Wasit Journal for Pure Science

Journal Homepage: https://wjps.uowasit.edu.iq/index.php/wjps/index
e-ISSN: 2790-5241 p-ISSN: 2790-5233

Using ARCH and GARCH mathematical models to predict the general budget deficit for the period from (2008) to (2023)

Saad Obaid Jameel AL-Masoodi[1]*

[1]First Author Affiliation,
ASE University, Bucharest, Romania

*Corresponding Author: Saad Obaid Jameel Al-masoodi

DOI: https://doi.org/ 10.31185/wjps.284
Received 20 November 2023; Accepted 3 December 2023; Available online 30 December 2023

ABSTRACT: In this research, (ARCH) and (GARCH) were used, which are mathematical models for time series, as the study of time series is considered one of the topics of great importance in analyzing the behavior of various phenomena and interpreting them by analyzing their change according to time, adopting different time periods. Among the most prominent time series are series Finance, which is volatile, that is, characterized by the characteristic of instability (volatility), which makes the use of (ARMA) models inaccurate in its predictive results, which work on the assumption that the variance of the random error, which is distributed normally, and which is constant over time. As for time series, this condition is This is not true and it shows variation and fluctuations at different periods of the series. This research aims to choose the best model for the conditional variance of the remainder of the time series model in the process of studying the surplus and deficit in the general financial budget of Iraq on a monthly basis for the period from January 2008 to September 2023. Accordingly, we will build a statistical model using the (ARCH) family and also know the risk element or Uncertainty) by using preference criteria to build time series models and then using the estimated model to predict the variance fluctuations of that model, The study showed that the best forecasting model is GARCH (2,0).

Keywords: ARCH, GARCH, Time series, Homogeneity, forecasting.

1. INTRODUCTION

The study and analysis of ordinary time series works on the assumption that the conditional variance of the model error is constant (Homoscedasticity), but financial time series do not fulfill this assumption, so researchers since 1982 have developed an analysis mechanism that differs from traditional ordinary analysis, in which the variance of the conditional error varies with time. Various time series models have emerged specialized in this aspect, including the Autoregressive Conditional Heteroscedasticity model (ARCH). Which was circulated in 1986 to the Generalized ARCH models and is written as an abbreviation (GARCH), which represents the random error term. The term autoregressive means that it is a process that depends on past data and information, and the heterogeneity of the conditional variance means that the variance conditional on the available information depends on the values. prior to the operation.

GARCH models are one of the useful tools in analyzing and forecasting the fluctuations of time series over time, Multivariate GARCH models are considered among the modern models that have the ability to contain multiple time series fluctuations, given the presence of heterogeneity of variance, and facilitate the possibility of modeling the joint movement of time series that is Multivariate with time-lagged conditional covariance matrix.

The topic of time chains has been introduced in many areas, especially in the economic fields, especially in the financial markets. Recent years have witnessed a significant development in the area of the stock market or the so-called stock exchange. This is the importance of studying time chains because they are characterized by instability, which means that there are volatile periods of time followed by periods of relative calm. This makes them experience extreme fluctuations and. Because of that, some prediction methods fail to analyze, such as a model. This requires the use of quantitative analytical models that can formulate these fluctuations with mathematical models that allow future plans to

*Corresponding author: sjameel@uowasit.edu.iq

be made, because we know that most financial markets, exchange markets and economic variables have a characteristic called stability. This characteristic means very large fluctuations that are out of the ordinary at times, such as stock prices or the number of shares traded daily. In any case, these fluctuations are undeserved by investors and financial transactions. In order to address such financial and economic problems, statistical models have been used that take account of these fluctuations. These models are ARCH and GARCH.

2. TIME SERIES

Time chains are widely applied in many areas of working life. Perhaps the most important of these areas are the economic areas in general and the financial area in particular, Which in our study includes the value of the deficit in the Iraqi financial budget, and it has been noted that one of the main features of economic and financial time chains is characterized by the characteristic of instability, which means periods of extraordinary high volatility followed by periods of relative calm, In other words, because of these fluctuations, the variability of the financial time chain varies over time and therefore the forecast periods of the chain levels also vary over time. and this contradicts familiar standard economics models that assume constant variability to the extent of error over time, in such cases, the imposition of constant variation is inappropriate.

2.1 Models Time Series

It is divided into two sections:

2.1.1 Linear Models

Linear models of time series have played a major role in modeling many phenomena. And I was able to give several theories a mathematical picture that helps predict their future values, but what is taken from these linear formulas is that they often cannot translate the kinetic character of these phenomena. This has hampered the development of chain aspects of modeling in the time chain model. "The linear hypothesis of these models requires time components to be time-bound. These models are built using regression analysis, which is a set of methods for studying relationships between variables, in order to estimate and predict single variable values using values of other variables in a common time chain.

A- autoregressive model (rank p)

The autoregressive (p) model, written as AR (p), is the current value of the time series depending on the sum of the previous values and the error of the current value, and its mathematical formula is:

$$y_t = \emptyset_1 y_{t-1} + \cdots + \emptyset_p y_{t-p} + \varepsilon_t \quad \ldots (1)$$

B- Moving Average model (rank q)

The moving average model is used to model the time series, and this process depends on the error to represent the series, and it can be expressed in the following formula:

$$y_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} \quad \ldots (2)$$

C- Mixed autoregressive and moving averages model rank (p,q)

In some applications, the AR model or the MA model becomes complex, and the reason is that we may need a higher order model with many parameters to adequately describe the dynamic structure of the data, and to overcome this difficulty, the autoregressive model - moving average, and the ARMA(p,q) model were introduced. It is written in the following format:

$$y_t = \sum_{i=1}^{p} \emptyset_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \quad \ldots (3)$$

D- The Mixed Integrative Model is written as an abbreviation ARIMA(p,d,q)

The time series (y_t) is said to follow the autoregressive model and the integrated moving media in the case of taking the difference of the series, which is defined by the following formula:

$$W_t = \nabla^d y_t \quad \ldots (4)$$

Using the back displacement indicator, the general shape of the model is given as follows:

$$\left(1 - \emptyset_1 \beta - \cdots - \emptyset_p \beta^p\right)(1-\beta)^d y_t = \left(1 - \theta_1 \beta - \cdots - \theta_q \beta^q\right)\varepsilon_t \quad \ldots (5)$$

3. ARCH AND GARCH MODELS

They are models aimed at modeling variance, most frequently used in financial data models, because the modern trend of investors is not only to study and predict the expected returns of stocks and bonds in the financial markets, but also to pay attention to risk elements or uncertainty.

When we study uncertainty, we need special models that deal with the fluctuation of stock values over a time series or what we might call "chain variance ", and the models that deal with this type of variance belong to what can be called ARCH models.

3.1 ARCH Model

It is an acronym for (autoregressive conditional on the inhomogeneity of variance), and it consists of two parts, the first (Autoregressive) AR is the autoregressive of the mean, and the second part is CH (Heteroscedasticity Conditioning), which is the instability of the variance, meaning that the variance has a dependence on the past of the series and that it changes with the change of time.

The ARCH model gains practical significance from the fact that the uncertainty situation is precisely the uncertainty associated with a particular investment over time.
The uncertainty of prediction changes with time intervals and not just with forecasting prospects. When it comes to time chains that represent the evolution of financial variables, most financial variables are characterized by a non-linear dynamic, unstable variability over time and asymmetric phenomenon not taken by Box-Jenkins models. (ARMA) considers and thus justifies the use of non-linear variability models (ARCH).

The first to propose ARCH models is the Angel researcher (Anjel) in 1982 in an article entitled "Self-conditioned heterogeneity regression to estimate inflation variability in the UK", where the article was published in a journal (Econometrica) of the same year and this model means that the variability is not constant in the current period (t) Is linked to its inconsistency in previous periods of time and may be written in the following mathematical form: -

$\sigma_t^2 = a_t + a_1 a_{t-1} + a_2 a_{t-2} + \ldots\ldots + a_p a_{t-p} \quad , where\ \mathrm{p}, 1\ldots\ldots, 0\mathrm{i}, > 0 \quad \boldsymbol{a_i}$ ... (6)

So when p = 1 is based on the above equation, we get the mathematical formula of the ARCH model: -

$\boldsymbol{\sigma_t^2 = a_0 + a_1 a_{t-1}}$ ... (7)

It has an unconditional variance feature known as: $\boldsymbol{\sigma_x^2 \frac{a_0}{1-a_1}}$

3.2 GARCH Model

The ARCH model can be expanded as (Bollerslev) in 1986 expanded the ARCH model with a more generalized model called the Generalized Heteroscedastic Autoregressive Conditional model with different variations. It did not rely solely on the error box in (t) where in the previous period the variance was:

$\boldsymbol{\sigma_t^2 = a_t + a_1 a_{t-1} + a_2 a_{t-2} + \ldots + a_p a_{t-p} + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + \ldots + \beta_q \sigma_{t-q}^2}$ ... (8)

The above formula can be written as follows:

$$\boldsymbol{\sigma_t^2 = a_0 + \sum_{i-1}^{p} a_i a_{t-1} + \sum_{j-1}^{q} \beta_i \sigma_{t-j}^2 \quad where \sum_{i-1}^{p} a_i + \sum_{j-1}^{q} \beta_j < 1} \quad \ldots (9)$$

And stages of building non-linear models is Identification, Estimation, Diagnostic checking and

4. FORECASTING

Which will be explained now:

4.1 Identification:

The diagnosis of ARCH models is an important step to build the model based on available data so that we have a perception of the problem of Autoregressive conditional on heterogeneity of the discrepancy in the studied data. For the diagnosis of ARCH models, the following steps are used:

4.2 Data Plot

The first step in the analysis of the time series and through the drawing of the series to form an idea while the series contains a general trend or instability leading to transfers on the data, and the drawing of the series shows its need for appropriate conversion to stabilize its average or variations before any analysis.

4.3 ARCH Test

This test was proposed by Engle in 1982 to detect disorders following the ARCH model. This test is based on the (LM) multiplier, and the test count is based on the determination factor (R^2) of the known regression model according to the following method:

$\eta_t = \varepsilon_t^2 - \sigma_t^2$ …(10)

and

$E(\eta_t) = E(\varepsilon_t^2 - \sigma_t^2) = 0$ ... (11)

With compensation in formula (3) we get:

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \eta_t \quad , t = 1,2,..k \qquad \ldots.(12)$$

The above version represents Autoregressive Model of rank (p) and k represents the number of views and η_t be an unrelated series, The hypothesis of nowhere:

$\mathrm{H}_0: \alpha_1 = \alpha_2 = \cdots . = \alpha_p = 0$

$H_1: \alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_p \neq 0$

p represents the number of variables in the Autoregressive equation that represent the threshold square at displacement (p).
The test statistic $(k \times R^2)$ squared for the approved model and is distributed alignigly to freely distribute the chi square for the degrees of freedom (p), i.e., the lacrang multiplier:

$$LM = k \times R^2 \sim x_p^2 \quad \ldots (13)$$

Where R^2 is the coefficient of determination in Autoregressive Model $R^2 = SSR/SST$, the Sum of Squares Total (SST) for the residual square η_t.
And the Sum of Squares Autoregressive (SSR) The calculated value is compared with the tabulated value with a degree of freedom (p) and the level of significance α. In the case of accepting the null hypothesis, it means that there is evidence of homogeneity and stability of the variance, but in the case of accepting the alternative hypothesis, the variance is not homogeneous over time.

5. ESTIMATION

After matching the appropriate model to the data series using the statistical program (Eviews) and after the results are known, we estimate the parameters of the diagnosed model according to a table dedicated to the estimated values of the model's parameters, and the stability condition of the model (GARCH), The test is carried out on data representing the Iraqi budget deficit divided monthly for the period from January (2008) to September (2023), can be tested using the following formula: $\boldsymbol{\alpha_1 + \beta_1 < 1}$

5.1 Examining the suitability of the model:

This stage includes testing the suitability of the model (GARCH) after it is estimated according to the studied data by examining the standard series of remainders $(\tilde{\varepsilon}_t)$, which is known by the following mathematical formula: $\tilde{\varepsilon}_t = \frac{\hat{x}_t}{\hat{\sigma}_t}$, As well as examining the standard square remainder series for $(\tilde{\varepsilon}_t)$.
The examination process is carried out by using (Ljung - Box) test on the standard series of remainders to examine the suitability of the mean equation and on the square of the standard series of remainders to examine the suitability of the variance equation of the model.
The (Ljung-Box) test or called the (portmanteau Q statistics) test was proposed by the two researchers (Ljung-Box) in 1978 by calculating the autocorrelation coefficients of the residuals.
It is tested under the following hypothesis:

$$\boldsymbol{H_0: P_1 = P_2 = \ldots\ldots = P_K = P_M = 0}$$
$$\boldsymbol{H_1: P_1 \neq P_2 \neq \ldots\ldots \neq P_K \neq P_M \neq 0}$$

5.2 Forecasting

The last goal of time series analysis using (ARCH) models is to forecast while achieving the conditional variance stability of the model in order to reach the best possible results in time series analysis.
The problem of heterogeneity of variance: analysis of variance is one of the most important branches of statistics with wide applications, as it is based on several basic assumptions, and if one of these assumptions is not available, it leads to incorrect results, and the most important of these assumptions is the problem of heterogeneity of variance, which comes from the difference in samples that follow communities with variances Different, or perhaps as a result of the exposure of the observations of those samples to pollution, which leads to inaccurate or shady decisions when testing hypotheses, as the level of significance rises automatically, and therefore the random differences must be equal for the different samples, with which it is possible to obtain a common variance of random error for all samples.

5.3 Homogeneity of variance: -

To obtain correct results and decisions, there are some conditions that must be met in the data, and the researcher must ensure that these conditions are met before analyzing them. In which the groups being compared have equal variances.
There are many tests that reveal the condition of homogeneity of variance, and one of the most important statistical tests to detect this problem is O'Brien's test.

6. STATISTICAL ANALYSIS

In order to reach the desired goal of the study, the program (Eviews) was resorted to, and based on the previously mentioned statistical models and methods, we built a time series model, and after several attempts, and based on the differentiation criteria in choosing the best model, we reached to determine the best model.

It is known that the series of surplus and deficit values in the federal budget of Iraq are exposed to severe fluctuations as a result of their influence by political factors, wars, and the changes that occur at the level of the world as a whole. Therefore, these fluctuations will negatively affect the construction of forecasting models for the series of surplus and deficit in the financial budget, which arises due to the instability of the variance of those series models, which is a basic condition for building such models. Therefore, studying these oscillations and building models for their effects on the residual series and on the variations of the residual series has a positive impact on building these models and then the possibility of using them for prediction because of their great importance in formulating countries' policies. This has been done in this The research examined fluctuations in the surplus and deficit in Iraq's monthly budget for the period from December 2008 to September 2023 for the purpose of building a fluctuations model from the ARCH family. We found that the appropriate model is the GARCH (0,1) model.

6.1 ARCH effect Test

$H0$: there is no ARCH effect.
$H1$: there is ARCH effect.

The presence of an effect in Table (1) shows the results of the ARCH test, which shows that the value of the (OBS * R-Square) statistic reached (26.255) with a probability value (p-value=0.0000) and since this value is less than the level of significance ∝=0.01, therefore We reject the null hypothesis and accept the alternative hypothesis and then judge the presence of an unconditional autoregressive effect of variance (ARCH family) in the model.

Table (1) Results of the heterogeneity of variance test (ARCH TEST)

Heteroskedasticity Test: ARCH

F-statistic	30.56568	Prob. F(1,172)	0.0000
Obs*R-squared	26.25533	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/23/23 Time: 01:05
Sample (adjusted): 2009M04 2023M09
Included observations: 174 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.15E+13	2.48E+13	2.078461	0.0392
RESID^2(-1)	0.388396	0.070252	5.528624	0.0000

R-squared	0.150893	Mean dependent var	8.41E+13
Adjusted R-squared	0.145956	S.D. dependent var	3.44E+14
S.E. of regression	3.18E+14	Akaike info criterion	69.63403
Sum squared resid	1.74E+31	Schwarz criterion	69.67034
Log likelihood	-6056.160	Hannan-Quinn criter.	69.64876
F-statistic	30.56568	Durbin-Watson stat	1.853103
Prob(F-statistic)	0.000000		

6.2 Diagnosis rank (GARCH – ARCH)

Multiple models were tested for the purpose of diagnosing the degree of influence in the model and whether this influence can be represented by the ARCH model only or extends to the other models of the family as well as the rank of each of them, as is known, the choice will be on the model that achieves the lowest value for the three comparison criteria (AIC, SC, and HQC).

It was noted that the best model representing the effect of ARCH is the GARCH(1,0) model because it achieved the lowest values for the three criteria.

The results in Table (2) also showed the estimates of the best model based on the values of the three criteria, where all the parameters of the model were significant, because the probability values of the Z test for each parameter were less than the level of significance ∝ = 0.01. All the parameters of the ARCH effect were all significant at the level of 0.01. It came with a probability value of less than 0.01.

Table (2) Maximum likelihood estimation results for the ARCH(2) diagnostic model

Sample (adjusted): 2009M04 2023M09
Included observations: 174 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	241979.5	697067.4	0.347139	0.7285
DCL(-1)	-0.147223	0.014478	-10.16892	0.0000
Variance Equation				
C	6.29E+13	2.60E+12	24.19813	0.0000
RESID(-1)^2	0.191115	0.040415	4.728812	0.0000
RESID(-2)^2	-0.030799	0.002999	-10.26935	0.0000

R-squared	0.044950	Mean dependent var	97008.84
Adjusted R-squared	0.039398	S.D. dependent var	9844864.
S.E. of regression	9648983.	Akaike info criterion	34.75482
Sum squared resid	1.60E+16	Schwarz criterion	34.84560
Log likelihood	-3018.669	Hannan-Quinn criter.	34.79165
Durbin-Watson stat	2.182403		

6.3 Testing the efficiency c

Now we verify some properties to demonstrate the accuracy and efficiency of the model selection through three tests of the residuals, which are:

A. Residue series correlation test

This test is conducted by testing the following hypothesis:

$H0$: there is no series correlation in the model.

$H1$: there is a series correlation.

The test results appear in Figure (3), which shows that all the confidence limits for the autocorrelation series and the partial autocorrelation of the standard error values are within the confidence limits, and that all the probability values for these correlations were each of them greater than the level of significance ∝=0.05, which leads to Not rejecting the null hypothesis, that is, accepting the absence of a series of correlations for the rest of the model, which is a good characteristic of the model and reflects the possibility of preferring it over other models.

Table (3) Results of testing correlations in series residuals for (36) time lags

Sample: 2009M03 2023M09
Included observations: 174

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.045	0.045	0.3647	0.546
		2	-0.018	-0.020	0.4244	0.809
		3	0.041	0.043	0.7203	0.868
		4	0.007	0.003	0.7291	0.948
		5	-0.033	-0.032	0.9298	0.968
		6	-0.033	-0.032	1.1276	0.980
		7	0.007	0.008	1.1360	0.992
		8	0.091	0.092	2.6587	0.954
		9	-0.013	-0.018	2.6907	0.975
		10	-0.028	-0.025	2.8338	0.985
		11	-0.022	-0.031	2.9254	0.992
		12	-0.001	0.001	2.9254	0.996
		13	-0.033	-0.025	3.1300	0.997
		14	-0.029	-0.019	3.2913	0.998
		15	-0.026	-0.028	3.4188	0.999
		16	-0.021	-0.029	3.5061	1.000
		17	-0.002	0.004	3.5066	1.000
		18	-0.031	-0.028	3.7007	1.000
		19	-0.033	-0.028	3.9173	1.000
		20	-0.023	-0.026	4.0182	1.000
		21	-0.034	-0.030	4.2493	1.000
		22	-0.036	-0.031	4.5163	1.000
		23	-0.019	-0.015	4.5893	1.000
		24	-0.008	-0.009	4.6034	1.000
		25	-0.033	-0.038	4.8305	1.000
		26	-0.033	-0.031	5.0562	1.000
		27	-0.011	-0.012	5.0834	1.000
		28	-0.024	-0.026	5.2087	1.000
		29	-0.036	-0.035	5.4793	1.000
		30	-0.037	-0.039	5.7752	1.000
		31	0.091	0.088	7.5509	1.000
		32	-0.023	-0.039	7.6602	1.000
		33	-0.033	-0.029	7.8992	1.000
		34	0.043	0.031	8.3091	1.000
		35	0.221	0.215	19.044	0.987
		36	-0.012	-0.029	19.075	0.991

*Probabilities may not be valid for this equation specification.

B. ARCH impact test

The effect of ARCH is tested to indicate whether the final estimated model still suffers from the effect of the presence of ARCH family models in the residual series, and by testing the following hypothesis:

$H0$: there is no ARCH effect.

$H1$: there is ARCH effect.

The test results were presented in Table (4), which shows that the probability value of the OBS R-Squared test statistic was 0.5505, and since it is greater than the significance level ∝=0.05, it can be said to accept the null hypothesis at a significance level ∝=0.05 and rule that there is no effect of the ARCH family. In the estimated model, which is another good feature of the model.

Table (4) Results of testing the effect of ARCH on the residual series for the estimated model

Heteroskedasticity Test: ARCH

F-statistic	0.353091	Prob. F(1,171)	0.5532
Obs*R-squared	0.356484	Prob. Chi-Square(1)	0.5505

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/29/23 Time: 17:14
Sample (adjusted): 2009M05 2023M09
Included observations: 173 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.008331	0.387940	2.599189	0.0102
WGT_RESID^2(-1)	0.045388	0.076384	0.594214	0.5532

R-squared	0.002061	Mean dependent var	1.056054
Adjusted R-squared	-0.003775	S.D. dependent var	4.982619
S.E. of regression	4.992016	Akaike info criterion	6.065050
Sum squared resid	4261.358	Schwarz criterion	6.101504
Log likelihood	-522.6268	Hannan-Quinn criter.	6.079839
F-statistic	0.353091	Durbin-Watson stat	1.998396
Prob(F-statistic)	0.553154		

C. Testing the normality of the residual distribution

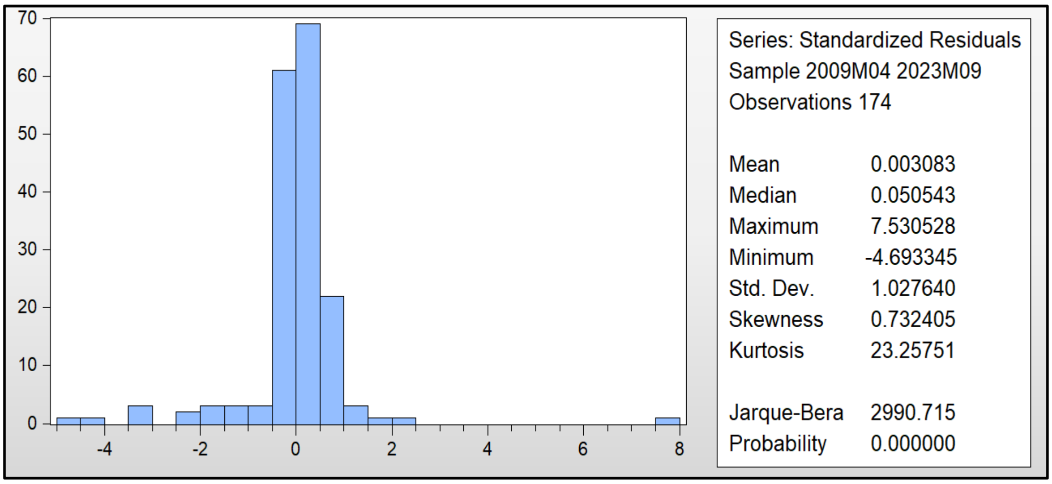
We now conduct the Gausses test to determine whether the residuals are normally distributed or not, by testing the following hypothesis:

$H0$: Residual are normally distributed.

$H1$: Residual are not normally distributed.

Figure (1) shows the results of the Jarque-Bera test, which shows that the test statistic reached 328.1523. The probability value, p-value=0.0000, and since the probability value is less than the level of significance ∝=0.05, the null hypothesis is rejected and the alternative is accepted. Consequently, the series of residuals is not normally distributed. Which is not considered desirable, but most statisticians agreed that the estimate would be acceptable if the remainder passed two tests, which is what we obtained in this research. Therefore, it can be said, in general, that the results of the research are acceptable and are the best that can be obtained.

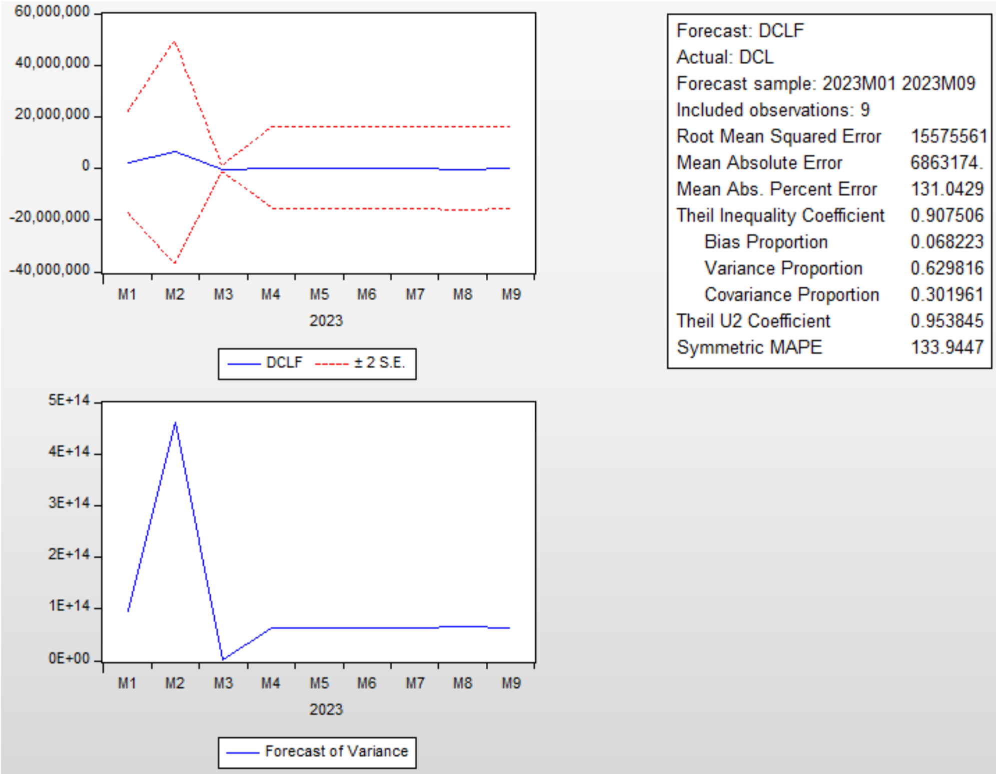
Figure (1) represents the results of the normal distribution test for the residuals



7. FORECASTING AND DISCUSSION

The final step of each time series model is to predict the values of the studied phenomenon. Figure (1) shows the prediction of the variances of the residuals for the estimated model, from which it becomes clear that the estimators were within the confidence limits at a significance level ∝=0.05, which reflects the quality of the model's estimators, which is also enhanced by the average The prediction squares were the lowest among the models that were tested, with a value of 15575561, as well as the average absolute error, which had a value of 6863174, which was overall lower than its counterpart for the other estimated models, which were sufficiently summarized in Table (3).

Figure (2) shows the prediction of model values and the variances of the residuals for the estimated model



REFERENCES

[1] Aielli, G. P. and Caporin, M. (2015). "Dynamic Principal Components: a New Class of Multivariate GARCH Models", Faculty of Economics and Business. Working Paper .194

[2] Bauwens, L. , Laurent, S. , and Rombouts, J.V., (2006), "Multivariate GARCH Models : A Survey", Journal of applied Econometrics, Vol. 21,No. 1, PP. 79 - 109 .

[3] Dhaene, G. and Wu, J. (2016). "Mixed–Frequency Multivariate GARCH", Faculty of Economics and Business Discussion Paper Series DPS 16.12

[4] Doan , T. A. (2014), "RATS Handbook for ARCH/GARCH and Volatility Models", Draft version. Estima, Evanston, IL.

[5] Engle, R. (2002), "Dynamic Conditional Correlation : Asimple Class of multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", Journal of Business and Economic Statistics , Vol. 20 , NO.3 PP.339-350.

[6] Fengler, M. R., and Herwartz, H. (2015), "Measuring Spot Variance Spillover when (Co)variances are time – varying – the case of Multivariate GARCH Model " University of St Gallen , School of Economics and Political Science Department of Economics , Discussion paper no 2015-17.

[7] Francq, C. , Horvath, L. and Zakoian, J. M. (2014), "Variance Targeting Estimation of Multivariate GARCH Modes", Journal of Financial Econometrics , MPERA paper No 57794 16:28 UTC vol. 14. 353-382.

[8] Francq, C. and zakoian, J. M. (2014). "Estimating Multivariate GARCH and Stochastic Correlation models equation by equation", MPRA Paper No. 54250. posted 9. March 23:47 UTC

[9] Kubilay, M. M. (2012), "The Volatility Spillover Among Acountrys Foreign exchange Bound and Stock Markets Amultivariate GARCH analysis", Master Thesis, The Graduate School of Applied Mathematics of Middle East Technical University.

[10] Ngailo, E. , Luvanda, E. and Massawe, E. S. (2014),"Time Series Modelling with Application toTanzania Inflation", Journal of Data Analysis and Information Processing,Vol. 2, No. 02, PP. 49.

[11] Nortey, E. N., Ngoh, D. D., Doku-Amponsah, K., and Ofori-Boateng, K. (2015), "Modeling inflation rates and exchange rates in Ghana : application of multivariate GARCH Models", Springer plus, Vol. 4, No, 1. PP.1-10.

[12] Pedelil, X. , Fokianos, K. , and Pourahmadi, M.(2015)," TwoCholesky log-GARCH models for multivariate volatilites", Journal of Statistical Modelling,Vol. 15, no. 3. pp 233-255

[13] Pelinescu, E. (2014), "Volatility analysis of the Romanian exchange rate", Procedia Economics and Finance,Vol. 8, 543-549

[14] Raddant, M. and Wagner, F. (2016), "Multivariate GARCH for a large number of stocks ",arXiv preprint arXiv:1609.07051..

[15] Sakthivel, P. , Bodkhe, N. and Kamaiah, B. (2012), "Correlation and Volatility Transmission across International Stock Markets: ABivariate GARCH Analysis", International Journal of Economics and Finance, Vol. 4, NO. 3. PP.253.

[16] Sedlak, J. , Hartman, J. (2013), "Forecasting Conditional Correlation for Exchange Rate using Multivariate GARCH Models with Historical Value- AT-Risk application ", Bachelor Thesis ,Uppsala University .

[17] Silvennoinen, A . and Terasvirta, T. (2008) " Multivariate GARCH models"Working Paper Series in Economics and Finance No 669

[18] Yang, S. Y. and Doong, S. C. (2004), "Price and Volatility Spillovers between Stock Prices and Exchange Rates : Empirical Evidence from The G-7 Countries ", International Journal of Business and Economics, Vol. 3, No.2, PP, 139 – 153 .

[19] Jameel, S. O., Salih, A. M., Jaleel, R. A., & Zahra, M. M. (2022). On The Neutrosophic Formula of Some Matrix Equations Derived from Data Mining Theory and Control Systems. *International Journal of Neutrosophic Science (IJNS)*, *19*(1).

[20] Yonis, M. (2011), "Stock Market Co- Movement and Volatility Spillover between USA and South Africa", Master Thesis, UMEA University .