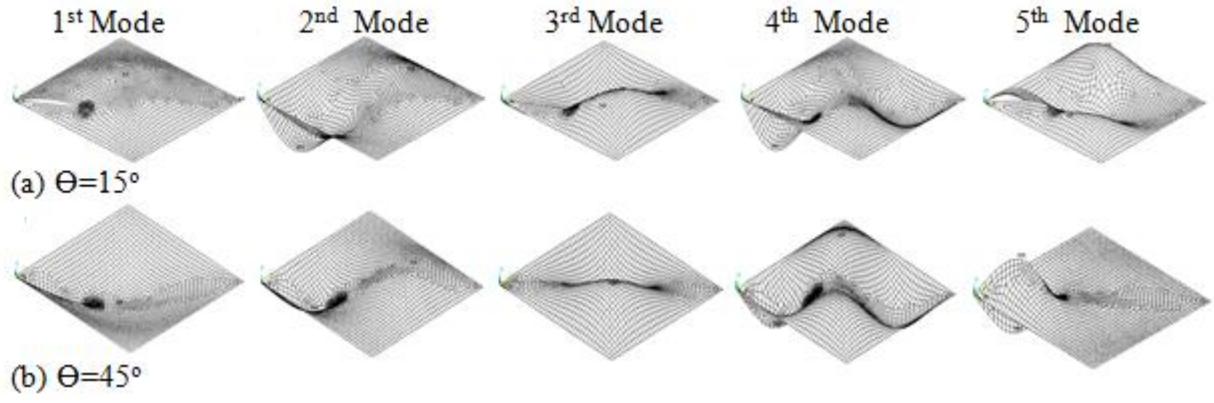


Figure 11 first five deformed mode shapes for simply supported corner cracked plate with relative crack length c/a=0.5.