CONTRACTIBILITY OF BIPARTITE GRAPHS

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Abstract

Akram[2006] introduced the concept of contractible class. He proved that the classes of Hamiltonian and 3-connected graphs as well as the class of trees are contractible classes. Further he introduced the concept of contractibility number and characterized regular graphs having contractibility number less equal two. In this work we characterized bipartite graphs having contractibility number less equal two.

Key words: Contraction, Bipartite graphs, $n-$connected graphs, Reducibility, and Connectivity

قابـــليـــة انكـــماش البيـــانات ثنـــائية التجـــزئة

اكـــرم بــرزان عطـــار

جـــامـعــــة ذي قـــــار

كلـيـــــة الـعـلـــوم

قســـم الحاسـبـات

عراق ذي قار

المـلـــخـص

في هذا البحث اثبتنا بان البيانات ثنائية التجزئة هي بيانات غيرقابلة للانكماش ثم ميزنا البيانات ثنائية التجزئة التي لها عدد انكمـاش اقل او يساوي اثنـــان.

1. Introduction

A *graph* $G=(V(G),E(G))$ consists of two finite sets, $V(G)$, the *vertex set* of the graph, often denoted by just V, which is a nonempty set of elements called *vertices*, and E(G), the *edge set* of the graph, often denoted by just E, which is a possibly empty set of elements called *edges*, such that each edge e in E is assigned an unordered pair of vertices (u,v) called the *end vertices* of e.The number of vertices of G will be called the *order* of G, and will usually be denoted by p; the number of edges of G will generally be denoted by q. If for a graph G, $p=1$ then G is called *trivial graph*; if $q=0$ then G is called a *null graph*. We shall usually denote the edge corresponding to (v,w) where (v and w are vertices of G) by vw.

If e is an edge of G having end vertices v,w then e is said to *join* the vertices v and w, and these vertices are then said to be *adjacent*. In this case, we also say that e is *incident* to v and w, and that w is a *neighbor* of v. The *open neighborhood* $N(v)$ of the vertex v consists of the set of vertices adjacent to v, that is $N(v)=\{w\in V: vw\in E\}$. An *independent set of vertices* in G is a set of vertices of G no two of which are adjacent. If two distinct edges are incident with a common vertex, then they are *adjacent edges.* An *independent set of edges* in G is a set of edges of G no two of which are adjacent.

Let v be a vertex of the graph G. If v joined to itself by an edge, such an edge is called *loop.* The degree $d(v)$ is the number of edges of G incident with v, counting each loop twice. If two (or more) edges of G have the same end vertices then these edges are called *parallel.* A graph is called *simple* if it has no loops and parallel edges.

The *underlying simple graph* of G is obtain by deleting from a graph G all loops and in each collection of parallel edges all edges but one in the collection. A simple graph in which every two vertices are adjacent is called a *complete graph*; the complete graph with p vertices is denoted by K_p.

A *bipartite graph* is one whose vertex set can be partitioned into two subsets in such away that each edge joins a vertex of the first subset to a vertex of the second subset.

A *walk* in a graph G is a finite sequence $W=v_0e_1v_1\ e_2...v_{k-1}e_kv_k$ whose terms are alternatively vertices and edges such that for $1\le i\le k,$ the edge e_i has ends v_{i-1} and v_i. The vertex v_0 is called the *origin* of the walk W, while v_k is called the *terminus* of W. The vertices $v_1,...,v_{k-1}$ in the above walk W are called *internal vertices.* If the edges $e_1,e_2,...,e_k$ of the walk $W=v_0e_1v_1\ e_2...v_{k-1}e_kv_k$ are distinct then W is called a *trail* and if $v_0=v_k$ then W is called a *closed trail*. If the vertices $v_0,v_1,...,v_k$ of the walk $W=v_0e_1v_1\ e_2...v_{k-1}e_kv_k$ are distinct then W is called a *path.* A closed trail in a graph G is called a *cycle* if its origin and internal vertices are distinct.

A graph G is *connected* if there is a path joining each pair of vertices of G; a graph which is not connected is called *disconnected*. A connected graph which contains no cycle

is called a ***tree.*** **A graph G is *Hamiltonian* if it has a cycle which includes every vertex of G. The *vertex connectivity* of G, denoted $\kappa(G)$ is the smallest number of vertices in G whose deletion from G leaves either a disconnected graph or K_1. A simple graph G is called n*-connected* (where $n \geq 1$) if $\kappa(G) \geq n$.**

For the undefined concepts and terminology we refer the reader to Wilson[1978], Clark[1991], Harary[1969], West[1999] and Tutte[1984].

All graphs throughout this paper are simple and connected.

2. Contractibility

Definition 2.1(Clark[1991]): Let $e = uv$ be an edge of the graph G, let d(u)=k and d(v)=l and let N(u)={v, u_1, u_2,..., u_{k-1}} and N(v)={u, v_1, v_2,..., v_{l-1}}. A *contraction* on the edge e change G to a new graph $G * e$ where

$$V(G * e) = (V(G) - \{u, v\}) \bigcup \{w\},$$

$$E(G * e) = E(G - \{u, v\}) \bigcup \{wu_1, ..., wu_{k-1}, wv_1, ..., wv_{l-1}\}$$

and w is a new vertex not belonging to G. If $N(u) \bigcap N(v) \neq \phi$ then the contraction on the edge $e = uv$ will create parallel edges incident with w. In this case we delete all but one of the edges in a collection of parallel edges. This amounts to taking the underlying simple graph. The definition is simple all that really happens is that u and v become fused (identified) as the one vertex w and the edge e is removed. Any edges joining u to u_i are replaced by edges joining w to u_i. Similarly, any edges joining v to v_j are replaced by edges joining w to v_j.

We can see from Definition 2.1 that the contraction preserve the connectedness. That is, if G is connected graph, $e = uv \in E(G)$. Then $G * e$ is connected graph. This fact was proved in Akram[2006].

We need the following definitions in Akram[2006].

Definition 2.2: Let $\Re$ be a class of graphs satisfying certain property P.

The class $\Re$ is called *contractible class* if for any nontrivial graph $G \in \Re, \exists e \in E(G)$ such that $G * e \in \Re$.

He proved that the class of trees is contractible class; the class of Hamiltonian graphs is contractible class; and the class of 3-connected graphs with at least five vertices is contractible class. Further, he proved that the class of regular graphs is not contractible class.

Definition 2.3: Let $\Re$ be a class of graphs and $G \in \Re$. The contractibility number of G with respect to $\Re$ is the smallest positive integer number of edges m, if exists, such that G contains a subset S of edges with cardinality m and $G * S \in \Re$. We write $\underset{\Re}{cont}(G) = m$. If such a number does not exist for G, then we say that the corresponding contractibility number is ∞.

One can immediately see that the class of graphs is contractible if and only if $\underset{\Re}{cont}(G) = 1$, $\forall G \in \Re$.

3. Contractibility of Bipartite Graphs

In this section, we characterized bipartite graphs having contractibility number less than equal 2.

Proposition 3.1: The class of bipartite graphs is not contractible class.

Proof: Consider a bipartite graph G in which every edge belongs to some cycle. By using the fact that every cycle in any bipartite graph is even cycle, then the contraction on any edge in G gives a new graph contains an odd cycle. That is the new graph is not bipartite graph and the proof follows.

Lemma 3.2: Let G be a bipartite graph with order p >2. If $v \in V(G)$ and $d(v) = k$, then the sequence of contractions on the set of edges {$e_1, e_2, ..., e_k$} which are incident in v gives a bipartite graph.

Proof: As G is bipartite graph, the vertex set V of G can be partitioned into two subsets V_1 and V_2 such that every edge of G join V_1 with V_2. Suppose that $v \in V_1$ and adjacent to k vertices in V_2 by the edges $e_1, e_2, ..., e_k$. The contraction on the edge e_1 gives a new vertex w_1 which is adjacent to $k-1$ vertices in V_2 and some vertices in V_1. As such the contraction on the edge e_2 which is join w_1 to a vertex in V_2 gives a new vertex w_2. We can observe that w_2 is adjacent to $k-2$ vertices in V_2 and some vertices in V_1. By the contraction on the edge e_3 which is join w_2 to a vertex in V_2, we get a new vertex w_3. We can see that w_3 is adjacent to $k-3$ vertices in V_2 and some vertices in V_1. By continuing this process on the edges $e_4, ..., e_k$, we get a vertex w_k which is not adjacent to any vertex in V_2 and it is adjacent to some vertices in V_1. The new graph H which we got it from this process is bipartite graph.

Theorem 3.3: Let $\Re$ be the class of bipartite graphs, $G \in \Re$ with order p >2. Then $\underset{\Re}{cont}(G) = 1$ if and only if G contains an edge not belongs to any cycle.

Proof: Let G be a bipartite graph with order p >2.

Suppose that $\underset{\Re}{cont}(G) = 1$. There exists an edge $e \in E(G)$ such that $G * e \in \Re$.

As G is bipartite graph, the vertex set V of G can be partitioned into two subsets V_1 and V_2 such that every edge of G join V_1 with V_2.

Let $u \in V_1$, $v \in V_2$ be the end vertices of the edge $e = uv$ in G. The contraction on the edge $e = uv$ remove the vertices u, v and addition a new vertex w adjacent to those vertices to which u or v was adjacent. As G is connected bipartite graph with order $p > 2$, then we have two cases:

(i) the vertex w is adjacent to some vertices in V_1 or some vertices in V_2 but not both.

(ii) the vertex w is adjacent to some vertices in V_1 and some vertices in V_2.

Suppose that (i) holds. In this case, as $G * e$ is bipartite graph, then either u not adjacent to any vertex other than v and w belongs to V_2 or v not adjacent to any vertex other than u and w belongs to V_1. That is either u or v has a degree 1. Hence the edge $e = uv$ not belongs to any cycle.

Suppose that (ii) holds. In this case, if the edge $e = uv$ belongs to some cycle, then the new graph $G * e$ contains an odd cycle. That is $G * e$ is not bipartite graph (A graph is bipartite iff all it's cycle are even Hararry[1969, P.18]) a contradiction to our assumption. Hence the edge $e = uv$ not belongs to any cycle.

Conversely, suppose that the edge $e = uv$ not belongs to any cycle. As G is connected bipartite graph with order $p > 2$, then we have two cases:

(1) one of the end vertices of the edge $e = uv$ has a degree 1.

(2) each of the end vertices of the edge $e = uv$ has a degree greater than 1.

Suppose that (1) holds and G contains a vertex v of degree 1. Then by Lemma 3.2, the contraction on the edge $e = uv$ gives a bipartite graph. Hence $\underset{\Re}{cont}(G) = 1$.

Suppose that (2) holds. In this case, the contraction on the edge $e = uv$ remove the vertices u, v and addition a new vertex w adjacent to those vertices to which u or v was adjacent. As the edge $e = uv$ not belongs to any cycle, then $N(w)$ is independent set of vertices and for every vertex t in $V(G) - [N(u) \cup N(v)]$ each of the paths which are join t with a vertex in $N(w)$ has a length odd or each of them has a length even, otherwise we get a contradiction to our assumption.

Suppose that A_1 is a subset of vertices consists of w and every vertex h in $V(G) - [N(u) \cup N(v)]$ in which each of the paths which are join h with a vertex in

$N(w)$ **has a length odd;** A_2 **is a subset of vertices consists of** $N(w)$ **and every vertex** l **in** $V(G)-[N(u)\bigcup N(v)]$ **in which each of the paths which are join** l **with a vertex in** $N(w)$ **has a length even.**

We prove that A_1 **is independent set of vertices. Suppose that** A_1 **is not. Then there exist two adjacent vertices in** A_1**. But then we get a vertex in** A_1 **is join to a vertex in** $N(w)$ **by two paths one of them has a length even and the other has a length odd a contradiction to our assumption. Hence** A_1 **is independent set of vertices. By similar argument we get that** A_2 **is independent set of vertices. Hence** $G*e$ **is bipartite graph. Thus** $\underset{\Re}{cont}(G)=1$**.**

Theorem 3.4: Let $\Re$ **be the class of bipartite graphs,** $G\in\Re$ **with order** $p\geq 4$**. Then** $\underset{\Re}{cont}(G)=2$ **if and only if every edge in** G **belongs to some cycle and** G **contain two edges** $e_1=uv, e_2=xy$ **such that one of the following holds.**

1. **The smallest degree of** G **is 2.**
2. **The subset of vertices** $[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]-\{u,v,x,y\}$ **is independent set of vertices such that for every vertex** f **in** $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ **each of the paths which are join** f **with a vertex in** $[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]-\{u,v,x,y\}$ **has a length even or each of them has a length odd.**
3. **Each of the two subsets of vertices** $[N(u)\bigcup N(v)]-\{u,v\}$ **and** $[N(x)\bigcup N(y)]-\{x,y\}$ **is independent set of vertices such that for every vertex** h **in** $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ **each of the paths which are join** h **with a vertex in** $[N(u)\bigcup N(v)]-\{u,v\}$ **has a length even and each of the paths which are join** h **with a vertex in** $[N(x)\bigcup N(y)]-\{x,y\}$ **has a length odd or the converse.**

Proof: Let G **be a bipartite graph with order** $p\geq 4$**.**

Suppose that $\underset{\Re}{cont}(G)=2$**. There exists a set of two edges** $S=\{e_1,e_2\}\in E(G)$ **such that** $G*S\in\Re$ **and** S **is the smallest such set.**

Let u,v **be the end vertices of the edge** $e_1=uv$**; and** x,y **be the end vertices of the edge** $e_2=xy$**. We consider two cases:**

1. **the two edges** $e_1=uv, e_2=xy$ **are adjacent.**
2. **the two edges** $e_1=uv, e_2=xy$ **are independent.**

Case(1): The two edges $e_1=uv, e_2=xy$ **are adjacent.**

As G **is simple graph, then the two edges** $e_1=uv, e_2=xy$ **are adjacent in exactly one end vertex, say** $v=x$**.**

As G is bipartite graph, the vertex set V of G can be partitioned into two subsets V_1 and V_2 such that every edge of G join V_1 with V_2.

As $e_1 = uv, e_2 = xy$ are adjacent in the vertex $v = x$, the sequence of contractions on the set of edges $S = \{e_1, e_2\}$ remove the vertices u, v, y and addition a new vertex w adjacent to those vertices to which u, v or y were adjacent.

As G is connected bipartite graph with order $p \geq 4$, we have two subcases:

(i) the vertex w is adjacent to some vertices in V_1 or some vertices in V_2 but not both.

(ii) the vertex w is adjacent to some vertices in V_1 and some vertices in V_2.

Suppose that (i) holds. If the common end vertex $v = x$ of the two edges $e_1 = uv, e_2 = xy$ belongs to V_1, then the vertices u, y are in V_2. If $d(v) > 2$, then w is adjacent to some vertices in V_1 and some vertices in V_2 a contradiction to our assumption. Hence $d(v) = 2$. If G contains a vertex of degree 1, then G is bipartite graph contains an edge not belongs to any cycle, but then by Theorem 3.3, $\underset{\Re}{cont}(G) = 1$ a contradiction. Thus the smallest degree of G is 2 and (1) holds.

Suppose that (ii) holds. As $G * S$ is bipartite graph, the vertex set of $G * S$ can be partitioned into two subsets Z_1 and Z_2 such that every edge of $G * S$ join Z_1 with Z_2.

Suppose that $w \in Z_1$ and the set of vertices $N(w) \in Z_2$. As $G * S$ is bipartite graph, then the set of vertices $N(w) = [N(u) \cup N(v) \cup N(x) \cup N(y)] - \{u, v, x, y\}$ is independent set of vertices.

Since every edge in $G * S$ join Z_1 with Z_2, then each path join a vertex in Z_1 with a vertex in $N(w)$ has a length odd, and each path join a vertex in Z_2 with a vertex in $N(w)$ has a length even and (2) holds.

Case(2): Suppose that the two edges $e_1 = uv, e_2 = xy$ are independent. In this case, the sequence of contractions on the set of edges $S = \{e_1, e_2\}$ remove the vertices u, v and addition a new vertex w_1 adjacent to those vertices to which u or v was adjacent and remove the vertices x, y and addition a new vertex w_2 adjacent to those vertices to which x or y was adjacent.

As $G * S$ is bipartite graph, the vertex set of $G * S$ can be partitioned into two subsets Z_1 and Z_2 such that every edge of $G * S$ join Z_1 with Z_2.

As the two edges $e_1 = uv, e_2 = xy$ are independent, we have two subcases:

(α) **the vertices w_1, w_2 are adjacent.**

(β) **the vertices w_1, w_2 are nonadjacent.**

Subcase (α): the vertices w_1, w_2 are adjacent.

Suppose that the vertex $w_1 \in Z_1$, then $w_2 \in Z_2$ and the set of vertices $N(w_1) = [N(u) \cup N(v)] - \{u, v\}$ is in Z_2; the set of vertices $N(w_2) = [N(x) \cup N(y)] - \{x, y\}$ is in Z_1.

As $G * S$ is bipartite graph, then each of the two subsets of vertices $N(w_1), N(w_2)$ is independent set of vertices. As every edge in $G * S$ join Z_1 with Z_2, then each path join the vertex h in Z_1 with a vertex in $N(w_1)$ has a length odd and each path join h with a vertex in $N(w_2)$ has a length even. Similarly for a vertex in Z_2 and (3) holds.

Subcase (β): the vertices w_1, w_2 are nonadjacent. We have two possibility:

(a) $w_1, w_2 \in Z_1$.

(b) $w_1 \in Z_1, w_2 \in Z_2$.

In case (a) $w_1, w_2 \in Z_1$ the vertices $N(w_1) = [N(u) \cup N(v)] - \{u, v\}, N(w_2) = [N(x) \cup N(y)] - \{x, y\}$ are in Z_2. As $G * S$ is bipartite graph, then the set of vertices $[N(u) \cup N(v) \cup N(x) \cup N(y)] - \{u, v, x, y\}$ is independent set of vertices. As every edge in $G * S$ join Z_1 with Z_2, then each path join a vertex in Z_1 with a vertex in $[N(u) \cup N(v) \cup N(x) \cup N(y)] - \{u, v, x, y\}$ has a length odd, and each path join a vertex in Z_2 with a vertex in $[N(u) \cup N(v) \cup N(x) \cup N(y)] - \{u, v, x, y\}$ has a length even and (2) holds.

In case (b) $w_1 \in Z_1, w_2 \in Z_2$, the vertices $N(w_1) \in Z_2$ and $N(w_2) \in Z_1$. As $G * S$ is bipartite graph, then each of the two subsets of vertices $N(w_1) = [N(u) \cup N(v)] - \{u, v\}$ and $N(w_2) = [N(x) \cup N(y)] - \{x, y\}$ is independent set of vertices.

As every edge in $G * S$ join Z_1 with Z_2, then each path join a vertex h in Z_1 with a vertex in $N(w_1)$ has a length odd and each path join h with a vertex in $N(w_2)$ has a length even. Similarly for a vertex in Z_2 and (3) holds.

Conversely, if G contains an edge not belongs to any cycle, then by Theorem 3.3, $\underset{\Re}{cont}(G) = 1$ a contradiction.

Suppose that (1) holds.

If G contains a vertex v of degree 2, then by Lemma 3.2, the sequence of contractions on the two edges which are incident on v gives a bipartite graph. Hence $\underset{\Re}{cont}(G) \leq 2$.

If $\underset{\Re}{cont}(G) \neq 2$, then there exists an edge e in G such that $G*e \in \Re$. That is $\underset{\Re}{cont}(G) = 1$. But then by Theorem 3.3, G contains an edge not belongs to any cycle which is a contradiction. Hence $\underset{\Re}{cont}(G) = 2$.

Suppose that (2) holds.

Suppose that the two edges $e_1 = uv, e_2 = xy$ are adjacent.

As G is simple graph, the two edges $e_1 = uv, e_2 = xy$ are adjacent in exactly one end vertex, say $v = x$. Then the sequence of contractions on the set of edges $S = \{e_1, e_2\}$ remove the vertices u, v, y and addition a new vertex w adjacent to those vertices to which u, v or y were adjacent.

Suppose that A_1 is a subset of vertices consists of the vertex w and every vertex t in $V(G) - [N(u) \cup N(v) \cup N(x) \cup N(y)]$ in which each of the paths which are join t with a vertex in $N(w)$ has a length odd; A_2 is a subset of vertices consists of the vertices $N(w)$ and every vertex k in $V(G) - [N(u) \cup N(v) \cup N(x) \cup N(y)]$ in which each of the paths which are join k with a vertex in $N(w)$ has a length even.

We prove that A_1 is independent set of vertices. Suppose that A_1 is not. Then there exist two adjacent vertices in A_1. But then we get a vertex in A_1 is join to a vertex in $N(w)$ by two paths one of them has a length even and the other has a length odd which is a contradiction to our assumption. Hence A_1 is independent set of vertices. By similar argument we get that A_2 is independent set of vertices. Then the new graph $G*S \in \Re$. Hence $\underset{\Re}{cont}(G) \leq 2$. If $\underset{\Re}{cont}(G) \neq 2$, then by similar argument as above we get that $\underset{\Re}{cont}(G) = 2.$

Suppose that the two edges $e_1 = uv, e_2 = xy$ are independent.

In this case, the sequence of contractions on the set of edges $S = \{e_1, e_2\}$ remove the vertices u, v and addition a new vertex w_1 adjacent to those vertices to which u or v was adjacent and remove the vertices x, y and addition a new vertex w_2 adjacent to those vertices to which x or y was adjacent and w_1, w_2 are nonadjacent.

Suppose that B_1 is a subset of vertices consists of w_1, w_2 and every vertex l in $V(G) - [N(u) \cup N(v) \cup N(x) \cup N(y)]$ in which each of the paths which are join l with a vertex in $[N(u) \cup N(v) \cup N(x) \cup N(y)] - \{u, v, x, y\}$ has a length odd; B_2 is a subset of vertices consists of the vertices $N(w_1), N(w_2)$ and every vertex m in $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ in which each of the paths which are join m with a vertex in $[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]-\{u,v,x,y\}$ has a length even. By using similar argument as above we prove that each of the two subset of vertices B_1 and B_2 is independent set of vertices and $\underset{\Re}{cont}(G)=2.$

Suppose that (3) holds.

In this case, the two edges $e_1=uv, e_2=xy$ are adjacent is impossible.

Suppose that the two edges $e_1=uv, e_2=xy$ are independent. In this case, the sequence of contractions on the set of edges $S=\{e_1,e_2\}$ remove the vertices u,v and addition a new vertex w_1 adjacent to those vertices to which u or v was adjacent and remove the vertices x,y and addition a new vertex w_2 adjacent to those vertices to which x or y was adjacent. Then either the new vertices w_1, w_2 are adjacent or not.

In case w_1, w_2 are adjacent. Suppose that D_1 is a subset of vertices consists of $w_1, N(w_2)$ and every vertex h in $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ in which each of the paths which are join h with a vertex in $N(w_1)$ has a length odd and each of the paths which are join h with a vertex in $N(w_2)$ has a length even; D_2 is a subset of vertices consists of $w_2, N(w_1)$ and every vertex j in $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ in which each of the paths which are join j with a vertex in $N(w_1)$ has a length odd and each of the paths which are join j with a vertex in $N(w_1)$ has a length even.

We prove that D_1 is independent set of vertices. By assumption $N(w_2)$ is independent set of vertices. Suppose that D_1 contains two adjacent vertices. In this case, we get a path with a length even join a vertex h in D_1 with a vertex in $N(w_1)$ and a path with a length even join h with a vertex in $N(w_2)$ which is a contradiction to our assumption. Hence D_1 is independent set of vertices. By similar argument we prove that D_2 is independent set of vertices. Thus $G*S\in\Re$ and $\underset{\Re}{cont}(G)\le 2$. If $\underset{\Re}{cont}(G)\ne 2$, then there exists an edge e in G such that $G*e\in\Re$. That is $\underset{\Re}{cont}(G)=1.$ But then by Theorem 3.3, G contains an edge not belongs to any cycle which is a contradiction. Hence $\underset{\Re}{cont}(G)=2.$

In case w_1, w_2 are nonadjacent. Suppose that F_1 is a subset of vertices consists of the vertices $w_1, N(w_2)$ and every vertex d in $V(G)-[N(u)\bigcup N(v)\bigcup N(x)\bigcup N(y)]$ in which each of the paths which are join d with a vertex in $N(w_1)$ has a length odd and each of the paths which are join d with a vertex in $N(w_2)$ has a length even; F_2 is a

subset of vertices consists of the vertices $w_2, N(w_1)$ **and every vertex** t **in** $V(G)-[N(u)\cup N(v)\cup N(x)\cup N(y)]$ **in which each of the paths which are join** t **with a vertex in** $N(w_2)$ **has a length odd and each of the paths which are join** t **with a vertex in** $N(w_1)$ **has a length even. By similar argument as above we prove that each of the two subset of vertices** F_1 **and** F_2 **is independent set of vertices and** $\underset{\Re}{cont}(G)=2.$

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