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Some Results of Representation of the Character Table of S_6

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A B S T R A C T

n this essay we use standard Young tableaux to compute each equivalent irreducible representation of the Symmetric group S_6 by determining the matrices for adjacent transpositions (12) , (23) , (34), (45) and (56) of S_6 .

Keywords: Symmetric group S6 , Standard Young tableaux , equivalent irreducible representation of S6

1. Introduction

Young diagram is considered as the main tool to study the representation of S_α. it is the Young diagram which is an array of boxes for a given partition $\alpha = \mu_1 + \mu_2 + \cdots + \mu_t$ and Which denoted by $\mu = (\mu_1, \mu_2, \dots, \mu_t)$,where $\mu_i \geq \mu_{i+1}$ for every $1 \leq i \leq t$. Thus, the Young diagrams corresponding to the partitions of 6 are:

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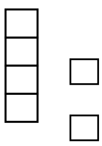
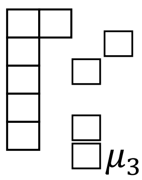
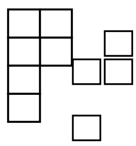
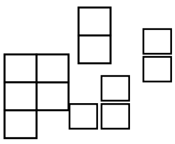
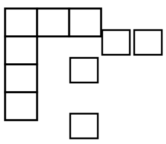
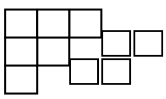
$\mu_1 = (1,1,1,1,1,1) \leftrightarrow$  $\mu_2 = (2,1,1,1,1) \leftrightarrow$  $\mu_3 = (2,2,1,1) \leftrightarrow$ 

$\mu_4 = (2,2,2) \leftrightarrow$ $\mu_5 = (3,1,1,1) \leftrightarrow$ $\mu_6 = (3,2,1) \leftrightarrow$

$\mu_7 = (3,3) \leftrightarrow$ $\mu_8 = (4,1,1) \leftrightarrow$ $\mu_9 = (4,2) \leftrightarrow$

$\mu_{10} = (5,1) \leftrightarrow$ $\mu_{11} = (6) \leftrightarrow$

It is correct in general that the irreducible representations of the S_α can be described using Young diagrams of boxes and hence there are eleven irreducible representations of S_6.

A Young tableau T of shape μ is obtained from the corresponding Young diagram by replacing the boxes by numbers 1, 2, 3… α , each number is used

exactly once, and if rows and columns increase it is called standard Young tableau.

The dimension of the irreducible representation module (called Specht module S^μ) corresponding to the shape μ , or the degree of the corresponding representation matrices ,is equal to the number of different standard Young tableau that can be obtained from the diagram of the representation, denoted by H^μ. This number can be calculated by hook – length formula.

$H^\mu = \alpha!/\pi$(hook length of each box)

A tabloid is an equivalence class of Young tableaux of shape μ , where two tableaux are equivalent if one is obtained from the other by per mutating the entries of each row. The Symmetric group S_α acts on the set of tabloids and therefore on the permutation module M^μ with the tabloid as abases [2] for each tableau T from the associated polytabloid

$$e_T = \sum_{\sigma \epsilon C_T} sgn(\sigma)\sigma(T)$$

Where C_T is the subgroup fixing all columns of T and $\sigma e_T = e_{\sigma T}$ [1] .

The Specht module, S^μ, is the sub module of M^μ spanned by the polytabloid e_T, where T is taken over all tableaux of shape μ .

In this paper, we compute matrices of the adjacent transpositions (12), (23), (34),(45) and (56) in S_6 for all Young's standard representations. These representations correspond to the irreducible representations and any other irreducible representation is equivalent to one of Young 's standard representations. Moreover, the adjacent transpositions generate all of S_6so that all the matrices of Young's standard representations are easily obtained by merely multiplying the appropriate generators.

Throughout this paper, we use the following convention in determining the representation of $(i \;\; i+1)e_T$ in terms of the basis of standard μ −polytabloids.

Case 1: if i and $i+1$ are in the same column of T. Then $(i \;\; i+1)e_T = -e_T$.

Case 2: if i and $i+1$ are in the same row of T . Then we apply the Garnir element [1] to obtain $(i \;\; i+1)e_T$ as a linear combination of standard μ −polytabloids .

Case 3: if i and $i+1$ are not in the same row or column of T. Then $(i\ i+1)e_T$ is a standard μ −polytabloids .

2. The S_6–module S^{μ_1}

There is exactly one standard μ_1 − table $T = \begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline 4\\ \hline 5\\ \hline 6\\ \hline\end{array}$

Thus, dim S^{μ_1} =1

Denoting the representation by X_{μ_1} we have immediately $X_{\mu_1}(I) = [1]$.

2. Matrix for (12), (23), (34), (45) and (56):

By case 1, $(12)e_T = (23)e_T = (34)e_T = (45)e_T = (56)e_T = -e_T$

Therefore, $X_{\mu_1}(12) = X_{\mu_1}(23) = X_{\mu_1}(34) = X_{\mu_1}(45) = X_{\mu_1}(56) = [-1]$

3. The S_6–module S^{μ_2}

There are exactly five standard μ_2 − tableaux

$T_1 = \begin{array}{|c|c|}\hline 1 & 2\\ \hline 3\\ \cline{1-1} 4\\ \cline{1-1} 5\\ \cline{1-1} 6\\ \cline{1-1}\end{array}$ $T_2 = \begin{array}{|c|c|}\hline 1 & 3\\ \hline 2\\ \cline{1-1} 4\\ \cline{1-1} 5\\ \cline{1-1} 6\\ \cline{1-1}\end{array}$ $T_3 = \begin{array}{|c|c|}\hline 1 & 4\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} 5\\ \cline{1-1} 6\\ \cline{1-1}\end{array}$ $T_4 = \begin{array}{|c|c|}\hline 1 & 5\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} 4\\ \cline{1-1} 6\\ \cline{1-1}\end{array}$ $T_5 = \begin{array}{|c|c|}\hline 1 & 6\\ \hline 2\\ \cline{1-1} 3\\ \cline{1-1} 4\\ \cline{1-1} 5\\ \cline{1-1}\end{array}$

Thus, dim $S^{\mu_2} = 5$

Denoting the representation by X_{μ_2} ,therefore $X_{\mu_2}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}$

3.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1} - e_{T_2} + e_{T_3} - e_{T_4} + e_{T_5}$

By case 1, $(12)\ e_{T_2} = -e_{T_2}$, $(12)e_{T_3} = -e_{T_3}$, $(12)e_{T_4} = -e_{T_4}$ and

$(12)e_{T_5} = -e_{T_5}$. Therefore the matrix for (12) is $X_{\mu_2}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ -1 & -1 & 0 & 0 & 0\\ 1 & 0 & -1 & 0 & 0\\ -1 & 0 & 0 & -1 & 0\\ 1 & 0 & 0 & 0 & -1\end{bmatrix}$

3.2 Matrix for (23)

By case 3, $(23)e_{T_1} = e_{T_2}$ and $(23)e_{T_2} = e_{T_1}$

By case 1, $(23)e_{T_3} = -e_{T_3}$, $(23)e_{T_4} = -e_{T_4}$ and $(23)e_{T_5} = -e_{T_5}$

Therefore, the matrix for (23) is $X_{\mu_2}(23) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

3.3 Matrix for (34)

By case 1, $(34)e_{T_1} = -e_{T_1}$, $(34)e_{T_4} = -e_{T_4}$ and $(34)e_{T_5} = -e_{T_5}$

By case 3, $(34)e_{T_2} = e_{T_3}$ and $(34)e_{T_3} = e_{T_2}$

Therefore, the matrix for (34) is $X_{\mu_2}(34) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

3.4 Matrix for (45)

By case 1, $(45)e_{T_1} = -e_{T_1}$, $(45)e_{T_2} = -e_{T_2}$ and $(45)e_{T_5} = -e_{T_5}$

By case 3, $(45)e_{T_3} = e_{T_4}$ and $(45)e_{T_4} = e_{T_3}$

Therefore, the matrix for (45) is $X_{\mu_2}(45) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

3.5 Matrix for (56)

By case 1, $(56)e_{T_1} = -e_{T_1}$, $(56)e_{T_2} = -e_{T_2}$ and $(56)e_{T_3} = -e_{T_3}$

By case 3, $(56)e_{T_4} = e_{T_5}$ and $(56)e_{T_5} = e_{T_4}$

Therefore, the matrix for (56) is

$$X_{\mu_2}(56) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

4. The S_6–module S^{μ_3}

There are exactly nine standard $\mu_3 -$ tableaux

$$T_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array} \quad T_2 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array} \quad T_3 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 6 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array} \quad T_4 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array} \quad T_5 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$$

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 6 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array} \quad \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array} \quad \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 6 \\ \hline \\ \cline{1-1} 5 \\ \cline{1-1}\end{array} \quad \begin{array}{|c|c|}\hline 1 & 5 \\ \hline 2 & 6 \\ \hline 4 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array}$$

$T_6 =$ $T_7 =$ $T_8 =$ $T_9 =$

3

Thus, $\dim S^{\mu_3} = 9$
Denoting the representation by X_{μ_3} ,

Therefore , $$X_{\mu_3}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4.1 Matrix for (12)

By case 1, $(12)e_{T_4} = -e_{T_4}$, $(12)e_{T_5} = -e_{T_5}$, $(12)e_{T_6} = -e_{T_6}$
$(12)e_{T_7} = -e_{T_7}$, $(12)e_{T_8} = -e_{T_8}$, $(12)e_{T_9} = -e_{T_9}$
By case 2, $(12)e_{T_1} = e_{T_1} - e_{T_4}$, $(12)e_{T_2} = e_{T_2} - e_{T_5} + e_{T_7}$ and
$(12)e_{T_3} = e_{T_3} - e_{T_6} + e_{T_8} - e_{T_9}$
Therefore, the matrix for (12) is

$$X_{\mu_3}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

4.2 Matrix for (23)

By case 3, $(23)e_{T_1} = e_{T_4}$, $(23)e_{T_4} = e_{T_1}$, $(23)e_{T_2} = e_{T_5}$,
$(23)e_{T_2} = e_{T_5}$, $(23)e_{T_3} = e_{T_6}$, $(23)e_{T_6} = e_{T_3}$
By case 1, $(23)e_{T_7} = -e_{T_7}$, $(23)e_{T_8} = -e_{T_8}$, $(23)e_{T_9} = -e_{T_9}$

Therefore the matrix for (23) is

$$X_{\mu_3}(23) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

4.3 Matrix for (34)

By case 2, $(34)e_{T_1} = e_{T_1} - e_{T_2} + e_{T_3} - e_{T_4} - e_{T_5} + e_{T_7}$

By case 1, $(34)e_{T_2} = -e_{T_2}$, $(34)e_{T_3} = -e_{T_3}$, $(34)e_{T_4} = -e_{T_4}$,
$(34)e_{T_9} = -e_{T_9}$

By case 3, $(34)e_{T_5} = e_{T_7}$, $(34)e_{T_7} = e_{T_5}$, $(34)e_{T_6} = e_{T_8}$, $(34)e_{T_8} = e_{T_6}$

Therefore the matrix for (34) is

$$X_{\mu_3}(34) = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

4.4 Matrix for (45)

By case 3, $(45)e_{T_1} = e_{T_2}$, $(45)e_{T_2} = e_{T_1}$, $(45)e_{T_4} = e_{T_5}$
$(45)e_{T_5} = e_{T_4}$, $(45)e_{T_8} = e_{T_9}$, $(45)e_{T_9} = e_{T_8}$

By case 1, $(45)e_{T_3} = -e_{T_3}$, $(45)e_{T_6} = -e_{T_6}$, $(45)e_7 = -e_{T_7}$

Therefore the matrix for (45) is

$$X_{\mu_3}(45) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

4.5 Matrix for (56)

By case 1, $(56)e_{T_1} = -e_{T_1}$, $(56)e_{T_4} = -e_{T_4}$ and $(56)e_{T_9} = -e_{T_9}$

By case 3, $(56)e_{T_2} = e_{T_3}$, $(56)e_{T_3} = e_{T_2}$, $(56)e_{T_5} = e_{T_6}$,
$(56)e_{T_6} = e_{T_5}$, $(56)e_{T_7} = e_{T_8}$, $(56)e_{T_8} = e_{T_7}$

Therefore the matrix for (56) is $X_{\mu_3}(56) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

5. The S_6–module S^{μ_4}

There are exactly five standard $\mu_4 -$ tableaux

$T_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline\end{array}$ $T_2 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 & 6 \\ \hline\end{array}$ $T_3 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline 5 & 6 \\ \hline\end{array}$ $T_4 = \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 5 \\ \hline 4 & 6 \\ \hline\end{array}$ $T_5 = \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 & 6 \\ \hline\end{array}$

Thus, $\dim S^{\mu_4} = 5$

Denoting the representation by X_{μ_4} , therefore $X_{\mu_4}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

5.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1} - e_{T_3}$, $(12)e_{T_2} = e_{T_2} - e_{T_4} + e_{T_5}$
By case 1, $(12)e_{T_3} = -e_{T_3}$, $(12)e_{T_4} = -e_{T_4}$ and $(12)e_{T_5} = -e_{T_5}$

Therefore the matrix for (12) is

$$X_{\mu_4}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

5.2 Matrix for (23)

By case 3, $(23)e_{T_1} = e_{T_3}$, $(23)e_{T_3} = e_{T_1}$, $(23)e_{T_2} = e_{T_4}$, $(23)e_{T_4} = e_{T_2}$

By case 1, $(23)e_{T_5} = -e_{T_5}$

Therefore the matrix for (23) is $X_{\mu_4}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}$

5.3 Matrix for (34)

By case 2, $(34)e_{T_1} = e_{T_1} - e_{T_2} + e_{T_4}$
By case 1, $(34)e_{T_2} = -e_{T_2}$, $(34)e_{T_3} = -e_{T_3}$
By case 3, $(34)e_{T_4} = e_{T_5}$, $(23)e_{T_5} = e_{T_4}$

Therefore the matrix for (34) is $X_{\mu_4}(34) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

5.4 Matrix for (45)

By case 1, $(45)e_{T_5} = -e_{T_5}$

By case 3, $(45)e_{T_1} = e_{T_2}$, $(45)e_{T_2} = e_{T_1}$, $(45)e_{T_3} = e_{T_4}$, $(45)e_{T_4} = e_{T_3}$

Therefore the matrix for (45) is $X_{\mu_4}(45) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

5.5 Matrix for (56)

By case 2, $(56)e_{T_1} = e_{T_1} - e_{T_2}$
$(56)e_{T_3} = e_{T_3} - e_{T_4}$

By case 1, $(56)e_{T_2} = -e_{T_2}$, $(56)e_{T_4} = -e_{T_4}$, $(56)e_{T_5} = -e_{T_5}$

Therefore the matrix for (56) is $X_{\mu_4}(56) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

6. The S_6–module S^{μ_5}

There are exactly ten standard $\mu_5 -$ tableaux

$T_1 = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$ $T_2 = \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$ $T_3 = \begin{array}{|c|c|c|} \hline 1 & 2 & 5 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$ $T_4 = \begin{array}{|c|c|c|} \hline 1 & 2 & 6 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1} 5 \\ \cline{1-1} \end{array}$ $T_5 = \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1} 5 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$

$T_6 = \begin{array}{|c|c|c|} \hline 1 & 3 & 5 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$ $T_7 = \begin{array}{|c|c|c|} \hline 1 & 3 & 6 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1} 5 \\ \cline{1-1} \end{array}$ $T_8 = \begin{array}{|c|c|c|} \hline 1 & 4 & 5 \\ \hline 2 \\ \cline{1-1} 6 \\ \cline{1-1} \end{array}$ $T_9 = \begin{array}{|c|c|c|} \hline 1 & 4 & 6 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} 5 \\ \cline{1-1} \end{array}$ $T_{10} = \begin{array}{|c|c|c|} \hline 1 & 5 & 6 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array}$

Thus, $\dim S^{\mu_5} = 10$

Denoting the representation by X_{μ_5} . Therefor

$$\boldsymbol{X_{\mu_5}(I)} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

6.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$,$(12)e_{T_2} = e_{T_2} - e_{T_5}$,

$(12)e_{T_3} = e_{T_3} - e_{T_6} + e_{T_8}$,$(12)e_{T_4} = e_{T_4} - e_{T_7} + e_{T_9} - e_{T_{10}}$

By case 1, $(12)e_{T_5} = -e_{T_5}$, $(12)e_{T_6} = -e_{T_6}$, $(12)e_{T_7} = -e_{T_7}$

$(12)e_{T_8} = -e_{T_8}$, $(12)e_{T_9} = -e_{T_9}$, $(12)e_{T_{10}} = -e_{T_{10}}$

Therefore the matrix for (12) is

$$\boldsymbol{X_{\mu_5}(12)} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&-1&0&0&-1&0&0&0&0&0\\ 0&0&-1&0&0&-1&0&0&0&0\\ 0&0&0&-1&0&0&-1&0&0&0\\ 0&0&1&0&0&0&0&-1&0&0\\ 0&0&0&1&0&0&0&0&-1&0\\ 0&0&0&-1&0&0&0&0&0&-1 \end{bmatrix}$$

6.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}$

By case 3, $(23)e_{T_2} = e_{T_5}$, $(23)e_{T_5} = e_{T_2}$, $(23)e_{T_3} = e_6$,

$(23)e_{T_6} = e_{T_3}$, $(23)e_{T_4} = e_{T_7}$, $(23)e_{T_7} = e_{T_4}$

By case 1, $(23)e_{T_8} = -e_{T_8}$, $(23)e_{T_9} = -e_{T_9}$, $(23)e_{T_{10}} = -e_{T_{10}}$

Therefore the matrix for (23) is $\boldsymbol{X_{\mu_5}(23)} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&0&0&-1 \end{bmatrix}$

6.3 Matrix for (34)

By case 2, $(34)e_{T_5} = e_{T_5}$

By case 3, $(34)e_{T_1} = e_{T_2}$, $(34)e_{T_2} = e_{T_1}$, $(34)e_{T_6} = e_{T_8}$,

$(34)e_{T_8} = e_{T_6}$, $(34)e_{T_7} = e_{T_9}$, $(34)e_{T_9} = e_{T_7}$

By case 1, $(34)e_{T_3} = -e_{T_3}$, $(34)e_{T_4} = -e_{T_4}$, $(34)e_{T_{10}} = -e_{T_{10}}$

Therefore, the matrix for (34) is $X_{\mu_5}(\mathbf{34}) = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} \end{bmatrix}$

6.4 Matrix for (45)

By case 2, $(45)e_{T_8} = e_{T_8}$

By case 3, $(45)e_{T_2} = e_{T_3}$, $(45)e_{T_3} = e_{T_2}$, $(45)e_{T_5} = e_{T_6}$,

$(45)e_{T_6} = e_{T_5}$, $(45)e_{T_9} = e_{T_{10}}$, $(45)e_{T_{10}} = e_{T_9}$

By case 1, $(45)e_{T_1} = -e_{T_1}$, $(45)e_{T_4} = -e_{T_4}$, $(45)e_{T_7} = -e_{T_7}$

Therefore the matrix for (45) is

$$X_{\mu_5}(\mathbf{45}) = \begin{bmatrix} -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

6.5 Matrix for (56)

By case 2, $(56)e_{T_{10}} = e_{T_{10}}$

By case 3, $(56)e_{T_3} = e_{T_4}$, $(56)e_{T_4} = e_{T_3}$, $(56)e_{T_6} = e_{T_7}$,

$(56)e_{T_7} = e_{T_6}$, $(56)e_{T_8} = e_{T_9}$, $(56)e_{T_9} = e_{T_8}$

By case 1, $(56)e_{T_1} = -e_{T_1}$, $(56)e_{T_2} = -e_{T_2}$, $(56)e_{T_5} = -e_{T_5}$

Therefore the matrix for (56) is $X_{\mu_5}(56) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

7. The S_6–module S^{μ_6}

There are exactly sixteen standard $\mu_6 -$ tableaux

$T_1 = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array}$ $T_2 = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 4 & 6 \\ \cline{1-2} 5 \\ \cline{1-1} \end{array}$ $T_3 = \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array}$ $T_4 = \begin{array}{|c|c|c|} \hline 1 & 2 & 4 \\ \hline 3 & 6 \\ \cline{1-2} 5 \\ \cline{1-1} \end{array}$ $T_5 = \begin{array}{|c|c|c|} \hline 1 & 2 & 5 \\ \hline 3 & 4 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array}$

$T_6 = \begin{array}{|c|c|c|} \hline 1 & 2 & 5 \\ \hline 3 & 6 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array}$ $T_7 = \begin{array}{|c|c|c|} \hline 1 & 2 & 6 \\ \hline 3 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} \end{array}$ $T_8 = \begin{array}{|c|c|c|} \hline 1 & 2 & 6 \\ \hline 3 & 5 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array}$ $T_9 = \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 & 5 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array}$ $T_{10} = \begin{array}{|c|c|c|} \hline 1 & 3 & 4 \\ \hline 2 & 6 \\ \cline{1-2} 5 \\ \cline{1-1} \end{array}$

$T_{11} = \begin{array}{|c|c|c|} \hline 1 & 3 & 5 \\ \hline 2 & 4 \\ \cline{1-2} 6 \\ \cline{1-1} \end{array}$ $T_{12} = \begin{array}{|c|c|c|} \hline 1 & 3 & 5 \\ \hline 2 & 6 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array}$ $T_{13} = \begin{array}{|c|c|c|} \hline 1 & 3 & 6 \\ \hline 2 & 4 \\ \cline{1-2} 5 \\ \cline{1-1} \end{array}$ $T_{14} = \begin{array}{|c|c|c|} \hline 1 & 3 & 6 \\ \hline 2 & 5 \\ \cline{1-2} 4 \\ \cline{1-1} \end{array}$

$T_{15} = \begin{array}{|c|c|c|} \hline 1 & 4 & 5 \\ \hline 2 & 6 \\ \cline{1-2} 3 \\ \cline{1-1} \end{array}$ $T_{16} = \begin{array}{|c|c|c|} \hline 1 & 4 & 6 \\ \hline 2 & 5 \\ \cline{1-2} 3 \\ \cline{1-1} \end{array}$

Thus, $\dim S^{\mu_6} = 16$

Denoting the representation by X_{μ_6} ,therefor $X_{\mu_6}(I) = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$

7.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$, $(12)e_{T_2} = e_{T_2}$, $(12)e_{T_3} = e_{T_3} - e_{T_9}$

$(12)e_{T_4} = e_{T_4} - e_{T_{10}}$, $(12)e_{T_5} = e_{T_5} - e_{T_{11}}$

$(12)e_{T_6} = e_{T_6} - e_{T_{12}} + e_{T_{15}}$,$(12)e_{T_7} = e_{T_7} - e_{T_{13}}$

$(12)e_{T_8} = e_{T_8} - e_{T_{14}} + e_{T_{16}}$

By case 1, $(12)e_{T_9} = -e_{T_9}$, $(12)e_{T_{10}} = -e_{T_{10}}$, $(12)e_{T_{11}} = -e_{T_{11}}$
$(12)e_{T_{12}} = -e_{T_{12}}$, $(12)e_{T_{13}} = -e_{T_{13}}$, $(12)e_{T_{14}} = -e_{T_{14}}$
$(12)e_{T_{15}} = -e_{T_{15}}$, $(12)e_{16} = -e_{T_{16}}$

Therefore the matrix for (12) is

$$X_{\mu_6}(\mathbf{12}) = \begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&1&-1&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&-1
\end{bmatrix}$$

7.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}$, $(23)e_{T_2} = e_{T_2}$
By case 3, $(23)e_{T_3} = e_{T_9}$, $(23)e_{T_9} = e_{T_3}$, $(23)e_{T_4} = e_{T_{10}}$, $(23)e_{T_{10}} = e_{T_4}$
$(23)e_{T_5} = e_{T_{11}}$, $(23)e_{T_{11}} = e_{T_5}$, $(23)e_{T_6} = e_{T_{12}}$,
$(23)e_{T_7} = e_{T_{13}}$ $(23)e_{T_8} = e_{T_{14}}$
By case 1, $(23)e_{T_{15}} = -e_{T_{15}}$, $(23)e_{T_{16}} = -e_{T_{16}}$
Therefore the matrix for (23) is

$$X_{\mu_6}(\mathbf{23}) = \begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&-1
\end{bmatrix}$$

7.3 Matrix for (34)

By case 3, $(34)e_{T_1} = e_{T_3}$, $(34)e_{T_3} = e_{T_1}$, $(34)e_{T_2} = e_{T_4}$, $(34)e_{T_4} = e_{T_2}$
$(34)e_{T_{12}} = e_{T_{15}}$, $(34)e_{T_{15}} = e_{T_{12}}$, $(34)e_{T_{14}} = e_{T_{16}}$, $(34)e_{T_{16}} = e_{T_{14}}$
By case 1, $(34)e_{T_6} = -e_{T_6}$, $(34)e_{T_8} = -e_{T_8}$, $(34)e_{T_{11}} = -e_{T_{11}}$
$(34)e_{T_{13}} = -e_{T_{13}}$
By case 2, $(34)e_{T_5} = e_{T_5} - e_{T_{11}} + e_{T_{12}}$, $(34)e_{T_7} = e_{T_7} - e_{T_{13}} + e_{T_{16}}$
$(34)e_{T_9} = e_{T_9} - e_{T_{11}}$, $(34)e_{T_{10}} = e_{T_{10}} - e_{T_{13}}$
Therefore the matrix for (34) is

$$X_{\mu_6}(34) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

7.4 Matrix for (45)

By case 2, $(45)e_{T_1} = e_{T_1} - e_{T_2}$, $(45)e_{T_{15}} = e_{T_{15}} - e_{T_{16}}$
By case 3, $(45)e_{T_3} = e_{T_5}$, $(45)e_{T_5} = e_{T_3}$, $(45)e_{T_4} = e_{T_6}$, $(45)e_{T_6} = e_{T_4}$
$(45)e_{T_7} = e_{T_8}$, $(45)e_{T_8} = e_{T_7}$, $(45)e_{T_9} = e_{T_{11}}$, $(45)e_{T_{11}} = e_{T_9}$
$(45)e_{T_{10}} = e_{T_{12}}$, $(45)e_{T_{12}} = e_{T_{10}}$, $(45)e_{T_{13}} = e_{T_{14}}$,
$(45)e_{T_{14}} = e_{T_{13}}$
By case 1, $(45)e_{T_2} = -e_{T_2}$, $(45)e_{T_{16}} = -e_{T_{16}}$

Therefore the matrix for (45) is

$$X_{\mu_6}(45) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

7.5 Matrix for (56)

By case 3, $(56)e_{T_1} = e_{T_2}$, $(56)e_{T_2} = e_{T_1}$, $(56)e_{T_3} = e_{T_4}$,
$(56)e_{T_4} = e_{T_3}$, $(56)e_{T_5} = e_{T_7}$, $(56)e_{T_7} = e_{T_5}$, $(56)e_{T_6} = e_{T_8}$
$(56)e_8 = e_{T_6}$, $(56)e_{T_9} = e_{T_{10}}$, $(56)e_{T_{10}} = e_{T_9}$, $(56)e_{T_{11}} = e_{T_{13}}$
$(56)e_{T_{13}} = e_{T_{11}}$, $(56)e_{T_{12}} = e_{T_{14}}$, $(56)e_{T_{14}} = e_{T_{12}}$
$(56)e_{T_{16}} = e_{T_{15}}$, $(56)e_{T_{15}} = e_{T_{16}}$

Therefore the matrix for (56) is $X_{\mu_6}(56) = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}$

8. The S_6–module S^{μ_7}

There are exactly five standard $\mu_7 -$ tableaux

$T_1 = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}$ $T_2 = \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}$ $T_3 = \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}$ $T_4 = \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}$ $T_5 = \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}$

Thus, 5

Denoting the representation by X_{μ_7} ,therefore

$$X_{\mu_7}(I) = \begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$$

8.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$, $(12)e_{T_2} = e_{T_2} - e_{T_4}$, $(12)e_{T_3} = e_{T_3} - e_{T_5}$

By case 1, $(12)e_{T_4} = -e_{T_4}$, $(12)e_{T_5} = -e_{T_5}$

Therefore the matrix for (12) is $X_{\mu_7}(12) = \begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&-1&0&-1&0 \\ 0&0&-1&0&-1 \end{bmatrix}$

8.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}$

By case 3, $(23)e_{T_2} = e_{T_4}$, $(23)e_{T_4} = e_{T_2}$, $(23)e_{T_3} = e_{T_5}$, $(23)e_{T_5} = e_{T_3}$

Therefore the matrix for (23) is $X_{\mu_7}(23) = \begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \end{bmatrix}$

8.3 Matrix for (34)

By case 3, $(34)e_{T_1} = e_{T_2}$, $(34)e_{T_2} = e_{T_1}$

By case 2, $(34)e_{T_3} = e_{T_3} - e_{T_5}$, $(34)e_{T_4} = e_{T_4} - e_{T_5}$
By case 1, $(34)e_{T_5} = -e_{T_5}$

Therefore the matrix for (34) is $X_{\mu_7}(34) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix}$

8.4 Matrix for (45)

By case 2, $(45)e_{T_1} = e_{T_1}$
By case 3, $(45)e_{T_2} = e_{T_3}$, $(45)e_{T_3} = e_{T_2}$, $(45)e_{T_4} = e_{T_5}$, $(45)e_{T_5} = e_{T_4}$

Therefore the matrix for (45) is $X_{\mu_7}(45) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

8.5 Matrix for (56)

By case 2, $(56)e_{T_1} = e_{T_1}$, $(56)e_{T_2} = e_{T_2} - e_{T_3}$, $(56)e_{T_4} = e_{T_4} - e_{T_5}$

By case 1, $(56)e_{T_3} = -e_{T_3}$, $(56)e_{T_5} = -e_{T_5}$

Therefore the matrix for (56) is $X_{\mu_7}(56) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

9.TheS_6- moduleS^{μ_8}

There are exactly ten standard μ_8 − tableaux

$T_1 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline 5 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$ $T_2 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 4 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$ $T_3 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 6 \\ \hline 4 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array}$ $T_4 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 4 & 5 \\ \hline 3 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$ $T_5 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 4 & 6 \\ \hline 3 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array}$

$T_6 = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 5 & 6 \\ \hline 4 \\ \cline{1-1}\end{array}$ $T_7 = \begin{array}{|c|c|c|c|}\hline 1 & 3 & 4 & 5 \\ \hline 2 \\ \cline{1-1} 6 \\ \cline{1-1}\end{array}$ $T_8 = \begin{array}{|c|c|c|c|}\hline 1 & 3 & 4 & 6 \\ \hline 2 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array}$ $T_9 = \begin{array}{|c|c|c|c|}\hline 1 & 3 & 5 & 6 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}$ $T_{10} = \begin{array}{|c|c|c|c|}\hline 1 & 4 & 5 & 6 \\ \hline 3 \\ \cline{1-1}\end{array}$

Thus, $\dim S^{\mu_8} = 10$

Denoting the representation by X_{μ_8} ,therefore $X_{\mu_8}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

9.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$, , $(12)e_{T_2} = e_{T_2}$, $(12)e_{T_3} = e_{T_3}$

$(12)e_{T_4} = e_{T_4} - e_{T_7}$, $(12)e_{T_5} = e_{T_5} - e_{T_8}$, $(12)e_{T_6} = e_{T_6} - e_{T_9} + e_{T_{10}}$

By case 1, $(12)e_{T_7} = -e_{T_7}$, $(12)e_{T_8} = -e_{T_8}$, $(12)e_{T_9} = -e_{T_9}$

$(12)e_{T_{10}} = -e_{T_{10}}$

Therefore the matrix for (12) is $X_{\mu_8}(\mathbf{12}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$

9.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}$, $(23)e_{T_2} = e_{T_2}$, $(23)e_{T_3} = e_{T_3}$

By case 3, $(23)e_{T_4} = e_{T_7}$, $(23)e_{T_7} = e_{T_4}$, $(23)e_{T_5} = e_{T_8}$,

$(23)e_{T_8} = e_{T_5}$, $(23)e_{T_6} = e_{T_9}$, $(23)e_{T_9} = e_{T_6}$

By case 1, $(23)e_{T_{10}} = -e_{T_{10}}$ Therefore the matrix for (23) is

$$X_{\mu_8}(\mathbf{23}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

9.3 Matrix for (34)

By case 2, $(34)e_{T_1} = e_{T_1}$, $(34)e_{T_7} = e_{T_7}$, $(34)e_{T_8} = e_{T_8}$

By case 3, $(34)e_{T_2} = e_{T_4}$, $(34)e_{T_4} = e_{T_2}$, $(34)e_{T_3} = e_{T_5}$,

$(34)e_{T_5} = e_{T_3}$, $(34)e_{T_9} = e_{T_{10}}$, $(34)e_{10} = e_{T_9}$

By case 1, $(34)e_{T_6} = -e_{T_6}$. Therefore the matrix for (34) is

$$X_{\mu_8}(\mathbf{34}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

9.4 Matrix for (45)

By case 2, $(45)e_{T_4} = e_{T_4}$, $(45)e_{T_7} = e_{T_7}$, $(45)e_{T_8} = e_{T_8}$, $(45)e_{T_{10}} = e_{T_{10}}$

By case 3, $(45)e_{T_1} = e_{T_2}$, $(45)e_{T_2} = e_{T_1}$, $(45)e_{T_5} = e_{T_6}$,

$(45)e_{T_6} = e_{T_5}$, $(45)e_{T_8} = e_{T_9}$, $(45)e_9 = e_{T_8}$

By case 1, $(45)e_{T_3} = -e_{T_3}$. Therefore the matrix for (45) is

$$X_{\mu_8}(45) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

9.5 Matrix for (56)

By case 2, $(56)e_{T_6} = e_{T_6}$, $(56)e_{T_9} = e_{T_9}$, $(56)e_{T_{10}} = e_{T_{10}}$

By case 3, $(56)e_{T_2} = e_{T_3}$, $(56)e_{T_3} = e_{T_2}$, $(56)e_{T_4} = e_{T_5}$,

$(56)e_{T_5} = e_{T_4}$, $(56)e_{T_7} = e_{T_8}$, $(56)e_8 = e_{T_7}$

By case 1, $(56)e_{T_1} = -e_{T_1}$. Therefore the matrix for (56) is

$$X_{\mu_8}(56) = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

10. The S_6–module S^{μ_9}

There are exactly nine standard $\mu_9 -$ tableau

$T_1 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 4 \\ \hline 5 & 6 \\ \cline{1-2} \end{array}$ $T_2 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 \\ \cline{1-2} \end{array}$ $T_3 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 6 \\ \hline 4 & 5 \\ \cline{1-2} \end{array}$ $T_4 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 4 & 5 \\ \hline 3 & 6 \\ \cline{1-2} \end{array}$ $T_5 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 4 & 6 \\ \hline 3 & 5 \\ \cline{1-2} \end{array}$

$T_6 = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 5 & 6 \\ \hline 3 & 4 \\ \cline{1-2} \end{array}$ $T_7 = \begin{array}{|c|c|c|c|} \hline 1 & 3 & 4 & 5 \\ \hline 2 & 6 \\ \cline{1-2} \end{array}$ $T_8 = \begin{array}{|c|c|c|c|} \hline 1 & 3 & 4 & 6 \\ \hline 2 & 5 \\ \cline{1-2} \end{array}$ $T_9 = \begin{array}{|c|c|c|c|} \hline 1 & 3 & 5 & 6 \\ \hline 2 & 4 \\ \cline{1-2} \end{array}$

Thus, $\dim S^{\mu_9} = 9$

Denoting the representation by X_{μ_9} ,therefore $X_{\mu_9}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

10.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$, $(12)e_{T_2} = e_{T_2}$, $(12)e_{T_3} = e_{T_3}$,

$(12)e_{T_4} = e_{T_4} - e_{T_7}$, $(12)e_{T_5} = e_{T_5} - e_{T_8}$, $(12)e_{T_6} = e_{T_6} - e_{T_9}$

By case 1, $(12)e_{T_7} = -e_{T_7}$, $(12)e_{T_8} = -e_{T_8}$, $(12)e_{T_9} = -e_{T_9}$

Therefore the matrix for (12) is

$$X_{\mu_9}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

10.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}, (23)e_{T_2} = e_{T_2}, (23)e_{T_3} = e_{T_3} - e_{T_6}$

By case 3, $(23)e_{T_4} = e_{T_7}$, $(23)e_{T_7} = e_{T_4}$, $(23)e_{T_5} = e_{T_8}$,

$(23)e_{T_8} = e_{T_5}$, $(23)e_{T_6} = e_{T_9}$, $(23)e_{T_9} = e_{T_6}$

Therefore the matrix for (23) is

$$X_{\mu_9}(23) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

10.3 Matrix for (34)

By case 2, $(34)e_{T_1} = e_{T_1}, (34)e_{T_6} = e_{T_6} - e_{T_9}, (34)e_{T_7} = e_{T_7},$

$(34)e_{T_8} = e_{T_8} - e_{T_9}$

By case 3, $(34)e_{T_2} = e_{T_4}$, $(34)e_{T_4} = e_{T_2}$, $(34)e_{T_3} = e_{T_5}$, $(34)e_{T_5} = e_{T_3}$

By case 1, $(34)e_{T_9} = -e_{T_9}$

Therefore the matrix for (34) is

$$X_{\mu_9}(34) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix}$$

10.4 Matrix for (45)

By case 2, $(45)e_{T_3} = e_{T_3}$, $(45)e_{T_4} = e_{T_4}, (45)e_{T_7} = e_{T_7}$

By case 3, $(45)e_{T_1} = e_{T_2}, (45)e_{T_2} = e_{T_1}, (45)e_{T_5} = e_{T_6}, (45)e_{T_6} = e_{T_5}$

$(45)e_{T_8} = e_{T_9}, (45)e_{T_9} = e_{T_8}$

Therefore the matrix for (45) is

$$X_{\mu_9}(45) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

10.5 Matrix for (56)

By case 2, $(56)e_{T_1} = e_{T_1}$, $(56)e_{T_6} = e_{T_6}$, $(56)e_{T_9} = e_{T_9}$
By case 3, $(56)e_{T_2} = e_{T_3}$, $(56)e_{T_3} = e_{T_2}$, $(56)e_{T_4} = e_{T_5}$,
$(56)e_{T_5} = e_{T_4}$, $(56)e_{T_7} = e_{T_8}$, $(56)e_{T_8} = e_{T_7}$
Therefore the matrix for (56) is

$$X_{\mu_9}(56) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

11. The S_6–module $S^{\mu_{10}}$

There are exactly five standard μ_{10} − tableaux

$$T_1 = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 3 & 4 & 5 \\ \hline 6 \\ \cline{1-1} \end{array} \quad T_2 = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 3 & 4 & 6 \\ \hline 5 \\ \cline{1-1} \end{array} \quad T_3 = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 3 & 5 & 6 \\ \hline 4 \\ \cline{1-1} \end{array}$$

$$T_4 = \begin{array}{|c|c|c|c|c|} \hline 1 & 2 & 4 & 5 & 6 \\ \hline 3 \\ \cline{1-1} \end{array} \quad T_5 = \begin{array}{|c|c|c|c|c|} \hline 1 & 3 & 4 & 5 & 6 \\ \hline 2 \\ \cline{1-1} \end{array}$$

Thus, $\dim S^{\mu_{10}} = 5$

Denoting the representation by $X_{\mu_{10}}$, therefore $X_{\mu_{10}}(I) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

11.1 Matrix for (12)

By case 2, $(12)e_{T_1} = e_{T_1}$,$(12)e_{T_2} = e_{T_2}$,$(12)e_{T_3} = e_{T_3}$,$(12)e_{T_4} = e_{T_4} - e_{T_5}$ by case 1, $(12)e_{T_5} = -e_{T_5}$

Therefore the matrix for (12) is $X_{\mu_{10}}(12) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

11.2 Matrix for (23)

By case 2, $(23)e_{T_1} = e_{T_1}$,$(23)e_2 = e_2$,$(23)e_{T_3} = e_{T_3}$

By case 3, $(23)e_{T_4} = e_{T_5}$,$(23)e_{T_5} = e_4$

Therefore the matrix for (23) is $X_{\mu_{10}}(23) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

11.3 Matrix for (34)

By case 3, $(34)e_{T_3} = e_{T_4}$, $(34)e_{T_4} = e_{T_3}$

By case 2, $(34)e_{T_1} = e_{T_1}$, $(34)e_{T_2} = e_{T_2}$, $(34)e_{T_5} = e_{T_5}$

Therefore the matrix for (34) is $X_{\mu_{10}}(34) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

11.4 Matrix for (45)

By case 2, $(45)e_{T_1} = e_{T_1}$,$(45)e_{T_4} = e_{T_4}$,$(45)e_{T_5} = e_{T_5}$

By case 3, $(45)e_{T_2} = e_{T_3}$, $(45)e_{T_3} = e_{T_2}$

Therefore the matrix for (45) is $X_{\mu_{10}}(45) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

11.5 Matrix for (56)

By case 2, $(56)e_{T_3} = e_{T_3}$,$(56)e_{T_4} = e_{T_4}$, $(56)e_{T_5} = e_{T_5}$
By case 3, $(56)e_{T_1} = e_{T_2}$, $(56)e_{T_2} = e_{T_1}$

Therefore the matrix for (56) is $X_{\mu_{10}}(56) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

12. The S_6–module $S^{\mu_{11}}$

There is exactly one standard $\mu_{11} -$ tableau

$T =$

1	2	3	4	5	6

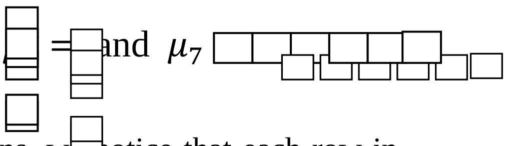
Thus, dim $S^{\mu_{11}}$ =1.

Denoting the representation by $X_{\mu_{11}}$,therefore $X_{\mu_{11}}(I) = [1]$

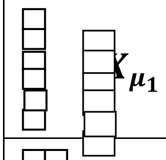
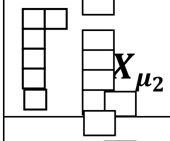
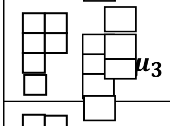
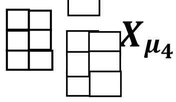
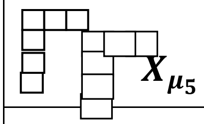
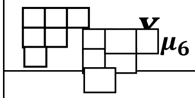
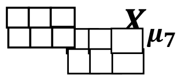
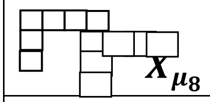
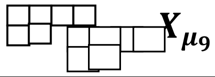
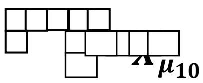
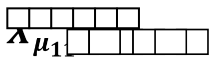
12.1 Matrix for (12), (23), (34) ,(45) and (56)

In every case , we have case 2 :
$(12)e_T = e_T, (23)e_T = e_T, (34)e_T = e_T, (45)e_T = e_T$ and $(56)e_T = e_T$
Therefore the matrices are: $X_{\mu_{11}}(12) = [1]$, $X_{\mu_{11}}(23) = [1]$,$X_{\mu_{11}}(34) = [1]$, $X_{\mu_{11}}(45) = [1]$ and $X_{\mu_{11}}(56) = [1]$.

13. Conclusion

We found that the representation corresponding to  = and μ_7 are the sign and trivial representations, respectively.

To obtain the characters from Young's standard representations, we notice that each row in the table completely characterized by its values in the first and second columns, which correspond to the character on the identity and on an adjacent transposition. Using the lexicographic order on the columns then, we have a standard presentation of the character table of S_6.

Class	1^6	$2\,1^4$	$3\ 1^3$	$4\ 1^2$	$2^2\,1^2$	$3\,2\,1$	$5\,1$	6	$4\,2$	2^3	3^2
Order	1	15	40	90	45	120	144	120	90	15	40
 χ_{μ_1}	**1**	**-1**	**1**	**-1**	**1**	**-1**	**1**	**-1**	**1**	**-1**	**1**
 χ_{μ_2}	**5**	**-3**	**2**	**-1**	**1**	**0**	**0**	**1**	**-1**	**1**	**-1**
 χ_{μ_3}	**9**	**-3**	**0**	**1**	**1**	**0**	**-1**	**0**	**1**	**-3**	**0**
 χ_{μ_4}	**5**	**-1**	**-1**	**1**	**1**	**-1**	**0**	**0**	**-1**	**3**	**2**
 χ_{μ_5}	**10**	**-2**	**1**	**0**	**-2**	**1**	**0**	**-1**	**0**	**2**	**1**
 χ_{μ_6}	**16**	**0**	**-2**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**-2**
 χ_{μ_7}	**5**	**1**	**-1**	**-1**	**1**	**1**	**0**	**0**	**-1**	**-3**	**2**
 χ_{μ_8}	**10**	**2**	**1**	**0**	**-2**	**-1**	**0**	**1**	**0**	**-2**	**1**
 χ_{μ_9}	**9**	**3**	**0**	**-1**	**1**	**0**	**-1**	**0**	**1**	**3**	**0**
 $\chi_{\mu_{10}}$	**5**	**3**	**2**	**1**	**1**	**0**	**0**	**-1**	**-1**	**-1**	**-1**
 $\chi_{\mu_{11}}$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

14.References

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