ARRAY RADIATION PATTERN RECOVERY UNDER RANDOM ERRORS USING CLUSTERED LINEAR ARRAY

Ahmed J. Abdulqader[1]   *Raad H. Thaher[2]   Jafar R. Mohammed[1]

1) Electrical Engineering Department, College of Engineering, Ninevah University, Mosul, Iraq.
2) Electrical Engineering Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq

Abstract: In practice, random errors in the excitations (amplitude and phase) of array elements cause undesired variations in the array patterns. In this paper, the clustered array elements with tapered amplitude excitations technique are introduced to reduce the impact of random weight errors and recover the desired patterns. The most beneficial feature of the suggested method is that it can be used in the design stage to count for any amplitude errors instantly. The cost function of the optimizer used is restricted to avoid any unwanted rises in sidelobe levels caused by unexpected perturbation errors. Furthermore, errors on element amplitude excitations are assumed to occur either randomly or sectionally (i.e., an error affecting only a subset of the array elements) through the entire array aperture. The validity of the proposed approach is entirely supported by simulation studies.

1. Introduction

In the past years, a wide range of array assembly techniques aimed at determining appropriate values of the antenna parameters (especially excitation values) to obtain the required radiation characteristics has been proposed by antenna designers [1-4]. However, in practice, the fabrication of the antenna array is characterized by uncertainties resulting in differences in the radiation patterns from the actual theoretical patterns. To overcome this problem, it is necessary to perform some complex calibrations and consume more time after the manufacturing process of correctly adjust the control points in the array to reach the desired pattern. One of the most important problems that occur in arrays is the errors that may affect the feeding network. Thus, there are two types of errors usually at hand: systematic and random errors. Systematic error can be adequately eliminated after discovering it, while, random error is difficult to identify because it can vary from one component to another. Consequently, random errors can be of two types [5]: Mechanical errors, such as errors in the positions of the array's elements; examples of such an error are seen in [6-7]. The second type includes electrical errors that randomly scatter the amplitudes and phases of the elements in the array [8]. The existence of these failures creates several conflicts for the array pattern including a change in field strength, unwanted (or unexpected) increase in the sidelobes, and changes in the direction and depth of the nulls. As a result, the initial pattern must be recovered after destruction, and these faults should be fixed.

Corresponding Author: Raadthaher55@gmail.com

In the literature [9-14], there are several solutions proposed by researchers. In [9], Biggelaar and colleagues explored the correction of random errors to which element excitation (amplitude and phase) is subjected by suggesting a statistical method relies on the Rician and Beckmann distributions, as well as the use of Monte Carlo optimization to evaluate and analyze the affected pattern and recover the original pattern. At [10], Another team of researchers employed Monte Carlo optimization to investigate the impact of errors on amplitude only, and its effect on the radiation pattern. Roca and others [4] proposed an observational approach focused on the interval model to estimate unpredictable distinction in the beam pattern because of the random errors that were subjected to the excitation.

Also, the interval analysis method described in [4] was used to study the performance of electromagnetic Radom under the influence of thickness errors on amplitude and phase and its applications in the design of robust Radom [11]. Keizer [12] had proposed another analytical approach that uses a Fourier transform to recreate the required array pattern and correct failed elements due to quantization errors. Some researchers resorted to exploiting the undamaged elements to reconstruct the complex weights of the elements (amplitudes and phases) to reduce the sidelobes in the sum and difference patterns as in [13], where the conjugate gradient method was used in that. In [14], the sensitivity of changing position and depth of null steering was studied under the influence of random error.

In [15], the construction of a subarray (clusters) antenna based on two different architectures was proposed by the researchers, where the elements of the subarray were selected to either it will be regular (uniform) or irregular (nonuniform). These proposed clusters have been added to

linear arrays. In this paper, the methods proposed in [15] are expanded to address the problem of random errors that exciting elements may be exposed to. Thus, here, two goals are achieved: errors correcting and reducing the complexity of the feeding network. Correcting errors is by recalculating the weights of the damaged elements so that the algorithm produces new weights in the form of subarrays according to the method described in [15]. However, the errors that may be exposed to the weights of the elements are either random or sectional (i.e., the errors affecting specific sections of the array aperture). To minimize and correct the random errors, then create clusters, the genetic algorithm (GA) mentioned in [16] was used.

2. The proposed Method

In this section, the idea of this paper consists of two parts: the first is the formulation of the array factor for the clustered linear array. Here, the element weights are selected before any errors are subjected to, according to the Dolph distribution (this is before clustering). The array pattern, in this case, is assumed the original pattern (i.e., it is considered a reference error-free pattern). Next, the effect of errors on the array pattern by damaging the amplitude only of the elements is studied. The second section of the article discusses how to use a GA to restore the original radiation pattern. The errors are either random or sectional.

2.1. The Clustered Linear Array

In this section, two types of clusters, regular clustered array (RCA) and irregular clustered array (ICA), are presented. In the RCA method, all the elements N of the array are divided into several clusters, say Q, of equal size M (i.e., each q has the same number of elements) such that always M<N. The value of Q must be an integer, meaning that the remainder of the N/M is zero. Figure 1 illustrates the RCA architecture. As it can be seen from this figure that each subarray

can contain a specific number of elements starting from 2,3, ... etc., according to the quotient of the division of N/M$_q$. In an ICA method, all array elements are also divided into several subarrays, but of different sizes. Figure 2 illustrates the architecture of this method. It's clear from this figure, that each cluster maybe contains 2,3, ... etc. elements.
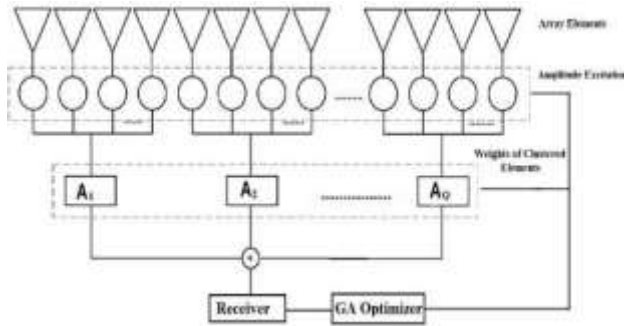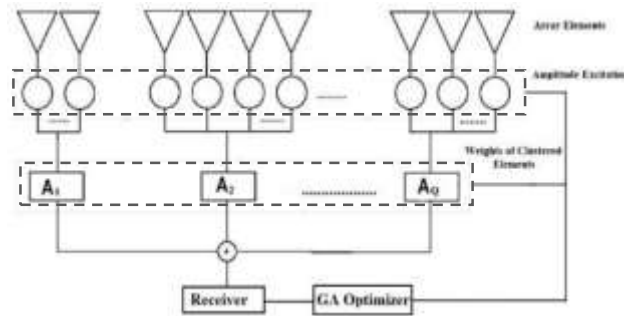

Figure 1. Architecture of RCA

Figure 2. Architecture of ICA

One of the most important benefits of using the clustered technique is to reduce the complexity of the feeding network. However, it is not always possible to simplify the feeding network without other problems occurring in the array system. One of the most common problems that may appear in the pattern is an increase in the sidelobes level (i.e., a high quantization level). On this basis, in the first method (RCA), it is observed that increasing the number of elements in Q is at the cost of generating high quantization lobes in the array pattern, meaning that the increase is not always beneficial. To overcome

this problem, the ICA method is used by choosing different sizes of clusters so that more degrees of freedom are provided than the RCA method. In order to construct the radiation pattern of the clustered array in two methods, the array factor for a clustered array can be written as follows [15]:

$$AF(u) =$$

$$2 \sum_{q=1}^{Q/2} A_q \sum_{n=1}^{N/2} \delta_{cnq} \, w_n \cos\{\frac{(2n-1)}{2} kdu\} \quad (1)$$

where

$$\delta_{cnq}$$
$$= \begin{cases} 1 \ if \ the \ nth \ element \ belong \ to \ the \ mth \ cluster \\ 0 \end{cases}$$

Where A_q and w_n are the element weights in each cluster and the element weights in the ordinary array respectively. $wn = a_n e^{jp_n}$, where a_n is the amplitude and p_n is the phase, k is the wavenumber equal to $2\pi/\lambda$, where λ is the wavelength, d is the spacing distance between any two elements in ordinarily array, u=sin(θ), where θ is the direction of the main beam. It can be seen through Eq.1; the N elements are distributed symmetrically around the center. Thus, only half of the elements are optimized, and this leads to the simplification of the feeding network in half as an initial step. To further simplify the feeding network, amplitude only is used with clustered arrays, meaning that the phase is equal to zero. Then, Eq. 1 can be rewritten as follows:

$$AF(u) =$$

$$2 \sum_{q=1}^{Q/2} |A_q| \sum_{n=1}^{N/2} \delta_{cnq} \, |w_n| \cos\{\frac{(2n-1)}{2} kdu\} \quad (2)$$

It is evident from this equation that the complex weights of both the clustered elements level and the individual elements level become $A_q = |A_q|$ and $w_n = |w_n| = a_n$ respectively. Individual element amplitudes can be chosen based on GA optimization or Dolph distribution, with the

beam pattern of the Dolph distribution serving as a default pattern for restoring after the feeding has encountered errors.

2.2. Error Effect on Amplitude Clustered Elements

In this section, the effect of random errors on clustered element levels $|A_q|$ is considered. The presence of these errors causes the amplitude levels to be randomly perturbed, either increase or decrease. The amount of error that can affect any level of the elements is ΔA_q. Consequently, the array factor shown in Eq. 2 will suffer undesired variances that differ from its original form (i.e., in the absence of errors), so Eq. 2 can be rewritten as follows [5]:

$$AF(u)_{error} = 2 \sum_{q=1}^{Q/2} |A_q -$$

$$\Delta A_q|^2 \sum_{n=1}^{N/2} \delta_{cnq}\, a_n \cos\{\frac{(2n-1)}{2} kdu\} \qquad (3)$$

The term $|A_q - \Delta A_q|$ is expressed of $A_q \delta_{A_q}$, where the factor δ_{A_q} expresses the error factor and is calculated by:

$$\delta_{Aq} = 1 - \frac{\Delta A_q}{A_q} \qquad (4)$$

Then, this factor can be calculated using $\bar{\delta} = \frac{1}{2\gamma} \int_{-\gamma}^{\gamma} \delta_{Aq}\, d\delta$, γ can be calculated by $\gamma = \frac{\gamma_r}{2(2^a-1)}$, Where γ_r expresses the specific values used in the digital attenuator, a is the number of bits. The factor γ_r is calculated in decibels, so γ is also calculated in decibels. Finally, the error factor is also in decibels and in the range $-\frac{\gamma_r}{2(2^a-1)} \leq \delta_{dB} \leq \frac{\gamma_r}{2(2^a-1)}$. Moreover, the variance in the errors of clustered elements level can be calculated using the following:

$$\sigma_\delta^2 = \frac{1}{2\gamma} \int_{-\gamma}^{\gamma} (10^{\frac{\delta_{dB}}{20}} - \bar{\delta})^2\, d\delta_{dB} \qquad (5)$$

Thus, this can be defined as:

$$\sigma_\delta^2 =$$

$$5 \frac{10^{-\gamma/10}}{\gamma\, ln(10)} \left[(10^{\gamma/5} - 1) - 20 \frac{(10^{\gamma/10}-1)^2}{\gamma\, ln(10)} \right] \qquad (6)$$

Obviously, large errors in element weights occur when digital attenuators have a finite range. In general, the range of amplitudes is calculated by the ratio between the weights of the elements in the center and the weights on the sides. Also, quantization errors can be reduced if the clustered levels are used in the optimization instead of the individual element level. In the presence of random errors, the variance in the directivity can be calculated through the following equation:

$$D_{deviation} = \frac{4\pi}{\sum_Q |A_q|^2} E[|AF(u)_{error}|^2] \qquad (7)$$

Where E is the expectation (or average), and this equation can be further simplified, as follows:

$$D_{deviation} = \bar{\delta}^2 D_{original} + 4\pi\sigma_\delta^2 \qquad (8)$$

It is clear from this equation that the variance in the directivity differs from the original directivity (error-free directivity). The value of this directivity is equal to the original directivity plus σ_δ^2, and this factor is considered a small value in the clustered array. Therefore, directivity is one of the parameters that have little effect on errors. From the other side, the variance of errors in the level of sidelobes can be calculated through the following equation:

$$SLL_{error} = \frac{\sigma_\delta^2 \sum_Q |A_q|^2}{\bar{\delta}^2 |\sum_Q A_q|^2} \qquad (9)$$

As it can be observed through this equation, since SLL_{error} is based on the amplitudes of the clustered level rather than the amplitudes of individual components, then the subarrays are more tolerant of weight errors.

3. Simulation Results

To examine the effectiveness of the proposed methods in reducing the effect of random errors of the elements amplitude, thus addressing the damaged pattern, simultaneously, the complexity of the feeding network is being reduced, large-

scale simulations are performed under the suggestion of various scenarios. The original fully linear array consists of 2N=48 isotropic elements with a distance of d=0.5 between any two elements within the array. The errors that can be added to the excitation of amplitude clusters are of the type of real random numbers with zero average value. The locations of the affected elements' weights are either randomly within the array aperture or sectional (i.e., affecting only a portion of the elements located in certain sections of the array aperture). GA with a population size of 20, 0.15 mutation rate, and single crossover is considered to improve the amplitudes of clusters by using the following objective (cost) function:

$CostFunction =$
$\sum |AF - Constraints \text{ (Mask limit)}|^2$ (10)
$Constraints \text{ (Mask limit)} =$
$\begin{cases} uppermasklimit = -30 dB ExemptMainBeam \\ lowermasklimit = -60 \ dB \end{cases}$

Where the $Constraints$ (Mask limit) represents the restrictions required to be imposed on the pattern of the linear clustered array for reducing sidelobes errors SLL_{error} as shown in figure 3.
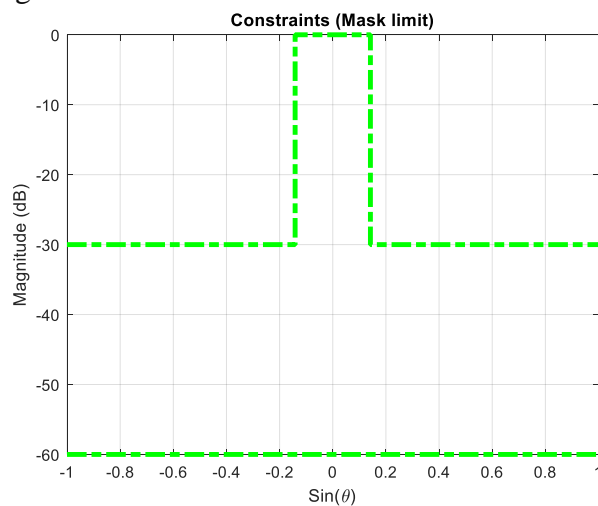
Figure 3. The proposed mask constraints

The boundaries for cluster amplitudes values are set between (0 and 1). Both types of RCA and ICA are constructed and examined in this paper.

In the RCA method, all clusters will contain the same number and values of elements with sizes 2, 3, ... etc. while in the ICA method, the number of elements is irregular, but evenly distributed among the clusters, so the size of each cluster is different. The GA uses the parameter M+Q for optimization. The following examples are considered to check and verify the performance of the proposed clusters.

3.1. Example1: Errors with RCA

Figure 4(a) shows the distribution of the weight of the elements without errors calculated by the Dolph distribution. As figure 4(b), it is noticed that the weights of the elements are damaged by random errors, which led to a variance in the distribution of the weights from the original. While figure 4(c)shows the result of adding regular clusters with size M = 3 to overcome the problem of the damaged sidelobe pattern. Finally, in figure 4(d), a comparison of the patterns in the three cases (original, damaged, regular cluster). It can be seen from the figure 4 that the clustered array is able to reduce the random errors and restore the original pattern by preserving its properties.

Next, the effect of random errors on placing broad null steering in the desired pattern is studied. Figure 5 shows comparison patterns for the three cases (original, damaged, regular cluster) with a broad null at 0.55 rad. The clustered array, in this case, was also considered as M=3. It is evident from this figure that the clustered method is also able to restore the original pattern that has a broad null.
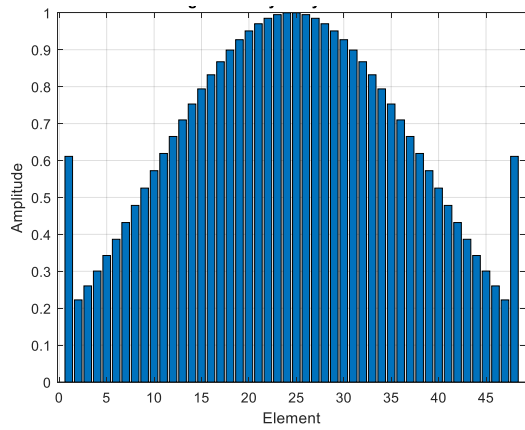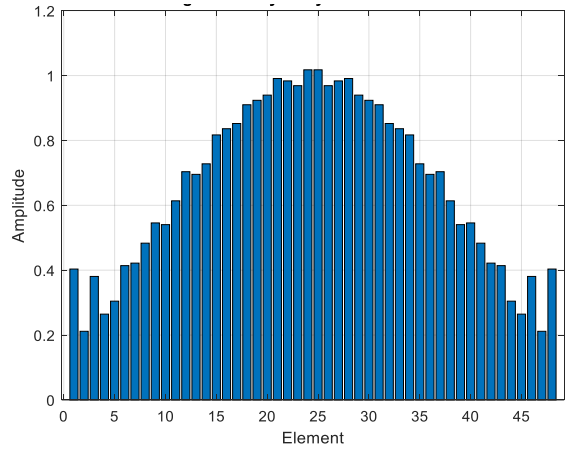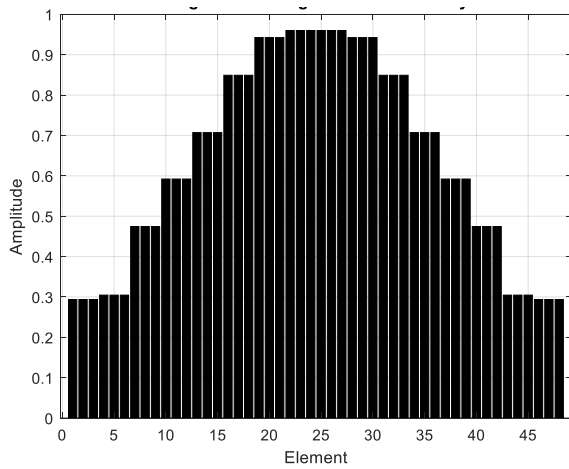
a. Error-free amplitudes of fully array

d. Radiation patterns

Figure 4 Results of applying RCA with M=3 and for original linear array with 2N=48
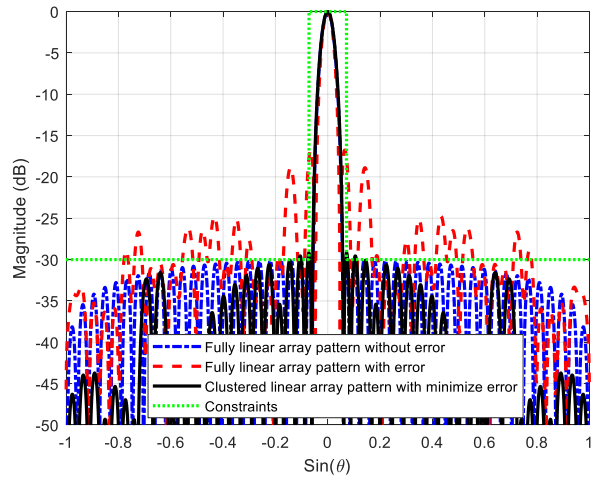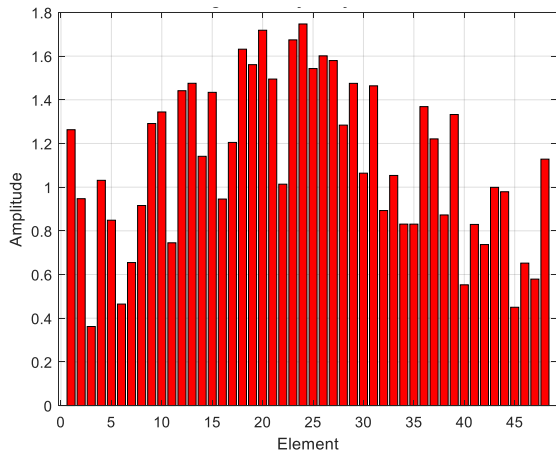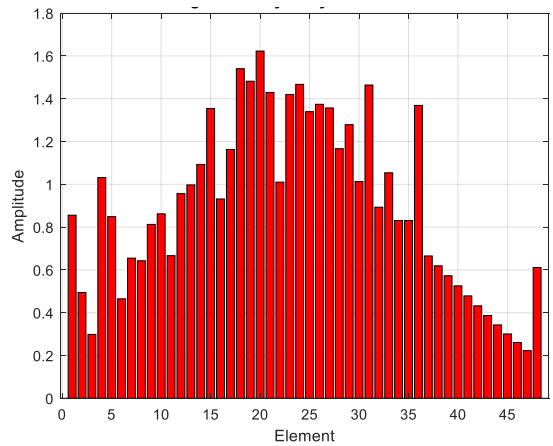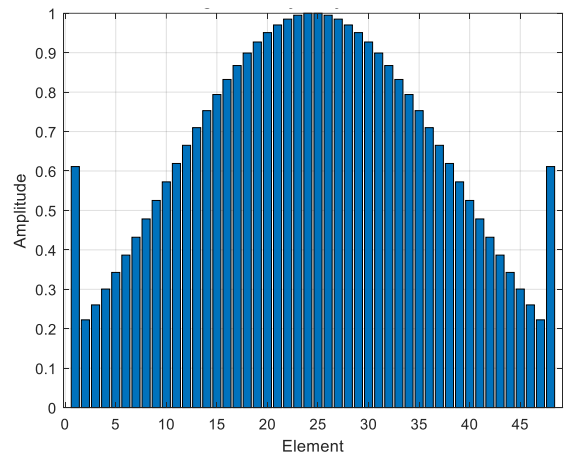

b. Amplitudes of fully array with error

a. Error-free amplitudes of fully array

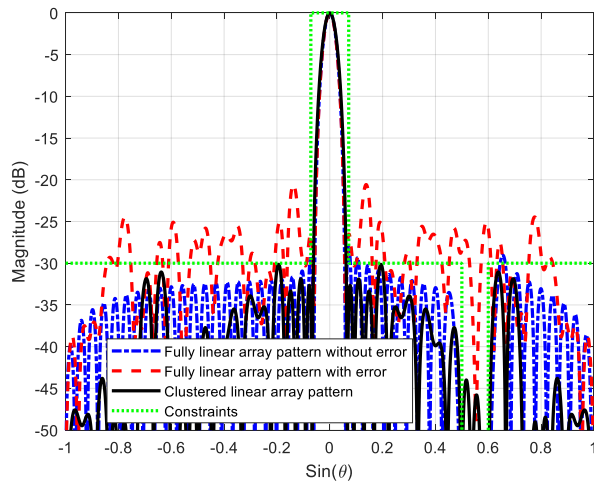c. Amplitudes of the RCA

b. Amplitudes of fully array with error
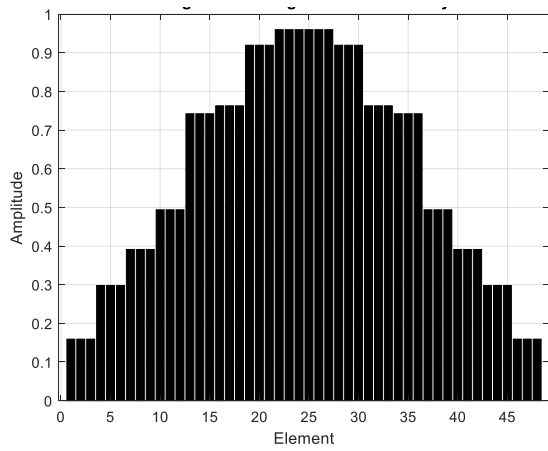
c. Amplitudes of the RCA

a. Error-free amplitudes of fully array

d. Radiation patterns
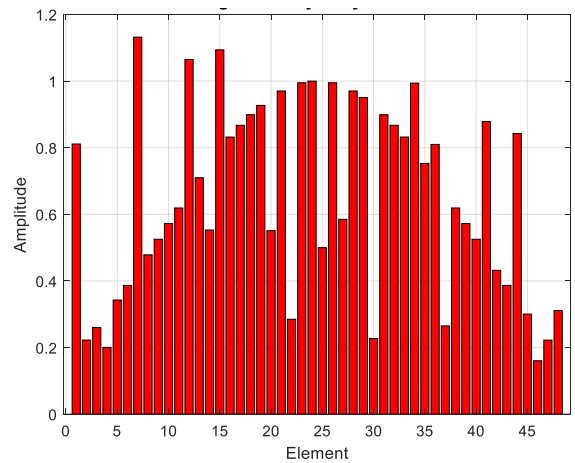
Figure 5 Results of applying RCA with a broad null

b. Amplitudes of fully array with error

3.2. Example2: Errors with ICA

Figure 6 illustrates the benefit of employing the different sizes of ICA method. This figure contains the patterns for the three cases with the weight distribution for each one. Again, the clustered method demonstrates its ability to bypass the problem of exposing weights to random errors.
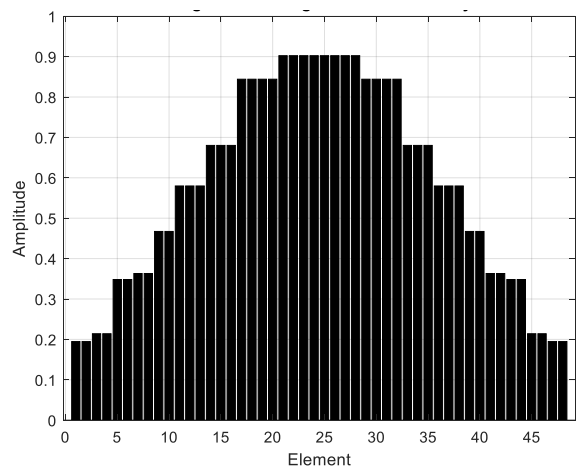
c. Amplitudes of the ICA
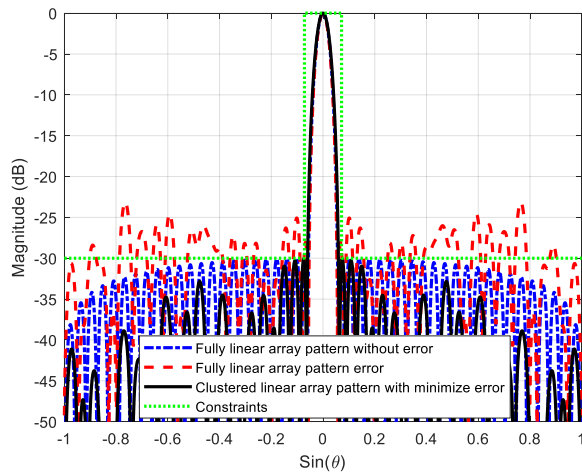
d. Radiation patterns

Figure 6 Results of applying ICA with M=2, 3 and 4 and for original linear array with 2N=48

3.3. Example3: Sectional Errors with Clustered Linear Array

In this example, a Dolph array of 2N=48 with specified constraints SLL=-30dB is imposed as the reference array, and its radiation pattern is set as the original pattern to be recovered. Next, the use of the GA with the proposed clustered array to solve the problem of random errors. In the first test, 25% of the elements (the first section of the array aperture elements) are affected by random errors are considered as shown in figure 7. It is noticed through this figure that the sidelobes level was affected, so that it exceeded the specified mask and became at -24 dB. To restore the original pattern in this case, RCA with size M=3 was used.

In the second test, 50% of the elements are damaged due to random errors. For this case, a RCA of size M=3 was also used. Figure 7 shows the details of this case. Finally, the percentage of the elements exposed to random errors was increased to 75%. The handle of the damaged pattern, in this case, is by the use of an ICA with different sizes (see figure 9).
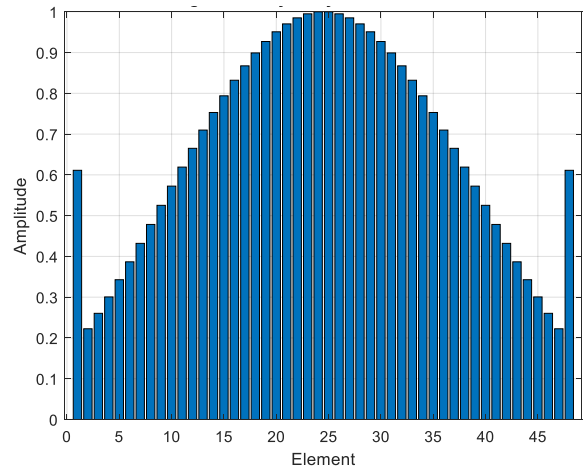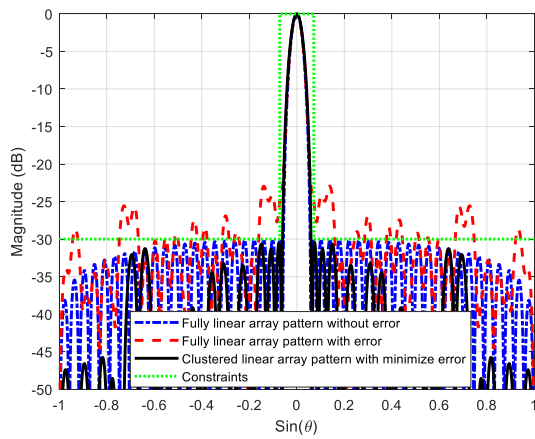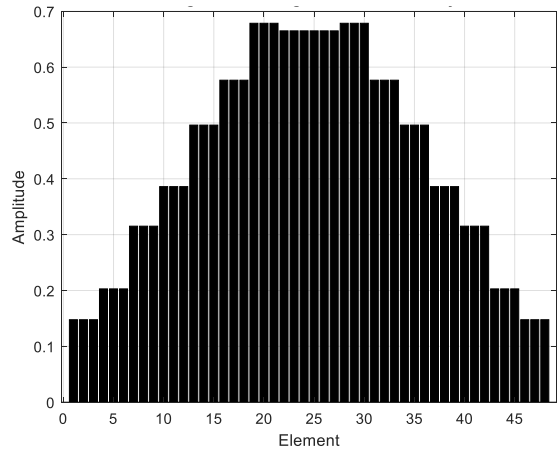

a. Error-free amplitudes of fully array
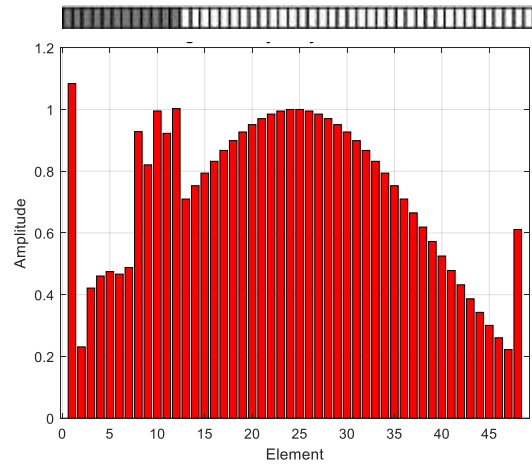
b. Amplitudes of fully array with 25% sectional error
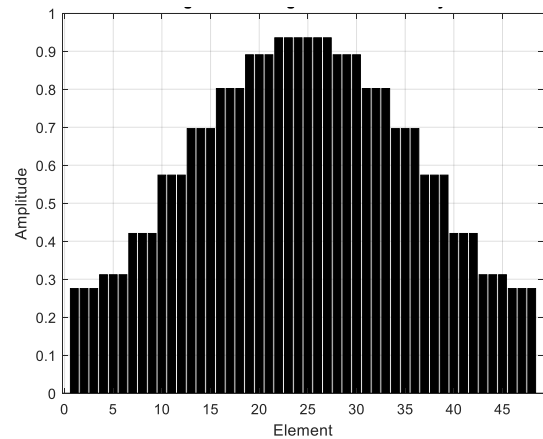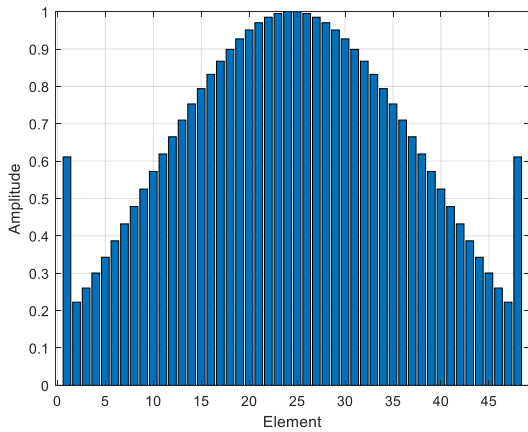

c. Amplitudes of the RCA

d. Radiation patterns
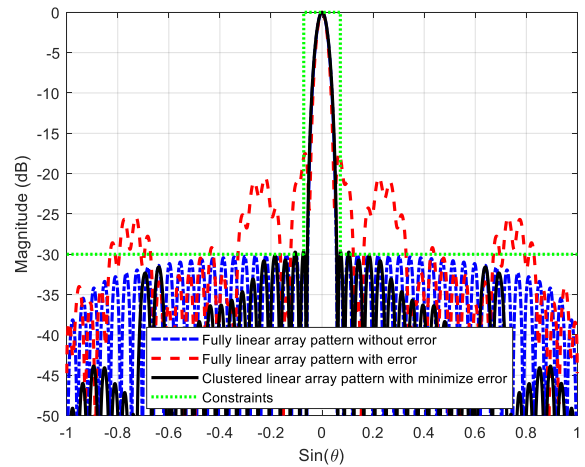
Figure 7. Results of RCA with 25% sectional error for an original Dolph linear array with 2N=48

a. Error-free amplitudes of fully array

c. Amplitudes of the RCA

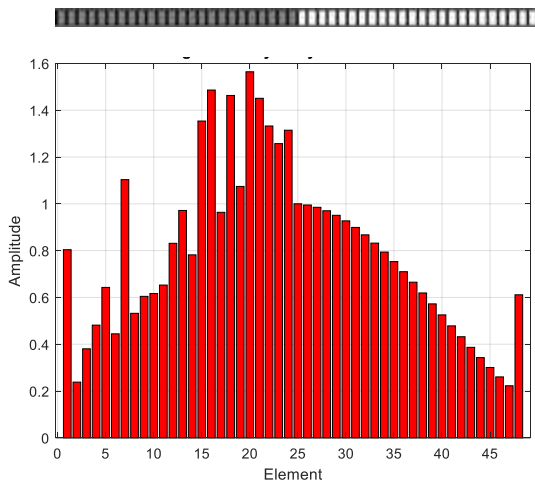c. Amplitudes of the RCA

d. Radiation patterns

Figure 8. Results of RCA with 50% sectional error for an original Dolph linear array with 2N=48

a. Error-free amplitudes of fully array

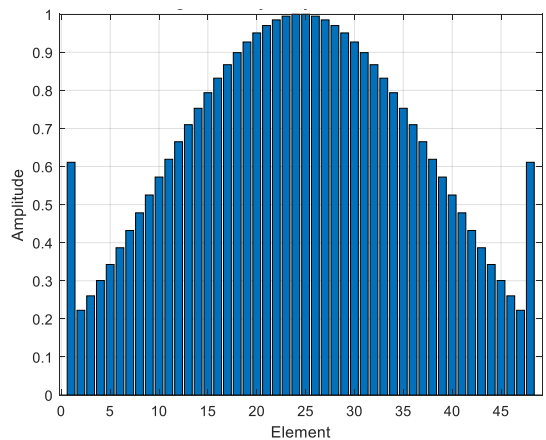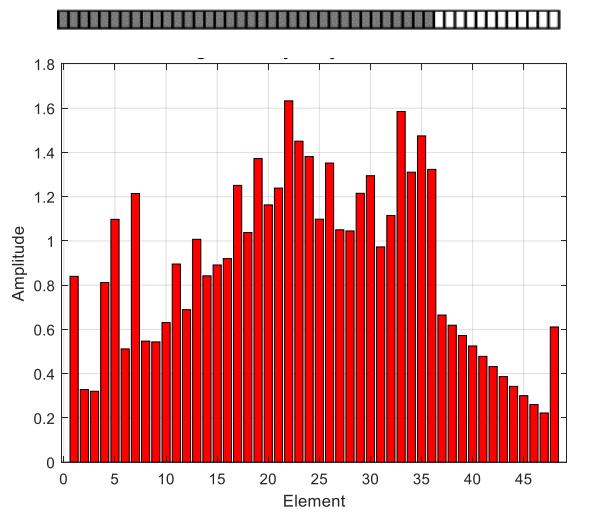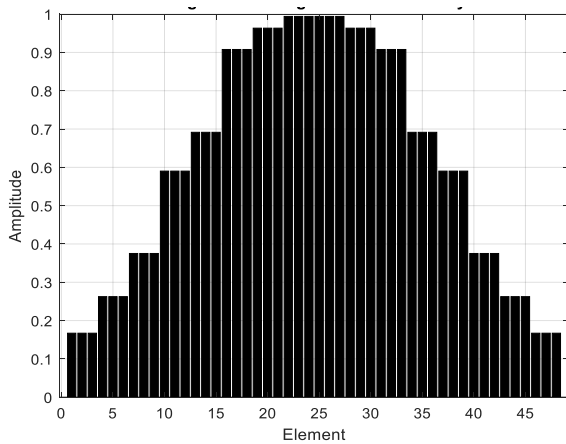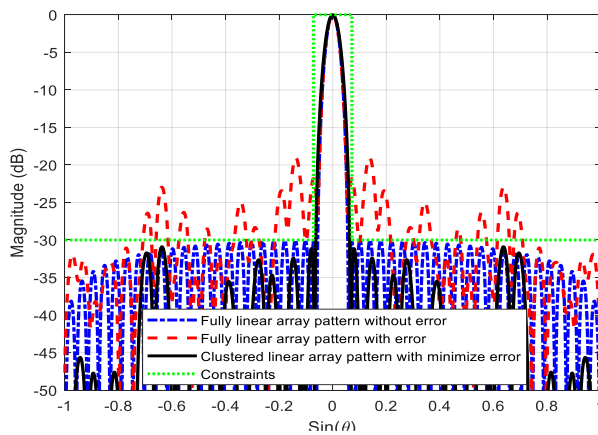b. Amplitudes of fully array with 75% sectional error

c. Amplitudes of RCA

d. Radiation patterns

Figure 9. Results of regular clustered arrays with 75% sectional error for an original Dolph linear array with 2N=48

To demonstrate the effect of random error on the array pattern, the relationship between the error variance with the sidelobe pattern and directivity is achieved. The sizes of the clusters in these comparisons are taken from the previous examples. Also, M=1, a fully linear array (array without clustering) was taken for the purpose of comparison. Figures 10 and 11 show the variations of sidelobe and directivity with error variance. As it can be seen through these two figures, the sidelobes of the fully linear array are more susceptible and changes of random errors than the clustered patterns. As for the directivity, it is less affected by random errors, and this is evident from Eq. 8. Then, all the above results demonstrate the effectiveness of the proposed RCA and ICA methods.

4. Conclusions

The results indicate that in the presence of sectional or random errors, the sidelobe pattern of traditional fully linear antenna arrays are significantly impacted. The suggested clustered (regular and irregular) arrays effectively fixed the issue by preserving the sidelobe error pattern within acceptable bounds and minimizing the error effects. The directivities of RCA and ICA arrays were found to be roughly within 11.7 dB and 11.9 dB, with a fluctuation of variance of error within 0.1 and 1, whereas peak sidelobe deviations were determined to be within 29 and 31 dBs, then it is very close to the required depth of 30 dB. As mentioned above, clustered arrays are a better choice for both error and complexity reductions.
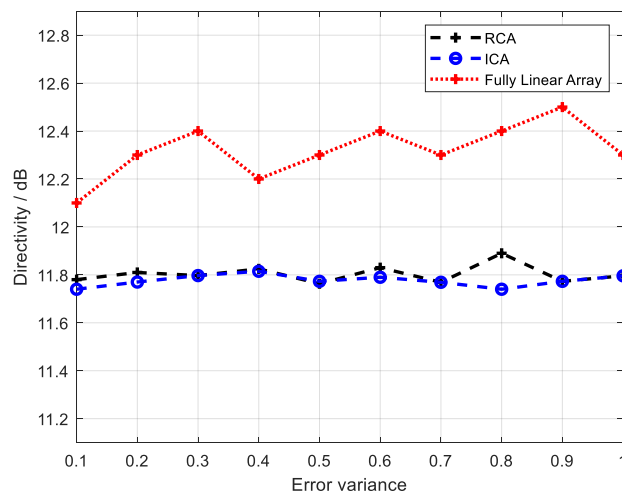
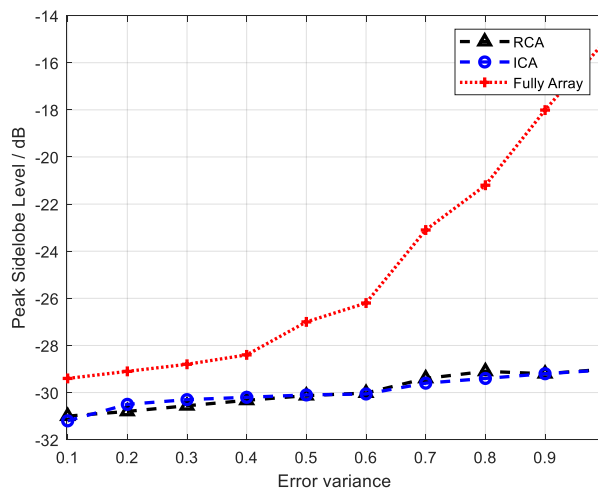**Figure 10.** SLL versus error variance in linear array

Figure 11. Directivity versus error variance in linear array

Conflict of Interest

The authors confirm that the publication of this article causes no conflict of interest.

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