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nc- Sets in Topological Space

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A B S T R A C T

Type of sets that we will introduce in this research, called nc-open set and present its properties which represent the topological properties of this type . More precisely, we post the sets with each of these properties . At the first , we present the definition of the property and then an illustrative example of this definition . Mean while , we move to the theorems with their proofs and give a counter example to the case that the opposite of the some theorems are not achieved .

Keywords *: n-open, n-closed, nc-open, nc-closed.*

1. Introduction

In 2010 [1] , if ($F \subseteq K$ such that $K \in SO(H) \forall h \in H, F \in \tau^c$) the a subset K of a top.space H_τ is defined by Alias and Zanyar to Ss-open, where top.spacedenoted to topological space . After that, Zanyar [4] in 2011 improved the notation of Pc-open and Pc-closed via introducing the idea of P-open sets and Bc-open sets are a new class of sets which developed by Hariwan [3] in 2013. Additionally ,C.W.Baker [2] explored the characteristics of the group of the subset of top.space H_τ.Which known as n-open sets in 2012. In fact , these sets meet up with provided that its interior and closure are not equal . In the present paper , we introduce a new type of open sets called nc-open and defined some top.properties such as nc-neighborhood, nc-interior($(K^\circ)^{nc}$) ,nc-derived ($ncD(K)$) and nc-closure sets $(\overline{K})^{nc}$. Also, we explore the relation between our type and n-open set which developed by C.W.Baked.

<u>Definition1.1</u> [4] IfH_τ is top.space , then the subset K of H_τ is n-open if $Int(K) \neq Cl(K)$ and it is n-closed if K^c is n-open . Where the sets of all n-open subsets of H_τ denoted by $nO(H_\tau)$or $(nO(H))$.

2.nc-Open Sets

This section contain the main definitions with some results .

Definition2.1 If H_τ is top.space , then the subset K of H_τ called nc-open if $\forall h \in K \in nO(H), \exists F: h \in F \subseteq K$ and F is closed . Where the sets of all nc-open subsets of H_τ denoted by $ncO(H_\tau)$ or $(ncO(H))$

Example2.2 Consider $H = \{h_1, h_2, h_3\}$and $\tau = \{\varphi, H, \{h_1\}, \{h_2\}, \{h_1, h_2\}\}$. Then the family of closed set are :$\{\varphi, H, \{h_3\}, \{h_1, h_3\}, \{h_2, h_3\}\}$.

So,$nO(H) = \{\{h_1\}, \{h_2\}, \{h_3\}, \{h_1, h_3\}, \{h_1, h_2\}, \{h_2, h_3\}\}$and
$ncO(H) = \{\{h_3\}, \{h_1, h_3\}, \{h_2, h_3\}\}$.

Remark2.3 From Definition 2.1 , every nc-open subset of H_τ is n-open , but the opposite is not true , see Example 2.4.

Example2.4 Considering the space H_τ as defined in Example 2.2 ,$\{h_1\} \in nO(H)$ but $\{h_1\} \notin ncO(H)$.

Proposition2.5 If H_τ is top.space then the subset K of H_τ is nc-open iff K is n-open and $K = \cup F_\alpha$, where F_α closed sets for each α.

Proof: Since K is nc-open set . Then K is n-open set. Let $h \in K \in nO(H)$, by definition of nc-open, $F; h \in F \subseteq K, K = \cup F_\alpha$, where K is n-open and F is closed. Now , let $h \in K \in nO(H)$, since $K = \cup F_\alpha$ where F_α is closed sets then $h \in F \subseteq K \rightarrow K$ is nc-open set .

Remark2.6nc-open set does not have to be closed .

Example2.7 The real number R with ray topology, such that $K \subseteq R$, if $K = (0, \infty)$ such that $(0, \infty) = \cup_{n=1}^{\infty} [\frac{1}{n}, \infty)$, then K is nc-open, but not closed.

Remark2.8 The union of two nc-open need not to be nc-open .

Example2.9 Consider $H = \{h_1, h_2, h_3\}$with $\tau = \{\varphi, H, \{h_2\}, \{h_3\}, \{h_2, h_3\}\}$. Thus the family of closed set are :$\{\varphi, H, \{h_1\}, \{h_1, h_3\}, \{h_1, h_2\}\}$. Hence, we obtain

$nO(H) = \{\{h_1\}, \{h_2\}, \{h_3\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_2, h_3\}\}$and
$ncO(H) = \{\{h_1\}, \{h_1, h_2\}, \{h_1, h_3\}\}$.

There $\{h_1, h_2\} \in ncO(H)$and $\{h_1, h_3\} \in ncO(H)$, but $\{h_1, h_2\} \cup \{h_1, h_3\} = H \notin ncO(H)$.

Remark2.10 If we have two nc-open, their intersection is not necessarily an nc-open .

Example2.11 Consider $H = \{h_1, h_2, h_3\}$with $\tau = \{\varphi, H, \{h_3, h_2\}, \{h_1, h_3\}, \{h_3\}\}$. Then the family of closed set are:$\{\varphi, H, \{h_1\}, \{h_2\}, \{h_1, h_2\}\}$. Thus, we deduce that

$nO(H) = \{\{h_1\}, \{h_2\}, \{h_3\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_2, h_3\}\}andncO(H) = \{\{h_1\}, \{h_2\}, \{h_1, h_2\}\}$$\{h_1\} \in ncO(H)$ with $\{h_2\} \in ncO(H)$, but $\{h_1\} \cap \{h_2\} = \varphi \notin ncO(H)$.

Remark2.12 The sets of all nc-open set is not topology on H since $H, \varphi \notin ncO(H)$.

Proposition2.13 If K is nc-open in H_τ then $\forall h \in K$,$\exists$ nc-open B such that $h \in B \subseteq K$.

Proof: Let K be nc-open in H_τ , then $\forall h \in K$, putting $K = B$ is nc-open containing h such that $h \in B \subseteq K$.

Theorem2.14 If$V \subseteq H$and $V^* \subseteq H^*$, then $V \times V^*$ is nc-open in $H \times H^*$iff V is nc-open in V or V^*nc-open in H^* .

Proof: Suppose $V \times V^*$ is not nc-open in $H \times H^*$ iff $V \times V^*$ is clopen in $H \times H^*$ iff $V$is clopen in H and V^* is clopen in H^* iff V is not nc-open in H and V^* is not nc-open in H^*.Thus, $V \times V^*$ is nc-open in $H \times H^*$ iff $V$is nc-open in H or $V^*$is nc-open in H^* .

Corollary2.15 If $V \subseteq H$and $V^* \subseteq H^*$.Then $V \times V^*$ is nc-open in $H \times H^*$ for all V and V^* are nc-open

Definition2.16A subset M of H_τ is nc-closed if M^c is nc-open. The sets of all nc-closed subset of H_τ is denoted by $ncC(H_\tau)$or $(ncC(H))$.

Example2.17 Considering the space H_τ as defined in Example 2.2 then $ncC(H) = \{\{h_2, h_1\}, \{h_1\}, \{h_2\}\}$.

Proposition2.18 A subset M of H_τ is nc-closed iff M is the intersection of open sets and it is n-closed

Proof: Obvious .

Remark2.19 The intersection of two nc-closed does not have to be nc-closed .

Example2.20 Considering H_τ as defined in Example 2.9 .Then $ncC(H) = \{\{h_2, h_3\}, \{h_2\}, \{h_3\}\}$,and$\{h_3\}, \{h_2\} \in ncC(H)$, but $\{h_2\} \cap \{h_3\} = \varphi \notin ncC(H)$.

Remark2.21 The union of two nc-closed does not have to be nc-cloesdset .

Example 2.22 Considering H_τ as defined in Example 2.11.Then $ncC(H) = \{\{h_3\}, \{h_1, h_3\}, \{h_2, h_3\}\}$we have .$\{h_2, h_3\}, \{h_1, h_3\}$are nc-closed, but $\{h_2, h_3\} \cup \{h_1, h_3\} = H \notin ncC(H)$.

Lemma2.23 If K is nc-open and$K = N \cup M$, then either N is nc-open or M is nc-open .

Proof: We have $= N \cup M$, K is not clopen , then either N is not clopen or M is not clopen . Thus either N is nc-open or M is nc-open .

3- The Property of nc-Open Sets

Now we will study and defined top.properties of nc-neighborhood ,nc-interior , nc-closure and nc-derived based on the concept of nc-open .

Definition3.1 If H_τ is top.space and $h \in H$, then $N \subseteq H$ is nc-neighborhood (shortly write nc-neighb.) of h, if $\exists$ nc-open U in H such that $h \in U \subseteq N$.

Example3.2 InspaceR_{τ_u} every open interval is nc-neighb.for any point in this interval φ,R.

Proposition3.3A subset K of H_τ is nc-open if it is nc-neighb of each of its points .

Proof: Let $K \subset H$ be nc-open , since $\forall h \in K, h \in K \subseteq K$ and K is nc-open . This shows K is nc-neighb.of each of its points .

Proposition3.4 For any two subset K and M of $H_\tau$and $K \subset M$, if K is nc-neighb. of a point $h \in H$, then M is also nc-neighb. of h.

Proof: Let K be nc-neighb of a point $h \in H$, and $K \subset M$, then by Definition 3.1, ∃ nc-open U such that $h \in U \subseteq K \subset M \rightarrow M$is also nc-neighb. of h.

Remark3.5 Every nc-neighb .of any points is n-neighb. Since every nc-open is n-open .

Definition3.6 If$K \subseteq H_\tau$, and $h \in H_\tau$ then h is called nc-interior point of K, if there exist nc-open U such that $h \in U \subseteq K$.The set of all nc-interior points of K is called nc-interior of K and symbolizes it $(K^\circ)^{nc}$.

Example3.7 Considering H_τ as defined in Example 2.2.If we take $K = \{h_1, h_3\}$.Then$(K^\circ)^{nc} = \{h_1, h_3\}$.

Using Definition (3.1 and 3.6) , we can conclude the following result .

Proposition3.8 InH_τ and$K \subset H, h \in H$.The point h is nc-interior of K iff K is nc-neighb . of h.

Proposition3.9 In $H_\tau$and$K \subset H, h \in H$, if $h \in (K^\circ)^{nc}$, then ∃ F closed set ,such that $h \in F \subset K$.

Proof: Let $h \in (K^\circ)^{nc}$ then ∃ nc-open U of H such that $h \in K \subset K$.Since U is nc-open , so ∃ F which is closed such that $h \in F \subset U \rightarrow h \in F \subset K$.

Next theorem give the properties of nc-interior.

Theorem3.10 For a subsets K and M of H_τ , the following statements hold .

(i)$(K^\circ)^{nc} \subset K$,

(ii)if$K \subset M$ then $(K^\circ)^{nc} \subset (M^\circ)^{nc}$,

(iii)if K is nc-open then $K = (K^\circ)^{nc}$,

(iv)$((K \cap M)^\circ)^{nc} \subset (K^\circ)^{nc} \cap (M^\circ)^{nc}$,

(v)$(K^\circ)^{nc} \cup (M^\circ)^{nc} \subset ((K \cup M)^\circ)^{nc}$,

(vi)$ncInt((K^\circ)^{nc}) = (K^\circ)^{nc}$and K is nc-open set .

Proof: Obvious .

Proposition3.11 If K is a subset of H_τ , then $(K^\circ)^{nc} \subset (K^\circ)^{n}$.

Proof: Since all nc-open is n-open . In general, $(K^\circ)^{nc} \neq (K^\circ)^{nc}$ which is shown in 3.12

Example3.12 Let $H = \{h_1, h_2, h_3, h_4\}$ then $\tau = \{H, \varphi, \{h_1\}, \{h_2\}, \{h_1, h_2\}, \{h_1, h_2, h_3\}$ .Then the closedsets are$\{\varphi, H, \{h_2, h_3, h_4\}, \{h_1, h_3, h_4\}, \{h_3, h_4\}, \{h_4\}\}$ Thus $nO(H) = \{\{h_1\}, \{h_2\}, \{h_3\}, \{h_4\}, \{h_1, h_2\}, \{h_1, h_3\}, \{h_1, h_4\}, \{h_2, h_3\}, \{h_2, h_4\}, \{h_3, h_4\},$

$\{h_1, h_2, h_3\}, \{h_1, h_3, h_4\}, \{h_1, h_2, h_4\}, \{h_2, h_3, h_4\}\}andncO(H) = \{\{h_2, h_3, h_4\}, \{h_1, h_3, h_4\}, \{h_3, h_4\}, \{h_4\}\}$. Let $K = \{h_2, h_4\}$, then $(K^\circ)^{nc} = \{h_4\}$ and $(K^\circ)^{n} = K$ .This shows that $(K^\circ)^{nc} \neq (K^\circ)^{n}$.

If $(K^\circ)^{nc} = (M^\circ)^{nc} \nRightarrow K = M$, as it is shown in 3.13

Example3.13 If we have H_τ as defined in Example 3.12

Such that $K = \{h_1, h_4\}$ and $M = \{h_2, h_4\}$, then we obtain that, $(K^\circ)^{nc} = (M^\circ)^{nc} = \{h_4\}$.

Definition3.14 Let $K \subseteq H_\tau$ then$h \in H$ is nc-limit point of K if for all nc-open U containing h and$U \cap K\backslash\{h\}) \neq \varphi$. Then nc-derived of K are the set of all nc-limit points of K and symbolizes it $ncD(K)$.

Example3.15 Considering the space H_τ as defined in Example 2.9.

$Z = \{h_1, h_2\}, V = \{h_1, h_3\}$.Then we see that $ncD(Z) = \{h_2, h_3\}$and $ncD(V) = \{h_2, h_3\}$.

Proposition3.16 Let$F \subset H_\tau$ be any containing h such that $F \cap (K\backslash\{h\}) \neq \varphi$, then h is nc-limit point of K.

Proof: Let $h \in U$ be any nc-open , then for all $h \in U \in nO(H), \exists$ closed set F such that $h \in F \subseteq U$ .Since we have $F \cap (K\backslash\{h\}) \neq \varphi$. Thus $U \cap (K\backslash\{h\}) \neq \varphi$. So a point $h \in H$ is nc-limit point of K .

Next theorem gives the properties of nc-derived .

Theorem3.17 For subset K and M of H_τ , the following statements hold .

(i) If $K \subset M$then $ncD(K) \subset ncD(M)$,

(ii)$ncD(K) \cup ncD(M) \subset ncD(K \cup M)$,

(iii)$ncD(K \cap M) \subset ncD(K) \cap ncD(M)$,

(iv)$ncD(K \cup ncD(K)) \subset K \cup ncD(K)$,

(v) If $h \in ncD(K)$, then $h \in ncD(K\backslash\{h\})$ and $ncD(\varphi) = \varphi$.

Proof:(iv) Let $h \in ncD(K \cup ncD(K))$ if $h \in K$, then result is obvious . Now let $h \in ncD(K \cup ncD(K))\backslash K$, there for nc-open U containing h and $U \cap (K \cup ncD(K))\backslash\{h\} \neq \varphi$. Thus, $U \cap (K\backslash\{h\}) \neq \varphi$or $\cap (ncD(K)\backslash\{h\}) \neq \varphi. U \cap (K\backslash\{h\}) \neq \varphi$, hence $h \in ncD(K)$. Therefore, in any case $ncD(K) \cup ncD(K) \subset K \cup ncD(K))$.

The proof of other parts is obvious.

If $ncD(K) = ncD(M) \not\Rightarrow K = M$, as it shown in the following example .

Example3.18 Considering H_τ as defined in Example 3.12.

If $K = \{h_1, h_3, h_4\}$and $= \{h_2, h_3, h_4\}$. Then we obtain that $ncD(K) = ncD(M) = \{h_1, h_2, h_3\}$.

Corollary3.19 If$K \subset H_\tau$, then $nD(K) \subset ncD(K)$.

Proof: It is enough to remember that every nc-open is n-open .

In general, the converse may not be true as shown in following example .

Example3.20 Considering H_τ as defined in Example 3.12.

If $K = \{h_1, h_2, h_3\}$. So $ncD(K) = \{h_1, h_2\}$ and $ncD(K) = \varphi$.Hence, $ncD(K) \nsubseteq nD(K)$.

Definition3.21 Let K be a subset of H_τ. The nc-closure of a set K is $K \cup ncD(K)$ and denoted by $\overline{K}^{nc}$ i.e.$\overline{K}^{nc} = K \cup ncD(K)$.

Example3.22 Considering the space H_τ as defined in Example 2.9. Then

$ncC(H) = \{\{h_2\},\{h_3\},\{h_2,h_3\}\}, N = \{h_2\}., (\overline{N})^{nc} = \{h_2\}$.

Proposition3.23 A subset K of H_τ is nc-closed iff it contains the set of its nc-limit points .

Proof: Let K be nc-closed and if h is a nc-limit point of K and $h \in K^c$, then K^c is nc-open containing nc-limit points of K. Therefore $K \cap K^c \neq \varphi$, which is a contradiction .

Conversely, suppose that K contains all of its nc-limit points .$\forall h \in K^c$, there exists nc-open U containing h such that $K \cap U = \varphi$, thus $h \in U \subset K^c$ by Proposition 2.13, K^c is nc-open and K is nc-closed .

Proposition3.24 Let $K \subset H_\tau$ if $K \cap F \neq \varphi$ for all closed F of $H_\tau$containing h, then $h \in \overline{K}^{nc}$.

Proof: Let $h \in U$ such that U any nc-open , then by 2.1, $\exists F$ which is closed such that $h \in F \subseteq U$ .We have $K \cap F \neq \varphi$ implies $K \cap U \neq \varphi, \forall$ nc-open U containing h. Therefore $h \in \overline{K}^{nc}$.

We show the properties of nc-closure of sets .

Theorem3.25 For subsets K and M of H_τ , the following statements are true .

(i)$K \subset \overline{K}^{nc}$,

(ii)if $K \subset M$ then $\overline{K}^{nc} \subset \overline{M}^{nc}$,

(iii)$(\overline{K})^{nc} \cup (\overline{M})^{nc} \subset (\overline{K \cup M})^{nc}$,

(iv)$(\overline{K \cap M})^{nc} \subset (\overline{K})^{nc} \subset (\overline{M})^{nc}$,

(v)if K is nc-closed then $(\overline{K})^{nc} = K$,

(vi) The nc-closure of K is the intersection of all nc-closed sets containing h.

Proof: Obvious.

Proposition3.26let $K \subset H_\tau$then the following statement are true .

(i)$H\backslash ncCL(K) = ncInt(H\backslash K)$,

(ii)$H\backslash ncInt(K) = ncCl(H\backslash K)$,

(iii)$\overline{K}^{nc} = H\backslash ncInt(H\backslash K)$,

(iv)$(K^{\circ})^{nc} = H\backslash ncCl(H\backslash K)$.

Proof: (i) For any point $h \in H$ then $h \in H\backslash \overline{K}^{nc}$ implies that $h \notin \overline{K}^{nc}$.Then for each $G \in ncO(H)$containing h, we find that $K \cap G = \varphi$, then $h \in G \subset H\backslash K$ .Thus, $h \in ncInt(H\backslash K)$.

Conversely, we can prove these part by reverse the above steps .

Similarly, the other branch can be proved .

<u>Conclusion</u> As a factual information , this article has some important result which wrote as follows (i) Every nc-open subset of H is n-open but the opposite is not always true.

(ii) The union of two nc-open does not need to be nc-open .

(iii) The intersection of two nc-closed does not have to be nc-closed .

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