

# Construction of an Ideal Topological Spaces from Undirected Graphs

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**ABSTRACT:** Graph theory, which is used effectively in many fields from science to liberal arts, has very important place in our lives. As a result of this, the ideal topological structure of the graphs is studied by many researchers. In this paper, investigate the topological spaces generated by cycle graph. The ideal topological space is obtained by deleting one of the edges of the cycle graph, then we take the largest tree graph through its vertices, we get the ideal topological space.

**Keywords:** Keyword 1, keyword 2, number of keywords is usually 3-7, but more is allowed if deemed necessary



## 1. INTRODUCTION

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. Emphasizing their application to real-world systems, the term network is sometimes defined to mean a graph in which attributes are associated with the vertices and edges, and the subject that expresses and understands real-world systems as a network is called network science. Data structures uses graphs to represent networks of communication, data organization, computational devices, the flow of computation [1]. Graph-theoretic methods, in various forms, have proven particularly useful in linguistics, since natural language often lends itself well to discrete structure. Graph theory is also used to study molecules in chemistry and physics. In condensed matter physics, the three-dimensional structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms [7]. Topology is an interesting and important field of mathematics because it is a powerful tool that leading to such beneficial concepts as connectivity, continuity, and homotopy. Its influence in most other branches of mathematics is evident [6]. Topologizing discrete structures is a problem that many publications concerned with. One of these discrete structures is graph theory. The investigation of topology on graphs is inspired by the representation of the digital image using a graph model; the points of the image and the connectivity between them are represented by the vertices and the edges of the graph respectively. Therefore, topological properties of the digital images can be studied through topologies on the vertices of graphs [5]. An interesting research topic in graph theory is to study graph theory by means of topology. Some researchers have created topologies from graphs using various methods. In 2013, M. Amiri et. al. has created a topology using vertices of an undirected graph [3]. In this paper aims at study to create a topological space by using cycle undirected graph. We present some properties of the topology that we create by using graphs, the ideal topological space is obtained through the tree graph resulting from deleting one of the edges of the cycle graph and taking the largest tree graph whose vertices represent the ideal topological space.

## 2. BASIC CONCEPTS

## 2.1 Graph theory

**Definition 2.1.1 [1]** A graph is a pair  $(V, E)$ , where  $V$  is a set of objects called vertices of the graph  $G$  is denoted by  $V(G)$ , and  $E \subseteq V \times V$  is a set of pairs  $(x, y)$  called edges of the graph  $G$ , is denoted by  $E(G)$ . Loops and multiple edges are forbidden.

**Definition 1.1.2 [9]** : A finite graph is a graph that has finite number of vertices and finite number of edges. Otherwise, a graph is called infinite graph .

**Definition 2.1.3[4]** (Cycle).  $C_n, n \geq 3$  is a cycle of length  $n$ ; it has vertices  $\{v_1, v_2, \dots, v_n\}$  and edges  $\{v_1v_2, v_2v_3, \dots\}$

**Definition 2.1.4[8]** (Sub graph). A sub graph of a graph  $G$  is a graph  $H$  with  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ .

**Definition 2.1.5 [1]** A graph  $G=(V,E)$  is connected if any pair of vertices are joined by at least one path, otherwise the graph is called disconnected.

**Definition 2.1.6 [9]** (Tree). A tree is an undirected connected graph with no cycles.

**Theorem 2.1.7 [4]** (Properties of a tree). The following are equivalent

1.  $G$  is a tree (connected, acyclic)
2.  $G$  is minimal connected
3.  $G$  is maximal acyclic

## 2.2 Topological spaces

**Definition 2.2.1[17]** Let  $X$  is a non-empty set of objects and  $T$  be collection of a subsets of  $X$ . Then  $(X, T)$  is called a topological space if the following conditions are fulfilled:

1.  $X, \emptyset \in \tau$ .
2. if  $A, B \in T$  then  $A \cap B \in \tau$ .
3. if  $A_i$  then  $\cup_{i \in I} A_i \in \tau$ .

**Definition 2.2 .2[19]** For a topological space  $(X, T)$  and  $A \subseteq X$ . The closure (respectively interior) of  $A$  briefly  $cl(A)$ , respectively  $int(A)$  is defined by

$$cl(A) = \cap \{V \subseteq X : V \text{ is a closed set where } A \subseteq V\}$$

$$int(A) = \cup \{W \subseteq X : W \text{ is an open set where } W \subseteq A\}$$

respectively.

**Remark 2.2.3 [17]** For a topological space  $(X, \tau)$  and  $A \subseteq X$  is called

1.  $A$  is an open if and only if  $A = int(A)$ .
2.  $A$  is closed if and only if  $A = cl(A)$ .

**Definition 2.2.3 [19]** For a topological space  $(X, \tau)$  and  $A \subseteq X$ ,

1. The set  $A$  is called pre-open  $A \subseteq int(cl(A))$ .

The family of all pre-open sets in a space  $(X, \tau)$  will be symbolized by  $\mathfrak{PO}(X)$ .

2. The set  $A^c$  is called pre-closed set.

The family of all pre-closed set in a space  $(X, \tau)$  will be symbolized by  $\mathfrak{PC}(X)$ .

**Example 2.2.4** Let  $X = \{1, 2, 3\}$ ,  $\tau = \{\emptyset, X, \{1, 2\}\}$  then

the family of all pre-open sets is  $\mathfrak{PO}(X) = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

and the family of all pre-closed sets is  $\mathfrak{PC}(X) = \{X, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}\}$

It is obvious that the set  $\{2, 3\}$  is a pre-open set; but it is not open, and the set  $\{2\}$  is a pre-closed set but it is not closed.

## 2.3 Ideal topological spaces

**Definition 2.3.1[14]** An ideal  $I$  is a nonempty collection of sets in a topological space  $(X, T)$ ; such that  $I$  is closed under heredity ( if  $A \subseteq B$  and  $B \in I$  then  $A \in I$ ), and finite additivity property ( $A$  and  $B \in I$  then  $A \cup B$ ).

**Example 2.3.2** Let  $X = \{1, 2, 3, 5, 7\}$ , then  $I = \{\emptyset, \{1\}, \{7\}, \{1, 7\}\}$ ; it is clear that satisfy the two condition of ideal, then it is ideal on  $X$ .

**Remark 2.3.3[11]** Let  $X$  be a nonempty collection; the simplest ideal on  $X$  are the improper ideal  $I = P(X)$ ; so  $I = \{\emptyset\}$  is called trivial ideal.

**Definition 2.3.4[13]** Let  $(X, \tau)$  be a topology space and let  $A \subseteq X$  then  $A$  is called:

1. Countable, if there exists an injective function  $F : A \rightarrow \mathbb{N}$
2. Nowhere dense in  $X$ , if  $(int(clA)) = \emptyset$ .

**Definition 2.3.5 [12]** Let  $X$  be a nonempty collection and let  $P(X)$  equal all subsets of  $X$ ; the set operator  $(.)^*: P(X) \rightarrow P(X)$  is called a local function of a set  $A \subseteq X$  with respect to the topology  $\tau$  and ideal  $I$  is defined as

$$A^* = \{W \in X : W \cap A \notin I, \forall W \in \tau(x)\}$$

where  $\tau(x) = \{W \in \tau : x \in W\}$ .

A Kuratowski closure operator  $cl^*(\cdot)$  for a topology  $\tau^*(I, \tau)$ ; called the topology; finer than  $\tau$ ; is defined by  $cl^*(A) = A \cup A^*(I, \tau)$

**Example 2.3.6** Let  $X = \{1, 2, 3\}$ ;  $\tau = \{X, \emptyset, \{2\}\}$ ,  $I = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$ ; we can be found  $\tau^*$  by the following table 2.1

Table 2.1 Closure operator

$(X)P$	$A^*$	$cl^*(A)$
$X$	$\{1,3\}$	$X$
$\emptyset$	$\emptyset$	$\emptyset$
$\{1\}$	$\{1,3\}$	$\{1,3\}$
$\{2\}$	$\emptyset$	$\{2\}$
$\{3\}$	$\emptyset$	$\{3\}$
$\{1,2\}$	$\{1,3\}$	$X$
$\{1,3\}$	$\{1,3\}$	$\{1,3\}$
$\{2,3\}$	$\emptyset$	$\{2,3\}$

Note from the above table;  $\tau^* = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\}$ .  $\tau^*$  also be found in a second way by using base of  $\tau^*$  as the follows

$$B = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}\} = \tau^*$$

this signifies in genera  $\tau^* \neq \tau$ .

**Remark 2.3.7[11]** Let  $(X, T, I)$  be an ideal topological space if  $I = \emptyset$  then  $\tau = \tau^*$ , but the opposite of is not right by the following example :

**Example 2.3.8** Let  $X = \{1, 2, 3\}$ ,  $\tau = \{X, \emptyset, \{2, 3\}\}$ ,  $I = \{\emptyset, \{1\}\}$ , then likely to find  $\tau^*$  of the base as follows ;

$$\mathfrak{B} = \{X, \emptyset, \{2, 3\}\} = \tau^* ; \text{ this signify } \tau = \tau^* ; \text{ but } I = \{\emptyset, \{1\}\} \neq \emptyset$$

### 3. TOPOLOGICAL SPACE INDUCED BY CYCLE GRAPH

**Definition 3.1.** Let  $G = (V, E)$  be a graph. Then the set of vertices becoming adjacent to a vertices  $v$  is called adjacency of  $v$  and it is denoted  $M_G(v)$ . Minimal adjacency of  $v$  is define as

$$[v]_G = \bigcap_{v \in M_G(v)} M_G(v).$$

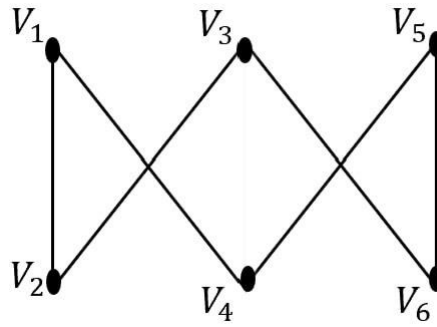
**Theorem 3.2** Let  $G = (V, E)$  be a simple connected undirected graph. Then the class  $\beta_G = \{[v]_G : v \in V\}$  is base for a topology on  $V$ .

**Corollary 3.3** Each simple connected undirected graph creates a topology on vertices set of the graph.

**Definition 3.4** Let  $G = (V, E)$  be simple connected graph. Then the topology generated by  $\beta_G = \{[v]_G : v \in V\}$  is called the topology generated by the graph. This topology is in the form of,

$$\tau_G = \{G \subseteq V : G \bigcup_{[v]_G \in \beta_G} [v]_G, v \in V\}$$

**Example 3.5** The graph whose vertices set is  $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$  is given in figure 1.



**Figure 1.** Connected bipartite graph.

The adjacencies of each vertex are as follows.

$$M_G(v_1) = \{v_2, v_4\}, M_G(v_2) = \{v_1, v_3\}, M_G(v_3) = \{v_2, v_6\}, M_G(v_4) = \{v_1, v_5\}, M_G(v_5) = \{v_4, v_6\}, M_G(v_6) = \{v_3, v_5\}$$

The minimal adjacencies of each vertex are as follow.

$$[v_1]_G = \bigcap_{v \in M_G(v)} = \{\{v_1, v_3\}, \{v_1, v_5\}\} = \{v_1\}$$

$$[v_2]_G = \bigcap_{v \in M_G(v)} = \{\{v_2, v_4\}, \{v_2, v_6\}\} = \{v_2\}$$

$$[v_3]_G = \bigcap_{v \in M_G(v)} = \{\{v_1, v_3\}, \{v_1, v_5\}\} = \{v_3\}$$

$$[v_4]_G = \bigcap_{v \in M_G(v)} = \{\{v_2, v_4\}, \{v_4, v_6\}\} = \{v_4\}$$

$$[v_5]_G = \bigcap_{v \in M_G(v)} = \{\{v_1, v_5\}, \{v_3, v_5\}\} = \{v_5\}$$

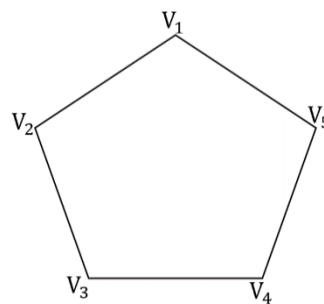
$$[v_6]_G = \bigcap_{v \in M_G(v)} = \{\{v_2, v_6\}, \{v_4, v_6\}\} = \{v_6\}$$

$$\beta_G = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}\}$$

and

$$\tau_G = \left\{ V, \emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_6\}, \{v_2, v_6\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\} \right\}$$

**Example 3.1.6** Let us investigate the topological space generated by cycle given figure 2 whose vertices set is  $V = \{v_1, v_2, v_3, v_4, v_5\}$



**Figure 2** Graph cycle  $C_5$

The adjacencies of vertices of cycle are as follows.

$$M_G(v_1) = \{v_2, v_5\}, M_G(v_2) = \{v_1, v_3\}, M_G(v_3) = \{v_2, v_4\}, M_G(v_4) = \{v_3, v_5\}, M_G(v_5) = \{v_1, v_4\}$$

The minimal adjacencies of each vertices are as follows.

$$[v_1]_G = \{v_1\}, [v_2]_G = \{v_2\}, [v_3]_G = \{v_3\}, [v_4]_G = \{v_4\}, [v_5]_G = \{v_5\}, [v_6]_G = \{v_5\}$$

$$\beta_G = \{\{v_3\}, \{v_4\}, \{v_1\}, \{v_2\}, \{v_5\}\}$$

$\tau_G$  is discrete topological space

### 3.2 Ideal induced by Edges Deletion of cycle graph

**Definition 3.2.1** Let  $G(V, E)$  be a cycle graph and  $H_{ij} \subseteq G/e$  is a tree sub graph generated by cycle graph  $G$ . Each vertex of the vertices of tree graph is taken adjacent to it from vertices of other, and then the intersection of those adjacent totals and what they contain on each vertex, as they represent the ideal topological space.

**Notation 3.2.2** The maximal tree of a graph can be used as an ideal for topological space

**Theorem 3.2.3** Any connected,  $V$ -node graph with  $V - 1$  edges are a tree

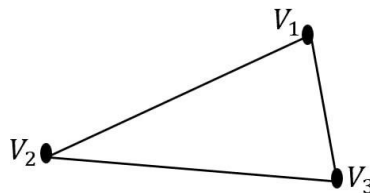
**Algorithm 3.1**

Input: A simple undirected graph  $G$ .

1. Label the set of vertices  $V(G)$  as  $\{v_1, v_2, \dots, v_n\}$ .
2. Label the set of edges  $E(G)$  as  $\{e_1, e_2, \dots, e_m\}$
3. Detect any cycle in  $G$ .
4. Name the element of the cycle as  $\mathcal{C} = \{c_1, c_2, \dots, c_k\}$
5. For  $c$  in  $\mathcal{C}$  do
  - 5.1. Remove one edge  $e$  in  $\mathcal{C}$
  - 5.2. Set  $\mathcal{C} = \mathcal{C}/\{e\}$

Output: Tree  $T$  of the graph  $G$ .

**Example 3.2.4** We construct a ideal topological space for the connected undirected graph in the figure 3.



**Figure 3.** Simple connected graph  $C_3$

The minimal adjacencies of each vertices are as follows.

$$\left[ v_1 \right]_G = \bigcap_{v \in M_G(v)} = \{ \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\} \} = \{v_1\}$$

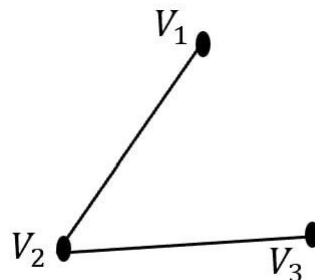
$$\left[ v_2 \right]_G = \bigcap_{v \in M_G(v)} = \{ \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\} \} = \{v_2\}$$

$$\left[ v_3 \right]_G = \bigcap_{v \in M_G(v)} = \{ \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\} \} = \{v_3\}$$

$$\beta_G = \{ \emptyset, \{v_1\}, \{v_2\}, \{v_3\} \}$$

$$\tau_G = \{ V, \emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\} \}.$$

Now, we find maximum tree of graph to construct an ideal topological space.



**Figure. 4** Tree of graph that given in Figure 2.

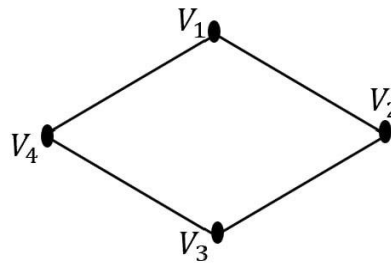
$$\begin{aligned} \left[ v_1 \right]_G &= \bigcap_{v \in M_G(v)} = \{ \{v_2\}, \{v_1, v_3\}, \{v_2\} \} = \{v_1, v_3\} \\ \left[ v_2 \right]_G &= \bigcap_{v \in M_G(v)} = \{ \{v_2\}, \{v_1, v_3\}, \{v_2\} \} = \{v_2\} \\ \left[ v_3 \right]_G &= \bigcap_{v \in M(v)} = \{ \{v_2\}, \{v_1, v_3\}, \{v_2\} \} = \{v_1, v_3\} \\ \beta_G &= \{ \emptyset, \{v_2\}, \{v_1, v_3\} \} \\ I &= \{ V, \emptyset, \{v_2\}, \{v_1, v_3\} \}. \\ P(V) &= \{ V, \emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\} \} \end{aligned}$$

The ideal topological space are

$P(V)^*(I, \tau) = \{ v \in V : P(V) \cap u \notin I \}$  where  $u \in N\tau_G$  set of all neighbor hoods of  $V$

$$\begin{aligned} \{v_1\}^* &= \{v_1, v_3\} \\ cl^*(\{v_1\}) &= \{v_1\} \cup \{v_1\}^* = \{v_1, v_3\} \\ \{v_2\}^* &= \emptyset, cl^*(\{v_1\}) = \{v_2\} \cup \{v_2\}^* = \{v_2\} \\ \{v_3\}^* &= \{v_1, v_3\}, cl^*(\{v_3\}) = \{v_3\} \cup \{v_3\}^* = \{v_1, v_3\} \\ \{v_1, v_2\}^* &= \{v_1, v_3\}, cl^*(\{v_1, v_2\}) = \{v_1, v_2\} \cup \{v_1, v_2\}^* = V \\ \{v_1, v_3\}^* &= \emptyset, cl^*(\{v_1, v_3\}) = \{v_1, v_3\} \cup \{v_1, v_3\}^* = \{v_1, v_3\} \\ \{v_2, v_3\}^* &= \{v_1, v_3\}, cl^*(\{v_2, v_3\}) = \{v_2, v_3\} \cup \{v_2, v_3\}^* = V \\ V^* &= \emptyset, cl^*(V) = V \cup V^* = V \end{aligned}$$

**Example 3.2.5** Let  $G$  be a graph shown in the figure 4 below.

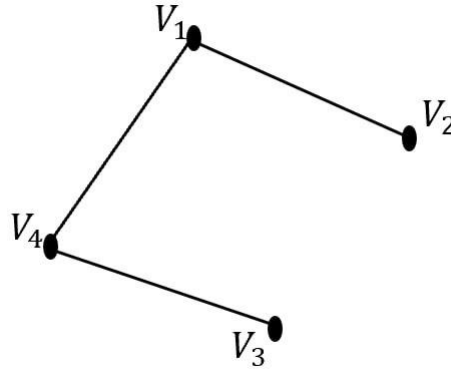


**Figure 4.** Simple graph  $C_4$ .

The minimal adjacencies of each vertices are as follows.

$$\begin{aligned} \left[ v_1 \right]_G &= \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\} \} = \{v_1, v_3\} \\ \left[ v_2 \right]_G &= \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\} \} = \{v_2, v_4\} \\ \left[ v_3 \right]_G &= \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\} \} = \{v_1, v_3\} \\ \left[ v_4 \right]_G &= \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\} \} = \{v_2, v_4\} \\ \beta_G &= \{ \emptyset, \{v_1, v_3\}, \{v_2, v_4\} \} \\ \tau_G &= \{ \emptyset, V, \{v_1, v_3\}, \{v_2, v_4\} \} \end{aligned}$$

Now, we find maximum tree of graph to construct an ideal topological space



**Figure .5** Tree of graph given in Figure 4.

The minimal adjacencies of each vertices are as follows.

$$\left[ v_1 \right]_G = \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\}, \{v_1\}, \{v_4\} \} = \{v_1\}$$

$$\left[ v_2 \right]_G = \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\}, \{v_1\} \} = \{v_2, v_4\}$$

$$\left[ v_3 \right]_G = \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\}, \{v_1\} \} = \{v_1, v_3\}$$

$$\left[ v_4 \right]_G = \bigcap_{v \in A_G(v)} = \{ \{v_2, v_4\}, \{v_1, v_3\}, \{v_1\}, \{v_4\} \} = \{v_4\}$$

$$\beta_G = \{ \emptyset, \{v_1\}, \{v_4\}, \{v_1, v_3\}, \{v_2, v_4\} \}$$

$$I = \{ \emptyset, V, \{v_1\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_4\}, \{v_1, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\} \}$$

$$P(V) = \left\{ V, \emptyset, \{v_1\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_4\}, \{v_1, v_4\}, \{v_1, v_3, v_4\}, \{v_2\}, \{v_3\}, \{v_2, v_4, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_3\}, \{v_3, v_4\}, \{v_2, v_3\}, \{v_1, v_2\} \right\}$$

$P(V)^*(I, \tau) = \{ v \in V : P(V) \cap u \notin I \}$  where  $u \in N\tau_G$  set of all neighbor hoods of  $V$

$$\{v_1\}^* = \emptyset, cl^*(\{v_1\}) = \{v_1\} \cup \{v_1\}^* = \{v_1, v_3\}$$

$$\{v_2\}^* = \{v_2\}, cl^*(\{v_2\}) = \{v_2\} \cup \{v_2\}^* = \{v_2\}$$

$$\{v_3, v_4\}^* = \{v_3\}, cl^*(\{v_3, v_4\}) = \{v_3, v_4\} \cup \{v_3, v_4\}^* = \{v_3\}$$

$$\{v_3\}^* = \{v_3\}, cl^*(\{v_3\}) = \{v_3\} \cup \{v_3\}^* = \{v_3\}$$

$$\{v_1, v_2, v_3\}^* = \{v_2\}, cl^*(\{v_1, v_2, v_3\}) = \{v_1, v_2, v_3\} \cup \{v_1, v_2, v_3\}^* = \{v_1, v_2, v_3\}$$

$$\{v_4\}^* = \emptyset, cl^*(\{v_4\}) = \{v_4\} \cup \{v_4\}^* = \{v_4\}$$

$$\{v_1, v_2, v_4\}^* = \emptyset, cl^*(\{v_1, v_2, v_4\}) = \{v_1, v_2, v_4\} \cup \{v_1, v_2, v_4\}^* = \{v_1, v_2, v_4\}$$

$$\{v_1, v_2\}^* = \{v_2\}, cl^*(\{v_1, v_2\}) = \{v_1, v_2\} \cup \{v_1, v_2\}^* = \{v_1, v_2\}$$

$$\{v_2, v_4, v_3\}^* = \{v_3\}, cl^*(\{v_2, v_4, v_3\}) = \{v_2, v_4, v_3\} \cup \{v_1, v_2, v_3\}^* = \{v_2, v_4, v_3\}$$

$$\{v_1, v_3\}^* = \emptyset, cl^*(\{v_1, v_3\}) = \{v_1, v_3\} \cup \{v_1, v_3\}^* = \{v_1, v_3\}$$

$$\{v_3, v_4\}^* = \{v_3\}, cl^*(\{v_3, v_4\}) = \{v_3, v_4\} \cup \{v_3, v_4\}^* = \{v_3, v_4\}$$

$$\{v_1, v_4\}^* = \emptyset, cl^*(\{v_1, v_4\}) = \{v_1, v_4\} \cup \{v_1, v_4\}^* = \{v_1, v_4\}$$

$$\{v_1, v_3, v_4\}^* = \emptyset, cl^*(\{v_1, v_4, v_3\}) = \{v_1, v_4, v_3\} \cup \{v_1, v_3, v_4\}$$

$$\{v_2, v_3\}^* = \{v_2, v_3\}, cl^*(\{v_2, v_3\}) = \{v_2, v_3\} \cup \{v_3, v_2\}^* = \{v_3, v_2\}$$

$$\{v_2, v_4\}^* = \emptyset, cl^*(\{v_4, v_2\}) = \{v_4, v_2\} \cup \{v_4, v_2\}^* = \{v_4, v_2\}.$$

**Theorem 2.3.6** Let  $C_n = (V, E)$  be a cycle whose vertices set is  $\{v_1, v_2, \dots, v_n\}$ , Then the topological space generated by the cycle  $C_n = (V, E)$  is a discrete topological space.

**corollary 2.3.7.** Let  $T = (V, E)$  be a tree whose vertices set is where  $V = \{v_1, v_2, v_3, v_4, v_5\}$ , Then the ideal topological space generated by the tree  $T = (V, E)$  is not discrete ideal topological space.

## 4. CONCIUSION

The main result in this paper is to explain the relation between graph theory and ideal topological space. It is studied topologies generated by certain graphs. Therefore, it is seen that there is a topology generated by every cycle graph. The ideal topological space is obtained by deleting one of the edges of the cycle graph and taking largest tree resulting from the deletion process, its vertices represent ideal topological space .

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