Iraqi Journal of Computers, Communications, Control & Systems Engineering (IJCCCE), Vol. 23, No. 1, March 2023

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

Spectral And Energy Efficiency Analysis For Beamforming Optimization Objectives For Wireless Massive MIMO Systems

Zaid Hashim Jaber[1], Dheyaa Jasim Kadhim[2], Ahmed Sabah Al-Araji[3]
[1]*Department of Computer Science, College of Science, University of Baghdad, Baghdad, Iraq*
[2]*Department of Electrical Engineering, College of Engineering, University of Baghdad, Baghdad, Iraq*
[3]*Department of Computer Engineering, University of Technology, Baghdad, Iraq*
[1]*ce.19.18@grad.uotechnology.edu.iq*, [2]*dheyaa@coeng.uobaghdad.edu.iq*,
[3]*ahmed.s.alaraji@uotechnology.edu.iq*

Abstract— *Massive Multiple-Input Multiple-Output (MIMO) is an extension of the conventional MIMO in the wireless systems which improves both of the access density and the spectral efficiency by adding a massive number of antenna array at the base station (BS). Massive MIMO increases the spectral efficiency by using the beamforming. Besides, the beamforming in massive MIMO improves the energy efficiency by focusing the energy in the desired direction instead of the omnidirectional propagation. In this paper, we propose and discuss different beamforming objectives in both the uplink and the downlink channels. These proposed objectives can be either use the beamforming of the desired signal without nulling the interference or use the beamforming with interference nulling. The beamforming with nulling objectives have better performance than those without nulling but this leads to a higher computational complexity as well. The results of this paper show and compare the performance of these objective including the spectral efficiency and energy efficiency as well as the computational complexity.*

Index Terms— *Beamforming, Interefernce nulling, Energy efficiency, Massive MIMO, Spectral efficiency.*

I. INTRODUCTION

Massive Multiple-Input Multiple-Output (MIMO) is one of the main technologies proposed for 5G and beyond to allow multiple users' access and higher density cells [1]-[3]. It is the ultimate version of MIMO [4]-[6] technology where the number of antenna array at the base station (BS) is much larger than the number of user equipments (UEs) [7]-[9]. Hundreds or thousands antennas can serve tens or hundreds UEs using the same time-frequency resources [7]-[9]. Beamforming in massive MIMO improves both the spectral and energy efficiencies as follows [7], [10]-[12]:

- Spectral efficiency is improved by transmitting and receiving multiple different independent beams from multiple UEs using the same time-frequency resources. Spectral efficiency can be increased by increasing both the multiplexing gain and beamforming gain.
- Energy efficiency is improved by focusing a narrow beam toward a desired UE instead of omnidirectional signal. Besides, it can be improved even more by nulling the signals from other UEs. The energy efficiency can be increased by

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

reducing the transmitted power while keeping the same amount of spectral efficiency.

Other benefits of massive MIMO are [7], [13]-[15]:

- Channel estimation is done using Time Division Duplexing (TDD) operation. Because the uplink and the downlink channels are reciprocal, only the pilot sequences from the UEs to the BS are required for channel estimation process. The BS does not need to send any pilot signals to the UEs.
- Channel hardening because of the large number of the BS antennas. The effect of fading of the channel becomes much smaller which improves the quality of the link.
- All the channel estimation and signal processing calculations are done at the BS without any calculations complexity at the UEs.
- The process of beamforming is linear and has nearly optimal results.

Some related papers [12], [16]-[18] classified massive MIMO beamforming to either analog beamforming, digital beamforming or hybrid beamforming which is a combination of both of the analog and the digital beamforming. Digital beamforming using the signal precoding is considered in this work.

The contribution of this paper is to propose different optimization objectives for massive MIMO in both the uplink and the downlink propagation and compare them with respect to their spectral efficiency and energy efficiency. Some of these objectives include beamforming the desired signal without nulling the interferences while other objectives include both the signal beamforming and the interferences nulling. The objectives that include beamforming without nulling have much simpler processing computations but less performance than the objectives that include both beamforming and nulling. As a future work, these optimized beamforming techniques can be enhanced so they can be envolved in the wireless cooperative communication systems [19] and the decentralized and distributed massive MIMO networks [20].

The outline of this paper is as follows, The mathematical model of both uplink and downlink of the massive MIMO systems is introduced in section II. Different beamforming optimization objectives for massive MIMO uplink and downlink propagations are proposed in section III. The results are discussed in section IV, we compare the performance of these different objectives in the results. This paper is concluded in section V.

II. SYSTEM MODEL

Massive MIMO consists of the BS which has M antenna units and K UEs such that $M \gg K$. Each UE is equipped with one antenna. If the channel gain vector of the k^{th} UE (UE_k) is $\boldsymbol{g}_k \in \mathbb{C}^{M\times 1}$ and it is independent and identically distributed (i.i.d.) $\sim CN(0, \boldsymbol{I}_M)$. And the channel gain matrix for all UEs is $\boldsymbol{G} \in \mathbb{C}^{M\times K}$. Then the uplink transmission from UE_k to the BS can be represented as in equation (1) [21].

$$\boldsymbol{y}_{ul,k} = \sqrt{p_{ul}}\boldsymbol{g}_k x_{ul,k} + \boldsymbol{n}_{ul} \tag{1}$$

And the uplink transmission from all UEs to the BS can be represented as in equation (2) [21].

$$\boldsymbol{y}_{ul} = \sqrt{p_{ul}}\sum_{k=1}^{K}\boldsymbol{g}_k x_{ul,k} + \boldsymbol{n}_{ul} = \sqrt{p_{ul}}\boldsymbol{G}\boldsymbol{x}_{ul} + \boldsymbol{n}_{ul} \tag{2}$$

where $\boldsymbol{y}_{ul,k} \in \mathbb{C}^{M\times 1}$ is the received signal vector by the BS from the UE_k and $\boldsymbol{y}_{ul} \in \mathbb{C}^{M\times 1}$ is the received signal vector by the BS from all UEs, p_{ul} is the average uplink signal-to-noise

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

ratio (SNR), $x_{ul,k}$ is the transmitted signal by the UE$_k$ where $\mathbb{E}\{|x_k|^2\} = 1$ and $\boldsymbol{x}_{ul} \in \mathbb{C}^{K\times 1} = \left[x_{ul,1} \dots x_{ul,K}\right]^T$, $\boldsymbol{n}_{ul} \in \mathbb{C}^{M\times 1}$ is the additive noise vector and it is i.i.d. $\sim CN(0, \boldsymbol{I}_M)$.

On the other hand, the downlink transmission from the BS to the UE$_k$ can be represented as in equation 3 [21].

$$y_{dl,k} = \sqrt{p_{dl}}\boldsymbol{g}_k^T \boldsymbol{x}_{dl} + n_{dl,k} \tag{3}$$

And the downlink transmission from the BS to all UEs can be represented as in equation (4).

$$\boldsymbol{y}_{dl} = \sqrt{p_{dl}}\boldsymbol{G}^T \boldsymbol{x}_{dl} + \boldsymbol{n}_{dl} \tag{4}$$

where $y_{dl,k}$ is the received signal by the UE$_k$ from the BS and $\boldsymbol{y}_{dl} \in \mathbb{C}^{M\times 1} = \left[y_{dl,1} \dots y_{dl,K}\right]^T$ is the received vector by all UEs from the BS, p_{dl} is the average downlink SNR, $\boldsymbol{x}_{dl} \in \mathbb{C}^{M\times 1}$ is the transmitted vector by the BS where $\mathbb{E}\{\|\boldsymbol{x}_{dl}\|^2\} = 1$. $n_{dl,k}$ is the additive noise and it is i.i.d. $\sim CN(0,1)$ and $\boldsymbol{n}_{dl} \in \mathbb{C}^{M\times 1} = \left[n_{dl,1} \dots n_{dl,K}\right]^T$.

III. MASSIVE MIMO BEMFORMING OPTIMIZATION

A. Channel Estimation

The channel conditions must be estimated accuratly before data transmission in both uplink and downlink channel [22]. Since the uplink and downlink channels are reciprocal, the channel estimation can be done using TDD operation. Thus, only the UEs need to send their pilot sequence to the BS to estimate the channel gain and all the calculations can be done in the BS without any calculations complexity needed at the UEs [7], [13]. The pilot signals from different UEs must be orthogonal to avoid interference. The received pilot sequences by BS from K UEs can be represented as in equation (5).

$$\boldsymbol{Y}_p = \sqrt{\tau_p p_u}\sum_{k=1}^{K} \boldsymbol{g}_k \boldsymbol{\Phi}_k^H + \boldsymbol{N} \tag{5}$$

where $\boldsymbol{\Phi}_k \in \mathbb{C}^{\tau_p \times 1}$ is the pilot sequence from the UE$_k$, $\boldsymbol{Y}_{\boldsymbol{p}} \in \mathbb{C}^{M\times \tau_p}$ is the received orthogonal pilot sequences from all UEs such that $\tau_p \geq K$, $\boldsymbol{N} \in \mathbb{C}^{M\times \tau_p}$ is the additive noise.

Since the pilot sequence of different UEs are orthogonal, then $\boldsymbol{\Phi}_k^H \boldsymbol{\Phi}_k = 1$ and $\boldsymbol{\Phi}_k^H \boldsymbol{\Phi}_j = 0$ for $k \neq j$. For that, the recieved pilot sequence from UE$_k$ is represented as in equation (6).

$$\boldsymbol{y}_{p,k} = \boldsymbol{Y}_p \boldsymbol{\Phi}_k = \sqrt{\tau_p p_u}\boldsymbol{g}_k + \boldsymbol{N}\boldsymbol{\Phi}_k \tag{6}$$

From equation (6) and by using the minimum mean square error (MMSE), the actual channel gain can be obtained as in equation (7).

$$\boldsymbol{g}_k = \sqrt{\rho}\,\widehat{\boldsymbol{g}}_k + \sqrt{1-\rho}\,\widetilde{\boldsymbol{g}}_k \tag{7}$$

where $\rho = \frac{\tau_p p_u}{\tau_p p_u + 1}$, $\widehat{\boldsymbol{g}}_k \sim CN(0, \rho \boldsymbol{I}_M)$ is the estimated channel gain and $\widetilde{\boldsymbol{g}}_k \sim CN0,(1-\rho)\boldsymbol{I}_M$ is the estimated channel error.

From equation 7, the accurate channel estimation can be achieved by either increasing the uplink SNR p_u or increasing the size of pilot sequence τ_p. For that, lower uplink SNR needs larger pilot sequence size. Perfect channel estimation is considered in this work.

B. Uplink Beamforming

The uplink signal that is received by the BS from the UE$_k$ with beamforming can be represented as in equations (8) and (9).

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

$$y_{ul,k} = \boldsymbol{w}_{ul,k}^{H}\left(\sqrt{p_{ul}}\boldsymbol{G}\boldsymbol{x}_{ul} + \boldsymbol{n}_{ul}\right) \tag{8}$$

$$y_{ul,k} = \underbrace{\sqrt{p_{ul}}\boldsymbol{w}_{ul,k}^{H}\boldsymbol{g}_{k}x_{ul,k}}_{desired\ signal} + \underbrace{\sqrt{p_{ul}}\boldsymbol{w}_{ul,k}^{H}\sum_{k\prime\neq k}^{K}\boldsymbol{g}_{k\prime}x_{ul,k\prime}}_{interference} + \underbrace{\boldsymbol{w}_{ul,k}^{H}\boldsymbol{n}_{ul}}_{noise} \tag{9}$$

where $\boldsymbol{w}_{ul,k} \in \mathbb{C}^{M\times 1}$ is the uplink weight vector for the UE_k.

Various objectives are proposed to optimize the value of $\boldsymbol{w}_{ul,k}$. The first proposed beamforming objective case (J_1) given in equation (10) is to maximize the uplink SNR of the uplink signal from UE_k (or minimize the noise power) while maintaining a unity gain for the this signal.

$$J_1:\ \boldsymbol{w}_{ul,k} = \underset{\boldsymbol{w}_{ul,k}\in\mathbb{C}^{M\times 1}}{argmin}\left\|\boldsymbol{w}_{ul,k}\right\|^2\ \ subject\ to\ \ \boldsymbol{w}_{ul,k}^{H}\boldsymbol{g}_{k} = 1 \tag{10}$$

The Lagrange multiplier function for the objective (J_1) in equation (10) is shown in equation (11).

$$f(\boldsymbol{w}_{ul,k},\lambda) = \left\|\boldsymbol{w}_{ul,k}\right\|^2 + \lambda(1 - \boldsymbol{w}_{ul,k}^{H}\boldsymbol{g}_{k}) \tag{11}$$

By setting the gradient of equation (8) $\nabla_{\boldsymbol{w}_{ul,k}} f\left(\boldsymbol{w}_{ul,k},\lambda\right)$ to zero, the optimized weight vector is obtained in equation (12).

$$J_1:\ \boldsymbol{w}_{ul,k} = \frac{\boldsymbol{g}_{k}}{\|\boldsymbol{g}_{k}\|^2} \tag{12}$$

The interferences from other UEs are not considered in the first proposed objective case (J_1). To minimize the effect of both the interference and the noise, we must first obtain the combined noise and interference vector in equation (13).

$$\widetilde{\boldsymbol{n}}_{ul,k} = \underbrace{\sqrt{p_{ul}}\boldsymbol{w}_{ul,k}^{H}\sum_{k\prime\neq k}^{K}\boldsymbol{g}_{k\prime}x_{ul,k\prime}}_{interference} + \underbrace{\boldsymbol{w}_{ul,k}^{H}\boldsymbol{n}_{ul}}_{noise} \tag{13}$$

Then, the second proposed objective case (J_2) is to minimize the combined noise and interference power while maintaining a unity gain of the desired signal as in equation (14).

$$J_2:\ \boldsymbol{w}_{ul,k} = \underset{\boldsymbol{w}_{ul,k}\in\mathbb{C}^{M\times 1}}{argmin}\ \mathbb{E}\{\left|\boldsymbol{w}_{ul,k}^{H}\widetilde{\boldsymbol{n}}_{ul,k}\right|^2\}\ \ subject\ to\ \ \boldsymbol{w}_{ul,k}^{H}\boldsymbol{g}_{k} = 1 \tag{14}$$

Using the Langrage multiplier function in equation (15) to obtain the optimized weight vector of the objective equation (14).

$$f(\boldsymbol{w}_{ul,k},\lambda) = \mathbb{E}\{\left|\boldsymbol{w}_{ul,k}^{H}\widetilde{\boldsymbol{n}}_{ul,k}\right|^2\} + \lambda(1 - \boldsymbol{w}_{ul,k}^{H}\boldsymbol{g}_{k}) \tag{15}$$

To Find the optimal weight vector for the objective case (J_2), the conditions of uncorrelated different signals and noise components in equations (16), (17) and (18) must be satisfied.

$$Condition\ 1:\ \ \mathbb{E}\left\{x_{ul,i}x_{ul,j}^{*}\right\} = \begin{cases} p_{ul} & if\ i = j \\ 0 & if\ i \neq j \end{cases} \tag{16}$$

$$Condition\ 2:\ \ \mathbb{E}\left\{n_{ul,i}n_{ul,j}^{*}\right\} = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases} \tag{17}$$

$$Condition\ 3:\ \ \mathbb{E}\left\{x_{ul,i}n_{ul,j}^{*}\right\} = \mathbb{E}\left\{n_{ul,i}x_{ul,j}^{*}\right\} = 0\quad \forall i,j \tag{18}$$

With the conditions stated in equations (16), (17) and (18), the expected power matrix of the different interference plus noise components is given in equation (19).

$$\widetilde{\boldsymbol{N}}_{ul,k} = \mathbb{E}\left\{\widetilde{\boldsymbol{n}}_{ul,k}\widetilde{\boldsymbol{n}}_{ul,k}^{H}\right\} = p_{ul}\sum_{k\prime\neq k}^{K}\boldsymbol{g}_{k\prime}\boldsymbol{g}_{k\prime}^{H} + \boldsymbol{I}_{M} \tag{19}$$

Received 12/March/2022; Accepted 22/May/2022

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

By setting the gradient of equation (15) $\nabla_{\boldsymbol{w}_{ul,k}} f\left(\boldsymbol{w}_{ul,k}, \lambda\right)$ to zero, the optimized weight vector is obtained in equation (20).

$$J_2\!: \ \boldsymbol{w}_{ul,k} = \frac{(\widetilde{\boldsymbol{N}}_{ul,k})^{-1}\boldsymbol{g}_k}{\boldsymbol{g}_k^H(\widetilde{\boldsymbol{N}}_{ul,k})^{-1}\boldsymbol{g}_k} \tag{20}$$

The third proposed objective case (J_3) is to minimize the noise power while maintaining the unity gain signal with zero gain (nulling) the interference. The objective function is given by equation (21).

$$J_3\!: \ \boldsymbol{w}_{ul,k} = \underset{\boldsymbol{w}_{ul,k} \in \mathbb{C}^{M\times 1}}{argmin} \left\|\boldsymbol{w}_{u,k}\right\|^2 \ \ subject\ to \ \ \boldsymbol{w}_{ul,k}^H \boldsymbol{g}_{k\prime} = \begin{cases} 1 & if\ k' \in K, k' = k \\ 0 & \forall k' \in K, \ \ k' \neq k \end{cases} \tag{21}$$

The Lagrange function for the objective case (J_3) is shown in equation (22).

$$f(\boldsymbol{w}_{ul,k}, \boldsymbol{\lambda}) = \left\|\boldsymbol{w}_{u,k}\right\|^2 + (\boldsymbol{\gamma}^T - \boldsymbol{w}_{ul,k}^H \boldsymbol{G})\boldsymbol{\lambda} \tag{22}$$

where $\boldsymbol{\lambda} = [\lambda_1 \ ... \ \lambda_K]^T$ is the Lagrange multiplier vector and $\boldsymbol{\gamma} = \left[\gamma_1 \ ... \ \gamma_K\right]^T$ is the gain constraint vector given in equation (23).

$$\gamma_{k\prime} = \begin{cases} 1 & if\ k' = k \\ 0 & if\ k' \neq k \end{cases} \tag{23}$$

By setting the gradient of equation (19) $\nabla_{\boldsymbol{w}_{ul,k}} f\left(\boldsymbol{w}_{ul,k}, \lambda\right)$ to zero, the optimized weight vector is obtained in equation (24).

$$J_3\!: \ \boldsymbol{w}_{ul,k} = \boldsymbol{G}(\boldsymbol{G}^H\boldsymbol{G})^{-1}\boldsymbol{\gamma} \tag{24}$$

Also, weight vector can be optimized by tuning between the minimum square error (MSE) for accuracy and the minimum noise power. Both of them can be used in one objective case (J_4) as given in equation (25).

$$J_4\!: \ \boldsymbol{w}_{ul,k} = \underset{\boldsymbol{w}_{ul,k} \in \mathbb{C}^{M\times 1}}{argmin} \left(\underbrace{\left\|\boldsymbol{w}_{ul,k}^H \boldsymbol{G} - \boldsymbol{\gamma}^T\right\|^2}_{Min.\ square\ error} + \alpha \underbrace{\left\|\boldsymbol{w}_{ul,k}\right\|^2}_{Min.\ noise} \right) \tag{25}$$

where α is the tuning factor between minimum square error and minimum noise power.

The optimized weight vector for objective case (J_4) when setting $\nabla_{\boldsymbol{w}_{ul,k}} f\left(\boldsymbol{w}_{ul,k}, \lambda\right)$ to zero is shown in equation (26).

$$J_4\!: \ \boldsymbol{w}_{ul,k} = (\boldsymbol{G}\boldsymbol{G}^H + \alpha \boldsymbol{I}_M)^{-1}\boldsymbol{G}\boldsymbol{\gamma} = \boldsymbol{G}(\boldsymbol{G}^H\boldsymbol{G} + \alpha\boldsymbol{I}_K)^{-1}\boldsymbol{\gamma} \tag{26}$$

The overall uplink weight matrix for all UEs and for all the objectives mentioned previously (J_1-J_4) can be written as $\boldsymbol{W}_{ul} \in \mathbb{C}^{M\times K} = \left[\boldsymbol{w}_{ul,1} \ ... \ \boldsymbol{w}_{ul,K}\right]$.

Finally, to avoid the matrix inversion calculations used in objectives J_2, J_3 and J_4, the overall optimized uplink weight matrix can be calculated using the gradient descent method. The weight matrix in each iteration is given in equation (27).

$$J_5\!: \ \boldsymbol{W}_{ul}^{i+1} = \boldsymbol{W}_{ul}^{i} - \eta\left[(\boldsymbol{G}^H\boldsymbol{G})\boldsymbol{W}_{ul}^{i} - \boldsymbol{G}^H\right] \ \ subject\ to \ \ \left\|\boldsymbol{w}_{ul,k}\right\|^2 \leq 1 \ \ \forall k \in K \tag{27}$$

where η is the step size for each iteration.

This method must be subjected to the noise power constraint ($\left\|\boldsymbol{w}_{ul,k}\right\|^2 \leq 1$) for all UEs and in each iteration. So, the weight matrix can be modified in each iteration as in equation (28).

Received 12/March/2022; Accepted 22/May/2022

ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

$$\boldsymbol{W}_{ul}^{i}=\begin{cases}\boldsymbol{W}_{ul}^{i} & if\ \forall\left\|\boldsymbol{w}_{ul,k}^{i}\right\|^{2}\leq 1,\ \ \forall k\in K\\ \boldsymbol{W}_{ul}^{i}/max(\left\|\boldsymbol{w}_{ul,k}^{i}\right\|^{2}) & if\ \exists\left\|\boldsymbol{w}_{ul,k}^{i}\right\|^{2}>1,\ \ \forall k\in K\end{cases} \tag{28}$$

C. Downlink Beamforming

For the download transmission, the beamforming can be achieved by sending the downlink beamforming vector shown in equation (29) from the BS to all UEs.

$$\boldsymbol{x}_{dl}=\boldsymbol{D}_{q}^{1/2}\boldsymbol{W}_{dl}\boldsymbol{x}_{dl}' \tag{29}$$

where $\boldsymbol{x}_{dl}\in\mathbb{C}^{M\times 1}$ is the transmitted vector by the BS such that $\mathbb{E}\{\|\boldsymbol{x}_{dl}\|^{2}\}=1$, $\boldsymbol{x'}_{dl}\in\mathbb{C}^{K\times 1}=\left[x_{dl,1}\ldots x_{dl,K}\right]^{T}$ is the original signal vector transmitted by the BS to the UEs such that $\mathbb{E}\{\left|x_{dl,k}\right|\}=1\ \ \forall k\in K$, $\boldsymbol{W}_{dl}\in\mathbb{C}^{M\times K}=[\boldsymbol{w}_{dl,1}\ldots\boldsymbol{w}_{dl,K}]$ is the beamforming downlink weight matrix of all UEs and $\boldsymbol{w}_{dl,k}\in\mathbb{C}^{M\times 1}$ is the downlink weight vector for the UE$_k$, $\boldsymbol{D}_{q}$ is a diagonal matrix whose diagonal elements $(q_{1}\ldots q_{K})$ is the downlink power constraint coefficients for the UEs. The downlink power constraint coefficients must be subjected to the constraints given in equations (30) and (31).

$$Constraint\ 1: q_{k}\geq 0 \tag{30}$$

$$Constraint\ 2: \sum_{k=1}^{K}q_{k}\leq K/tr(\boldsymbol{W}_{dl}\boldsymbol{W}_{dl}^{H}) \tag{31}$$

By substituting equation (29) in equation (3), the downlink received signal by UE$_k$ is given by equations (32) and (33).

$$y_{dl,k}=\sqrt{p_{dl}}\boldsymbol{g}_{k}^{T}\left(\boldsymbol{D}_{q}^{1/2}\boldsymbol{W}_{dl}\boldsymbol{x}_{dl}'\right)+n_{dl,k} \tag{32}$$

$$y_{dl,k}=\underbrace{\sqrt{p_{dl}\cdot q_{k}}\boldsymbol{g}_{k}^{T}\boldsymbol{w}_{dl,k}x'_{dl,k}}_{desired\ signal}+\underbrace{\sqrt{p_{dl}}\boldsymbol{g}_{k}^{T}\sum_{k'\neq k}^{K}\sqrt{q_{k'}}\boldsymbol{w}_{dl,k'}x'_{dl,k'}}_{interference}+\underbrace{n_{dl,k}}_{noise} \tag{33}$$

The first downlink objective case (J_6) is to maximize the desired signal power without exceeding the power constraint i.e. the downlink weight vector must be a unity vector. The objective function of objective case (J_6) is then written as given in equation (34).

$$J_{6}:\ \boldsymbol{w}_{dl,k}=\underset{\boldsymbol{w}_{dl,k}\in\mathbb{C}^{M\times 1}}{argmax}\left|\boldsymbol{g}_{k}^{T}\boldsymbol{w}_{dl,k}\right|^{2}\ \ subject\ to\ \ \left\|\boldsymbol{w}_{dl,k}\right\|^{2}\leq 1 \tag{34}$$

The Lagrange function for the objective case (J_6) is shown in equation (35).

$$f(\boldsymbol{w}_{dl,k},\lambda)=\left|\boldsymbol{g}_{k}^{T}\boldsymbol{w}_{dl,k}\right|^{2}+\lambda\left(1-\left\|\boldsymbol{w}_{dl,k}\right\|^{2}\right) \tag{35}$$

By setting the gradient of equation (32) $\nabla_{\boldsymbol{w}_{ul,k}}f\left(\boldsymbol{w}_{ul,k},\lambda\right)$ to zero, we get equation (36).

$$(\boldsymbol{g}_{k}^{*}\boldsymbol{g}_{k}^{T})\boldsymbol{w}_{dl,k}=\lambda\boldsymbol{w}_{dl,k} \tag{36}$$

Since the objective case (J_6) is a maximization optimization and both the objective and constraint functions are convex and the matrix $\left(\boldsymbol{g}_{k}^{*}\boldsymbol{g}_{k}^{T}\right)$ is a positive semidefinite matrix, then λ is the maximum eigenvalue of $\left(\boldsymbol{g}_{k}^{*}\boldsymbol{g}_{k}^{T}\right)$ and $\boldsymbol{w}_{dl,k}$ is its corresponding unit eigenvector. The eigenvalues of $\left(\boldsymbol{g}_{k}^{*}\boldsymbol{g}_{k}^{T}\right)$ are obtained by solving equation (37).

$$det\left|\boldsymbol{g}_{k}^{*}\boldsymbol{g}_{k}^{T}-\lambda\boldsymbol{I}_{M}\right|=0 \tag{37}$$

The solution of equation (37) is obtained by equation (38).

Received 12/March/2022; Accepted 22/May/2022

ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

$$(\|\boldsymbol{g}_k\|^2 - \lambda)\lambda^{M-1} = 0 \tag{38}$$

From equation (34), the maximum eigenvalue (λ_{max}) is $\|\boldsymbol{g}_k\|^2$ and its corresponding eigenvector is the optimal downlink weight vector which is given in equation (39).

$$J_6{:}\ \ \boldsymbol{w}_{dl,k} = \frac{\boldsymbol{g}_k^*}{\|\boldsymbol{g}_k\|} \tag{39}$$

Another objective case (J_7) is to minimize the mean square error while nulling the interference from other UEs. This objective function is shown in equation (40).

$$J_7{:}\ \ \boldsymbol{w'}_{dl,k} = \underset{\boldsymbol{w}_{ul,k}\in\mathbb{C}^{M\times 1}}{argmin} \left\|\boldsymbol{G}_k^T\boldsymbol{w'}_{dl,k} - \boldsymbol{\gamma}\right\|^2 \tag{40}$$

where $\boldsymbol{\gamma}$ is the gain vector and it is given in equation (23).

By setting the gradient of equation (40) $\nabla_{\boldsymbol{w}_{ul,k}} f(\boldsymbol{w}_{ul,k}, \lambda)$ to zero, the optimized downlink weight vector (denormalized) is obtained in equation (41).

$$J_7{:}\ \ \boldsymbol{w'}_{dl,k}(denormalized) = (\boldsymbol{G}^*\boldsymbol{G}^T)^{-1}\boldsymbol{G}^*\boldsymbol{\gamma} = \boldsymbol{G}^*(\boldsymbol{G}^T\boldsymbol{G}^*)^{-1}\boldsymbol{\gamma} \tag{41}$$

To avoid near singular matrix inversion, the optimal downlink beamforming vector can be modified by adding tuning factor β in the new objective case (J_8) as shown in equation (42).

$$J_8{:}\ \ \boldsymbol{w'}_{dl,k}(denormalized) = (\boldsymbol{G}^*\boldsymbol{G}^T + \beta\boldsymbol{I}_M)^{-1}\boldsymbol{G}^*\boldsymbol{\gamma} = \boldsymbol{G}^*(\boldsymbol{G}^T\boldsymbol{G}^* + \beta\boldsymbol{I}_K)^{-1}\boldsymbol{\gamma} \tag{42}$$

The optimal weight vectors of objectives (J_7) and (J_8) in equations (41) and (42) must be normalized to a unity vector because of the downlink power constraints as given in equation (43).

$$\boldsymbol{w}_{dl,k}(normalized) = \frac{\boldsymbol{w'}_{dl,k}(denormalized)}{\|\boldsymbol{w'}_{dl,k}(denormalized)\|} \tag{43}$$

The overall downlink weight matrix for all UEs for all objectives mentioned previously is $\boldsymbol{W}_{dl}\in\mathbb{C}^{M\times K} = [\boldsymbol{w}_{dl,1}\ \dots\ \boldsymbol{w}_{dl,K}]$.

Finally, the uplink weight matrix that uses the gradient descent method in equation (27) as well as other uplink objectives can be used to obtain the normalized downlink weight matrix as given in equation (44).

$$\boldsymbol{w}_{dl,k} = \frac{\boldsymbol{w}_{ul,k}^*}{\|\boldsymbol{w}_{ul,k}\|} \tag{44}$$

The duality relationship between the uplink and downlink beamforming weight vector in equation (41) can be used to obtain the downlink weight vectors from the uplink weight vectors and vice versa. This reduces the computation complexity of the weight vectors calculations of uplink and downlink transmission independently.

IV. RESULTS AND DISCUSSION

A. Spectral Efficiency

According to Shannon theory, the spectral efficiency of the uplink transmission is given by equation (45) [23].

$$SE_{ul} = log_2\, det(\boldsymbol{I}_k + P_u\boldsymbol{G}^H\boldsymbol{G}) \tag{45}$$

Similarly, the spectral efficiency of the downlink transmission is given by equation (46) [23].

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

$$SE_{dl} = \max_{\substack{q_k \geq 0 \\ \sum_{k=1}^{K} q_k \leq 1}} \; log_2\, det(\boldsymbol{I}_M + P_d \boldsymbol{G}^* \boldsymbol{D}_q \boldsymbol{G}^T) \tag{46}$$

For equally downlink power for all UEs, the downlink spectral efficiency in equation (46) can be rewritten as in equation (47).

$$SE_{dl} = log_2\, det\left(\boldsymbol{I}_M + \frac{P_d}{K}\boldsymbol{G}^*\boldsymbol{G}^T\right) = log_2\, det\left(\boldsymbol{I}_K + \frac{P_d}{K}\boldsymbol{G}^T\boldsymbol{G}^*\right) \tag{47}$$

Since the massive MIMO has a large number of antenna units (M) in the BS. This lead to two important advantages. The first one is the channel hardening [14] which can be represented by equation (48). The second one is the favorable propagation [14] which is given by equation (49).

$$\lim_{M \to \infty} \frac{\|\boldsymbol{g}_k\|^2}{M} = 1 \tag{48}$$

$$\lim_{M \to \infty} \frac{\boldsymbol{g}_k^H \boldsymbol{g}_{k'}}{M} = 0 \quad for\ k \neq k' \tag{49}$$

The channel hardening and the favorable propagation cause the ideal spectral efficiency for both the uplink and the downlink transmission. The ideal spectral efficiency for the uplink transmission is shown in equation (50) and the ideal spectral efficiency for the downlink transmission is shown in equation (51).

$$SE_{ul,ideal} = K\, log_2(1 + MP_u) \tag{50}$$

$$SE_{dl,ideal} = K\, log_2(1 + \frac{M}{K}P_d) \tag{51}$$

For the ideal uplink and downlink spectral efficiency in equations (50) and (51) respectively, massive MIMO improves the spectral efficiency by increasing the multiplexing gain (K) linearly and the antenna beamforming gain (M) logarithmically.

To find the actual spectral efficiency of both the uplink and the downlink transmissions using the beamforming objectives mentioned in the previous section, the signal-to-interference and noise ratio (SINR) for each UE must be calculated first. The uplink SINR of the UE_k is given by equation (52) while the downlink SINR of the UE_k is given by equation (53).

$$SINR_{ul,k} = \frac{p_u |\boldsymbol{w}_{ul,k}^H \boldsymbol{g}_k|^2}{p_u \sum_{k' \neq k}^{K} |\boldsymbol{w}_{ul,k}^H \boldsymbol{g}_{k'}|^2 + \|\boldsymbol{w}_{ul,k}^H\|^2} \tag{52}$$

$$SINR_{dl,k} = \frac{q_k p_d |\boldsymbol{g}_k^T \boldsymbol{w}_{dl,k}|^2}{p_d \sum_{k' \neq k}^{K} q_{k'} |\boldsymbol{g}_k^T \boldsymbol{w}_{dl,k'}|^2 + 1} \tag{53}$$

Then the overall uplink spectral efficiency is shown in equation (54) and the the overall downlink spectral efficiency is shown in equation (55).

$$SE_{ul} = \sum_{k=1}^{K} log_2(1 + SNIR_{ul,k}) \tag{54}$$

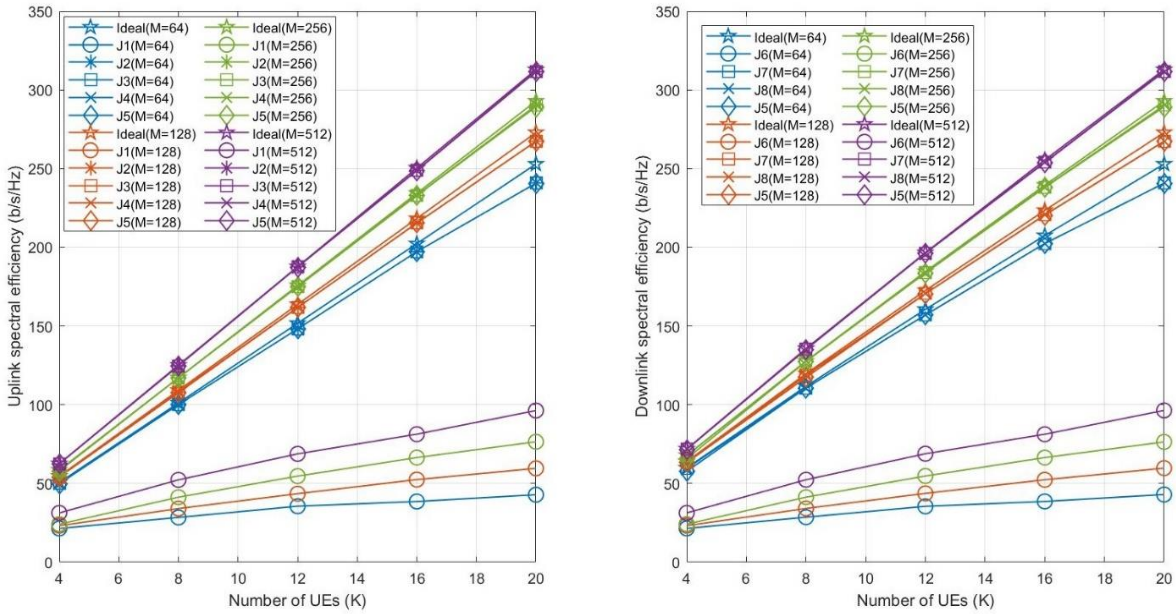
$$SE_{dl} = \sum_{k=1}^{K} log_2(1 + SNIR_{dl,k}) \tag{55}$$

Fig. 1 shows the uplink spectral efficiency using $p_u = 20\, dB$ and the downlink spectral efficiency using $p_d = 33\, dB$ for the different optimization objectives and various number of antenna units at the BS (*M* = 64, 128, 256, 512). The spectral efficiency of the most objectives (beamforming with nulling) become closer to the ideal spectral efficiency when increasing M. The optimization objectives that have high spectral efficiency (beamforming with nulling) also have higher computational complexity than the objectives

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

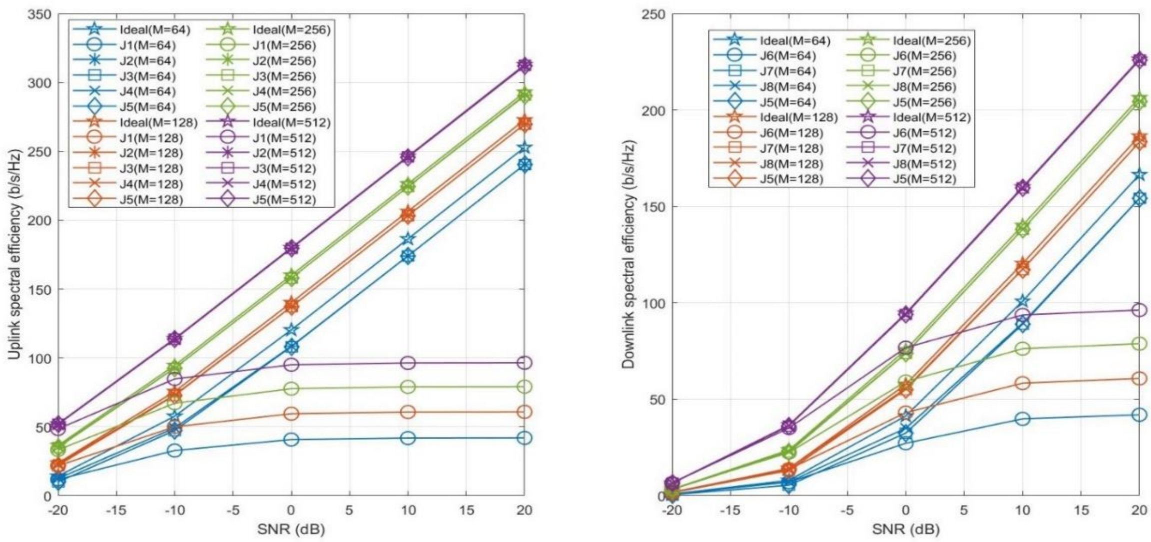
with low spectral efficiency (beamforming without nulling). So, there is a performance-complexity tradeoff.



(A) (B)

FIG.1. SPECTRAL EFFICIENCY (A) UPLINK TRANSMISSION (B) DOWNLINK TRANSMISSION.

Fig. 2 shows that the massive MIMO improves the spectral efficiency when increasing the number of the BS antenna units even with low SNR because of the beamforming gain.



(A) (B)

FIG.2. SPECTRAL EFFICIENCY VERSUS SNR (K = 20) (A) UPLINK TRANSMISSION (B) DOWNLINK TRANSMISSION.

B. Energy Efficiency

The energy efficiency is the number of the transmitted data (in bits) per the consumed energy (in Joule). The energy efficiency of the uplink transmission is shown in equation (56). Similarly, The energy efficiency of the uplink transmission is shown in equation (57).

$$EE_{ul} = \frac{B \cdot SE_{ul}}{P_{circuit} + \sum_{k=1}^{K} P_{ul,k} + (M+K)P_{ant}} = \frac{B \cdot SE_{ul}}{P_{circuit} + KP_{ul} + (M+K)P_{ant}} \tag{56}$$

Received 12/March/2022; Accepted 22/May/2022
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

$$EE_{dl} = \frac{B \cdot SE_{dl}}{P_{circuit} + \sum_{k=1}^{K} q_k P_{dl,k} + (M+K) P_{ant}} = \frac{B \cdot SE_{ul}}{P_{circuit} + P_{dl} + (M+K) P_{ant}} \quad (57)$$

where B is the channel bandwidth, $P_{circuit}$ is the fixed circuit power of the BS, P_{ant} is the circuit power per antenna per UE, P_{ul} is the power of a single UE during the uplink transmission, P_{dl} is the power of the BS during the downlink transmission.

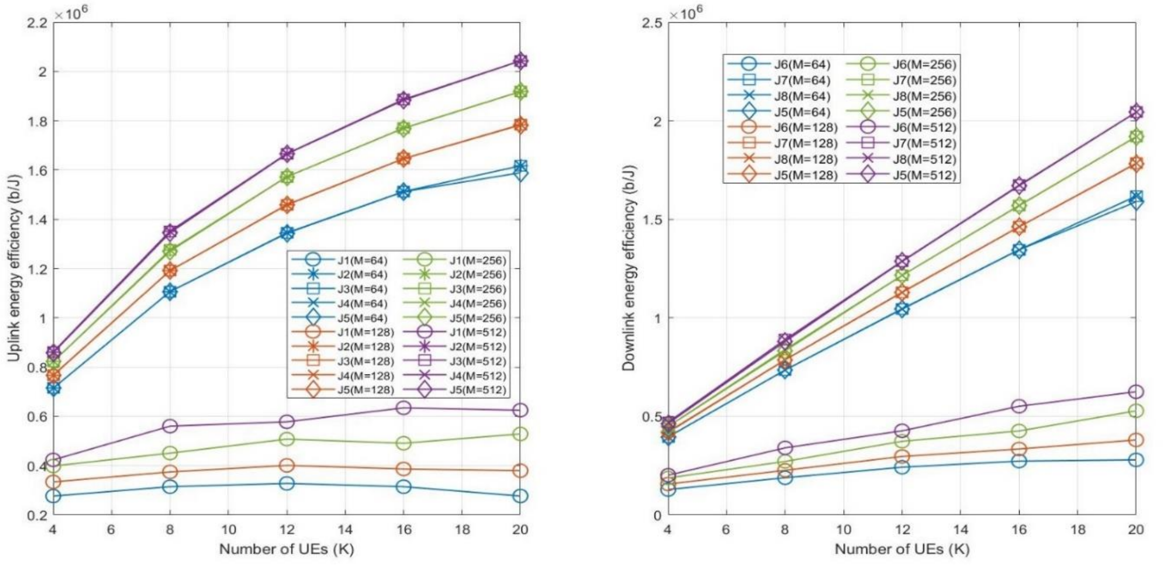
P_{ul} and P_{dl} can be obtained from equations (58) and (59).

$$P_{ul} = p_{ul} B N_o \quad (58)$$

$$P_{dl} = p_{dl} B N_o \quad (59)$$

where $N_o/2$ is the noise spectral density.

Fig. 3 shows the uplink and downlink energy efficiency using the parameters shown in Table I. Massive MIMO improves the uplink energy efficiency because of beamforming gain. Also, it improves the downlink energy efficiency by beamforming the signal toward the desired UE.



(A) (B)

FIG. 3. ENERGY EFFICIENCY (A) UPLINK TRANSMISSION (B) DOWNLINK TRANSMISSION.

TABLE I. ENERGY EFFICIENCY PARAMETERS

Parameter	Value
B	20 MHz
$P_{circuit}$	30 dBm
P_{ant}	-10 dBm
p_{ul}	20 dB
p_{dl}	33 dB
BN_o	0 dBm

C. Computational Complexity

Different uplink and downlink optimization objectives mentioned previously have different computation complexity. In general, objectives that include beamforming with interference nulling have better spectral and energy efficiency but higher computation complexity than of those which include beamforming without interference nulling.

Received 12/March/2022; Accepted 22/May/2022

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

Fig. 4 shows the complexity for different beamforming objectives. The objectives that have better spectral and energy efficiency have higher computational complexity.

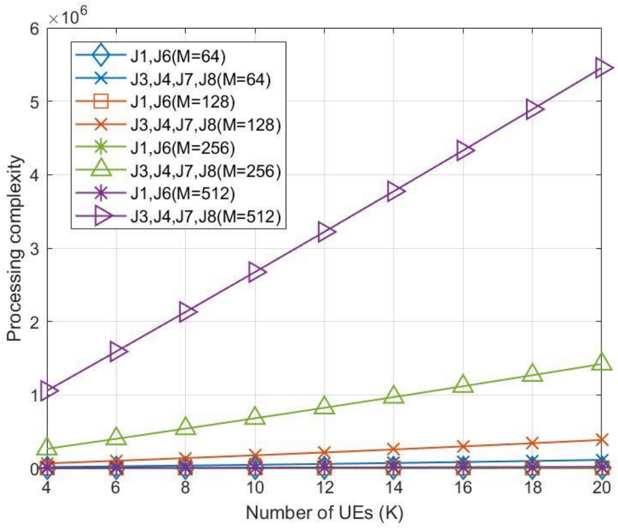


FIG. 4. COMPUTATIONAL COMPLEXITY

For the gradient descent method (J_5), the computation complexity depends on the number of the iterations that are required to find the optimal weight matrix. If $M \gg K$, the number of iterations decreases. *Fig. 5* shows the number of iterations versus the number of the UEs for different number of M. the number of iterations becomes smaller when the number of M increases which is the case of the massive MIMO system.

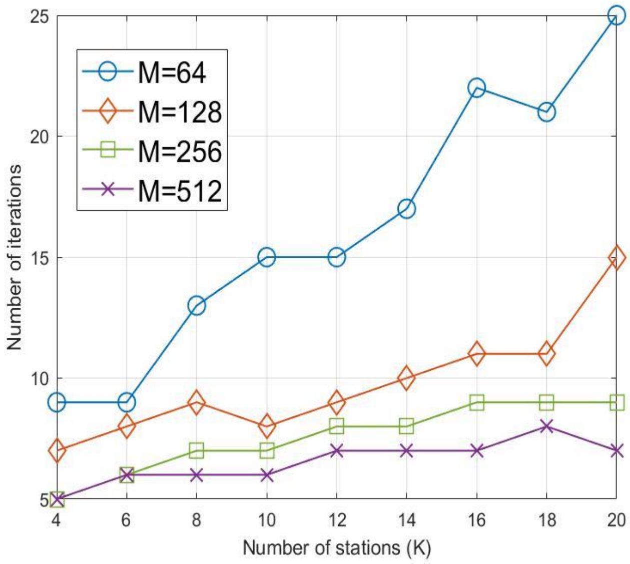


FIG. 5. GRADIENT DESCENT ITERATIONS

The comparison between the different beamforming objectives is summarized in Table II.

Received 12/March/2022; Accepted 22/May/2022

ISSN (Print) 1811-9212 *ISSN (Online) 2617-3352*

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

TABLE II. A BRIEF COMPARISON BETWEEN THE PROPOSED MASSIVE MIMO BEAMFORMING OBJECTIVES

Objective	Description	Transmission	Performance	Complexity
J_1	Min. noise power	Uplink	Lower than ideal	Very low O(MK)
J_2	Min. noise and interference	Uplink	Close to ideal	Very high $O(M^3+M^2(K-1)+2M((K-1)^2)$
J_3	Min. noise power and interference nulling	Uplink	Close to ideal	High $O(2MK^2+K^3)$
J_4	Tuning between min. square error and min. noise power	Uplink	Close to ideal	High $O(2MK^2+K^3)$
J_5	Iterative gradient descent	Uplink /Downlink	Close to ideal	Directly proportional to the number of iterations
J_6	Max. signal power	Downlink	Lower than ideal	Very low O(MK)
J_7	Min. square error with interference nulling	Downlink	Close to ideal	High $O(2MK^2+K^3)$
J_8	Tuning between min. square error and interference nulling	Downlink	Close to ideal	High $O(2MK^2+K^3)$

V. CONCULSIONS

The contribution of this paper is to propose and discuss different beamforming objectives for the wireless massive MIMO systems as one of the important technology for the recent wireless communications to improve the spectral efficiency and energy efficiency. These beamforming objectives are proposed for both uplink and downlink to find the optimal beamforming weight matrix. The results of this paper show that the massive MIMO improves the spectral efficiency because of the multiplexing and beamforming gain. Also, it improves the uplink energy efficiency due to the beamforming gain and improves the downlink energy efficiency by focusing the desired signal. The beamforming objectives that include the interference nulling have better performance and higher complexity computation than the objectives that do not have interference nulling included. The performance of the objectives with interference nulling becomes closer to the ideal performance while increasing the number of the antenna units at the BS. A brief comparison between these different types of the beamforming objectives is summarized at the end of this paper.

REFERENCES

[1] B. L. Bengtsson, F. Rusek, S. Malkowsky, F. Tufvesson, P. C. Karlsson, and O. Edfors, "A Simulation Framework for Multiple-Antenna Terminals in 5G Massive MIMO Systems," IEEE Access, vol. 5, pp. 26819-26831, 2017.
[2] R. Chataut, and R. Akl, "Massive MIMO Systems for 5G and beyond Networks—Overview, Recent Trends, Challenges, and Future Research Direction," Sensors, vol. 20, no. 10, pp. 2753, 2020.
[3] Z. H. Jaber, D. J. Kadhim, and A. S. Al-Araji. "Medium access control protocol design for wireless communications and networks review." International Journal of Electrical & Computer Engineering, vol. 12, no. 2, pp. 1711-1723, 2022.
[4] M. Y. Muhsin, A. J. Salim and J. K. Ali, "An Eight-Element MIMO Antenna system for 5G Mobile Handsets," In 2021 International Symposium on Networks, Computers and Communications (ISNCC), pp, 1-4, 2021.
[5] A. Salim, R. Fyath, and J. K. Ali, "A New Miniaturized Folded Fractal Based PIFA Antenna Design for MIMO Wireless Applications." In Proceedings of the International Conference on Information and Communication Technology, pp. 36-40, 2019.

Received 12/March/2022; Accepted 22/May/2022

DOI: https://doi.org/10.33103/uot.ijccce.23.1.2

[6] A. J. Salim, , R. S. Fyath, A. H. Ahmad, and J. K. Ali. "A New Fractal Based PIFA Antenna Design for MIMO Dual Band WLAN Applications." Progress In Electromagnetics Research, pp. 1526-1530, 2012.

[7] T. L. Marzetta, E. G. Larsson, H. Yang, and H. Q. Ngo, "Fundamentals of Massive MIMO," Cambridge University Press, 2016.

[8] E. Björnson, J. Hoydis, and L. Sanguinetti, "Massive MIMO Networks: Spectral, Energy, and Hardware Efficiency," Foundations and Trends in Signal Processing, vol. 11, no. 3-4, pp. 154-655, 2017.

[9] H.Q. Ngo, "Massive MIMO," In 5G and Beyond, pp. 101-127, Springer, Cham, 2021.

[10] M. Vaezi, Z. Ding, and H. V. Poor, "Multiple Access Techniques for 5G Wireless Networks and Beyond," Berlin, Springer, 2019.

[11] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An Overview of Massive MIMO: Benefits and Challenges," IEEE journal of selected topics in signal processing, vol. 8, no. 5 pp. 742-758, 2014.

[12] E. Ali, M. Ismail, R. Nordin, and N. F. Abdulah, "Beamforming techniques for massive MIMO systems in 5G: overview, classification, and trends for future research," Frontiers of Information Technology & Electronic Engineering, vol. 18, no. 6, pp. 753-772, 2017.

[13] H. Q. Ngo, and E. G. Larsson, "No Downlink Pilots are Needed in TDD Massive MIMO," IEEE Transactions on Wireless Communications, vol. 16, no. 5, pp. 2921-2935, 2017.

[14] Z. Chen, and E. Björnson, "Channel Hardening and Favorable Propagation in Cell-Free Massive MIMO with Stochastic Geometry," IEEE Transactions on Communications, vol. 66, no. 11, pp. 5205-5219, 2018.

[15] O. Elijah, C. Y. Leow, T. A. Rahman, S. Nunoo, and S. Z. Iliya. "A Comprehensive Survey of Pilot Contamination in Massive MIMO—5G System," IEEE Communications Surveys & Tutorials, vol. 18, no. 2, pp. 905-923, 2015.

[16] A. F. Molisch, V. V. Ratnam, S. Han, Z. Li, S. L. H. Nguyen, L. Li, and K. Haneda, "Hybrid Beamforming for Massive MIMO: A Survey," IEEE Communications Magazine, vol. 55, no. 9, pp. 134-141, 2017.

[17] B. Yang, Z. Yu, J. Lan, R. Zhang, J. Zhou, and W. Hong, "Digital Beamforming-Based Massive MIMO Transceiver for 5G Millimeter-Wave Communications," IEEE Transactions on Microwave Theory and Techniques, vol. 66, no. 7, pp. 3403-3418, 2018.

[18] T. E. Bogale, and L. B. Le. "Beamforming for multiuser massive MIMO systems: Digital versus hybrid analog-digital," In 2014 IEEE Global Communications Conference, pp. 4066-4071, 2014.

[19] T. A. Almohamad, M. F. M. Salleh, and M. N. Mahmud. "Partner Selection Technique for Wireless Cooperative Communication," Wireless Personal Communications, vol. 77, no. 1, pp. 255-267, 2014.

[20] M. A. Albreem, A. Alhabbash, A. M. Abu-Hudrouss, and T. A. Almohamad, "Data detection in decentralized and distributed massive MIMO networks," Computer Communications, vol. 189, pp. 79-99, 2022.

[21] H. Q. Ngo, "Massive MIMO: Fundamentals and System Designs," Linköping University Electronic Press, 2015.

[22] T. A. Almohamad, M. F. M. Salleh, M. N. Mahmud, İ. R. Karaş, N. S. M. Shah, and S. A. Al-Gailani, "Dual-Determination of Modulation Types and Signal-to-Noise Ratios Using 2D-ASIQH Features for Next Generation of Wireless Communication Systems," IEEE Access, vol. 9, pp. 25843-25857, 2021.

[23] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity Limits of MIMO Channels," IEEE Journal on selected areas in Communications, vol. 21, no. 5, pp. 684-702, 2003.

Received 12/March/2022; Accepted 22/May/2022

