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RESEARCH ARTICLE - PHYSICS

Numerical Solutions of Van Der Pol-Mathieu Type Nonlinear Ordinary Differential Equation in Complex Plasma

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Abstract

The description dust plasma behavior via a multi-fluid model, Where it represents that Boltzmann distributed electrons and ions, the dust momentum and Poisson's equations, in addition to the continuity equation that has a source term for the dust grains. The Runge-Kutta technique of fourth order was then used to numerically solve the equation. The solution shows a variety of behaviors under wide range of parameters changes. In particular, the solution of the equation shows a chaotic behavior at α= 1.0. In this study, the chaotic behavior in the plasma complex is graphically studied and discussed for some certain parameters. The aim of this study can be helped the researchers to investigate several nonlinear oscillations in different plasma and fluid mechanics models.

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Keywords: Dust Plasma, Runge – Kutta, Van der Pol – Mathieu equation, Chaos.

1. Introduction

Recently, the characteristics of dusty plasmas have drawn more attention [1,2]. Due to the developments of dusty plasma theory and observations in interstellar space, dust streams from Jupiter and in Saturn's rings, dust tails of comets, additionally, it was shown that noctilucent clouds in Earth's upper atmosphere have a substantial role in the particle contamination of semiconductor wafers., and adversely affect the operations of nuclear fusion at significant quantities, and they might be radioactive in fusion reactors [3]. The presence of relatively large micron (or submicron) sized masses of dust grains in the electron-ion plasma is what gives dusty plasma its name [4-6]. The accumulation of electrons and ions on the surface of the dust particles, which are submerged in the plasma, causes the dust particles to acquire charges, which are often negatively charged due to the greater mobility of electrons [7]. However, the dust charge of the particles is never fixed and temporally changed [8]. Thus, along with the other dynamic properties, it becomes a dynamically variable, which can be modified plasma properties strongly. Charged dust particle grains can change the dynamics of plasma in a variety of ways, most notably the emergence of low frequency dust-acoustic waves [9]. It is known that dust charge fluctuation damped dust – acoustic waves in a dusty plasma [10]. Saitou and Honzawa [11] proposed simplified model based on nonlinear dust charge fluctuation of van der Pol – Mathieu equation and showed that a solution of the equation possibly becomes chaotic. Momeni et al. [12,13] studied simplified model based on nonlinear dust charge fluctuation of van der Pol – Mathieu equation. They discussed stability regions of van der Pol – Mathieu equation and showed the existence of periodically stable and unstable orbits in different parameter space. They noted that oscillations that leads to chaos stems mainly from a balance between Paul van der equation and terms like Mathieu equation.

In this work, we have derived the simplified model of dust charge fluctuation in dusty plasma's van der Pol–Matthew equation. The derived equation is numerically analyzed using the fourth – order Runge-Kutta method (that approximates the curve of the function f between two points by a 4th degree polynomial. Geometrically, we can visualize the curve between two points as part of a 4th degree polynomial) and the solutions graphically represented.

2. Basic Background Formulation

We take into consideration spatially one dimensional an unmagnetized collisionless dusty plasma consisting of electrons, ions and dust grains. The dust grains, which are much heavier than the ion and electrons, are static and the mass of it, md, is taken to remain constant. The plasma ions and electrons surrounding dust grains have a density is large enough compared with those of the dust grains. As a result, the ion and electron density does not fluctuate during the interactions between plasma particles or between the plasma particles and dust grains. The charge neutrality always is verified. The charge of grain dust, q_d, create a variable that is time-dependent,$q_d\ (t)\ =\ -Z_d\ (t)e$ and is much larger than those of the ion or electron, and so continuously the charge q_d changing with the passage of time.

Assuming local thermodynamic equilibrium for electrons and ions, thus adopt Boltzmann type distribution for their number densities n_e and n_i, respectively [14-18].

$$n_e = n_{e0} \exp\left(\frac{e\phi}{k_B T_e}\right) \tag{1}$$

$$n_i = n_{i0} \exp\left(\frac{-Z_i e\phi}{k_B T_i}\right) \tag{2}$$

Where ϕ, T_e , T_i , n_{e0} and n_{i0} represent electrostatic plasma potential, electron (ion) temperature and equilibrium electron (ion) number density, respectively. The ion charge number denotes by Z_i and k_B is the Boltzmann constant. The following equations of continuity and motion with the dust grain source term and Poisson's equation are given:

$$m_d \frac{\partial u_d}{\partial t} = -q_d \nabla \cdot \phi \tag{3}$$

$$\frac{\partial n_d}{\partial t} + n_{d0} \nabla \cdot u_d = \alpha n_d - \frac{1}{3}\beta n_d^3 \tag{4}$$

$$\varepsilon_0 \nabla^2 \phi = -e(Z_i n_i - n_e - Z_d n_d) \tag{5}$$

The variables $n_d,\ u_d$, are perturbed dust density and velocity, respectively. The equilibrium dust number density is denoted by n_{d0} and ε_0 is permittivity of free space. The dust charge number denotes by Z_d.

In the continuity equation the convective derivative term $(u_d \cdot \nabla)u_d$ is neglected due to assumption that the average dust velocity is homogeneous throughout space and has a minimal spatial gradient. Further, we use approximation that the term $\nabla \cdot (n_{d0} u_d)$ is replaced by $n_{d0} \nabla \cdot u_d$ since we suppose that charged dust particles are distributed uniformly throughout space. In the motion equation, α shows the rate of dust absorption of electrons, dust particle charge buildup will be dominated by (electrons since they move through plasmas far more quickly than ions). β

indicates the rate at which dust particles lose charge. We've presumed that the ratio of charged dust particle generation to dust density. The principal reason of the cubic loss term n_d^3 appearance is the dispersion of dust because of recombination of three-body process. $X^+ + e^- + Z \rightarrow X^* + Z$, i.e., an electron may be loss from a dust particle to an ion as a result of an ion-dust particle interaction. [14,16].

Differentiate equations (3) and (4) with respect to space and time, respectively,

$$m_d \frac{\partial u_d}{\partial z \partial t} = -q_d \nabla^2 \phi \qquad (6)$$

$$\frac{\partial^2 n_d}{\partial t^2} + n_{d0} \frac{\partial u_d}{\partial z \partial t} = \alpha \frac{\partial n_d}{\partial t} - \beta n_d^2 \frac{\partial^2 n_d}{\partial t^2} \qquad (7)$$

By substituting Eq. (6) in Eq. (7) we obtain,

$$\frac{\partial^2 n_d}{\partial t^2} - n_{d0} \left(\frac{q_d}{m_d} \nabla^2 \phi \right) = \alpha \frac{\partial n_d}{\partial t} - \beta n_d^2 \frac{\partial^2 n_d}{\partial t^2}$$

$$\frac{\partial^2 n_d}{\partial t^2} - \alpha \frac{\partial n_d}{\partial t} + \beta n_d^2 \frac{\partial n_d}{\partial t} = \frac{n_{d0} q_d}{m_d} \nabla^2 \phi$$

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2) \frac{\partial n_d}{\partial t} = \frac{n_{d0} q_d}{m_d} \nabla^2 \phi \qquad (8)$$

We assume a harmonic potential to be changing harmonically in space and use,

$$\phi(z, t) = \hat{\phi} \exp(ikz)$$

Hence, $\nabla^2 \phi = -k^2 \phi$, where k is wave number. From Eq. (1) when $\frac{e\phi}{k_B T_e} \ll 1$ we can expand n_e , using Taylor expansion,

$$n_e \approx n_{e0} \left(1 + \frac{e\phi}{k_B T_e} + \frac{1}{2} \frac{e^2 \phi^2}{k_B^2 T_e^2} \right) + \mathcal{O} \left(\frac{e^3 \phi^3}{k_B^3 T_e^3} \right)$$

Taken into account only the first order, thus,

$$n_e \approx n_{e0} \left(1 + \frac{e\phi}{k_B T_e} \right) \qquad (9)$$

Similarly for the $\frac{Z_i e\phi}{k_B T_i} \ll 1$ in Eq. (2),

$$n_i \approx n_{i0} \left(1 + \frac{Z_i e\phi}{k_B T_i} + \frac{1}{2} \frac{Z_i^2 e^2 \phi^2}{k_B^2 T_i^2} \right) + \mathcal{O} \left(\frac{Z_i^3 e^3 \phi^3}{k_B^3 T_i^3} \right)$$

$$n_i \approx n_{i0} \left(1 + \frac{Z_i e\phi}{k_B T_i} \right) \qquad (10)$$

Substituting Eq. (9) and (10) into Eq. (5) and using $\nabla^2 \phi = -k^2 \phi$, we obtain,

$$\nabla^2\phi \approx -\frac{q_d n_d}{\varepsilon_0(k_D+k^2)}k^2 \tag{11}$$

In obtaining this equation, we use the electron and ion Debye radii, $\lambda_{De} = \sqrt{\varepsilon_0 k_B T_e/n_{e0}e^2}$ and $\lambda_{Di} = \sqrt{\varepsilon_0 k_B T_e/n_{i0}e^2}$ respectively. The Debye wave number has also been defined by$k_D \equiv \lambda_{\mathrm{Deff}}^{-1} = \sqrt{\lambda_{De}^{-2} + \lambda_{Di}^{-2}}$, where λ_{Deff} is effective Debye length. Substituting Eq. (11) into Eq. (8),

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2)\frac{\partial n_d}{\partial t} = -\frac{n_{d0}q_d}{m_d}\nabla^2\phi$$

$$\nabla^2\phi \approx \frac{q_d n_d}{\varepsilon_0(k_D+k^2)}k^2$$

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2)\frac{\partial n_d}{\partial t} = -\frac{n_{d0}q_d}{m_d}\frac{q_d n_d}{\varepsilon_0(k_D+k^2)}k^2 \tag{12}$$

We shall assume that the fluctuating dust charge to be changing harmonically with time with a frequency ν as [15],

$$q_d(t) = q_{d0}\sqrt{(\delta - \epsilon\,\lambda\cos\nu t)} \tag{13}$$

where (ϵλ) stands for the strength of charge fluctuation. Finally, for the dust density, we can obtain a closed evolution equation.

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2)\frac{\partial n_d}{\partial t} = -\frac{n_{d0}q_{d0}^2\, n_d\,(\delta - \epsilon\lambda\cos\nu t)}{\varepsilon_0 m_d}\frac{k^2}{(k_D+k^2)}$$

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2)\frac{\partial n_d}{\partial t} = -\omega_d^2\frac{k^2}{(k_D+k^2)}\, n_d\,(\delta - \epsilon\lambda\cos\nu t)$$

where the dust plasma frequency is given by,

$$\omega_d = \sqrt{\frac{n_{d0}q_{d0}^2}{\varepsilon_0 m_d}}$$

In the long – wavelength limit, the characteristic oscillation frequency,

$$\omega_0^2 \approx \omega_d^2\frac{k^2}{(k_D+k^2)}$$

Hence,

$$\frac{\partial^2 n_d}{\partial t^2} - (\alpha - \beta n_d^2)\frac{\partial n_d}{\partial t} + \omega_0^2\, n_d\,(\delta - \epsilon\,\lambda\cos\nu t) = 0 \tag{14}$$

Finally, by defining dimensionless variables $\tilde{t} = \omega_0 t$, $x = n_d/n_{d0}$, $\tilde{\alpha} = \alpha/\omega_0$, $\tilde{\beta} = \beta n_d^2/\omega_0$, and $\tilde{\gamma} = \gamma/\omega_0$, we have,

$$\frac{\partial^2 x}{\partial \tilde{t}^2} - \left(\tilde{\alpha} - \tilde{\beta}x^2\right)\frac{\partial x}{\partial \tilde{t}} + \tilde{\omega}_0^2\, x(\delta - \epsilon\,\lambda\cos\tilde{\gamma}\tilde{t}) = 0$$

For simplicity, we will drop the $\sim$ marks,

$$\frac{\partial^2 x}{\partial t^2} - (\alpha - \beta x^2)\frac{\partial x}{\partial \tilde{t}} + \omega_0^2\, x(\delta - \epsilon\,\lambda \cos\gamma t) = 0 \tag{15}$$

The simplest Mathieu equation, which is a model equation for response of many systems to sinusoidal parametric excitation, can be stated as,

$$\ddot{y} + (h - \psi\cos t)y = 0$$

with h and ψ as constants. The van der Pol equation, on the other hand, is a second order differential equation that originates from self-sustaining electric circuits and exhibits nonlinear damping. It can be written as follows:

$$\ddot{y} + \gamma(y^2 - \eta)\dot{y} + y = F(t) \tag{16}$$

with γ and η real. The left-hand side of Eq. (15) second nonlinear term is similar to that of the van der Pol equation, and the third parametric forcing term to the Mathieu one. Hence, the characteristics of both them are combined in this ordinary differential equation to create a hybrid equation. We note that when $\epsilon\lambda \to 0$, we recover the Van der Pol equation, and when $\alpha = \beta = 0$, we recover the Mathieu equation.

3. Results and Discussion

The oscillatory (periodic) solution of the Van der Pol–Mathieu equation is a periodic attractor. By changing α and β parameters, periodic solutions may be explored. In order to numerically integrate Eq. (15), the latter can be represented as a pair of linked ODEs in the form

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = (\alpha - \beta x^2)y + \omega_0^2 x(\delta - \epsilon\lambda\cos\gamma t) \tag{17}$$

The dynamical profile of Eq. (17) may investigate numerically by using a list of recommended fixed values for the variables of the system: $\omega_0 = \epsilon = \lambda = 1$, different values of α and β, besides the initial conditions $x_0 = 1$ and different y_0 for $t_0 = 0$. We solve equation (17) using fourth – order Runge – Kutta method provided by Maple 16 software package. Typical orbits in a phase space (x, y), (t, x) and (t, y) planes for some various values of β are depicted in Fig. (1). Figure (2) shows the size also changes with changing β. Different stable and unstable limit cycles were discovered in the system.

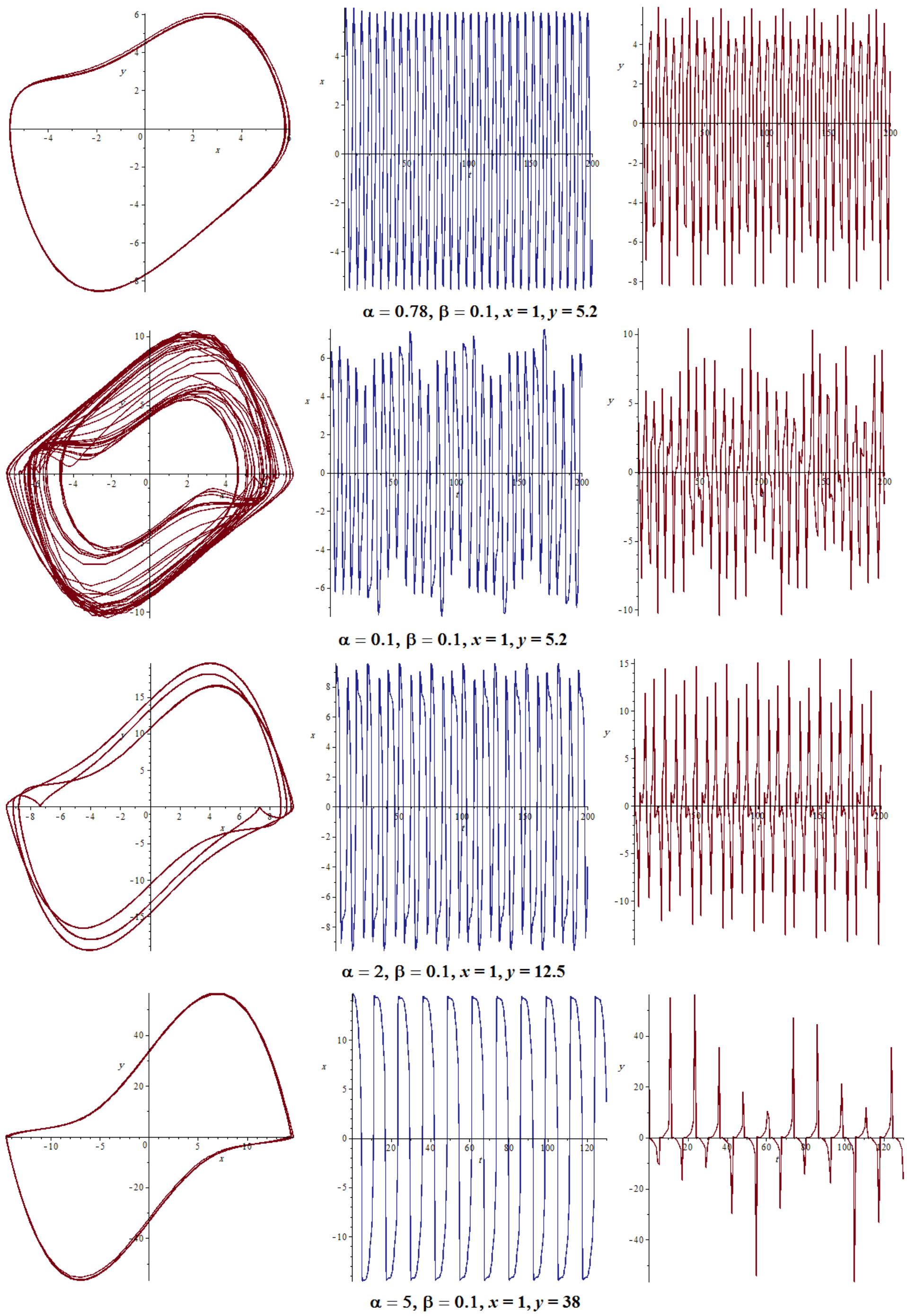

Fig. 1**:** The shape of the attractor changes with changing α and fixed β.

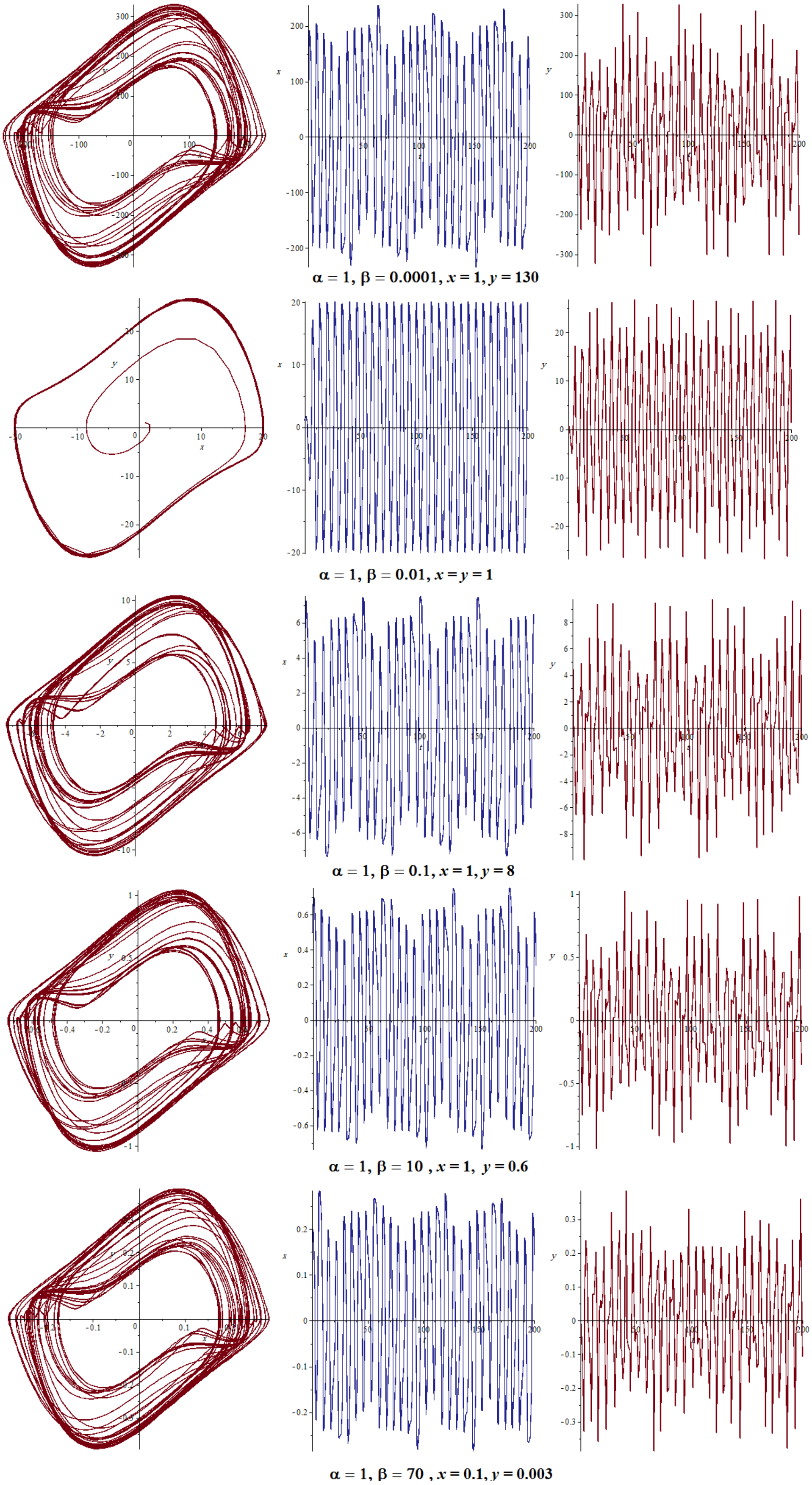

Fig. 2**:** The shape of the attractor changes with changing β and fixed α.

From figures (1 and 2), it is clear that the attractor shape shows quite complicated behavior if $\alpha = 1$ due to the chaos. The attractor is $\alpha -$ dependence. There is a balance between the stability depending on vanderPol and the instability depending on Mathieu equation which determines the shape of the attractor. These stability and instability are competitive at $\alpha = 1$, therefore, the system becomes chaotic. The shape of the attractor for small α is comparable until the last cycle of the stable Mathieu equation while the attractor form and the van der Pol equation's limit cycle exhibit similar characteristics for large α. Now, the dynamical profile of equation (16) may investigate numerically by using a brand-new proposed parameter values that are constant, indicated: $\omega_0 = 1$, $\epsilon = 1$ and $\lambda = 0.01$ for the same values of α and β, besides initial conditions $x_0 = y_0 = 1$ for $t_0 = 0$.

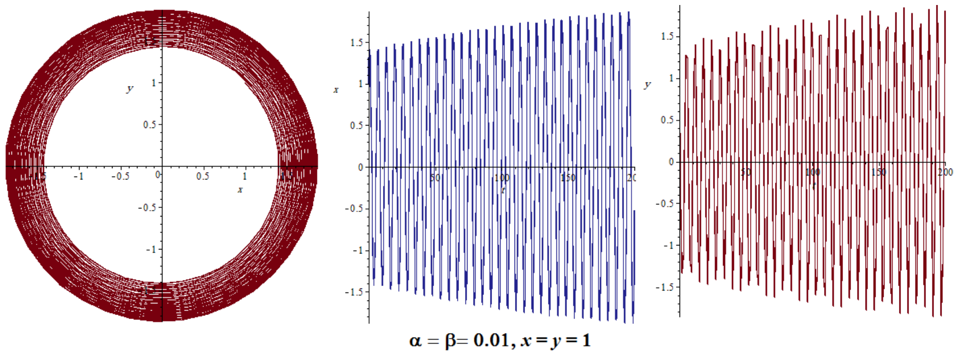

Fig. 3**:** Phase diagram in $x - y$, $t - x$ and $t - y$ planes for $\alpha = \beta = 0.01$

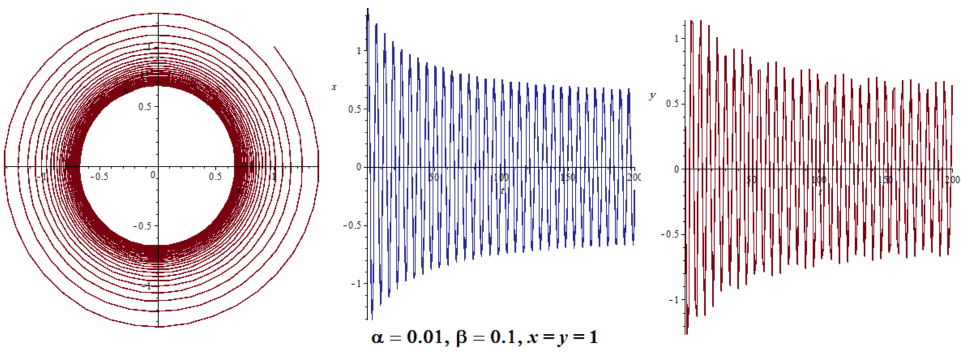

Fig. 4**:** Diagram of the phase$x - y$, $t - x$ and $t - y$ planes for $\alpha = 0.01$, $\beta = 0.1$

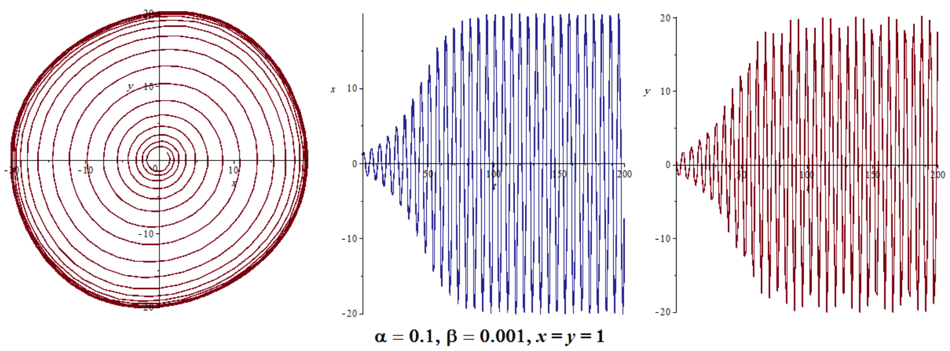

Fig. 5**:** Phase curve in $x - y$, $t - x$ and $t - y$ for $\alpha = 0.1$, $\beta = 0.001$

Eq. (17) can be expressed with effect of parametric excitation frequency as

$$\frac{dx^2}{dt^2}-(\alpha-\beta x^2)\frac{dx}{dt}+\omega_0^2(t)x=0$$

If we assume a solution

$$x=a(t)\cos\left(\omega_0+\frac{\varepsilon}{2}\right)t+b(t)\sin\left(\omega_0+\frac{\varepsilon}{2}\right)t \tag{18}$$

Such that a and b are a real coefficients which vary slowly with time. By substituting Eq. (18) into Eq. (17) for the only first order terms ε and $\epsilon\lambda$, we have a couple of first order, autonomous, ordinary differential equations.

$$\frac{da}{dt}=\frac{\alpha a}{2}-\frac{b}{2}\left(\varepsilon+\frac{h\omega^2}{2}\right)-\frac{\beta}{8}(a^3+ab^2)$$

$$\frac{db}{dt}=\frac{\alpha b}{2}+\frac{a}{2}\left(\varepsilon-\frac{h\omega^2}{2}\right)-\frac{\beta}{8}(a^3+b^2a) \tag{19}$$

Remembering that the vanderPol-Mathieu equation has a constant periodic solution, we take the Jacobian matrix.

$$\mathbf{J}=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}=\begin{bmatrix}\frac{\alpha}{2}-\frac{\beta}{8}(3a^2+b^2) & -\frac{1}{2}\left(\varepsilon+\frac{h\omega_0}{2}\right)+\frac{\beta}{4}ab\\ \frac{1}{2}\left(\varepsilon-\frac{h\omega_0}{2}\right)-\frac{\beta}{4}ab & \frac{\alpha}{2}-\frac{\beta}{8}(3a^2+b^2)\end{bmatrix}$$

Investigating the stability profile in the (α, β) plane with regard to amplitudes (a and b) and parameters ε and ϵλ with the determinant set to zero.

$$0=\frac{1}{4}\alpha^2-\frac{1}{4}\alpha\beta b^2-\frac{1}{4}\alpha\beta a^2+\frac{7}{32}\beta^2a^2b^2+\frac{3}{64}\beta^2a^4+\frac{3}{64}\beta^2b^4+\frac{1}{4}\varepsilon^2-\frac{1}{4}\varepsilon\beta ab-\frac{1}{16}(\epsilon\lambda)^2\omega_0^2$$

Figure (6) demonstrates the stability zone for a typical range of amplitude and parameter values in the (α – β) plane.

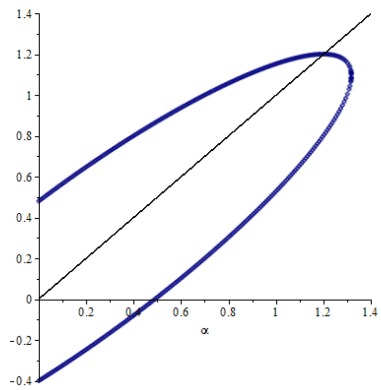

Fig. 6**:** Zones of stability and instability in the$\alpha\ -\ \beta$ plane, for $\varepsilon\ =\ 0.1$, $\epsilon = \lambda\ =\ \omega_0\ =\ 1$ and $a\ =\ b\ =\ 1$

The stable regions are the oval – shaped curve $a_{11}a_{22} - a_{21}a_{12} =\ 0$ and the enclosed below the line $a_{11} + a_{22} =\ 0$; all other regions are unstable. We can determine one of the critical boundaries by $a_{11} + a_{22} =\ 0$ and $a_{11}a_{22} - a_{21}a_{12} >\ 0$ due to two imaginary eigenvalues. As a result, we can observe that the periodic solution could become unstable due to a Hopf bifurcation near the critical limit, in which case $a_{11} + a_{22} =\ 0$. The interpretation of these results is clear and direct, where the charge is visible on the dust granules' surface, although it will be to start zero on the neutral grain, but it is growing due to a random disturbance and abandoning the area near the origin of very quickly. Then, a limit cycle represents a dynamically stable condition, and its physical properties rely on the relevant physical parameters. In relation to its neighboring states, this state serves as an attractor.

4. Conclusions

The generalized Van der Pol equation for investigating the nonlinear oscillations across a wide range of scientific disciplines has been analyzed and solved analytically. Then application in a simplified dusty plasma system with temporally varying dust charges, we numerically solve the equation using the Runge-Kutta method in fourth order. The solution shows a variety of behaviors under wide range of parameters changes. In particular, the solution of the equation shows a chaotic behavior only at α= 1.0. We conclude that this system is characterized by a balance between the region of instability and a stability region where it behaves according to the Mathieu equation, and the other follows the Van der-Pol equation profile (recall that the VdP equation always possesses a periodic solution) respective. These results agree previous findings on dust charging instabilities occurring in dusty plasmas. All estimation obtained may help a lot of researchers in different fields to illustrate and model many nonlinear fact that appear in physics of plasma and fluid technique.

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