Wasit Journal for Pure Science

Journal Homepage: https://wjps.uowasit.edu.iq/index.php/wjps/index
e-ISSN: 2790-5241 p-ISSN: 2790-5233

Stability of nonlinear impulsive higher order differential – fractional integral delay equations with nonlocal initial conditions

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DOI: https://doi.org/ 10.31185/wjps.261
Received 10 October 2023; Accepted 2 December 2023; Available online 30 December 2023

ABSTRACT: The aim of this paper is to investigate some types of stability such as generalized Hyers-Ulam- Rassias stability(G-H-U-R-stabile) and the relation with Hyers-Ulam(H-U-stabile) stable and Hyers-Ulam-Rassias stable (H-U-R-stabile) and generalized Hyers-Ulam stable (G-H-U- stable) to obtain which one guarantee to satisfy stability of equations included a nonlinear function some of them contains a delay time of solution and the other contain a vector of different order of derivatives for the solution to n-time and vector of fractional order of integrals with different fractional orders and that was the for using a calculus of fractional calculus to satisfies the issue of this techniques. Moreover, the nonlocal initial values for the proposal equation of nonlinear impulsive higher order differential – fractional integral delay time equations which are adding more interesting for nonlinear analytic object of nonlinear higher order integro – fractional order impulsive classes, and the impulsive difference of the equation has some necessary conditions to prove the results of solution to be stable with certain type has related with other types. The necessary and sufficient conditions which assumed on this nonlinear higher order integro-differential impulsive equation have been achieved the stability with interesting certain estimates obtain through the proving technique. Also, the uniqueness of solution has been studied with same conditions was presented for stability and used for that issue a contraction fixed point theorem.

Keywords: stability, impulsive, higher order, delay time, nonlocal initial conditions.

1. INTRODUCTION

The interesting object for analytic of qualitative equivalent approach to study the equivalent behaviors of trajectory for ordinary and partial differential equations is the stability theory and impulsive stability theory that which combine continuous of modules of ordinary differential equations with instantaneous state jumps or resets which referred to as impulsive.

Akhmetov and Zafer [4] studied the second method of Lyapunov for impulsive differential equations to be stable. The nonlinear impulsive Stochastic systems with dwell time condition in [9]. well as stability and stabilization of some fractions systems with delay function studied from Lazarevic in [11]. As For nonlinear system with delay impulsive have been presented in [12]. LIU and TEO in [14], interested on analysis stability of some of impulsive control systems

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and with time delay interested from LIU in [13]. Obloza [15,16] were among many approaches of studding the behaviors of H—U-stable one of stability types for interesting differential equations. [27], There are many authors studied with some problems a H-U-stable and H-U-R-stable as criteria for guarantee this issue. In [19,20], the application of impulsive differential equations in different fields such as engineering and natural sciences. There are different classes of impulsive differential equations with first order reportedin [22,25]. Also system of integro-differential equations with their stability explained in [17] with details information.

Some of the impulsive Fractional differential equations introduced as modeling real World phenomena descripted as some moments as a state change instantaneously, [1], also impulsive Riemann-Liouville fractional differential equations are useful in some dynamics model, [2]. The very important mathematical model for describing of particles which has random motion is Langevin equation, [28].

The existence and stability of impulsive stochastic integro-differential equations which has mild random solutions with some proprieties of semigroup and some operators in suitable space, [6], and integral equation of mild solutions of impulsive fractional differential equations presented in [18]. The impulsive fractional integro-differential equations with mixed boundary defined as Riemann–Liouville fractional integral conditions studied in [24], and some of papers studied the solutions by using fixed point theorems such as [5],also, the nonlocal boundary value of impulsive integro-differential equations as a system studied in [23], interesting existence result, and the concept of Ulam stability have studied in [8].

The initial value nonlinear delay differential problem of Riemann-Liouville fractional derivative has been studied with stability concept and, the approach of the delayed-type matrix Mittag-Leffler function with finite-time stability, [3], [10], [7].

Ulam's-type stabilities for class of bounded variable delays of first-order impulsive differential equations on compact interval, [21]. The Grönwall integral inequality fractional integrable impulses for piecewise continuous functions introduced in [26].

The aim of this paper is to present the analytics stability of the nonlinear impulsive higher order differential-fractional integral equations with nonlocal conditions and given the technical of the solution and conditions of stability. Consider the following nonlinear impulsive higher order differential-fractional integral delay equation:

$$\left\{\begin{array}{l} x^{(n)}(t) = F_1\big(s, x(t-\tau)\big) + F_2(t,\{x\})\, g(t,\{x\}^{\propto})\,) \\ \triangle\, x^{(i)}(t_k) = J_k\left(x^{(i)}(t_k^-)\right) \\ with\ nonlocal\ initial\ values \\ x(t_0) = f_0(t)\ ,\ \ x^{(1)}(t_0) = f_1(t_0)\ ,\dots,\ \ x^{(n-1)}(t_0) = f_{n-1}(t_0) \\ where \\ \{x\} = \left\{x, x^{(1)}, \dots, x^{(n-1)}\right\}, \{x\}^{\propto} = \{x, I^{\propto_1}x, I^{\propto_2}x, \dots, I^{\propto_n}x\} \end{array}\right\} \quad (1)$$

2. PRELIMINARIES

In this section, we present the suitable fractional space for the presented system and some definitions of U-stability and explain the auxiliary lemmas to prove interesting main results:

$PC^{n,\alpha}(I,R) = \left\{y: I \to R \middle| y^{(i)}\ PC(I,R),\ i = 0,1,\dots.,n\right\}$ is the Banach space with norm defined by $\|y\|_{PC^{n,\alpha}}$ max $\left\{\|y^{(i)}\|_{PC}: i = 0,1,\dots.,n\right\}$

let $R^+ = [0,+\infty), \{y\} = \left\{y, y^{(1)}, \dots, y^{(n-1)}\right\}, \{y\}^{\propto} = \{y, I^{\propto_1}x, I^{\propto_2}y, \dots, I^{\propto_n}y\}$

$$\left\{\begin{array}{c} \left|x^{(n)}(t) - F_1\big(s, x(t-\tau)\big) - F_2(t,\{x\})\, g(t,\{x\}^{\propto})\,)\right| \le \boldsymbol{\epsilon}\,,\ \ \boldsymbol{t} \in \boldsymbol{I'} \\ \left|\triangle\, x^{(i)}(t_k) - J_k\left(x^{(i)}(t_k^-)\right)\right| \le \boldsymbol{\epsilon},\ \ \boldsymbol{i} = 0,1,\dots,n-1 and\ k = 1,2,\dots,m \end{array}\right\} \quad (2)$$

and

$$\begin{Bmatrix} \left|x^{(n)}(t) - F_1\big(s, x(t-\tau)\big) - F_2(t,\{x\})\, g(t,\{x\}^{\alpha})\,\right| \le \theta(t)\,, \quad \boldsymbol{t \in I'} \\ \left|\triangle\, x^{(i)}(t_k) - J_k\left(x^{(i)}(t_k^-)\right)\right| \le \boldsymbol{\mu}, \quad \boldsymbol{i} = 0,1,\dots,n-1 and\ k = 1,2,..,m \end{Bmatrix} \tag{3}$$

and

$$\begin{Bmatrix} \left|x^{(n)}(t) - F_1\big(s, x(t-\tau)\big) - F_2(t,\{x\})\, g(t,\{x\}^{\alpha})\,\right| \le \epsilon\theta(t)\,, \quad \boldsymbol{t \in I'} \\ \left|\triangle\, x^{(i)}(t_k) - J_k\left(x^{(i)}(t_k^-)\right)\right| \le \boldsymbol{\epsilon\mu}, \quad \boldsymbol{i} = 0,1,\dots,n-1 and\ k = 1,2,..,m \end{Bmatrix} \tag{4}$$

The following definitions explain all types of stability for the system (1) with fractional integral related with some nonlinear and delay functions to make up a system which generalize of many as a classis of it.

2.1 Definition:

1. For real number $\widehat{\boldsymbol{K}}_{\boldsymbol{F_1,F_2,m}} > \boldsymbol{0}$ and every $\boldsymbol{\epsilon > 0}$, the solution $\boldsymbol{x} \in \ \boldsymbol{PC^{n,\alpha}(I,R)}$ of (2), Equation (1) is called H-U-stable on **I** if there exists a solution $\boldsymbol{y_0} \in \ \boldsymbol{PC^{n,\alpha}(I,R)}$ of (1) such that $|\boldsymbol{x(t) - y_0(t)}| < \widehat{\boldsymbol{K}}_{\boldsymbol{F_1,F_2,m}}\boldsymbol{\epsilon},\ \boldsymbol{t \in I}$.
2. For a function $\widehat{\boldsymbol{G}}_{\boldsymbol{F_1,F_2,m}} \in \boldsymbol{C(R^+,R^+)}$ with $\widehat{\boldsymbol{G}}_{\boldsymbol{F_1,F_2,m}}(\boldsymbol{0}) = \boldsymbol{0}$ and $\boldsymbol{\epsilon > 0}$, the solution of equation (1) is said G-H-U-stable on $\boldsymbol{I}$ if there a solution $\boldsymbol{y_0} \in \ \boldsymbol{PC^{n,\alpha}(I,R)}$ of (1) such that $|\boldsymbol{x(t) - y_0(t)}| < \widehat{\boldsymbol{G}}_{\boldsymbol{F_1,F_2,m}}(\boldsymbol{\epsilon}),\ \boldsymbol{t \in I}$.
3. For $\widehat{\mathbf{M}}_{\mathbf{F_1,F_2,,m,\theta}} > \mathbf{0}$, $\boldsymbol{\epsilon} > \mathbf{0}$ and for every solution $\mathbf{x} \in \ \mathbf{PC^{n,\alpha}(I,R)}$ of (4), there exists a solution $\mathbf{y_0} \in \ \mathbf{PC^{n,\alpha}(I,R)}$ of (1) such that $|\mathbf{x(t) - y_0(t)}| < \widehat{\mathbf{M}}_{\mathbf{F_1,F_2,,m,\theta}}\boldsymbol{\epsilon}(\boldsymbol{\theta}(\mathbf{t}) + \boldsymbol{\mu})$, **for t** $\in$ **I** with respect to $(\boldsymbol{\theta}, \boldsymbol{\mu})$**.** Then equation (1) is H-U-R-stable on I.
4. For $\hat{\mathbf{L}}_{\mathbf{F_1,F_2,,m,\theta}} > \mathbf{0}$ and for every solution $\mathbf{x} \in \ \mathbf{PC^{n,\alpha}(I,R)}$ of (3), if there exists a solution $\mathbf{y_0} \in \ \mathbf{PC^{n,\alpha}(I,R)}$ of (1) such that $|\mathbf{x(t) - y_0(t)}| < \hat{\mathbf{L}}_{\mathbf{F_1,F_2,,m,\theta}}(\boldsymbol{\theta}(\mathbf{t}) + \boldsymbol{\mu})$, **for t** $\in$ **I.** Then equation (1) is G-H-U-stable on I, with respect to $(\boldsymbol{\theta}, \boldsymbol{\mu})$.

2.2. Concluding Remark (1) $\Longrightarrow$ (2); (3) $\Longrightarrow$ (4); for $\boldsymbol{\theta(t) = \mu = 1}$; (3) $\Longrightarrow$ (1)

2.3. Lemma If

$$\boldsymbol{y(t) \le a(t) + \int_{t_0}^{t} b(s)y(s)ds + \sum_{t_0<t_k<t} \zeta_k y(t_k^-)}$$

for $\boldsymbol{t \ge t_0 \ge 0}$, where $\boldsymbol{y, a, b \in PC([t_0,\infty), R^+)}$, a is nondecreasing, $\boldsymbol{b(t) > 0}$, and $\boldsymbol{\zeta_k > 0}$, then

$$\boldsymbol{y(t) \le a(t) \prod_{t_0<t_k<t} (1+\zeta_k)\exp\left(\int_{t_0}^{t} b(s)ds\right),} \qquad \boldsymbol{for\ t \ge t_0}$$

3. ULAM'S STABILITY RESULTS FOR NONLINEAR HIGHER ORDER DIFFERENTIAL – FRACTIONAL INTEGRAL DELAY EQUATIONS

we need the following notations as a beginning to understand locally analytic for proving details.

Let B=$(I \times \prod_{i=0}^{n-1}[-N_i\,, N_i]) \times (I \times \prod_{i=0}^{n-1}[-\widehat{N}_i\,, \widehat{N}_i])$, where $N_i = \left\|y^{(i)}\right\|_{PC^{n,\alpha}}$ and $\widehat{N}_i = \left\|I^{i\alpha}y\right\|_{PC^{n,\alpha}}$.

Our interesting of this section to a achieve the U-stable for equation (1).

Lemma 3.1 If the conditions presented and hold it as follows:

i. $\left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) - F_2\left(t, x_2(t), \ldots, x_2^{(n-1)}(t)\right)\right| \leq S_1 \sum_{i=1}^{n}\left|\left(x_1^{(i)}(t) - x_2^{(i)}(t)\right)\right|$, $S_1 > 0$ is a constant.

ii. $\left|g\left(t, I^{\alpha}x_1(t), \ldots\ldots, I^{n\alpha}x_1(t)\right) - g\left(t, I^{\alpha}x_2(t), \ldots\ldots, I^{n\alpha}x_2(t)\right)\right| \leq S_2 \sum_{i=1}^{n}\left|I^{i\alpha}\left(x_1(t) - x_2(t)\right)\right|$, $S_2 > 0$ is a constant.

Then

$$(t-t_0)^{n-1}\left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_1(t), \ldots, I^{n\alpha}x_1(t)\right) - F_2\left(t, x_2(t), \ldots, x_2^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_2(t), \ldots, I^{n\alpha}x_2(t)\right)\right| \leq \left[M_1 h_1(t)\left(\sum_{i=1}^{n}\left|I^{i\alpha}\left(x_1(t)-x_2(t)\right)\right|\right) + M_2 h_2(t)\left(\sum_{i=1}^{n}\left|\left(x_1^{(i)}(t) - x_2^{(i)}(t)\right)\right|\right)\right]$$

Where $h_1, h_2\ I \to R^{+}$ are integrable functions.

Proof

For $t < t_0$

$$(t-t_0)^{n-1}\left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_1(t), \ldots, I^{n\alpha}x_1(t)\right) - F_2\left(t, x_2(t), \ldots, x_2^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_2(t), \ldots, I^{n\alpha}x_2(t)\right)\right|$$

$$\leq (t-t_0)^{n-1}\left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_1(t), \ldots, I^{n\alpha}x_1(t)\right) - F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_2(t), \ldots, I^{n\alpha}x_2(t)\right) + F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_2(t), \ldots, I^{n\alpha}x_2(t)\right) - F_2\left(t, x_2(t), \ldots, x_2^{(n-1)}(t)\right) g\left(t, I^{\alpha}x_2(t), \ldots, I^{n\alpha}x_2(t)\right)\right|$$

$$\leq (t-t_0)^{n-1}\left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right)\right| . \left|g\left(t, I^{\alpha}x_1(t), \ldots\ldots, I^{n\alpha}x_1(t)\right) - g\left(t, I^{\alpha}x_2(t), \ldots\ldots, I^{n\alpha}x_2(t)\right)\right| + \left|F_2\left(t, x_1(t), \ldots, x_1^{(n-1)}(t)\right) - F_2\left(t, x_2(t), \ldots, x_2^{(n-1)}(t)\right)\right| . \left|g\left(t, I^{\alpha}x_2(t), \ldots\ldots, I^{n\alpha}x_2(t)\right)\right|$$

$$\leq \left[M_1 h_1(t)\left(\sum_{i=1}^{n}\left|I^{i\alpha}\left(x_1(t)-x_2(t)\right)\right|\right) + M_2 h_2(t)\left(\sum_{i=1}^{n}\left|\left(x_1^{(i)}(t) - x_2^{(i)}(t)\right)\right|\right)\right]$$

Lemma 3.2

Let $\boldsymbol{y} \in \boldsymbol{PC}^{\boldsymbol{n},\boldsymbol{\alpha}}(\boldsymbol{I}, \boldsymbol{R})$ satisfies the following inequalities equation

$$\begin{cases} \left|\boldsymbol{y}^{(n)}(\boldsymbol{t}) - \boldsymbol{F_1}\left(\boldsymbol{t}, (\boldsymbol{t}-\boldsymbol{\tau})\right) - \boldsymbol{F_2}(\boldsymbol{t}, \{\boldsymbol{x}\}^{\Lambda})\boldsymbol{g}(\boldsymbol{t}, \{\boldsymbol{x}\}^{\propto})\right| \leq \boldsymbol{\epsilon} \quad , \boldsymbol{t} \in \boldsymbol{I}/\{\boldsymbol{t_0}, \ldots, \boldsymbol{t_m}\} \\ \left|\triangle\, \boldsymbol{y}^{(i)}(\boldsymbol{t_k}) - \boldsymbol{J_k}\left(\boldsymbol{y}^{(i)}(\boldsymbol{t_k^-})\right)\right| \leq \boldsymbol{\epsilon}, \qquad \boldsymbol{i} = \boldsymbol{0}, \boldsymbol{1}, \ldots, (\boldsymbol{n}-\boldsymbol{1})\ \boldsymbol{and}\ \boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{m} \end{cases}$$

if and only if there exist a function $\boldsymbol{h} \in C(\boldsymbol{I}, \boldsymbol{R})$ and $\boldsymbol{h_k^i}$ be a sequence on $\boldsymbol{y}$ **such that** $|\boldsymbol{h}(\boldsymbol{t})| \leq \boldsymbol{\epsilon}\ \boldsymbol{for}\ \boldsymbol{t} \in \boldsymbol{I}\ \boldsymbol{and}\ \left|\boldsymbol{f_k^i}\right| \leq \boldsymbol{\epsilon}\ \boldsymbol{for}\ \boldsymbol{i} = \boldsymbol{0}, \boldsymbol{1}, \ldots, (\boldsymbol{n}-\boldsymbol{1})\ \boldsymbol{and}\ \boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{m}$ and

$$\boldsymbol{y}^{(n)}(\boldsymbol{t}) = \boldsymbol{F_1}\left(\boldsymbol{t}, \boldsymbol{y}(\boldsymbol{t}-\boldsymbol{y})\right) + \boldsymbol{F_2}(\boldsymbol{t}, \{\boldsymbol{y}\}^{\Lambda})\boldsymbol{g}(\boldsymbol{t}, \{\boldsymbol{y}\}^{\alpha}) + \boldsymbol{h}(\boldsymbol{t}), \boldsymbol{t} \in \boldsymbol{I}/\{\boldsymbol{t_0}, \ldots, \boldsymbol{t_m}\} \qquad (\boldsymbol{5})$$

$$\begin{Bmatrix} \triangle\, \boldsymbol{y}^{(i)}(\boldsymbol{t_k}) = \boldsymbol{J_k}\left(\boldsymbol{y}^{(i)}(\boldsymbol{t_k^-})\right) + \boldsymbol{h_k^i}\ , \boldsymbol{i} = \boldsymbol{0}, \boldsymbol{1}, \ldots, (\boldsymbol{n}-\boldsymbol{1})\ \ \boldsymbol{and}\ \ \boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{m} \\ , \boldsymbol{0} < \boldsymbol{s} < \boldsymbol{t} < \boldsymbol{T} \end{Bmatrix} \qquad (\boldsymbol{6})$$

Lemma 3.3

If $\boldsymbol{y} \in \boldsymbol{PC}^{\boldsymbol{n},\boldsymbol{\alpha}}(\boldsymbol{I}, \boldsymbol{R})$ satisfy the following

$$\begin{cases} \left|\boldsymbol{y}^{(n)}(\boldsymbol{t}) - \boldsymbol{F_1}\left(\boldsymbol{t}, \boldsymbol{y}(\boldsymbol{t}-\boldsymbol{\tau})\right) - \boldsymbol{F_2}(\boldsymbol{t}, \{\boldsymbol{x}\}^{\Lambda})\boldsymbol{g}(\boldsymbol{t}, \{\boldsymbol{x}\}^{\propto})\right| \leq \boldsymbol{\epsilon} \quad , \boldsymbol{t} \in \boldsymbol{I} \\ \left|\triangle\, \boldsymbol{y}^{(i)}(\boldsymbol{t_k}) - \boldsymbol{J_k}\left(\boldsymbol{y}^{(i)}(\boldsymbol{t_k^-})\right)\right| \leq \boldsymbol{\epsilon} \quad , \boldsymbol{i} = \boldsymbol{0}, \boldsymbol{1}, \ldots, (\boldsymbol{n}-\boldsymbol{1})\boldsymbol{and}\ \boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{m} \end{cases}$$

then

$$\left| y^{(n-i)}(t) - \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_{n-i+j}(t)}{j!} - \sum_{j=1}^{k} J_j\left(y^{(n-1)}(t_j^-)\right) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_1(s, y(t-\tau)) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_2(s,\{x\}^{\Lambda}) g(s,\{x\}^{\propto}) ds \right| \leq \left(\frac{(t-t_0)^i}{i!} + m \right)$$

Proof:

From lemma (3.3), we have that and $t\epsilon(t_k, t_{k+1}]$

$$y^{(n-i)}(t) = \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_{n-i+j}(t)}{j!} - \sum_{j=1}^{k} J_j\left(y^{(n-1)}(t_j^-)\right) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_1(s, y(t-\tau)) + \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_2(s,\{x\}^{\Lambda}) g(s,\{x\}^{\propto}) ds + \int_{t_0}^{w_i=t} \dots \int_{t_0}^{w_2} \int_{t_0}^{w_1} h(s) ds$$

Therefore,

$$\left| y^{(n-i)}(t) - \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_{n-i+j}(t)}{j!} - \sum_{j=1}^{k} J_j\left(y^{(n-1)}(t_j^-)\right) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_1(s, y(t-\tau)) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_2(s,\{x\}^{\Lambda}) g(s,\{x\}^{\propto}) ds \right| \leq \int_{t_0}^{w_i=t} \dots \int_{t_0}^{w_2} \int_{t_0}^{w_1} |h(s)| ds + \sum_{j=1}^{k} \left| h_j^{n-i} \right|$$

Hence

$$\left| y^{(n-i)}(t) - \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_{n-i+j}(t)}{j!} - \sum_{j=1}^{k} J_j\left(y^{(n-1)}(t_j^-)\right) - \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_1(s, y(t-\tau)) + \int_{t_0}^{t} \frac{(t-s)}{(i-1)!}^{i-1} F_2(s,\{x\}^{\Lambda}) g(s,\{x\}^{\propto}) ds \right| \leq \left(\frac{(\boldsymbol{t}-\boldsymbol{s})^{\boldsymbol{i}-\boldsymbol{1}}}{(\boldsymbol{i}-\boldsymbol{1})!} + \widehat{m} \right) \boldsymbol{\varepsilon}.$$

The following result explained the necessary and sufficient conditions of H-U- R-stable for impulsive solution to be converge to the require state and study their uniqueness property of impulsive nonlinear fractional integral system.

4 THEOREM 4.1

If the necessary and sufficient following conditions are satisfied

i. For $t < t_0$

$$(t-t_0)^{n-1} \left| F_2\left(t, x_1(t), \dots, x_1^{(n-1)}(t)\right) g\left(t, I^{\alpha} x_1(t), \dots, I^{n\alpha} x_1(t)\right) - F_2\left(t, x_2(t), \dots, x_2^{(n-1)}(t)\right) g\left(t, I^{\alpha} x_2(t), \dots, I^{n\alpha} x_2(t)\right) \right|$$
$$\leq \left[M_1 h_1(t) \left(\sum_{i=1}^{n} \left| I^{i\alpha}\left(x_1(t) - x_2(t)\right) \right| \right) + M_2 h_2(t) \left(\sum_{i=1}^{n} \left| \left(x_1^{(i)}(t) - x_2^{(i)}(t) \right) \right| \right) \right]$$

ii. $\left| F_1\left(t, x_1(t-\tau)\right) - F_1\left(t, x_2(t-\tau)\right) \right| \leq M_0 \left| x_1(t) - x_2(t) \right|$

iii. $(t-t_0)^{n-1} \left| F_1\left(t, x_1(t-\tau)\right) - F_1\left(t, x_2(t-\tau)\right) \right| \leq h_3(t) \left(\sum_{i=1}^{n-1} \left| \left(x_1^{(i)}(t) - x_2^{(i)}(t) \right) \right| \right)$

iv. $J_k: R \to R$, such that $|J_k(x_1) - J_k(x_2)| \le L_k|x_1 - x_2|$ for $k = 1, \dots, m$. x_1, $x_1 \in R$
Then the nonlinear higher order differential – fractional integral delay equations (5), (6) is G- H-U- R-stable. If in addition

v. $\left[\sum_{j=1}^{k} L_j + \frac{1}{(n-1)!}\int_0^t [h_3(s) + h_1(s) + h_2(s)]ds\right] < 1$

vi. $\beta(s, \theta(s)) \in PC^{n,\alpha}(I, R)$ and $\theta(s) \in PC(I, R)$ are nondecreasing functions respectively such that $\int_{t_0}^{t} \beta(s, \theta(s))ds \le \rho_\beta \beta(t, \theta(t))$ where ρ_β is positive constant.Then (5), (6) has a unique solution.

Proof:

Let $y \in PC^{n,\alpha}(I, R) = \{f(t): f^{(i)}(t) \in C(I, R),\ \mathbf{I}^{i\alpha} f(t) \in C(I, R), i = 1, \dots, n \text{ and } 0 < \alpha < 1\}$ be a solution of the following inequality equation

$$\left|y^{(n)} - F_1(s, x(t-\tau)) - F_2(t, \{x\}).\, g(t, \{x\}^{\propto})\right| < \beta(t, \theta(t)), \quad t \in I \qquad (7)$$

$$\left|\triangle\, y^{(i)}(t_k) - J_k\left(y^{(i)}(t_k^-)\right)\right| < \sigma, \quad i = 1, \dots, n-1. \text{ and } k = 1, \dots, m \qquad (8)$$

The exact solution $x \in PC^{n,\alpha}(I, R)$ of initial value problem with nonlocal of initially values.

$$\left\{\begin{array}{l} x^{(n)}(t) = F_1(s, x(t-\tau)) + F_2(t, \{x\})\, g(t, \{x\}^{\propto})\,) \\ \triangle\, x^{(i)}(t_k) = J_k\left(x^{(i)}(t_k^-)\right) \\ \{x\} = \{x, x^{(1)}, \dots, x^{(n-1)}\}, \{x\}^{\propto} = \{x, I^{\propto_1}x, I^{\propto_2}x, \dots, I^{\propto_n}x\} \\ x(t_0) = f_0(t)\ ,\ x^{(1)}(t_0) = f_1(t_0)\ , \dots,\ x^{(n-1)}(t_0) = f_{n-1}(t_0) \end{array}\right\}$$

defined as the following,

$$x(t) = \begin{cases} \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_i(t)}{j!} + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1(s, x(t-\tau))ds \\ \qquad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_2(s, \{x\}^{\Lambda}) g(t, \{x\}^{\propto})ds, \qquad t \in [t_0, t_1] \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_i(t)}{j!} + J_1(x(t^-)) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1(s, x(t-\tau))ds \\ \qquad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_2(s, \{x\}^{\Lambda}) g(t, \{x\}^{\propto})ds, \qquad t \in [t_1, t_2] \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_i(t)}{j!} + \sum_{j=1}^{2} J_j(x(t^-)) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1(s, x(t-\tau))ds \\ \qquad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_2(s, \{x\}^{\Lambda}) g(t, \{x\}^{\propto})ds, \qquad t \in [t_2, t_3] \\ \vdots \\ \vdots \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_i(t)}{j!} + \sum_{j=1}^{m} J_j(x(t^-)) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1(s, x(t-\tau))ds \\ \qquad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_2(s, \{x\}^{\Lambda}) g(t, \{x\}^{\propto})ds, \qquad t \in [t_m, t_F] \end{cases}$$

From lemma (3.3), we have that

$$\begin{cases} \left| y(t) - \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_i(t)}{j!} + \sum_{j=1}^{k} J_j\left(y(t_j^-)\right) - \int_{t_0}^{t} \frac{(t-s)^{n-1}}{(n-1)!} F_1\left(s, y(s-\tau)\right) ds \right. \\ \left. + \int_{t_0}^{t} \frac{(t-s)^{n-1}}{(n-1)!} F_2(s, \{y\}^{\Lambda}) g(s, \{y\}^{\propto}) ds \right| \leq (m^{\Lambda} + \rho_{\sigma}^{n})\left(\beta(t, \theta(t)) + \mu\right) \end{cases} \quad (9)$$

For $t \in (t_k, t_{k+1})$ and form (9), and lemma (2.3), we get

$$| \; y(t) - x(t)| \leq \left| y(t) - \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_j(t)}{j!} - \sum_{j=1}^{k} J_j\left(y(t_j^-)\right) \right.$$
$$\left. - \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1\left(s, y(s-\tau)\right) ds - \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \; F_2(s, \{x\}^{\Lambda}) g(s, \{x\}^{\propto}) ds \right|$$
$$+ \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \left| F_1\left(s, y(s-\tau)\right) + F_2(s, \{y\}^{\Lambda}) g(s, \{y\}^{\propto}) - F_1\left(s, y(s-\tau)\right) \right.$$
$$\left. - \; F_2(s, \{x\}^{\Lambda}) g(s, \{x\}^{\propto}) \right| ds + \sum_{j=1}^{n} J_j\left(y(t_j^-)\right) - J_j\left(x(t_j^-)\right)$$
$$\leq (m^{\Lambda} + \rho_{\sigma}^{n})\left(\beta(t, \theta(t)) + \mu\right) \frac{1}{(n-1)!}\int_{t_0}^{t} [\boldsymbol{M_0 h_3(s)} |\boldsymbol{y(s) - x(s)}|$$
$$+ M_1 h_1(s)\left(\sum_{i=1}^{n} \left| I^{i\alpha}\left(y(s) - x(s)\right) \right| \right) + M_2 h_2(s)\left(\sum_{i=1}^{n} \left| \left(\boldsymbol{y^{(i)}(s) - \; x^{(i)}(s)}\right) \right| \right)] \mathrm{ds} + \mathrm{L_k} |y(t) - x(t)|$$
$$\leq (m^{\Lambda} + \rho_{\sigma}^{n})\left(\beta(t, \theta(t)) + \mu\right) \prod_{k=1}^{m} (1 + \mathrm{L_k}) exp\left(\frac{1}{(n-1)!}\int_{t_0}^{t} M_0 h_3(s) + M_1 h_1(s) + M_2 h_2(s) ds\right) \quad (10)$$

Therefore, equation (10) satisfy G-H-U-R- stable on I. The solution to be unique we need the following, for $g^{\Lambda} \in PC^{n,\alpha}(I, R)$ where define $\Lambda g^{\Lambda} : PC^{n,\alpha}(I, R) \to PC^{n,\alpha}(I, R)$ as follows:

$$\Lambda g^{\Lambda}(t) = \begin{cases} \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_j(t)}{j!} + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1\left(t, g^{\Lambda}(s-t)\right) ds \\ \quad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \, F_2\left(s, \{g^{\Lambda}\}^{\Lambda} g(t, \{g^{\Lambda}\}^{\propto})\right) ds, \qquad t \in [t_0, t_1] \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_j(t)}{j!} + J_1\left(g^{\Lambda}(t^-)\right) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1\left(t, g^{\Lambda}(s-t)\right) ds \\ \quad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \, F_2\left(s, \{g^{\Lambda}\}^{\Lambda} g(t, \{g^{\Lambda}\}^{\propto})\right) ds, \qquad t \in [t_1, t_2] \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_j(t)}{j!} + \sum_{j=1}^{2} J_j\left(g^{\Lambda}(t^-)\right) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1\left(t, g^{\Lambda}(s-t)\right) ds \\ \quad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \, F_2\left(s, \{g^{\Lambda}\}^{\Lambda} g(t, \{g^{\Lambda}\}^{\propto})\right) ds, \qquad t \in [t_1, t_2] \\ \vdots \\ \vdots \\ \sum_{j=0}^{n-1} \frac{(t-t_0)^j f_j(t)}{j!} + \sum_{j=1}^{m} J_j\left(g^{\Lambda}(t^-)\right) + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} F_1\left(t, g^{\Lambda}(s-t)\right) ds \\ \quad + \frac{1}{(n-1)!}\int_{t_0}^{t} (t-s)^{n-1} \, F_2\left(s, \{g^{\Lambda}\}^{\Lambda} g(t, \{g^{\Lambda}\}^{\propto})\right) ds, \qquad t \in [t_m, t_F] \end{cases}$$

Let $g_1^{\Lambda}, g_2^{\Lambda} \in PC^{n,\alpha}$

$$\left|(\Lambda g_1^{\Lambda})(t)-(\Lambda g_2^{\Lambda})(t)\right| = \left|\sum_{j=1}^{k}\left(J_j\left(g_1^{\Lambda}(t_j^-)\right)-J_j\left(g_2^{\Lambda}(t_j^-)\right)\right)\right.$$
$$\left.+\frac{1}{(n-1)!}\int_{t_0}^{t}(t-s)^{n-1}F_1\left(t,g_1^{\Lambda}(s-t)\right)ds+\frac{1}{(n-1)!}\int_{t_0}^{t}(t-s)^{n-1}F_2(s,\{g_2^{\Lambda}\}^{\Lambda}).\,g(t,\{g_1^{\Lambda}\}^{\alpha})ds\right|$$
$$\leq\left|\sum_{j=1}^{k}\left(J_j\left(g_1^{\Lambda}(t_j^-)\right)-J_j\left(g_2^{\Lambda}(t_j^-)\right)\right)\right|+\frac{1}{(n-1)!}\int_{t_0}^{t}(t-s)^{n-1}\left|F_1\left(t,g_1^{\Lambda}(s-t)\right)ds\right.$$
$$\left.-F_1\left(t,g_2^{\Lambda}(s-t)\right)ds\right|+\frac{1}{(n-1)!}\int_{t_0}^{t}(t-s)^{n-1}\left|F_2(s,\{g_1^{\Lambda}\}^{\Lambda}).\,g(t,\{g_1^{\Lambda}\}^{\alpha})ds\right.$$
$$\left.-F_2(s,\{g_1^{\Lambda}\}^{\Lambda}).\,g(t,\{g_1^{\Lambda}\}^{\alpha})ds\right|\leq\sum_{j=1}^{k}L_j\left|g_1^{\Lambda}(t_j^-)-g_2^{\Lambda}(t_j^-)\right|+\frac{1}{(n-1)!}\int_{t_0}^{t}h_3(t)\left|g_1^{\Lambda}(s)-g_2^{\Lambda}(s)\right|ds$$
$$+\frac{1}{(n-1)!}\int_{t_0}^{t}\left[h_1(t)\left(\sum_{i=1}^{n}\left|I^{i\alpha}\left(g_1^{\Lambda^{(i)}}(t)-g_1^{\Lambda^{(i)}}(t)\right)\right|\right)+h_2(t)\left(\sum_{i=1}^{n}\left|g_1^{\Lambda^{(i)}}(t)-g_1^{\Lambda^{(i)}}(t)\right|\right)\right]$$
$$\leq\left[\sum_{j=1}^{k}L_j+\frac{1}{(n-1)!}\int_{0}^{t}[h_3(s)+h_1(s)+h_2(s)]ds\right].\left\|g_1^{\Lambda}-g_2^{\Lambda}\right\|_{PC^{n,\alpha}}$$

Hence, by Banach contraction principle theorem, we get Λ has a unique fixed point and therefore unique solution of (1). The following result explain the G-H-U-R-stable of impulsive differential nonlinear delay fractional integral (1), and the result is an interesting in field of differential order- fractional order delay time system.

Theorem 5.1

Assume that conditions (i) -(vi) in theorem (4.1) are satisfied. If F_1 satisfied conditions of lemma (3.1), then the nonlinear higher order differential – fractional integral delay equations (1) has G-H-U-R-stable on I.

Proof:

$y \in PC^{n,\alpha}(I,R)$ is a solution of (1). For i=1,2…, n which defined as follows:

$$x^{(n-i)}(t)=\begin{cases}\displaystyle\sum_{j=0}^{i-1}\frac{(t-t_0)^j f_{n-i+j}(t)}{j!}+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_1\big(s,x(t-\tau)\big)ds \\ \displaystyle\qquad+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_2(s,\{x\}^{\Lambda})g(t,\{x\}^{\alpha})ds, \qquad t\in[t_0,t_1] \\ \displaystyle\sum_{j=0}^{i-1}\frac{(t-t_0)^j f_{n-i+j}(t)}{j!}+J_1\big(x(t^-)\big)+\frac{1}{(n-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_1\big(s,x(t-\tau)\big)ds \\ \displaystyle\qquad+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_2(s,\{x\}^{\Lambda})g(t,\{x\}^{\alpha})ds, \qquad t\in[t_1,t_2] \\ \displaystyle\sum_{j=0}^{i-1}\frac{(t-t_0)^j f_{n-i+j}(t)}{j!}+\sum_{j=1}^{2}J_j\big(x(t^-)\big)+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_1\big(s,x(t-\tau)\big)ds \\ \displaystyle\qquad+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_2(s,\{x\}^{\Lambda})g(t,\{x\}^{\alpha})ds, \qquad t\in[t_2,t_3] \\ \vdots \\ \vdots \\ \displaystyle\sum_{j=0}^{i-1}\frac{(t-t_0)^j f_{n-i+j}(t)}{j!}+\sum_{j=1}^{m}J_j\big(x(t^-)\big)+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_1\big(s,x(t-\tau)\big)ds \\ \displaystyle\qquad+\frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_2(s,\{x\}^{\Lambda})g(t,\{x\}^{\alpha})ds, \qquad t\in[t_m,t_F]\end{cases}$$

For $t \in (t_k, t_{k+1})$

$$\left|y^{(n-i)}(t) - x^{(n-i)}(t)\right| \leq \left|y^{(n-i)}(t) - \sum_{j=0}^{i-1} \frac{(t-t_0)^j f_{n-i+j}(t)}{j!} - \sum_{j=1}^{m} J_j\left(y(t_j^-)\right)\right.$$
$$\left. - \frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1} F_1(s, y(t-\tau))ds + \frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}F_2(s,\{y\}^{\Lambda})g(t,\{y\}^{\propto})ds\right|$$
$$+ \frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}\left|F_1(t, y(s-t)) - F_1(t, x(s-t))\right|ds$$
$$+ \frac{1}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}\left|F_2(s,\{y\}^{\Lambda})g(t,\{y\}^{\propto}) - F_2(s,\{x\}^{\Lambda})g(t,\{x\}^{\propto})\right|ds$$
$$+ \sum_{j=1}^{k}\left|J_j\left(y^{n-i}(t_j^-)\right) - J_j\left(x^{n-i}(t_j^-)\right)\right|$$
$$\leq (m^{\Lambda} + \rho_\sigma^n)(\theta(t) + \mu) + \frac{n}{(i-1)!}\int_{t_0}^{t}(t-s)^{i-1}\left[M_1 \sum_{j=1}^{n}\left|I^{j\alpha}\left(y^{n-i} - x^{n-i}(t)\right)\right|\right]$$
$$+ \frac{M_2}{n}\left(\left|y^{(n-i)}(t) - x^{(n-i)}(t)\right|\right) + L_k\left|y^{n-i}(t_j^-) - x^{n-i}(t_j^-)\right|$$

By lemma (2.6), we get

$$\left|y^{(n-i)}(t) - x^{(n-i)}(t)\right| \leq (m^{\Lambda} + \rho_\sigma^n)\left(\beta\left(t, \theta(t)\right) + \mu\right) \prod_{t_0 < t_k < t}(1 + L_k)$$
$$exp\left(\frac{M_1 + \frac{M_2}{n}}{(n-1)!}\left(t_f - t_0\right)^n\right)$$

Hence (1) is G-H-U-R-stable on I.

6. CONCLUSION

- **i.** We concluded that if we investigate G-H--U- R-stable it is approaches to other related types of stability such as H-U-stable and H-U-R-stable and G-H-U-stable.
- **ii.** The Lipchitz conditions of nonlinear functions of higher orders of derivatives and higher orders of integral made a good role for satisfy the stability and obtain a suitable estimation of interesting constants all work together in neighborhood of solution.
- **iii.** We concluded that by adding some necessary and sufficient conditions over the conditions of stability to achieve a unique solution.
- **iv.** The delay time function imbedded in technical of proving with conditions introduced in it for satisfied the target.
- **v.** We concluded that the nonlocal initial conditions are not made abstraction to satisfy the target for the aims of this paper.
- **vi.** The system was presented in this paper is a generalized of systems introduced as a fractional integro-differential nonlinear delay time with high orders of derivatives and different fractional order of integral with nonlocal initial values.

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