Effect of Enlarging the Aircraft Vertical Tail on State Variables of Stability Response during Dynamic Coupling

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ABSTRACT

In this paper the variables of high performance aircraft including the modification of vertical tail during the roll inertia cross coupling phenomenon is presented. To demonstrate the effects of the enlarging the area of vertical tail on stability of aircraft, the longitudinal-lateral equations of aircraft motion at steady roll rate are solved. The results indicate that damping of the modified system is increased and the longitudinal-lateral state variables response of the aircraft is improved. The increase in the area of the vertical fin is a necessary feature in the development of the aircraft to improve stability during roll and decrease the possibility of cross coupling occurrence.

Also the results indicated that the critical value of roll rate in case 1,at Mach no.=0.6, takes the range of $-1.29 < p_o < -1.49\, rad/sec$,while the values of critical roll rates in case 2,3 and 4 are $-1.54 < p_0 < -1.94$, $-1.49 < p_0 < -1.49$, $-1.485 < p_0 < -1.94$,Respectively. The numerical results shows 19% , 15.5% , and 15% gain in rolling margin for cases 2, 3, and 4 respectively, when compared with original (case 1) .

Keywords: Aircraft state variables, Cross coupling, Aircraft stability, Critical roll rates

تأثير زيادة مساحة الزعنفة العمودية للطائرة على متغيرات الحالة لإستجابة أستقراريتها أثناء الازدواج الديناامي

عبدالجبار جوير شامخ

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الجامعة التكنولوجية

الخلاصة:

يتم في هذا البحث استعراض متغيرات الحالة لطائرة عالية الاداء متضمنا تطوير الزعنفة العمودية خلال تعرضها لظاهرة الازدواج الكتلي التدحرجي. وللتعرف على تأثيرات الزيادة في مساحة الزعنفة العمودية على استقراريه الطائرة، تم حل معادلات الحركة الطوليةوالعرضية للطائرة لمعدل تدحرج مستقر. وأظهرت النتائج زيادة في تخميد النظام المعدل وتحسين الاستقرارية الطولية-العرضية لمتغيرات الحالة للطائرة. ان زيادة المساحة للزعنفة العمودية تعتبر ظرورية في تطوير الطائرة لتحسين الاستقرارية خلال التدحرج وتقلل من احتمالية حصول الازدواج .

النتائج تؤثر : إن القيمة هي roll في الحالة الاولى عند عدد ماخ = 0.6 $-1.29 < p_o < -1.49\ rad/sec$
بينما القيمة للتدوير الحرج في الحـــالات 2,3و 4 هي الحرجة لمعدل التدوير $-1.485 < p_0 < -1.94 -$
$-1.54 < p_0 < -1.94, , -1.49 < p_0 < -1.49$ بالمقارنة مع الحالة (1).

INTRODUCTION

Modern aircraft designers try to obtain better performance from jet fighters, huge and large engines are installed along with shorter and thinner wings. This trend results in significant changes in the moment of inertia of the aircraft, which lead to catastrophic results for some aircrafts. The culprit was “inertial cross-coupling”. This phenomenon happens when the aircraft rolls at high angular velocities **[1]**.

Many aircrafts have additional maneuverability requirements .The aerodynamic and stability derivatives must therefore be estimated from the geometry of design configuration **[2]**.

The common approaches use aerodynamic analysis of any flying body making use of design charts and tables such as, Datcom, Russian charts………etc. Due to the limitations of these methods, numerical methods were used for more generality in Mach number range and flying body configuration. Detailed dynamic stability and cross coupling associated problems are illustrated.

The panel method solver considers isotropic, linearized potential flow only. It is less accurate than the former solver, but, within the limited theoretical assumptions, they have provided results that are found to be in excellent agreement with other data obtained by the former solver. Such methods, when applied to airframes having only small amounts of separated flow, give reasonably accurate predictions of aerodynamic characteristics and stability derivative of complete configurations **[3]**.

Inertia roll coupling is a resonance divergence in pitch or yaw when roll rates equal the lower pitch or yaw natural frequency. (Longitudinal – lateral in stability). A Pilot Induced Oscillation (PIO) can be described as sustained or oscillations resulting from efforts of the pilot to control aircraft. If atypical military jet, with most mass in fuselage, is rolled at high rate the inertia in pitch tends to cause angle of attack to increase. It's like a dumbbell, which may be sufficient to override the aircraft's natural stability that is trying to keep it down. At the same time, the rotation caused by pitch – up is coupled with rotation in roll to induce an increase in side slip **[10, 11].**

Aerodynamic and Stability Derivatives Models

In the present work, the configuration and dimensions of the main parts of the selected aircraft such as (fuselage, wing, horizontal tail, and vertical tail) are summarized in Appendix A, The aerodynamic coefficients is obtained by using panel Method and stability derivatives required for roll coupling analysis were calculated by the semi-empirical formula of the Datcom software **[9]** and indicated in Appendix (B).

Linearized Equation of Motion Mathematical Model

The traditional approach is to assume that the disturbing forces and moments are due to aerodynamic effects, gravitational effects, the effects of atmospheric disturbances and the effects of the movement of aerodynamic controls power change. The first two assumptions fit with this analysis of roll coupling, U is considered constant $(U = U_o)$ during the rolling maneuver, so that the X force equation can be eliminated.This leaves only the Y and Z force equations, [4-8]

$$m(\dot{V} - pW + rU) = \sum F_y \qquad (1)$$
$$m(\dot{W} - qU + pV) = \sum F_z$$

Where:-
m: Aircraft mass
p: Roll rate
q : Pitch rate
r : Yaw rate
U, V, W :Aircraft velocity components

As it is desired to study the effects of high-roll rate, the second assumption is that, roll rate is constant. The changes in rolling moment must be zero, then the moment equations to be derived are:

$$I_y\dot{q} + (I_x - I_z)P_o r = \sum M \qquad (2)$$
$$I_z\dot{r} - (I_x - I_y)P_o q = \sum N$$

Also ifV_e = W_e= 0 thenV =V_e + , W = W_e+ w

Substituting in equations (1) and (2) the followings are obtained:

$$\sum F_y = m(\dot{v} + U_o r - wP_o)$$

$$\sum F_z = m(\dot{w} + V\,P_o - U_o q) \qquad (3)$$

$$\sum M = qI_y + P_o\dot{r}(I_x - I_z)$$

$$\sum N = I_z\dot{r} + P_o q(I_y - I_x)$$

Where
$I_x, I_{y,} I_z$ Roll, Pitch and Yaw moment of Inertia
M Pitching moment, N Yawing moment

Additional necessary assumptions can be made to simplify the equations of motions which are:
then, when $\Delta\beta = v\,/U_o,\quad \Delta\alpha = w\,/\,U_0$
Note that engine gyroscopic effects are neglected.

Then:-

Pitching: $\dot{q} - \left(\frac{I_z - I_x}{I_Y}\right) P_o r = \frac{M_q q}{I_Y} + \frac{M_\alpha}{I_Y}\Delta\alpha$

Yawing: $\dot{r} - \left(\frac{I_x - I_y}{I_z}\right) P_o q = \frac{N_r}{I_z} r + \frac{N_\beta}{I_z}\Delta\beta$

Side force: $\dot{\beta} + r - p_0\Delta\alpha = \frac{Y_\beta}{mV}\Delta\beta + p_0\alpha_0$ (4)

Normal force**:** $\dot{\alpha} - q + P_o\Delta\beta = \frac{L_\alpha}{mV}\Delta\alpha$

$N_\beta, M_q, M_\alpha, N_r, Y_\beta, L\alpha,$ in equation (4) are functions of vertical tail area For example

$M_q = f\left(C_{mq}\right)$

$$C_{mq} = -\frac{2\acute{X}}{C^2} C_{L\propto} - \frac{2l_t^2}{c^2} C_{L\propto t}\frac{S_t}{S_w}$$

Where:-

$\acute{X}$: - distance from c.g to wing quarter chord, negative for c.g behind quarter chord
c- Chord
$C_{L\propto}$- lift curve slope
l_t- Distance from a.c of tail to a.c of wing
$C_{L\propto t}$ $-$Tail lift curve slop
S_t $-$Tail aera
S_w $-$ Wing area
The two parameters related to vertical tail are $C_{L\propto t}$ and S_t
$N_\beta = f(C_{n\beta})$

$$C_{n\beta_{total}} = (C_{n\beta})_w + \left(C_{n\beta}\right)_{fus} + (C_{n\beta})_v + \Delta_1 C_{n\beta} + \Delta_2 C_{n\beta}$$

$(C_{n\beta})_v$ - Static directional derivative of vertical tail (change in yawing moment coefficient resulting from a change in side slip angle) .

$(C_{n\beta})_v = a_v \frac{S_V}{S_N}\frac{l_V}{b_w}\eta_V$ Where

a_v Lift curve slope of vertical tail.

S_v Area of vertical tail

$(C_{n\beta})_v$ Plays a main role in stability of aircraft to estimate the total static directional derivative.

The numerical solution of aircraft equations of motions stability boundaries can be illustrated according to the following formulas.

$$\omega_\varphi^2 = \frac{C_{n\beta}\ qSb}{I_z P_o^2}$$
$$\omega_\theta^2 = \frac{C_{m\alpha}\ qS\bar{c}}{I_y P_o^2}$$

Where:
Po: study roiling velocity
ω_θ : Natural frequency in Pitch
ω_φ : Natural frequency in Yaw

Fourth order (Runge Kutta) method is used in present work to solve the aircraft equations of motion and the program is written in Matlab.

For more details see [Ref [12]:

RESULTS AND DISCUSION

The high performance aircraft MIG-21with Mach no.=0.6 and 1.2, was taken into consideration in this analysis in order to demonstrate the cross coupling variables. The lift coefficient of aircraft at subsonic and supersonic speed is shown in Figs. (1 - 4) using panel method solver (Including only the linear part of lift coefficient).

In this study four cases were implemented to enlarge the vertical tail plan form geometrical parameters by 15% when the original vertical tail, namely,(case1),Increase in span (case 2) , increase tip chord (case 3) and increase root chord (case 4)as displayed in table(1).All cases are shown in (Fig.5). The numerical results indicate that enhancement in response of state variables. The change in angle of attack and the sideslip angle tend to remain in certain value to keep the aircraft stable, as demonstrated in Fig. (6)

Also, the enlargement in vertical tail is about 15% which are so effective because the change in yawing moment coefficient resulting from a change in side slip angle is increased (table 2).

In supersonic speed, the response of state variables of the original model of aircraft tend to produce rapid instability while the change in angle of attack and sideslip angle, cases (3,4) approach to the zero(Fig.7).

The same trend including the pitch rate and yaw rate is shown in Fig.(8).Divergent influence in above variables for the original is indicated and enhancement in stability appeared in other cases.

The enlarging of aircraft vertical causes an increase in static directional derivative $C_{n\beta}$.

The effect of $C_{n\beta}$ on spiral stability in vertical tail is more than its effect on or fuselage .

it is agreed that values of $C_{n\beta}$ as high as practically possible are desired for good flying qualities. All the previous effects are shown in figs.[6-8]and table[2].adds more positive lift to the tail unit and create a moment to oppose the pitching motion ,that causes high negative values of C_{mq}.This derivative is very important in longitudinal dynamics because it plays a major role in damping of the short mode .

Conclusion

The roll inertia cross coupling and its effects on stability and response of high performance aircraft has been predicated in present work.

The transient motion in angle of attack and sideslip during constant roll has also been analyzed. Solutions of coupled equations of motion are presented to determine the pertinent modes of motion. The following conclusions have been detected by the current work.

Violent cross coupled lateral and longitudinal motion subject to aircraft, rolling has been found to cause instability when the rolling frequency exceeds the lower of pitching and yawing natural frequencies of non-rolling aircraft.

The increase in the area of vertical fin is quite necessary feature in the development of the aircraft to improve the stability in roll and decrease the possibility of cross coupling occurrence.

The following conclusions are obtained:

1. Enhancement in total damping of the system is obtained.
2. From analysis presented in this paper, it appears that the transient motion in angle of attack and side slip consists chi felly of an oscillatory mode.
3. Improvement in directional stability is indicated due enlargement of vertical tail.

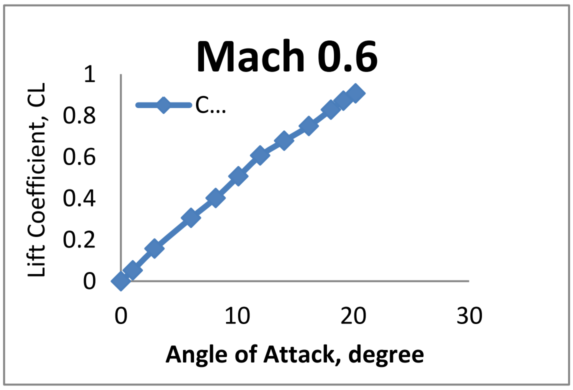


Fig.1 Aircraft lift coefficient as a function of angle of attack at Mach No. =0.6

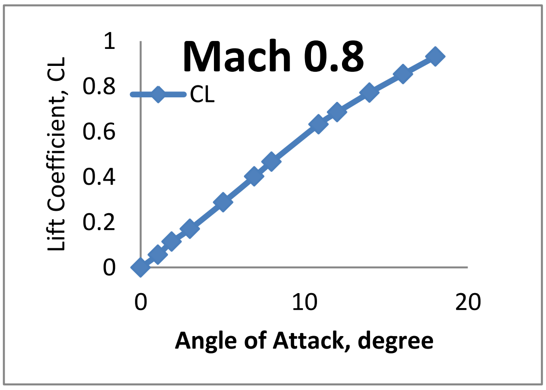


Fig.2 Aircraft lift coefficient as a function of angle of attack at Mach No. =0.8

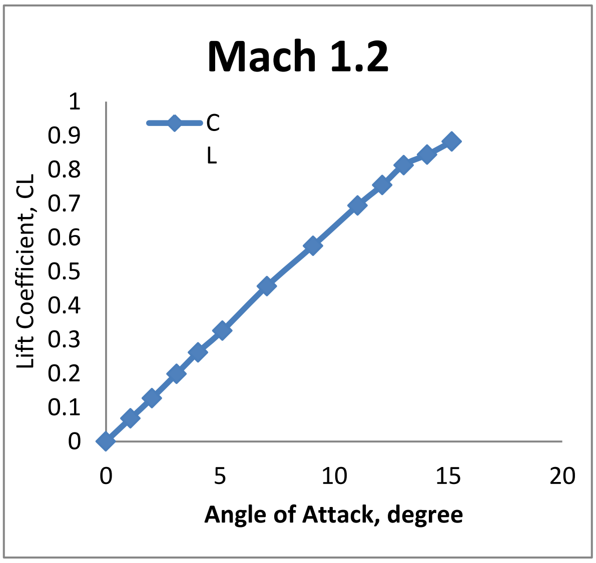


Fig.3 Aircraft lift coefficient as a function of angle of attack at Mach No. =1.2

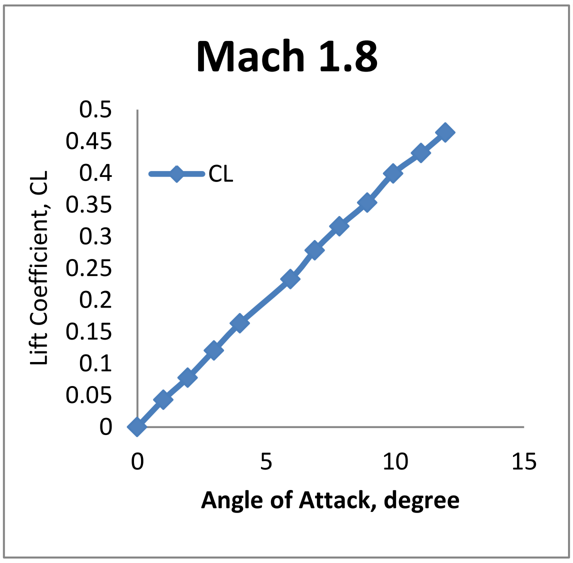
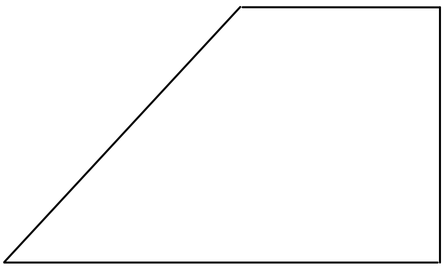
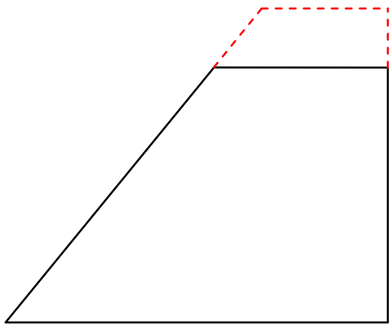


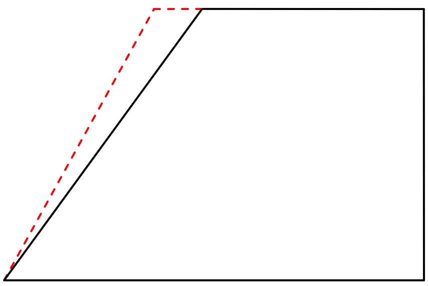
Fig.4 Aircraft lift coefficient as a function of angle of attack at Mach No. =1.8



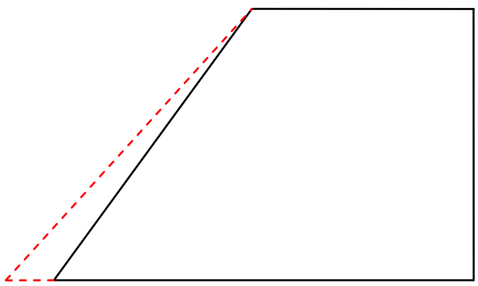
Case (1)
Original Case



Case (2)
Increasing in Span

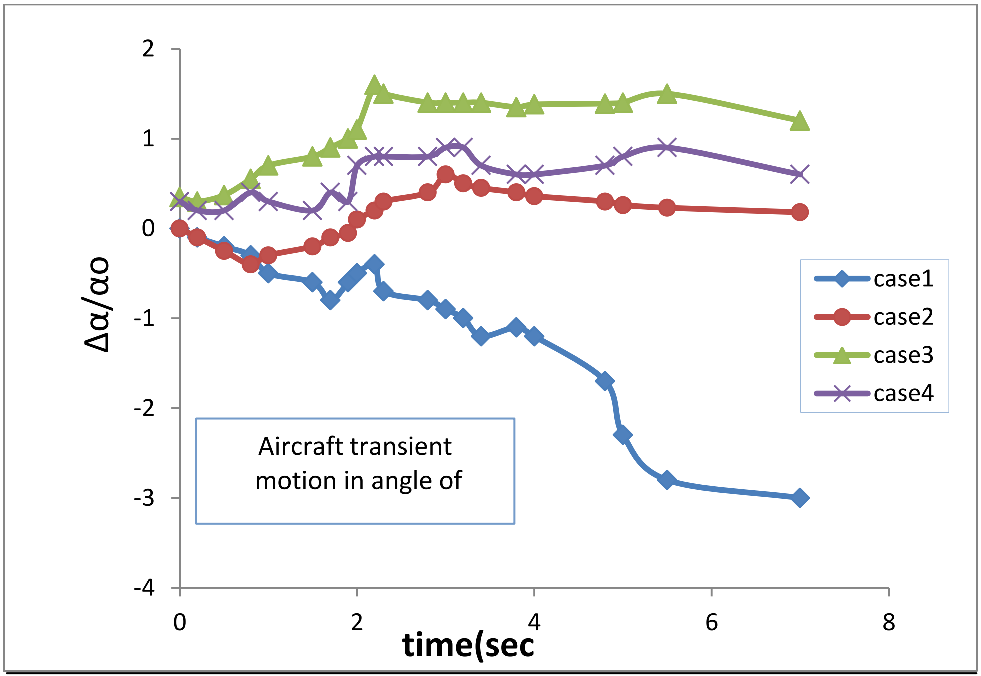


Case (3)
Increasing in Tip Chord

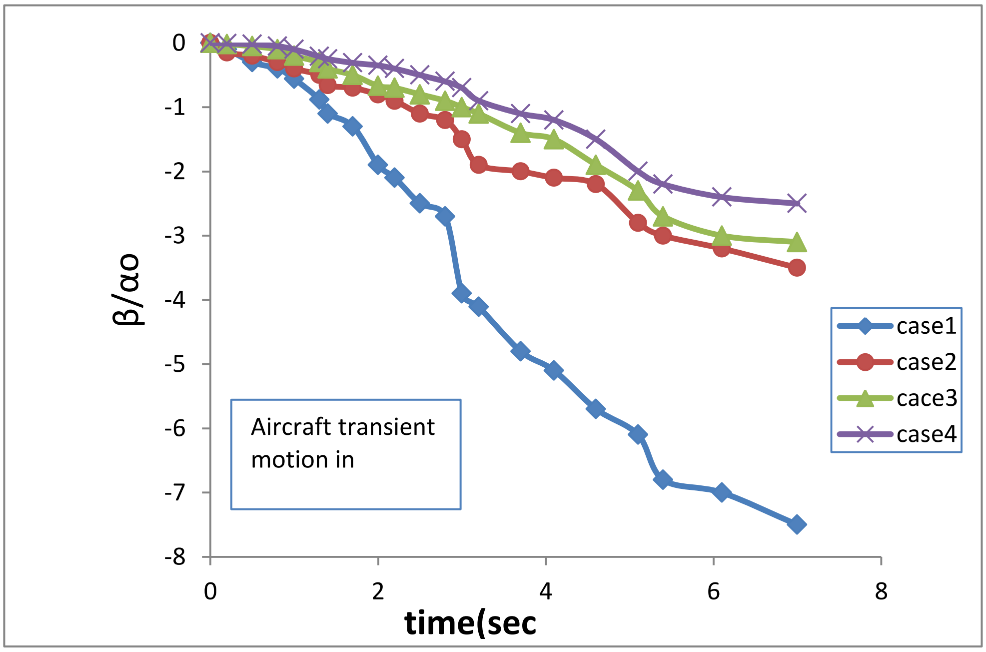


Case (4)
Increasing in Root Chord

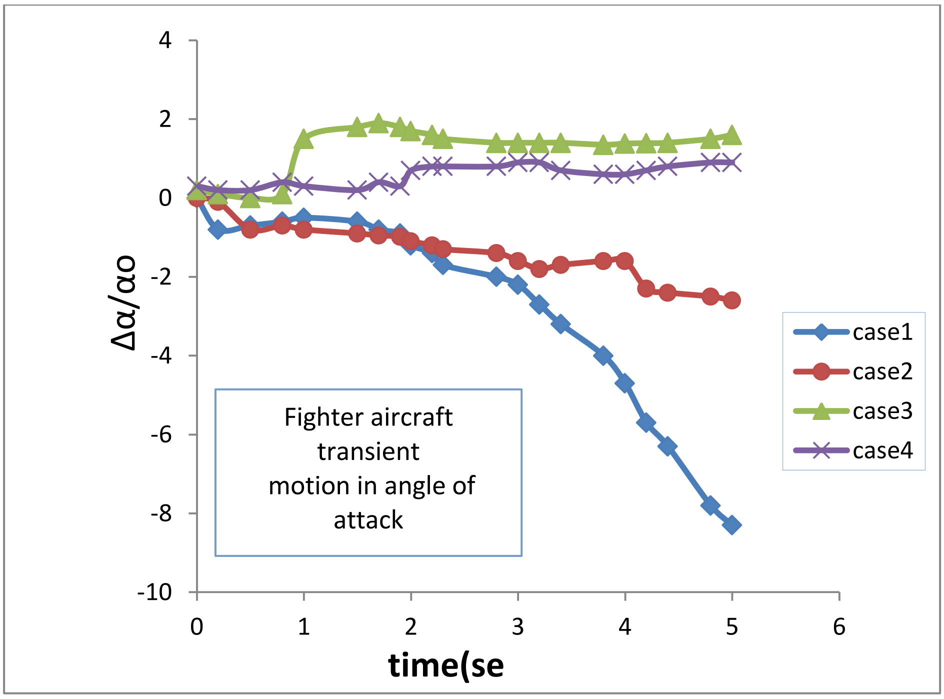
Fig. (5): Vertical tail plan form change types.



[A]



[B]
Fig.6
[A] Time history comparison results of motion in angle of attack of aircraft for cases 1,2,3&4 at roll rate= -1.5rad/sec :Mach No.=0.6
[B] Time history comparison results of motion in sideslip of aircraft for cases 1,2,3&4 at roll rate= -1.5rad/sec :Mach No.=0.6



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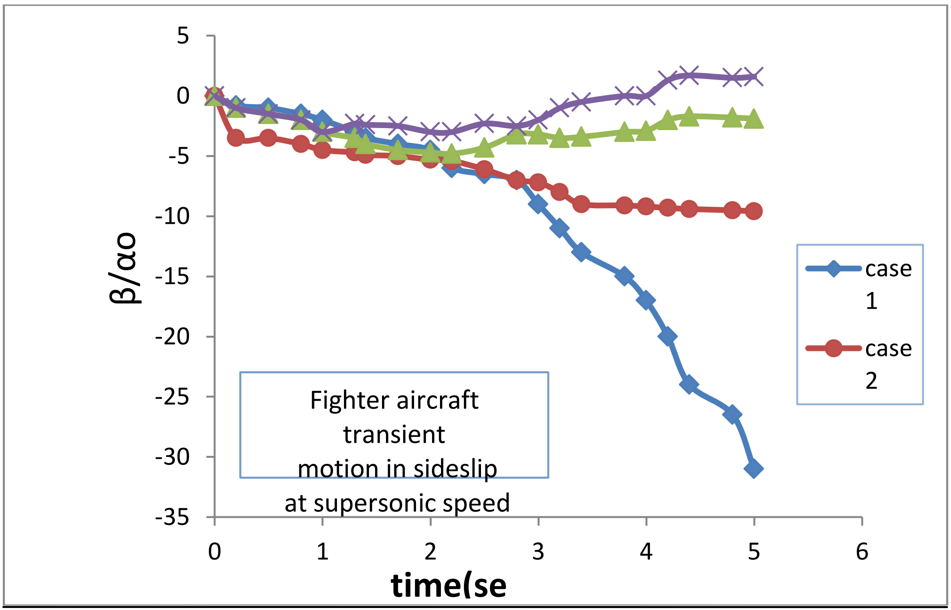
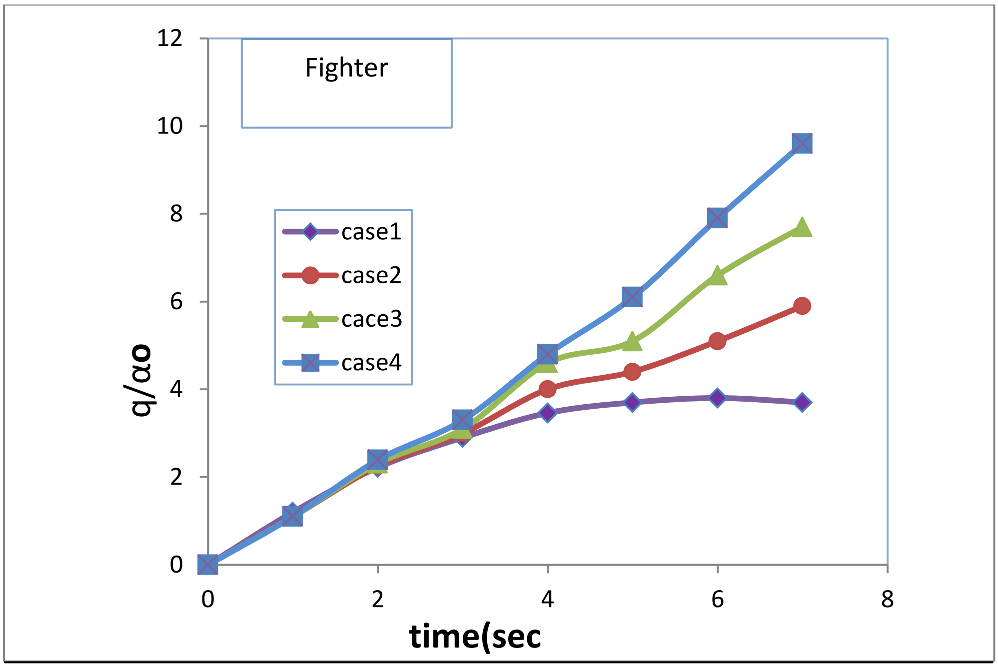


[B]

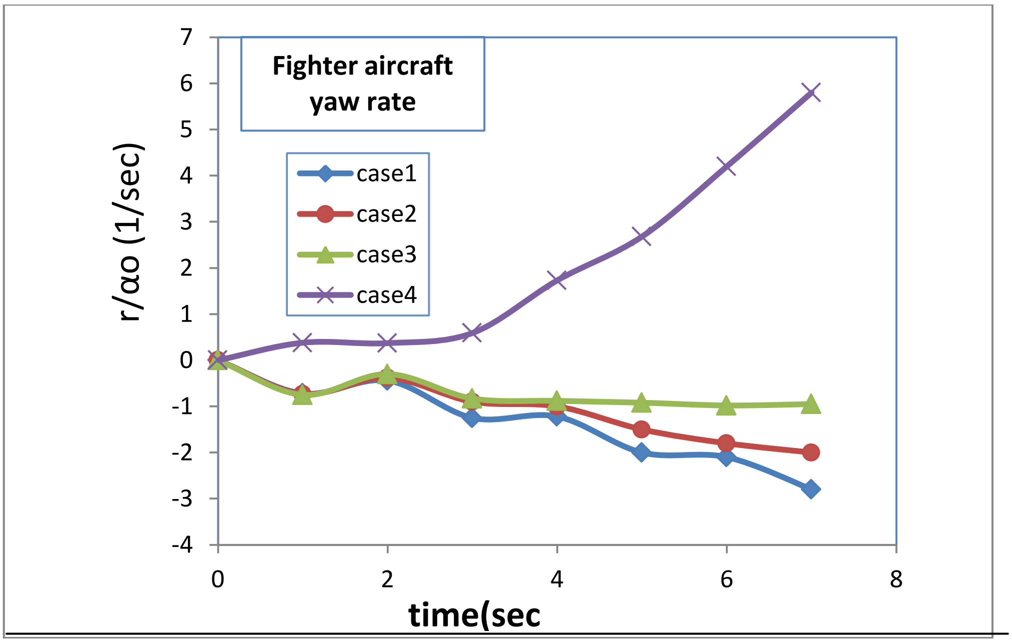
Fig.**7**

[A] Comparison results of transient motion in angle of attack for cases 1, 2, 3 & 4 at roll rate= -3.35 rad/sec: Mach No.=1.20.

[B] Comparison results of transient motion in sideslip for cases 1, 2, 3 & 4 at roll rate= -3.35rad/sec :Mach No.=1.20.



[A]



[B]

Fig.8

[A] Comparison Results of transient motion in angle of attack for cases 1, 2, 3, 4 at roll rate= -3.35: Mach no. =1.20

[B] Comparison Results of transient motion in angle of attack for cases 1, 2, 3, 4 at roll rate= -3.35: Mach no. =1.20

case	Span (b) (m)		Root Chord(C_r) (m)		Tip Chord(C_r) (m)		Aspect AR	Swept tail θ (deg)	Taper ratio of V. tail AR	Area of V. tail m^2
	Increase %	Value	Increase %	Value	Increase %	Value				
1	0	2.0	0	3.8	0	1.47	0.760	60	0.386	5.27
2	15	2.3	0	3.8	0	1.47	0.8728	60	0.386	6.0605
3	0	2.0	0	3.8	15	1.6905	0.7285	58.38	0.444	5.49
4	0	2.0	15	4.37	0	1.47	0.6849	63.628	0.336	5.84

Table (1) Values of geometrical parameters due to change in vertical tail plan form

Case	Mach	$C_{l\alpha V.T}$	Cn_β	Cy_β	Cn_r	Cm_q
1	0.6	1.47	0.103	-0.466	--0.0583	-3.600
	1.2	1.70	0.115	-.5195	-0.0743	-3.620
2	0.6	1.56	0.116	-0.5	-0.0702	-3.7125
	1.2	1.87	0.133	-0.579	-0.0909	-3.7295
3	0.6	1.478	0.104	-0.470	-0.0586	-3.6382
	1.2	1.782	0.120	-0.544	--0.773	-3.6552
4	0.6	1.4869	0.106	-0.482	-0.05615	-3.683
	1.2	1.78	0.122	-0.552	-0.0792	-3.7009

Table (2) Results of Aircraft Stability derivatives.

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Appendix A

Details of the selected aircraft configuration

Fuselage Wing

Overall length 12.80 m Airfoil section 0006

Wetted area 46.84m^{2}Area 23 m^2

Max.diameter1.165 mSpan 7.15m

Swept angle 57^o M.A.C 4.0 m

AR 2.22

Horizontal Tail Vertical Tail

Exposed Area 3.78m^2 Exposed area 5.27m^2

Span 3.6 m Span 2.00m

Root chord 1.9 mAR 0.76m

Tip chord 0.9 m Swept angle 60^o

Swept angle 55^{o}M.A.C 3.08m

Appendix B

$I_y = 75355.7\ Kg.m$ $cm_q\,(per\ rad) = -3.6$

$cm_\alpha\,(per\ rad) = -.4$ $I_z = 82359.9\ \ Kg.m$

$C_{L\propto}(per\ rad) = 2.844$

$C_{n\beta}(per\ rad) = .1030$

$C_{nr}(per\ rad) = -.583$

$I_x = 8696.5\ \ Kg.m$