Effect of Friction Forces in Robotic Mechanism Joints on the Dynamic Analysis

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ABSTRACT

Dynamic analysis of parallel robotic mechanisms plays a vital role in the design and control of such robotic mechanisms. To simplify the dynamic analysis formulation, many researches had been done assuming the robotic mechanism joints as ideal joints (without friction). This paper represents a novel mathematical model for evaluation of friction forces, torques acting in the spherical, prismatic joints of parallel robotic mechanism and their effect on the dynamic formulation of any parallel robotic mechanism.

The aim of this paper is to obtain a new mathematical formulation for evaluation of the dynamic analysis in non – ideal robotic mechanism.

The results show that the friction forces and torques acting on the robotic mechanism joints have actual effects on the actuators to implement the same tasks. The actuators forces shall be increased about ten percentages than the power of the actuators in case of ideal robotic joints

Keyword: Parallel robotics mechanisms, Dynamic analysis, Friction forces, Gough – Stewart manipulators.

تاثير قوى الاحتكاك في مفاصل منظومة الروبوت على الحسابات الديناميكية

الخلاصة

ان الحسابات الديناميكية لاي منظومة روبوت متوازي تلعب دورا كبيرا في تصميم الروبوت وحسابات منظومة السيطرة لهكذا نوع من الروبوتات. لغرض تبسيط معادلات الحسابات الديناميكية لهذه الروبوتات تم انجاز مجموعة من الابحاث مع الاخذ بالافتراض هو ان مفاصل هذه الروبوتات ذات طبيعة مثالية (بدون قوى احتكاك). هذا البحث يستعرض طريقة رياضية جديدة لحساب قوى الاحتكاك وعزوم الاحتكاك التي تؤثر في مفاصل الروبوت الكروية والمستقيمة وحساب تاثير هذه القوى والعزوم على الحسابات الديناميكية للروبوت بصورة عامة.

بينت النتائج بان لقوى الاحتكاك تاثير فعلي على المحركات لتنفيذ نفس المهمات للروبوت . ان القوى التي يسلطها المحركات يجب ان تزداد حوالي عشرة بالمائة عنها في حالة استخدام مفاصل مثالية للروبوت.

ان هدف هذا البحث هو لايجاد معادلة رياضية للحسابات الديناميكية للروبوت ذو المفاصل غير المثالية (ذات الاحتكاك).

1. Introduction

There are two types of robotic systems mechanisms, the first type is named open kinematics chain mechanisms and the second is the closed kinematics chain mechanisms. Dynamic analysis of any robotic mechanisms means the evaluation of the actuators forces (controlled forces) which are necessary to implement its task.
In this paper the Gough – Stewart robotic mechanism is chosen **(Fig.1**). It is the famous kind of the closed kinematics chain mechanisms. It consists of a set of serial links each connected to a fixed base from one end, and connected to a common moving platform (or end effecter) on the other end.
In general, Gough-Stewart platform manipulator is a six degree of freedom with two main bodies **[3].** The fixed body is called the base, while another body is regarded as movable and is called the moving plate (platform).
In this paper, to solve the dynamic of the robotic mechanism, every linkage (i) of its linkages (legs) will be divided into five parts as shown in **Fig.2**.
Zone (0i): global coordinate base.
Zone (1i): local coordinate (moving platform)
Zone (2i): the linkage body part located between spherical joint on the moving platform and the prismatic joint
Zone (3i): the prismatic joint
Zone (4i): the linkage body part located between spherical joint on the base and the prismatic joint.
In this paper, the motivation is to derive a mathematical formulation for the evaluation of the actuators forces in case of non ideal mechanism joints. The values will be evaluated by obtaining a novel formulation and using MATH CAD program.

2. Problem formulation

Forces acting on the linkage (i) of the proposed robotic mechanism will be defined as shown in the Fig.3 and Fig.4.as follow:
$R_x^{0i}, R_y^{0i}, R_z^{0i}, M_z^{0i}$: Reactions and moment acting on the spherical joint (with finger) connected the linkage with the base.
$R_x^{1i}, R_y^{1i}, R_z^{1i}$, : Reactions acting on the spherical joint connected the linkage with the moving platform.
$F_x^{2i}, F_y^{2i}, F_z^{2i}, M_x^{2i}, M_y^{2i}, M_z^{2i}$: Inertia forces and moments of the (2i) part of the robotic mechanism linkage
$R_x^{3i}, R_y^{3i}, F_Q^{3i}, M_x^{3i}, M_y^{3i}, M_z^{3i}$: Reactions and inertia force and moment acting on the prismatic joint (3i),F_Q^{3i}: is the actuator (controlled force) acting on this linkage of mechanism.
$F_x^{4i}, F_y^{4i}, F_z^{4i}, M_x^{4i}, M_y^{4i}, M_z^{4i}$: Inertia forces and moments of the (4i) part of the robotic mechanism linkage

R_{ai}: The coordinate vector of the spherical joints connected the fixed base with the legs of the manipulator (in the global coordinate system);
R_{bi}: The coordinate vector of the spherical with finger joints connected the moving platform with the legs of the manipulator (in the local coordinate system);
R_o: The coordinate vector of the moving platform center (in the global coordinate system);
T_{ol}: The matrix of the coordinate transformation from the local coordinate system to the global coordinate system;

s_i : Prismatic joints displacement (legs extension).

The robotic mechanism will be divided into six structures; each linkage (leg) of the manipulator mechanism is treated as an independent substructure. The forces equilibrium of the (i) linkage will be divided into two bodies. The first body is the linkage part located between the mechanism base (0i) and the prismatic joint (3i) as in Fig.3, the second body is the linkage part which located between the prismatic joint (3i) and the moving platform (2i) as in Fig.4. The equilibrium equation for the first body can be written:

$$
\begin{aligned}
&\sum F_x = 0 \ , R_x^{0i} + F_x^{4i} + R_x^{3i} = 0\\
&\sum F_y = 0, \ R_y^{0i} + F_y^{4i} + R_y^{3i} = 0\\
&\sum F_z = 0 \ , R_z^{0i} + F_z^{4i} + F_Q^{3i} = 0\\
&\sum M_x = 0\\
&\qquad M_x^{4i} - M_x^{3i} - F_y^{4i}(L_{c4i}) + R_y^{3i}(S_i - L_{1i}) = 0\\
&\sum M_y = 0\\
&\qquad M_y^{4i} - M_y^{3i} - F_x^{4i}(L_{c4i}) + R_x^{3i}(S_i - L_{1i}) = 0\\
&\sum M_z = 0\\
&\qquad M_z^{4i} + M_z^{3i} - M_z^{0i} = 0
\end{aligned}
\qquad (1)
$$

The equilibrium equation for the second body will similarly be written as follow:

$$
\begin{aligned}
&\sum F_x = 0 \ , R_x^{1i} + F_x^{2i} + R_x^{3i} = 0\\
&\sum F_y = 0, \ R_y^{1i} + F_y^{2i} + R_y^{3i} = 0\\
&\sum F_z = 0 \ , R_z^{1i} + F_z^{2i} + F_Q^{3i} = 0\\
&\sum M_x = 0\\
&\qquad M_x^{2i} + M_x^{3i} + R_y^{3i}(L_{c1i}) + F_y^{2i}(L_{c2i}) = 0\\
&\sum M_y = 0\\
&\qquad M_y^{2i} + M_y^{3i} - R_x^{3i}(L_{c1i}) - F_x^{2i}(L_{c2i}) = 0\\
&\sum M_z = 0 \ , M_z^{2i} + M_z^{3i} = 0
\end{aligned}
\qquad (2)
$$

Where:

$$L_{c4i} = |A_i C_{4i}|$$

$$L_{1i} = |B_i D_i|$$
$$S_{c4i} = |A_i B_i|$$
$$L_{c2i} = |B_i C_{2i}|$$

C_{2i} , C_{4i} are the mass centers of the bodies 2_i and 4_i respectively.

3. Robotic mechanism joints friction forces

Friction force in the spherical joint (A_i) between the moving platform and the linkage part (2_i) can be determined as a friction moment. This moment is acting proportional with the main reaction vector in the joint and in the opposite direction of the relative angular velocity of the moving platform (1_i) to the linkage (i). This moment can be written as follow:

$$M_{fr}^{1i,i} = -K_{Ai}\sqrt{(R_{Ai}^{T}.R_{Ai})}\cdot\frac{\omega_{1i,i}^{0}}{|\omega_{1i,i}^{0}|} \tag{3}$$

Where:

K_{Ai} : coefficient of friction in the joint B_i

$\omega_{1i,i}^{0}$: Projection of the relative angular velocity vector of the moving platform to the linkage.

Similarly, the friction moment in the spherical joint (B_i) in the mechanism base can be written as follow:

$$M_{fr}^{0i,i} = -K_{Bi}\sqrt{(R_{Bi}^{T}.R_{Bi})}\cdot\frac{\omega_{0i,i}^{0}}{|\omega_{0i,i}^{0}|} \tag{4}$$

Where:

K_{Bi} : coefficient of friction in the joint A_i

$\omega_{0i,i}^{0}$: Projection of the angular velocity vector of the linkage according to the base.

Friction force in the prismatic joint will be acted along the linkage length and in the opposite direction of the joint velocity($\dot{S}$). It can be written as follow:

$$F_{fr} = -K_i\sqrt{\left(\left(R_x^{3i}\right)^2+\left(R_y^{3i}\right)^2\right)}\cdot\frac{\dot{S}_i}{|\dot{S}_i|} \tag{5}$$

Where:

K_i: coefficient of the friction in the prismatic joint (3i)

$\dot{S}_i$: Linear velocity of the prismatic joint along the linkage length.

In additional there are other forces acting on the linkage F. The forces consist from gravity force of the mechanism elements, working force and inertia forces **[1].**

$F = (F_x \quad F_y \quad F_z \;\; M_x \quad M_y \quad M_z)$

4. Controlled forces evaluation

The equations (1) & (2) are in the local coordinates systems in which z-axis is parallel to the linkage length. In case of transferring these equations to the Global Coordinate System, the coordinate's transformation matrix T shall be used. When the linkage (i) is assumed in the position of angles α_i and β_i with the Global Coordinate System, the matrix T will be as follow:

$$T = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

Where:

$$a_{11} = \cos\alpha_i\cos\beta_i,\; a_{12} = -\sin\alpha_i,\;\; a_{13} = \cos\alpha_i\sin\beta_i,\; a_{21} = \sin\alpha_i\cos\beta_i$$

$a_{22} = \cos\alpha_i,\ a_{23} = \sin\alpha_i \sin\beta_i,\ a_{31} = -\sin\beta_i,\ a_{32} = 0,\ a_{33} = \cos\beta_i$

From the mathematical analysis it can be used

$\sin\alpha_i = \frac{L_{yi}}{\sqrt{L_{xi}^2+L_{yi}^2}}$,

$\cos\alpha_i = \frac{L_{xi}}{\sqrt{L_{xi}^2+L_{yi}^2}}$

$\sin\beta_i = \frac{\sqrt{L_{xi}^2+L_{yi}^2}}{|L_i|}$

When the linkage (i) will move to another position with the angles $\alpha_i + \delta\alpha_i$ and $\beta_i + \delta\beta_i$, the little angular displacement vector of the linkage (i) can be written as follow:

$$\in_i = k_0\, \delta\alpha_i + j\delta\beta_i$$

Transformation of the little angular displacement vector to the Global Coordinate System will be result:

$$\in_{xi}^0 = -\sin\alpha_i \delta\beta_i$$
$$\in_{yi}^0 = \cos\alpha_i \delta\beta_i$$
$$\in_{zi}^0 = \delta\alpha_i$$

In other words, the little angular displacement vector equal to:

$$\in_i^0 = \begin{pmatrix} -\sin\alpha_i \delta\beta_i \\ \cos\alpha_i \delta\beta_i \\ \delta\alpha_i \end{pmatrix}$$

From this vector, the angular velocity vector of the linkage (i) in the Global Coordinate System can be obtained:

$$\omega_i^0 = \begin{pmatrix} -\sin\alpha_i\, \dot{\beta}_i \\ \cos\alpha_i\, \dot{\beta}_i \\ \dot{\alpha}_i \end{pmatrix}$$

To use the virtual work equation, the robotic mechanism shall implement little linear and angular displacements for any force and moment. These displacements will be as follow:

Little linear displacement for the controlled forces $\left(F_Q^{3i}\right)$and friction force in the prismatic joint $\left(F_{fr}\right)$ is δs.

The little linear and angular displacement for the external forces $(F)$and the forces in the spherical joint of the moving platform (F_{Bi}) is$\delta\rho$.

Also the little angular displacement for the forces in the spherical joint of the base (F_{Ai}) is$\delta \in_i$.

Now, the virtual work on the robotic mechanism (all linkages) can be derived as in the relation below:

$$(F^T + \sum_{i=1}^{6} F_{Bi}^T).\delta\rho + \left(F_Q^T + F_{fr}^T\right).\delta s + \sum_{i=1}^{6} F_{Ai}^T\ .\delta\epsilon_i = 0 \qquad (6)$$

$$\boldsymbol{F_Q} = \left[\left(\frac{\partial \boldsymbol{U}}{\partial \boldsymbol{\rho}}\right)\left(\frac{\partial \boldsymbol{\rho}}{\partial \boldsymbol{s}}\right)\right]^{\boldsymbol{T}} .\boldsymbol{G^T}.\left[\boldsymbol{F} + \sum_{i=1}^{6} \boldsymbol{F_{Bi}}\right] - \boldsymbol{F_{fr}} - \sum_{i=1}^{6}\left(\frac{\partial \in_i}{\partial \boldsymbol{s}}\right)^{\boldsymbol{T}} .\boldsymbol{F_{Ai}} \qquad \textbf{(7)}$$

Where:

$\boldsymbol{F_Q}$: controlled forces in actuators
$\boldsymbol{F_{Ai}}$: All the forces and moments acting in spherical joints A_i
$\boldsymbol{F_{Bi}}$: All the forces and moments acting in spherical joints B_i
$\boldsymbol{F}$ **:** All external forces and moments acting on the moving platform

$$G = \begin{pmatrix} E & 0 \\ 0 & K \end{pmatrix} \quad , K = \begin{pmatrix} 0 & \cos\vartheta & \sin\vartheta\sin\theta \\ 0 & \sin\vartheta & -\cos\vartheta\sin\theta \\ 1 & 0 & \cos\theta \end{pmatrix}$$

5. Example of evaluation and discussion

In this example a Gough –Stewart mechanism has been chosen with the following parameters:
- The base of the mechanism is 2m diameter. The global coordinate system center is located in the base center with z=0. The spherical joints located on the outer diameter of the base with the angle $\gamma = 0, 45, 135, 180,\ 225, 315,$ with reference to the x-axis.
- The moving platform of the mechanism is 1.6m diameter. The local coordinate system center is located in the moving platform center with z=0. The spherical joints located on the outer diameter of the moving platform with the angle $\emptyset = 0, 30, 120, 180,\ 210, 300,$ with reference to the x-axis of the local coordinate system.
- Moving platform mass = 15 Kg
- The external forces and moments acting on the moving platform:

$$F = \begin{pmatrix} 0.5N & 0.25N & -0.75N \\ 0.5N.m & 1N.m & 1.25N.m \end{pmatrix}^T$$

- The inertia tensor is a unit matrix of 0.1 Kg.m^2
- the linear and angular displacement of the moving platform with 0.1m/s^2 and 0.01 degree / s^2 in all directions.
- Prismatic joints linear velocity is 0.02m/s for each linkage.
- Coefficient of friction is 0.03 in each prismatic and spherical joint.

The moving platform center implement a track begins from point 1 and finished in point 6. The track divided into six segments 1,2,3,4,5,6. It was assumed that the moving platform center stopped in each point of the six points. The actuators forces needed to implement the track have been evaluated in each point of the six points for each linkage. The results are as in shown (**Fig.5 a,b,c,d,e,f)** in curve.1 for the mechanism without friction and curve.2**.** The results show that the friction forces and torques acting on the robotic mechanism joints have actual effects on the actuators to implement the same tasks. The actuators forces shall be increased about ten percentages than the forces of the actuators in case of ideal robotic joints

6. Conclusion

In this paper, a novel method had been used to derive a mathematical formulation for the dynamic analysis of a parallel robotic mechanism. The dynamic analysis is proposed based on the virtual work method. The innovation in this paper is that, the joints of the robotic mechanism are no-ideal (with the friction forces evaluation). It has been proved that all the controlled forces can be evaluated and compared with the same forces when the joints without friction.

7. References

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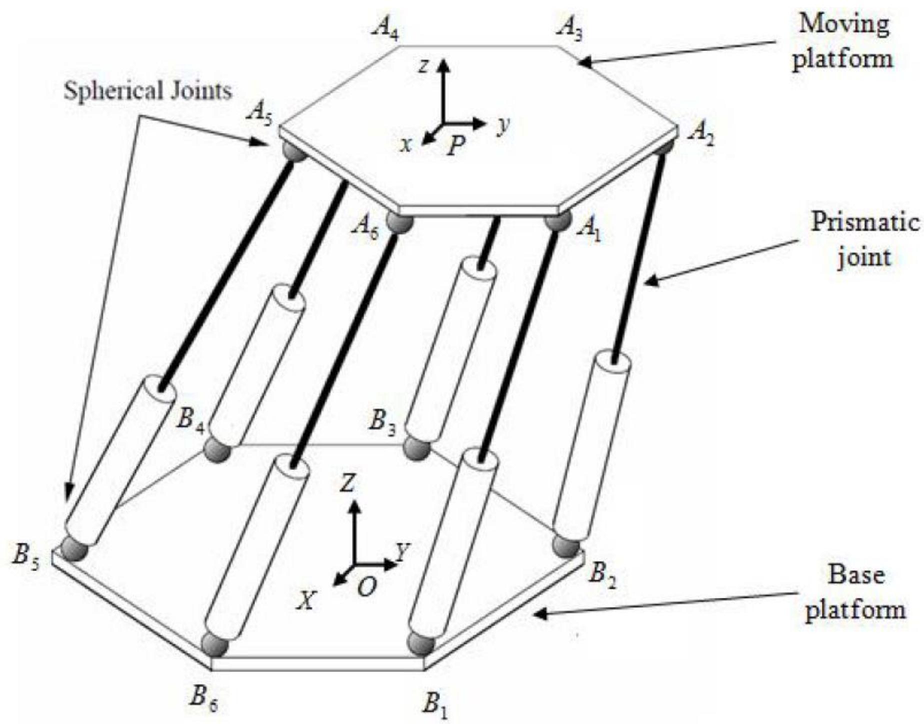

Fig ure 1: Gough – stewart mechanism

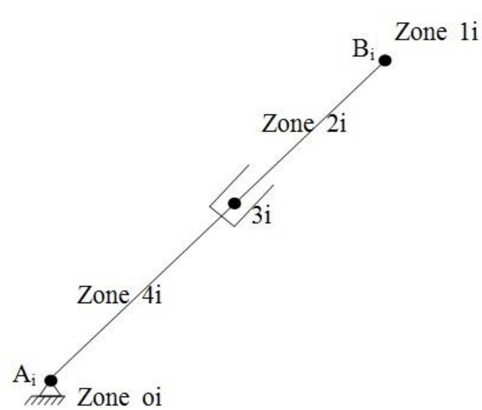

Figure 2: Robotic mechanism linkage (i)

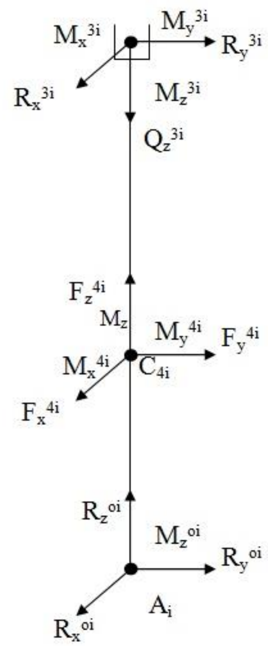

Figure 3: first linkage body

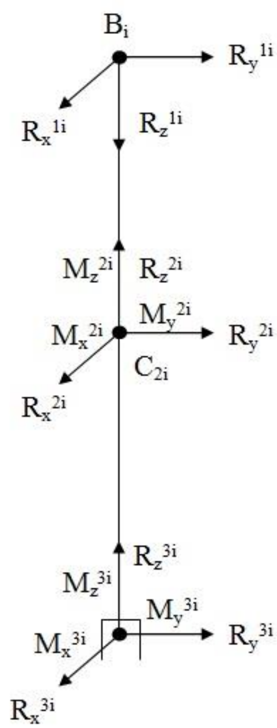

Figure 4: second linkage body

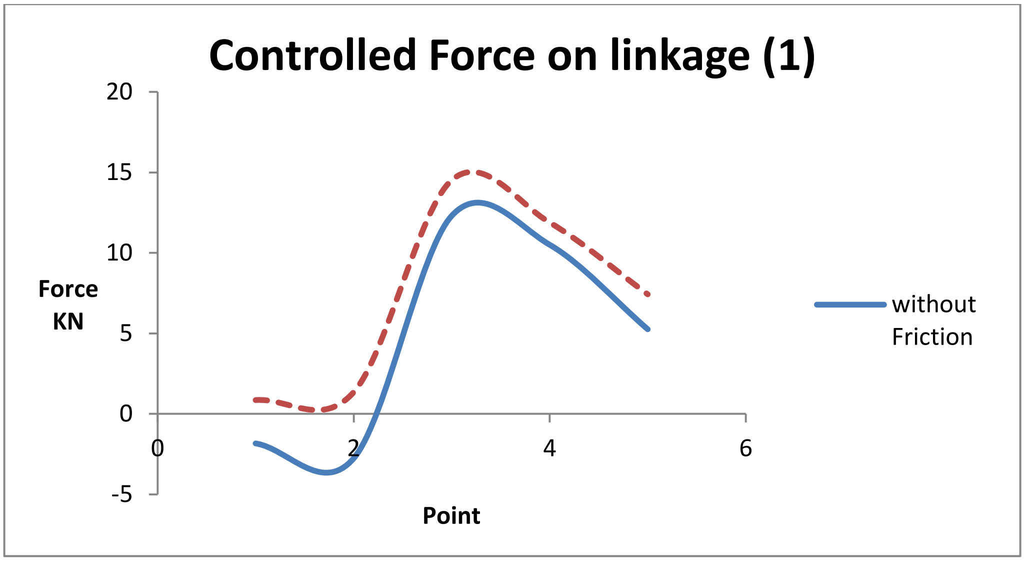

Figure 5 a

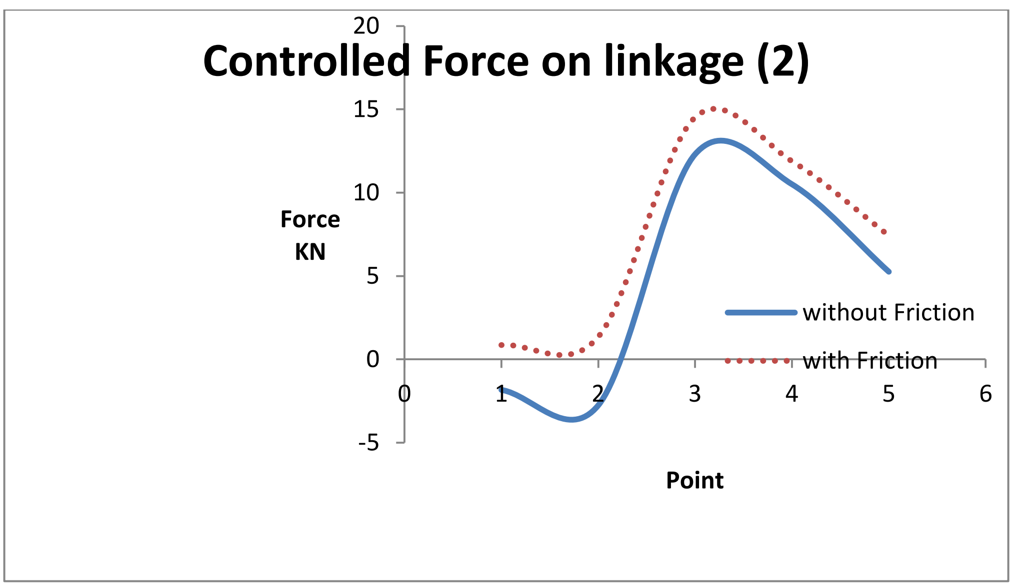

Figure 5 b

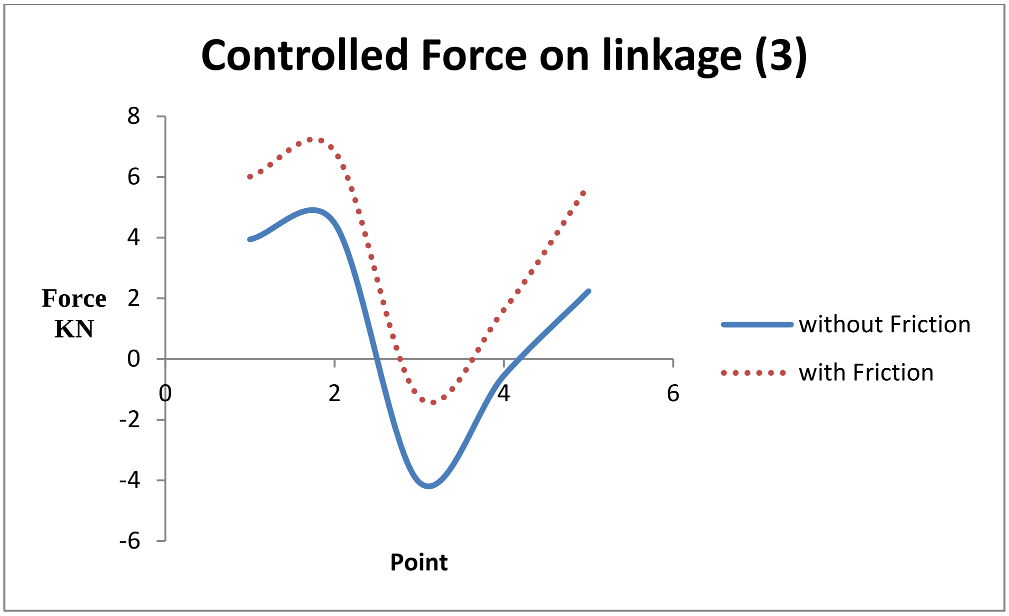

Figure 5 c

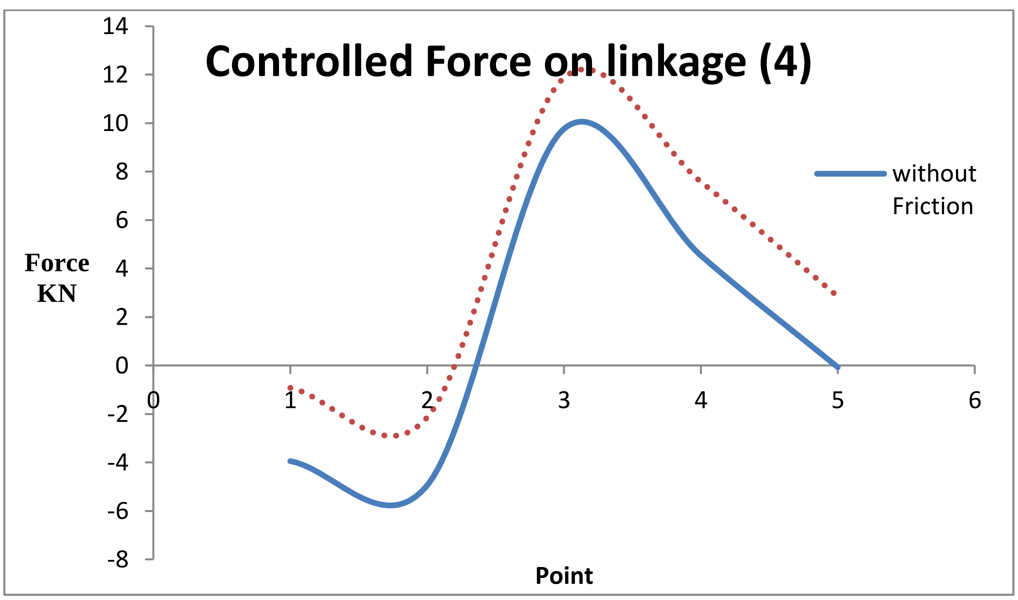

Figure 5 d

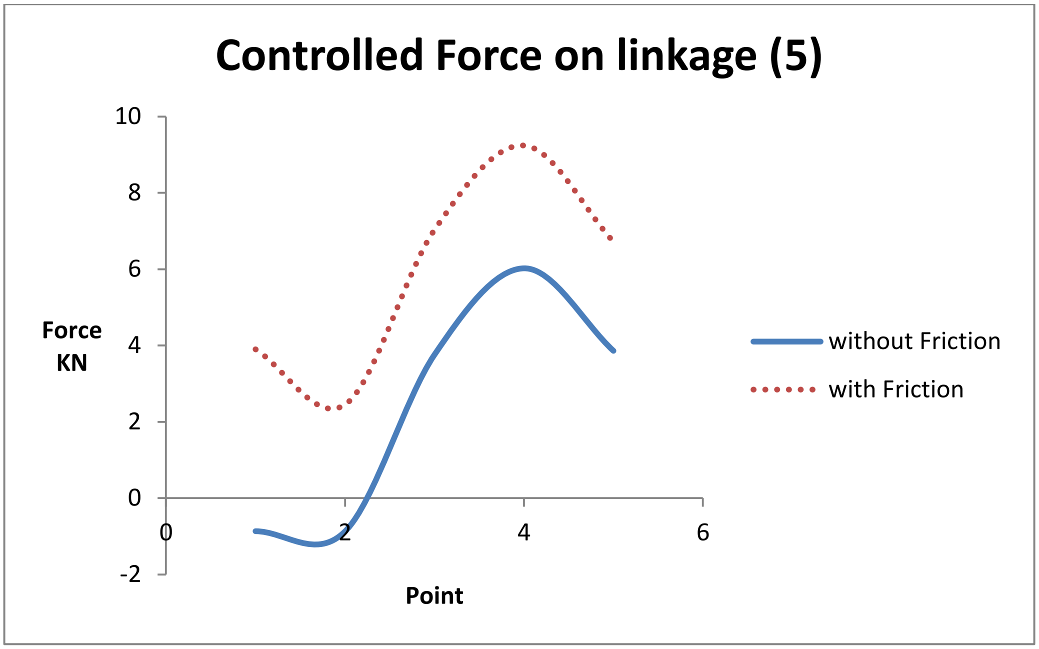

Figure 5 e

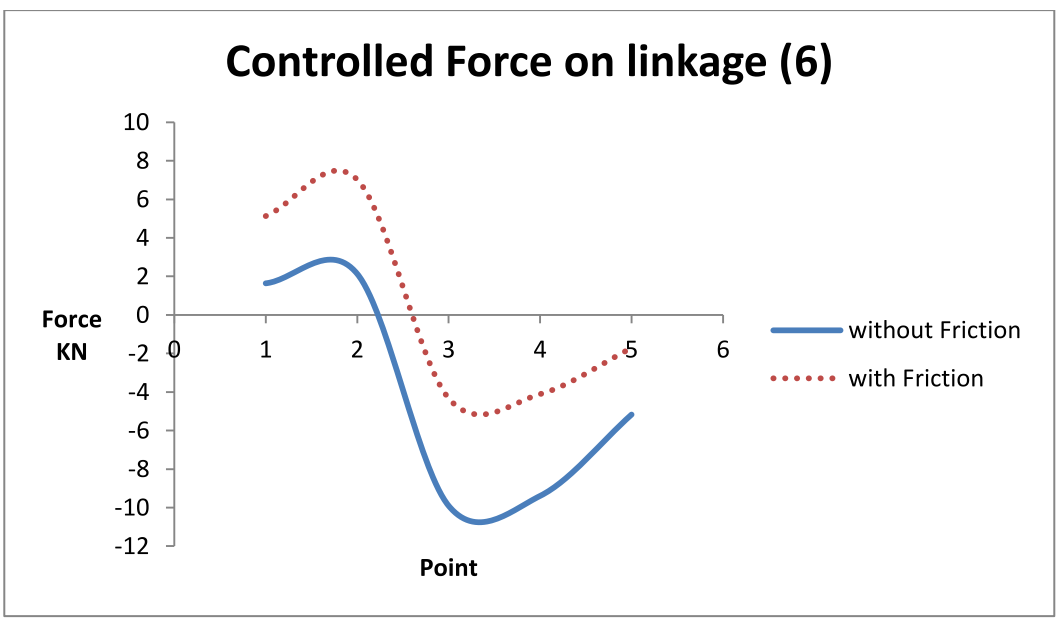

Figure 5 f