Finite Element Analysis of Fins with Convection and Radiation Heat Transfer

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Received 10 Novmber 2014 Accepted 25 March 2015

ABSTRACT

One dimensional analysis of heat transfer from fin is simple when only heat transfer by convection from fin surface. However, in some cases heat transfers by combined phenomena (convection and radiation), this makes the governing differential equation of heat transfer non-linear and the analytical solution becomes hard to solve. This paper uses an approximate method to solve the problem with high-quality results based on finite element technique. The method can be used for different types of fins and for different shapes. The results show suitable agreement between semi-exact solution and numerical solution.

KEY-WORDS: Heat Exchanger, Fins, Radiation, Finite Element.

تحليل انتقال الحرارة من الزعانف بالحمل والإشعاع باستخدام طريقة العناصر المحددة

الدكتور محمد حمزة عبد السادة

دكتوراه في الهندسة الميكانيكية

الخلاصة

التحليل ببعد واحد لانتقال الحرارة للزعنفة بسيط عندما تنتقل الحرارة من الزعنفة بواسطة الحمل فقط ولكن عندما تنتقل الحرارة من الزعنفة بواسطة التأثير المشترك للحمل والإشعاع، المعادلة التفاضلية لانتقال الحرارة سوف تصبح غير خطية وسوف يكون الحل التحليلي لها صعب.هذه الورقة استعملت طريقة تقريبية لحل هذه المشكلة مع ظهور نتائج جيدة. والتي هي تعتمد أساسا على تقنية العناصر المحددة. أن الطريقة المستخدمة يمكن أن تطبق على أنواع مختلفة وأشكال مختلفة من الزعانف. النتائج تشير الى توقف مناسيب بين الحل التحليلي والحل العددي

Notation

A	Area	m^2	T	Temperature	$^oC, K$
A_c	Cross section Area	m^2	T_{av}	Average Temperature	$^oC, K$

A_s	Surface Area	m^2	T_b	Base Temperature	$^oC, K$
h	Convection Heat Transfer Coefficient	$W/m^{2o}C$	T_L	End Temperature	$^oC, K$
h_r	Radiation Heat Transfer Coefficient	$W/m^{2o}C$	T_∞	Environment Temperature	$^oC, K$
k	Thermal Conductivity	W/m^oC	X	Space Coordinates in Cartesian system	m
L	Length	m			
P	Perimeter	m	ε	Emissivity	
Q	Heat Transfer rate	J/s	ξ	Shape Factor	
q_{conv}	Convection Heat Transfer rate	J/s	θ	Temperature Difference	$^oC, K$
q_{rad}	Radiation Heat Transfer rate	J/s	σ	Stefan Boltzmann constant	W/m^2K^4
Q	Heat Transfer	J	Subscript		
t	Thickness	m	X	Denotes some local position with respect to x coordinates	m

1.Introduction:

It has been recognized that use of fins can facilitate an augmentation of the heat exchange between two fluids which are separated by solid interface and they are used in many engineering applications to improve heat transfer from a surface to its surroundings. In this project a simple one-dimensional model will developed for the heat flow in a fin of arbitrary shape. The resulting governing differential equation will be solved approximately in some simple cases.Gaurav[1]has been shown method based on finite volume analysis with approximate results. This work tries to improve the results previously through the implementation of finite element method.

This paper treats situations for which heat is transferred by diffusion under one-dimensional steady-state conditions. The term “one-dimensional” refers to the fact that only one coordinates is needed to describe the spatial variation of the dependent variables.

Hence, in a one-dimensional system, temperature gradient exits along only a single coordinate direction and heat transfer occurs exclusively in that direction. The system is characterized by steady-state conditions if the temperature at each point is independent of time. Despite their inherent simplicity, one dimensional steady-state models may be used to accurately represent numerous engineering systems.

It can be assumed that the heat transfer from fins is one-dimensional steady-state heat transfer, and the heat dissipates from fins by convection and radiation to the environment, the governing differential equation can be solved exactly if heat transfer by convection to the surrounding but when heat transfer by both convection and radiation, exact solution of the non-linear differential equation will be so difficult and in some cases impossible according to the boundary condition of the case under analysis.

Finite element solution[2-3] is the method used here to find the temperature distribution a long fin length and the amount of heat transfer from the fin of the compound effect of convection and radiation heat transfer.

2.Theoretical Analysis :

To determine the heat transfer rate associated with a fin, firstly mustobtained the temperature distribution along the fin. Starting performing an energy balance on an appropriate differential element[4-5]. Consider the extent surface on Fig. (1).

The analysis is simplified if certain assumptions are made. Choosing one-dimension conditions in the longitudinal (x) direction, even though conduction within the fin is actually two dimensional.
The rate at which energy transfers to the environment by compound effect convection and radiation from any point on the fin surface must be balanced by the rate at which energy reaches that point due to the conduction in transverse (y, z) direction. Practically the fin is thin and temperature changes in the longitudinal direction are much larger than the transverse direction. Hence, Itmay be assumed one-dimensional conduction in the (x) direction. It will be considered steady-state conditions and also it assumed that the thermal conductivity is constant, that the heat generation effects are absent, the convection heat transfer coefficient (h) is uniform over the surface, the surface emitted as a blackbody, thus the emissivity is constant andequal unity ($\varepsilon = 1$).
Applying the conversation of energy requirement to the differential element of Fig. (1), obtain[4-5]:

$$q_x = q_{x+dx} + dq_{conv} + dq_{rad} \tag{1}$$

From Fourier's law

$$q_x = -kA_c \frac{dT}{dx} \tag{2}$$

Since the conduction heat rate at ($x + dx$) may be expressed as

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx \tag{3}$$

It follows that

$$q_{x+dx} = -kA_c \frac{dT}{dx} - kA_c \frac{d}{dx}\left(\frac{dT}{dx}\right)dx \tag{4}$$

The convection heat transfer rate may be expressed as

$$dq_{conv} = h\, dA_s (T - T_\infty) \tag{5}$$

The radiation heat transfer rate may be expressed as

$$dq_{rad} = \varepsilon\sigma\, dA_s \left(T^4 - T_\infty^4\right) \tag{6}$$

Substituting the forgoing rate equations into the energy balance equation (1), obtain

$$kA_c \frac{d^2T}{dx^2} - h\frac{dA_s}{dx}(T - T_\infty) - \varepsilon\sigma \frac{dA_s}{dx}\left(T^4 - T_\infty^4\right) = 0 \tag{7}$$

2.1. Semi- Exact Solution

Equation (7) is difficult to be solved exactly unless some assumptions are made to overcome this difficulty[6].
The heat transfer by radiation from the fin surface in the equation (6) can be written in the form

$$dq_{rad} = \varepsilon\sigma\ dA_s\left(T^2 + T_\infty^2\right)\left(T + T_\infty\right)\left(T - T_\infty\right) \tag{8}$$

and it can be assumed that the temperature on two first term approximately equal to the average temperature of the fin surface $(T = T_{av})$, thus equation (8) can be write

$$dq_{rad} = \varepsilon\sigma\ dA_s\left(T_{av}^2 + T_\infty^2\right)\left(T_{av} + T_\infty\right)\left(T - T_\infty\right) \tag{9}$$

Substituting equation (9) into equation (7)

$$kA_c\frac{d^2T}{dx^2} - \left(h + \varepsilon\sigma\left(T_{av}^2 + T_\infty^2\right)\left(T_{av} + T_\infty\right)\right)\frac{dA_s}{dx}\left(T - T_\infty\right) = 0 \tag{10}$$

It is necessary to be more specific about the geometry. Two cases of fins rectangular will analysis and pin of uniform cross sectional area. It can be observed:

1. Each fin is attached to a base surface of temperature $T(0) = T_b$ and extent into a fluid of temperature T_∞.
2. for the prescribed fins, A_c is a constant and,
3. $A_s = Px$ where A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly with $dA_s/dx = P$.
4. Radiation heat transfer coefficient $h_r = \varepsilon\sigma\left(T_{av}^2 + T_\infty^2\right)\left(T_{av} + T_\infty\right)$

Equation (10) can be modified know to the form

$$\frac{d^2T}{dx^2} - \left[\frac{(h + h_r)P}{kA_c}\right]\left(T - T_\infty\right) = 0 \tag{11}$$

To simplify the form of this equation, transforming the dependent variable by defining an excess temperature (θ) as

$$\theta(x) = T(x) - T_\infty \tag{12}$$

Where, since T_∞ is a constant $d\theta/dx = dT/dx$ substituting equation (12) into equation (11), obtain

$$d^2\theta/dx^2 - m^2\theta = 0 \tag{13}$$

Where

$$m^2 = \left[\frac{(h + h_r)P}{kA_c} \right] \tag{14}$$

Equation(13) is a linear, homogeneous second order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{15}$$

To evaluate constants C_1, C_2 of equation (15) it is necessary to specify appropriate boundary conditions. Such conditions may be specified in term of temperature
at $x = 0$ $\quad$ $\mathrm{T} = \mathrm{T_b}$ and $\theta_b = \theta(0) = T_b - T_\infty$
There are four cases:

CASE A

Convection and Radiation heat transfer from the tip at $x = L$ can be written in the form

$$(h + h_r)(T - T_\infty) = -k \left. \frac{dT}{dx} \right|_{x=L} = -k \left. \frac{d\theta}{dx} \right|_{x=L} \tag{16}$$

assume $h_t = (h + h_r)$ (17)

thus re-write equation (16) $h_t (T - T_\infty) = -k \left. \frac{dT}{dx} \right|_{x=L}$ (18)

by solving equation (15) the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h_t / mk) \sinh m(L - x)}{\cosh mL + (h_t / mk) \sinh mL} \tag{19}$$

applying Fourier's law at the fin base in order to find heat transfer from the fin

$$q_f = q_b = -kA_c \left. \frac{dT}{dx} \right|_{x=0} \tag{20}$$

thus,

$$q_f = M \frac{\sinh mL + (h_t / mk) \cosh mL}{\cosh mL + (h_t / mk) \sinh mL} \tag{21}$$

where $\quad M = \sqrt{h_t P k A_c}\,(T_b - T_\infty)$ (22)

CASE B

Convection and Radiation heat transfer from the tip at $x = L$ is negligible, which means thatthe tip is adiabatic and the second boundary condition is:

$$\left.\frac{dT}{dx}\right|_{x=L} = -k\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \tag{23}$$

by using the boundary conditions and solving equation (15), the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL} \tag{24}$$

and fin heat transfer rate is

$$q_f = M \tanh ml \tag{25}$$

CASE C

Prescribed Temperature at the tip of the fin $x = L$ and the second boundary condition is

$$\theta_{\mathrm{L}} = \theta(L) = T_L - T_\infty \tag{26}$$

using equation (15) with this boundary conditions, the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL} \tag{27}$$

and the fin heat transfer is

$$q_f = M\left(\frac{\cosh mx - (\theta_L/\theta_b)}{\sinh mL}\right) \tag{28}$$

CASE D

The very long fin (infinite fin) is an interesting extension of these results. In particular, as $x \to \infty, \theta_L \to 0$ and it is easy verify that

$$\frac{\theta}{\theta_b} = e^{-mx} \tag{29}$$

and the fin heat transfer is

$$q_f = M \tag{30}$$

After summarizing the general cases that occur in fins, a computer program was developed to calculate the temperature distribution along the fin and the rate of heat transfer at the end of the fin. The following flow chart Fig. (2) explain were followed:

2.2. Finite Element Solution

In the above solution using semi-exact solution, now consider the method of finite element solution for the same problem using 2-nodes element[2-3, 6-7].
The general equation of the heat transfer from the fins can be write in the following form

$$\chi = Q_2 T_2 - Q_1 T_1 + \int_0^L \frac{A_c k}{2} \left(\frac{dT}{dx} \right)^2 dx - \int_0^L H(x) T(x) dx \tag{31}$$

the general solution of one-dimensional rod by using 2-nodes elementas shown in Fig. (3) can be satisfy through the following analysis.

assume that the temperature distribution through the rod can be write

$$T(x) = \sum_{i=1}^{2} T_i N_i \tag{32}$$

$$N_i = \prod_{\substack{j=1 \\ i \neq j \\ i=1}}^{n} \frac{x - x_j}{x_i - x_j} \tag{33}$$

assume that the shape factor is $\xi = \frac{x}{L}$ (34)

substitution in equation (33) for three node, find the node element as a function of ξ

$$N_1(x) = N_1(\xi) = (1 - \xi) \tag{35}$$

$$N_2(x) = N_2(\xi) = \xi \tag{36}$$

the temperature distribution along fin can be write in matrix form as flows

$$T(x) = \underline{T}^t N = \underline{N}^t T = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{37}$$

from equation (34), can write $dx = L d\xi$
now substitute the above assumption into equation (31) and can be write in the form

$$\chi = -\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ -Q2 \end{bmatrix} + \frac{A_c k}{L} \int_0^1 \frac{1}{2} \left(\frac{dT}{d\xi} \right)^2 d\xi - L \int_0^1 H(\xi) T(\xi) d\xi \tag{38}$$

$$\frac{dT}{d\xi}=\begin{bmatrix}T_1 & T_2\end{bmatrix}\begin{bmatrix}\dfrac{dN_1}{d\xi}\\ \dfrac{dN_2}{d\xi}\end{bmatrix} \tag{39}$$

$$\left.\begin{aligned}\frac{dN_1}{d\xi}&=-1\\ \frac{dN2}{d\xi}&=1\end{aligned}\right\} \tag{40}$$

equation (38) can be write as flows

$$\chi=-\underline{T}^t\,\underline{Q}+\frac{A_ck}{L}\int_0^1\frac{1}{2}\underline{T}^t\,\underline{R}\underline{R}^t\,\underline{T}d\xi-L\int_0^1\underline{H}(\xi)\underline{T}^t\,\underline{N}d\xi \tag{41}$$

where $R=\begin{bmatrix}\dfrac{dN_1}{d\xi}\\ \dfrac{dN_2}{d\xi}\end{bmatrix}$ (42)

according to theorems

I. $\frac{\partial}{\partial\omega}\left(\underline{\omega}^t\,\underline{B}\right)=\underline{B}$ (43)

II. $\frac{\partial}{\partial\omega}\left(\frac{1}{2}\underline{\omega}^t\,\underline{B}\underline{\omega}\right)=\underline{B}\underline{\omega}$ (44)

where$B$ issquare matrix

when $\frac{\partial\chi}{\partial T}=0$ equation (41) will become $0=-\underline{Q}+\frac{A_ck}{L}\int_0^1\underline{B}\underline{T}d\xi-L\int_0^1\underline{H}(\xi)\underline{N}d\xi$

rearrange the equation above

$$\frac{A_ck}{L}\int_0^1\underline{B}\underline{T}d\xi=\underline{Q}+L\int_0^1\underline{H}(\xi)\underline{N}d\xi \tag{45}$$

having three terms in the above equation, will treat with them individually

$$L.H.T=\frac{A_ck}{L}\int_0^1\underline{B}\underline{T}d\xi \tag{46}$$

where matrix $\underline{B} = \underline{R}^t \underline{R}$ (47)

and thus the symmetric matrix equal to

$$B = \begin{bmatrix} \left(\frac{dN_1}{d\xi}\right)^2 & \frac{dN_1}{d\xi}\frac{dN_2}{d\xi} \\ \frac{dN_2}{d\xi}\frac{dN_1}{d\xi} & \left(\frac{dN_2}{d\xi}\right)^2 \end{bmatrix} \tag{48}$$

substituting the above matrix in equation (46) and after integration the equation become

$$L.H.T = \frac{A_c k}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{49}$$

the other term of the equation (45) can be write in the following equation

$$\underline{Q} = \begin{bmatrix} Q_1 \\ -Q_2 \end{bmatrix} \tag{50}$$

and the other term can be write

$$R.H.T = L\int_0^1 \underline{H}(\xi)\underline{N}d\xi \tag{51}$$

The heat transfer from the fin surface divided in two main methods convection and radiation, thus re-write the equation (51) in form

$$R.H.T = L\int_0^1 \{\underline{HC}(\xi) + \underline{HR}(\xi)\}\underline{N}d\xi \tag{52}$$

$$R.H.T = L\int_0^1 \left\{hP(T_\infty - T(\xi)) + \varepsilon\sigma P\left(T_\infty^4 - T^4(\xi)\right)\right\}\underline{N}d\xi$$

$$R.H.T = hPL\int_0^1 (T_\infty - T(\xi))\underline{N}d\xi + \varepsilon\sigma PL\left(\left(T_\infty^2 + T_{av}^2\right)(T_\infty + T_{av})\right)\int_0^1 (T_\infty - T(\xi))\underline{N}d\xi$$

$$R.H.T = \{hPL + h_r PL\}\int_0^1 (T_\infty - T(\xi))\underline{N}d\xi$$

$$\gamma = PLh_t \tag{53}$$

$$R.H.T = \gamma\left\{T_{\infty}\int_{0}^{1}\underline{N}d\xi - \int_{0}^{1}\underline{N}^{t}\,\underline{N}\underline{T}d\xi\right\}$$

after integration the above equation the right hand term can be write in the following form

$$R.H.T = \gamma\left\{\frac{T_{\infty}}{2}\begin{bmatrix}1\\1\end{bmatrix} - \frac{1}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}T_1\\T_2\end{bmatrix}\right\} \tag{54}$$

substitution the equations (49),(50)and(54) into equation(45), the final equation as follows

$$\frac{A_c k}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}T_1\\T_2\end{bmatrix} = \begin{bmatrix}Q_1\\-Q_2\end{bmatrix} + \gamma\left\{\frac{T_{\infty}}{2}\begin{bmatrix}1\\1\end{bmatrix} - \frac{1}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}T_1\\T_2\end{bmatrix}\right\} \tag{55}$$

$$\left\{\frac{A_c k}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix} + \frac{\gamma}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\right\}\begin{bmatrix}T_1\\T_2\end{bmatrix} = \begin{bmatrix}Q_1\\-Q_2\end{bmatrix} + \frac{\gamma T_{\infty}}{2}\begin{bmatrix}1\\1\end{bmatrix} \tag{56}$$

equation (56) the final equation represent the finite element solution for two node elements , after summarizingthe solution, below the general steps to find the temperature distribution and heat transfer from the fin explained in flow chart Fig.(4).

The steps above are general and can be slightly differingif use 3-nods element or more than two elements and the detail of the solution can be differ also. So, using the 3-nodes element,depend upon the same analysis above the final equation of three nodes element is

$$\left\{\frac{A_c k}{3L}\begin{bmatrix}7 & -8 & 1\\-8 & 16 & -8\\1 & -8 & 7\end{bmatrix} + \frac{\gamma}{60}\begin{bmatrix}8 & 4 & -2\\4 & 32 & 4\\-2 & 4 & 8\end{bmatrix}\right\}\begin{bmatrix}T_1\\T_2\\T_3\end{bmatrix} = \begin{bmatrix}Q_1\\Q_2\\-Q_3\end{bmatrix} + \frac{\gamma T_{\infty}}{6}\begin{bmatrix}1\\4\\1\end{bmatrix} \tag{57}$$

3.Case Study

Heat flow through a copper rectangular fin has a thermal conductivity 400W/mK, shown in Fig. (5), having a thickness 0.01m, width 0.01m and length 0.2m is attached to a boiler surface having a temperature of $100\,^{o}C$. The fin is exposed to ambient air and its convicting and radiating heat to the environment at temperature of $25\,^{o}C$, and convection heat transfer coefficient along the length and the end is 20W/m^2K, the fin emissivity 1 and the Stefan-Boltizmann constant 5.7E-08 W/m^2K^4. Determine the temperature distribution and the heat transfer from the fin.Moreover, change the fin metal by

1. Aluminum(k=180W/mK)
2. Stainless steel (k=14W/mK)

This change wouldaffectthetemperature distribution along fin length. To solve the case study in semi-exact using the general step in section 2.1 and according to the boundary conditions the problem is case A and in finite element solution has been used the steps in section 2.2 and solve in 2-nodes two elements and 3-nods one element.

4.Results and Discussions

After following the procedure above, first table (1) shows the temperatures along fin length in three points.These results represent all cases of our analysis and it can be seenthat exact method in first column with convection heat transfer only. The results shows the temperature distribution along the fin, it observed that the effect of radiation can be clear if temperature of the base is high and with a greater value of emissivity and this factis clear from equation (17). The case study indicted from table (1) there is a small difference between the temperature values with convection heat transfer and with those with compound effect of convection and radiation heat transfer and this can be shown in Fig. (6).
It is clear from table (1) that the methods used to calculate combined effect of convection and radiation heat transfer give good results especially between semi-exact method and with finite element method used 3-nods with those two methods slightly differ from the other finite element method used 2-nods with two elements and this clearly shown in table (1) and Fig. (6).
Fig. (7) shows the effect of changing fin materials and it is clear from Fig. the heat transferred from stainless steel fin is larger than those of copper and aluminum and this difference belong to thermal conductivity differences between materials of fins.
The effect of changing emissivity can be shown in Fig. (8) and it is clear the temperature decrease with increase of emissivity value because of increase of emissive heat by radiation from the fin surface especially when the distance from the base increase.

5. Conclusion

Radiation heat transfer is an important factor in heat transfer calculation and its value effect is differing according the amount of this value and the interest of application.
This paper trying to indicate the radiation effect on fins calculation, the value of radiation depend on the emissive of the body surface and temperatures of surface body and environment when any of those variables increase the radiation heat transfer coefficient increase that mean radiation heat transfer increase.
The method used in analysis is good and give good results and there is good approximation between semi-exact solution and numerical solution and from the result, notice the finite element technique with 3-nodes element give result nearest from exact results.
The value of thermal conductivity increase the heat transfer from the fins when decrease.
It can be neglected the radiation value if the base temperature of the fin is low and the emissivity of the fin surface is nears zero.
The important things in fins surface must be emissive because of high emissivity give a great amount of heat radiation transfer from the fin.

6. References:

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Table (1): shows the results of exact, semi-exact and numerical solutions.

	Convection	**Convection and Radiation Heat Transfer**		
Temperature	Exact solution without radiation	Semi-exact solution	FEM,2-nodes, 2 elements	FEM, 3-nodes, 1 element
$T_1(^oC), x_1 = 0m$	100	100	100	100
$T_2(^oC), x_2 = 0.1m$	82.6973	77.89197	77.53397	77.87332
$T_3(^oC), x_3 = 0.2m$	77.12766	70.96069	70.5105	70.99628
Total Heat Transfer from the Fin $Q(W)$	9.646632	12.55253	11.7958	12.55664

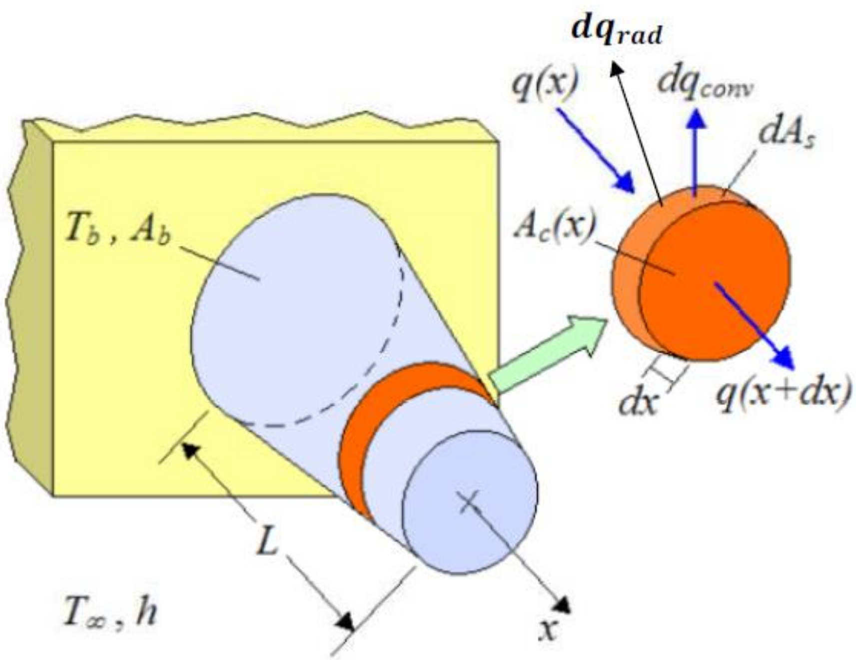


Figure (1): Energy Balance of one-Dimensional Conduction, Convection and Radiation through a fin.

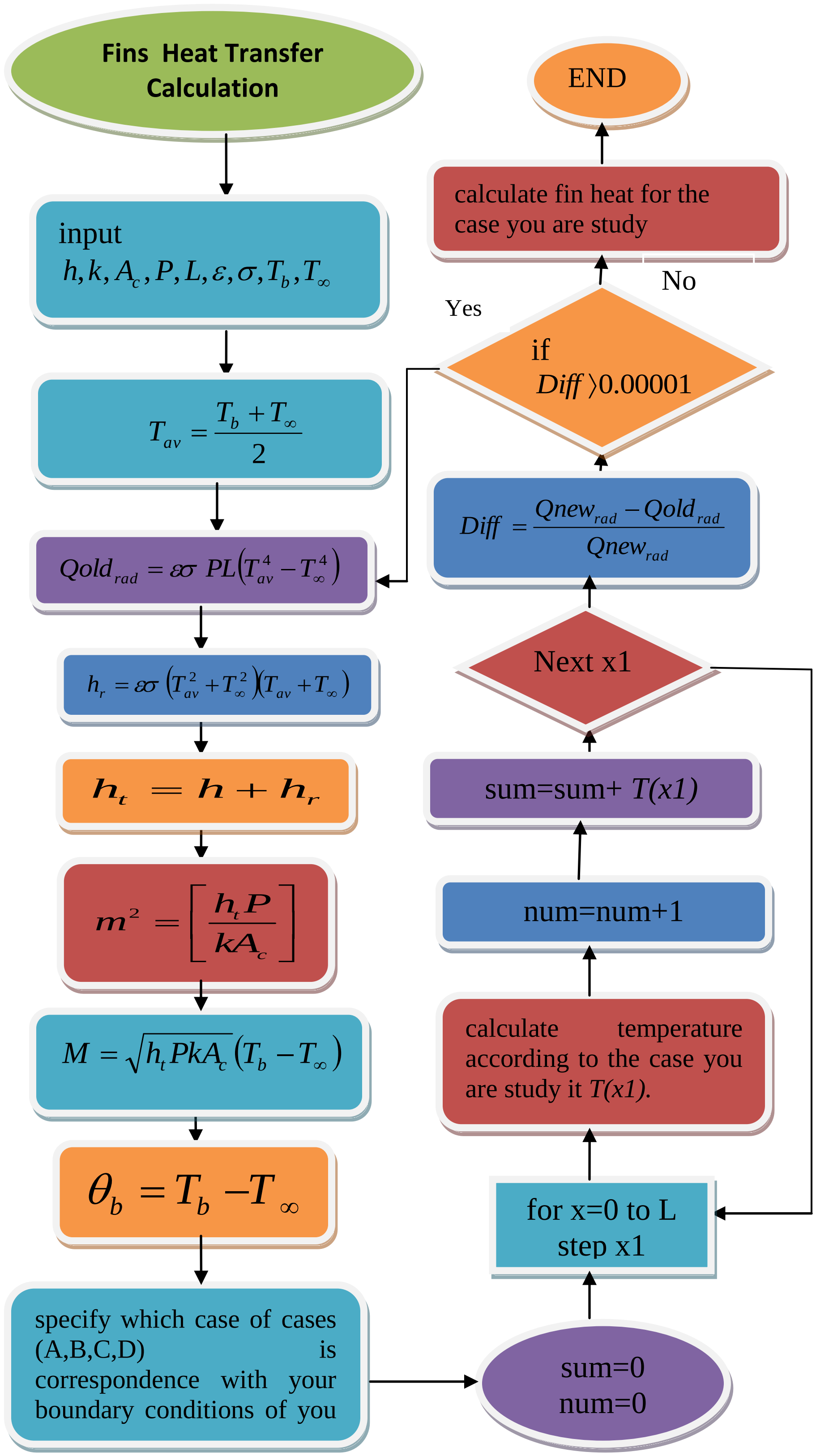


Figure (2): Flow Chart of Fins Heat Calculation

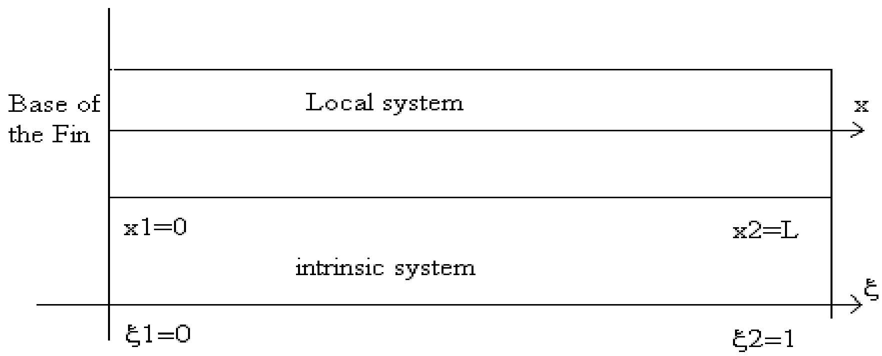


Figure (3): Local system and intrinsic system for one-dimensional rod

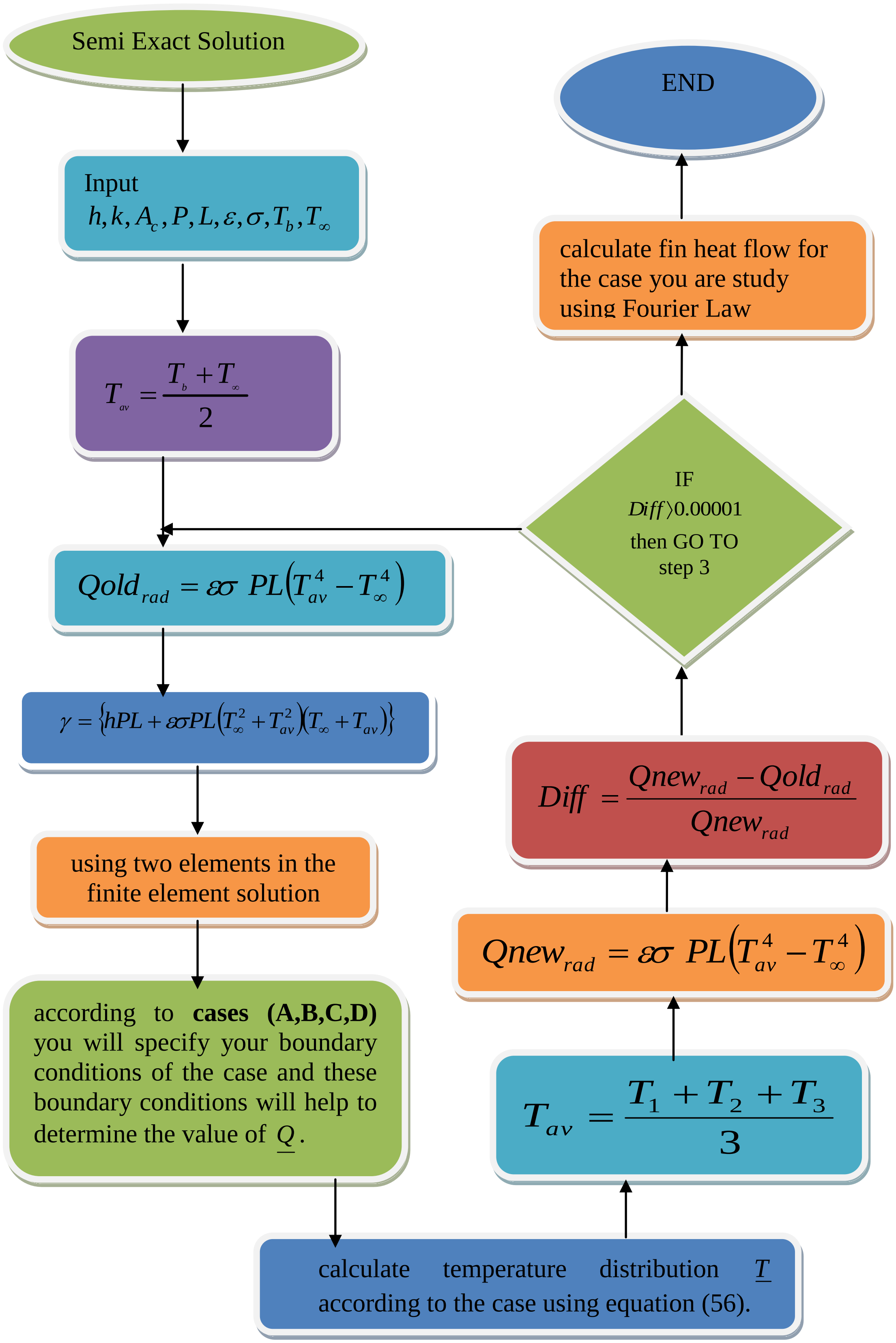
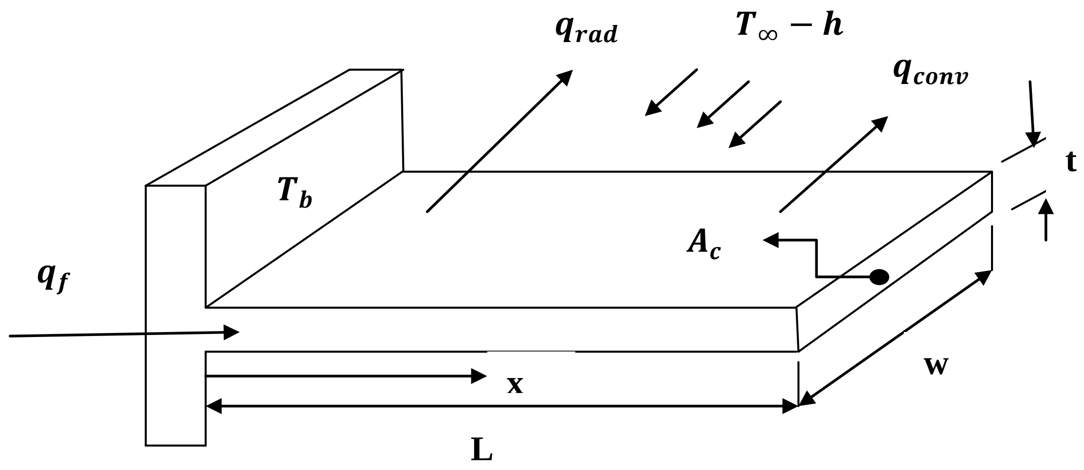


Figure (4): Flow Chart of Fins Heat Calculation (FEM)



$$P = 2w + 2t$$
$$A_c = wt$$

Figure (5): Rectangular Fin of the Case Study

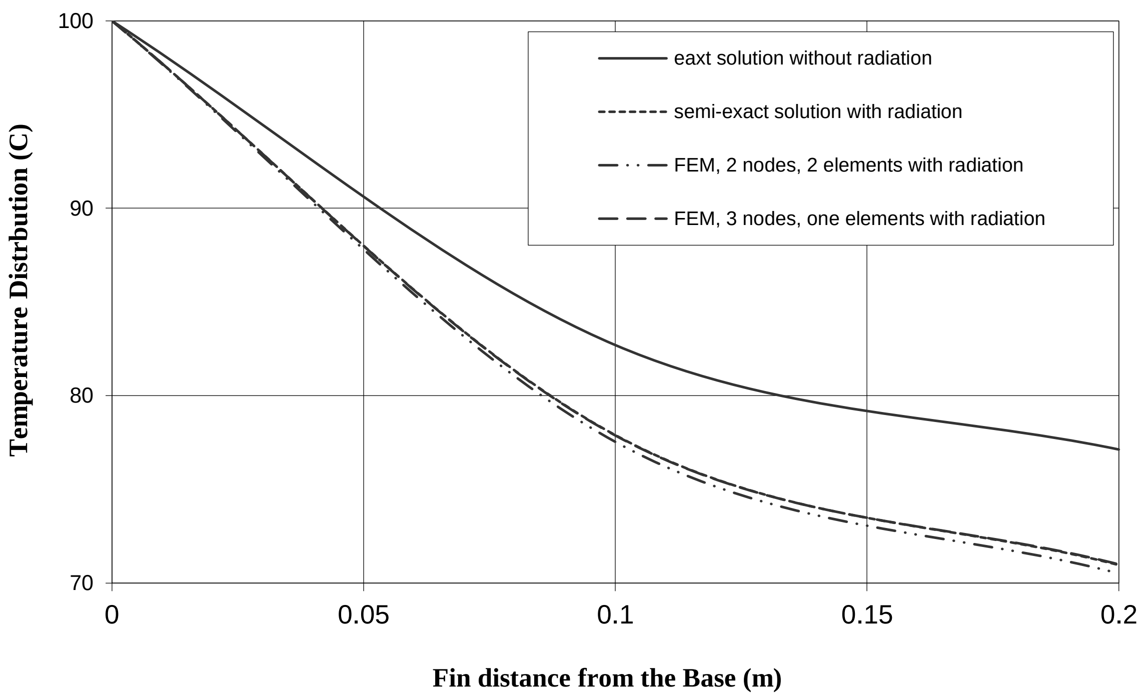


Figure (6): Temperature Distrbution Alonge The Fin Length

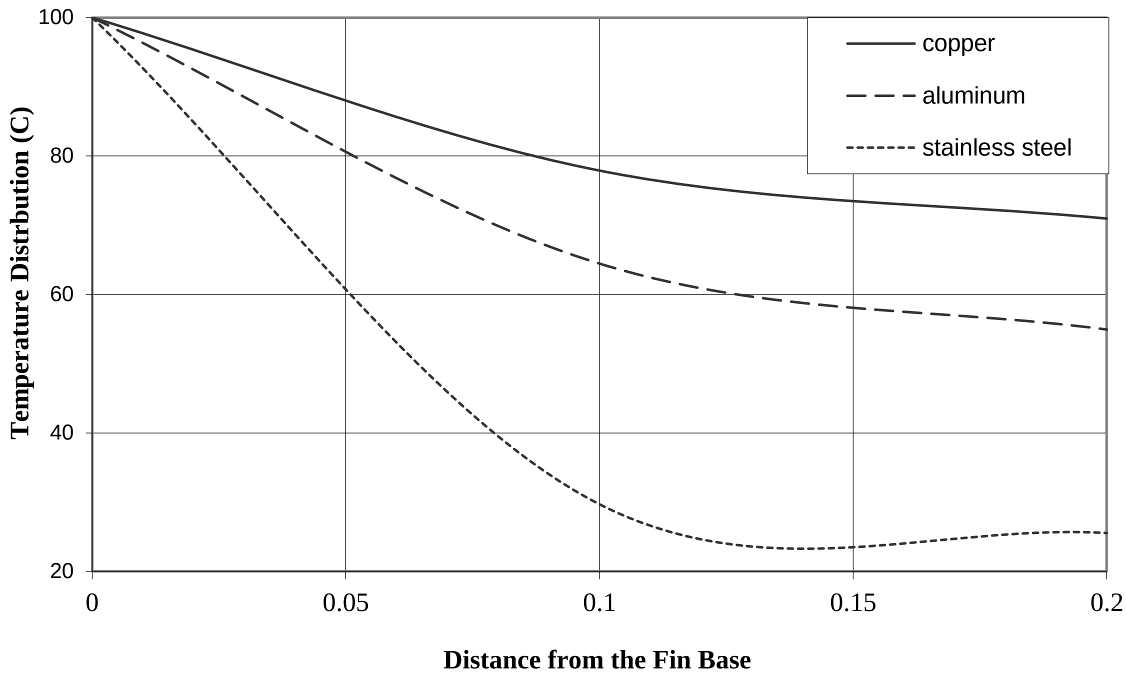


Figure (7): Temperature Distrbution of different fin materials

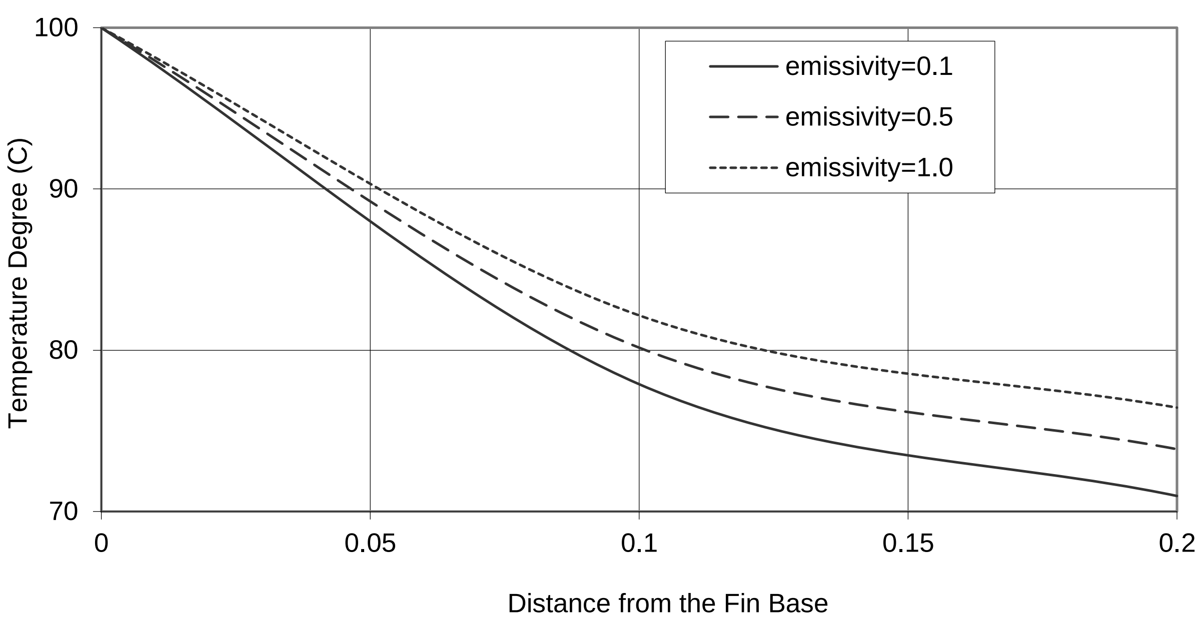


Figure (8): Temperature Distrbution of Different Value of Emissivity for Aluminum