Proton Momentum Distributions and Elastic Electron Scattering from ^{70}Zn, ^{72}Ge and ^{74}Se nuclei

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Abstract

In the framework of correlation method so-called coherent density fluctuation model (CDFM) the proton momentum distributions (PMD) of the ground state for some even mass nuclei of *fp*-shell like ^{70}Zn,^{72}Ge and ^{74}Se nuclei are examined. Proton momentum distributions are expressed in terms of the fluctuation function($|f(x)|^2$) and determined from theory and experiment. The main characteristic feature of the PMD obtained by CDFM is the existence of high-momentum components, for momenta $k \geq 2$ fm^{-1}. For completeness, also elastic electron scattering form factors F(q) are evaluated within the same framework.

Keywords: Proton momentum distributions, Charge density distributions, Coherent density fluctuation model.

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توزيعات زخم البروتون والاستطارة الالكترونية المرنة من النوى ^{70}Zn ، ^{72}Ge و ^{74}Se

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الخلاصة

تم استخدام أنموذج تموج الكثافة المترابط في حساب توزيعات زخم البروتون (PMD) للحالة الارضية لبعض النوى الزوجية الواقعة ضمن القشرة النووية *fp* مثل النوى ^{70}Zn ، ^{72}Ge و ^{74}Se. لقد تم التعبير عن PMD بدلالة دالة التموج ($|f(x)|^2$)والتي تحسب من خلال النتائج النظرية والعملية. في هذه الدراسة تم ايضاً حساب عوامل التشكل للاستطارة الالكترونية المرنة لهذه النوى.

الكلمات المفتاحية: توزيعات زخم البروتون، توزيعات كثافة الشحنة، أنموذج تموج الكثافة المترابط

Introduction

The nuclear properties such as nuclear binding energies and nuclear density distributions ρ(r) have been gain more attention in theoretical studies for the last few decades. These quantities have been investigated successfully using the zero range Hartree-Fock method since they are not sentient to the correlations of the short range nucleon-nucleon interactions. On the other hand, the high energy interactions between particles and a target nucleus more over some low energy phenomena, giant resonance for example, have been becomes the scope of the experimental works in the resent years. These works make some additional quantities able to be done such as the nucleon momentum distributions n(k) [1,2] of nuclei which expand the range of the nuclear ground state theory. The nucleon momentum distribution is connected to some process, which are concerning by the experimental works, such as nuclear photo effect and absorption of the mesons by nuclei. Since the short range of the nucleon-nucleon force dose not included in the Hartree-Fock method, it is becomes impossible to investigate density and momentum distributions at the same time using this method where n(k) is sensitive to the short range and tensor correlations. For this reason we need a nuclear correlation methods to make a correct simultaneous characterization of the relate ρ(r) and n(k) distributions. Different correlation methods [1,2,4-7] have been clearly indicates that the distinctive property of the nuclear momentum distribution is the presence of the high momentum component for (k> 2 fm^{-1}) because the short range and tensor nucleon correlations. This property of n(k) has been assured by exclusive and inclusive electron scattering measured data on nuclei. In general, for the nuclear interactions calculations of the cross-section it is important to know the momentum distribution of the interacting nucleons. The coherent density fluctuation model (CDFM) based on the local density distribution as a variable of the theory has been suggested in [1, 2] to investigate the nuclear structure and nuclear reactions using the essential results of the infinite nuclear matter theory.

Hamoudi *et al*. [8,9,10] have studied the NMD and elastic electron scattering form factors for *p*-shell [8], *sd*-shell [9] and *fp*- shell [10] nuclei using the framework of CDFM. They [8, 9,10] derived an analytical form for the NDD based on the use of the single particle harmonic oscillator wave functions and the occupation number of the states. The derived NDD's, which are applicable throughout the whole *p*-shell [8], *sd*-shell [9] and *fp*- shell [10] nuclei, have been used in the CDFM. The calculated NMD and elastic form factors of all considered nuclei have been in very good agreement with experimental data.

In this research, we follow the work of Hamoudi *et al*. [8,9,10] and utilize the CDFM with weight functions originated in terms of theoretical charge density distribution (CDD) of some *fp*-shell nuclei such as ^{70}Zn,^{72}Ge and ^{74}Se nuclei. It is found that the theoretical weight function $|f(x)|^2$ based on the derived CDD is capable to give information about the proton momentum distributions (PMD) and elastic charge form factors as do those of the experimental data [11].

Theory

The charge density distribution CDD of one body operator can be written as [12]:

$$\rho_c(r) = \frac{1}{4\pi}\sum_{n\ell} \xi_{n\ell}\, 2(2\ell+1)|R_{n\ell}|^2 \tag{1}$$

Where $\xi_{n\ell}$ is the proton occupation probability of the state $n\ell$ ($\xi_{n\ell} = 0$ or 1 for closed shell nuclei and $0 < \xi_{n\ell} < 1$ for open shell nuclei) and $R_{n\ell}$ is the radial part of the single particle harmonic oscillator wave function.

The CDD form of ^{70}Zn,^{72}Ge and ^{74}Se nuclei is derived on the assumption that there are filled $1s$, $1p$ and $1d$ orbitals and the proton occupation numbers in $2s$, $1f$ and $2p$ orbitals are equal to, respectively, $(2-\alpha_1)$, $(Z-20-\alpha_2)$ and $(\alpha_1+\alpha_2)$ and not to 2, $(Z-20)$ and 0 as in the simple shell model, where the parameters α_1 and α_2 are the occupation number of higher shells. Using this supposition in Eq. (1), the ground state CDD of ^{70}Zn, ^{72}Ge and ^{74}Se nuclei is obtained as:

$$\rho_c(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}\, b^3}\left[5-\frac{3}{2}\alpha_1+\left(\frac{11}{3}\alpha_1+\frac{5}{3}\alpha_2\right)\left(\frac{r}{b}\right)^2+\left(4-2\alpha_1-\frac{4}{3}\,\alpha_2\right)\left(\frac{r}{b}\right)^4 + \left(\frac{4}{21}\alpha_2+\frac{8}{105}(Z-20)+\frac{4}{15}\alpha_1\right)\left(\frac{r}{b}\right)^6\right] \tag{2}$$

Where Z is the atomic number, b is the harmonic oscillator size parameter.

The corresponding the mean square radius (MSR) is

$$\langle r^2\rangle = \frac{b^2}{Z}\left(\frac{9Z-60}{2}+\alpha_1\right) \tag{3}$$

The central CDD, $\rho_c(r=0)$ is obtained from Eq. (2) as

$$\rho_c(0) = \frac{1}{\pi^{3/2}\, b^3}\left[5-\frac{3}{2}\alpha_1\right] \tag{4}$$

Then α_1 is obtained from Eq. (4) as

$$\alpha_1 = \frac{2}{3}\left(5-\rho_c(0)\pi^{3/2}\, b^3\right) \tag{5}$$

The PMD, $n(k)$, of the considered nuclei is studied using two distinct methods. In the first, it is determined by the shell model using the single particle harmonic oscillator wave functions in momentum representation and is given by [13]:

$$n(k) = \frac{b^3}{\pi^{3/2}} e^{-b^2k^2} \left[5 + 4\,(bk)^4 + (Z-20)\frac{4}{105}\,(bk)^6\right] \tag{6}$$

k is the momentum of the particle.

Whereas in the second method, the $n(k)$ is determined by the Coherent Density Fluctuation Model (CDFM), where the Wigner distribution function (WDF) can be expressed as [1,2]:

$$w(k,r) = \int_0^\infty |f(x)|^2\, \theta(x-|\vec{r}|)\, \theta\big(k_F(x)-|\vec{k}|\big) dx \tag{7}$$

Which is the probability of finding a particle with momentum k at position r .

Using the basic relationships of the $\rho_c(r)$ and $n(k)$ with the WDF,

$$\rho_c(r) = 4\int w(k,r)\,\frac{d^3k}{(2\pi)^3}, \tag{8}$$

And

$$n_{CDFM}(k) = \int w(k,r)\, d^3r, \tag{9}$$

One can obtain the corresponding expressions for $\rho_c(r)$ and $n(k)$ using the WDF from Eq. (7), that is,

$$\rho_c(r) = \int_0^\infty |f(x)|^2 \rho_x(r)\, dx \tag{10}$$

And

$$n_{CDFM}(k) = \int_0^\infty |f(x)|^2\, \frac{4}{3}\pi x^3 \theta\big(k_F(x)-|\vec{k}|\big) dx \tag{11}$$

In eq. (10), $\rho_x(r)$ has the following form [1,2]:

$$\rho_x(r) = \rho_0(x)\theta(x-r) \tag{12}$$

In the case of monotonically-decreasing density distributions $(d\rho/dr < 0)$ one can obtain from (10) the relation of the weight function $|f(x)|^2$ with the density

$$|f(x)|^2 = -\frac{1}{\rho_0(x)}\frac{d\rho_c(r)}{dr}\bigg|_{r=x} \tag{13}$$

As a result the $n(k)$ can be obtained as a functional of the charge density distribution ρ_c [1,2]:

$$n_{CDFM}(k) = \left(\frac{4\pi}{3}\right)^2 \frac{4}{Z}\left[\int_0^{\alpha/k} 6\rho_c(x)x^5\,dx - \left(\frac{\alpha}{k}\right)^6 \rho_c\left(\frac{\alpha}{k}\right)\right] \tag{14}$$

Where $\alpha = (9\pi Z/8)^{1/3}$ with a normalization condition:

$$\int n_{CDFM}(k)\frac{d^3k}{(2\pi)^3} = Z \tag{15}$$

The form factor $F(q)$ of the target nucleus is also expressed in the CDFM as [6]:

$$F(q) = \frac{1}{Z}\int_0^\infty |f(x)|^2\, F(q,x)\,dx \tag{16}$$

Where $F(q,x)$ is the form factor of uniform charge density distribution given by [6]:

$$F(q,x) = \frac{3Z}{(qx)^2}\left[\frac{\sin(qx)}{(qx)} - \cos(qx)\right] \tag{17}$$

Inclusion the corrections of the nucleon finite size $F_{fs}(q)$ and the center of mass corrections $F_{cm}(q)$ in the calculations requires multiplying the form factor of equation (16) by these corrections. Here, $F_{fs}(q)$ is considered as free nucleon form factor which is assumed to be the same for protons and neutrons. This correction takes the form [14]:

$$F_{fs}(q) = e^{\left(\frac{-0.43q^2}{4}\right)} \tag{18}$$

The correction $F_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [14]:

$$F_{cm}(q) = e^{\left(\frac{b^2q^2}{4A}\right)} \tag{19}$$

It is important to point out that all physical quantities studied above in the framework of the CDFM such as PMD and $F(q)$, are expressed in terms of the weight function $|f(x)|^2$.

Therefore, it is worthwhile trying to obtain the weight function firstly from the CDDs of two parameter Fermi (2PF) and three parameter Fermi (3PF) models extracted from the analysis of elastic electron-nuclei scattering experiments and secondly from theoretical considerations. The CDDs of 2PF and 3PF, respectively are given by [11]

$$\rho_c(r) = \frac{\rho_0}{1+e^{(r-c)/z}} \tag{20a}$$

$$\rho_c(r) = \frac{\rho_0(1+wr^2/c^2)}{1+e^{(r-c)/z}} \tag{20b}$$

Introducing Eqs. (20) into Eq. (13), we obtain the experimental weight function $|f(x)|^2_{2PF}$ and $|f(x)|^2_{3PF}$ as

$$|f(x)|^2_{2PF} = \frac{4\pi x^3\rho_0}{3AZ}\left(1+e^{\frac{x-c}{z}}\right)^{-2} e^{\frac{x-c}{z}} \tag{21a}$$

$$|f(x)|^2_{3PF} = \frac{4\pi x^2\rho_0}{3A}\left[\frac{\left(1+\frac{wx^2}{c^2}\right)\left(1+e^{\frac{x-c}{z}}\right)e^{\frac{x-c}{z}}}{z} - \frac{2wx\left(1+e^{\frac{x-c}{z}}\right)^{-1}}{c^2}\right] \tag{21b}$$

Moreover, introducing the derived CDD of Eq. (2) into Eq. (13), we obtain the theoretical weight function $|f(x)|^2_{th}$ as

$$|f(x)|^2_{th} = \frac{8\pi x^4}{3Zb^2}\rho_C(x) - \frac{16x^4}{3Z\pi^{1/2}b^5}\left\{\frac{11}{6}\alpha_1 + \frac{5}{6}\alpha_2 + \left(4-2\alpha_1-\frac{4}{3}\alpha_2\right)\frac{x^2}{b^2} + \left(\frac{4}{35}(Z-20)+\frac{2}{5}\alpha_1+\frac{2}{7}\alpha_2\right)\frac{x^4}{b^4}\right\}e^{-x^2/b^2} \tag{22}$$

Results and Discussion

In the present work the CDFM has been used to investigate $n(k)$ and F(q) for some nuclei namely; ^{70}Zn, ^{72}Ge and ^{74}Se. In order to calculate the proton momentum distributions n(k), obtained from Eq. (14), we need to investigate the CDD for both experiment, such as, 2PF and 3PF [11] and theoretical consideration using Eq. (2) which includes some parameters needed for calculations. These parameters have been calculated and presented in Table (1) together with other parameters employed for the selected nuclei used in the present work. The parameter α_1 is determined by introducing the harmonic oscillator size parameter *b*, which gives the experimental root mean square (rms) radii and the experimental central density $\rho_{exp}(0)$ into Eq. (5), while the parameter α_2 is assumed as a free parameter to be adjusted to obtain agreement with the experimental CDD. It is important to remark that when $\alpha_1=\alpha_2=0$, Eq. (2) is reduced to simple shell model prediction. The $(2-\alpha_1)$, $(Z-20-\alpha_2)$ and $(\alpha_1+\alpha_2)$ proton occupations numbers for 2*s*, 1*f* and 2*p* orbitals, respectively, have been also calculated and tabulated in Table (2). For comparison the calculated rms $\langle r^2\rangle_{cal}^{1/2}$ and the experimental one $\langle r^2\rangle_{exp}^{1/2}$ are also displayed in Table (2). A remarkable agreement has been shown for all considered nuclei.

Table 1- Parameters for the CDD of the considered nuclei

Nuclei	Z	*Model*	*w*	*c (fm)*	*z (fm)*	$\rho_{exp}(0)$ (fm^{-3}) [11,15]	*b (fm)*	α_1	α_2
^{70}Zn	30	2PF	----	4.404	0.583	0.071426	2.155	0.678123	0.93102
^{72}Ge	32	2PF	----	4.45	0.573	0.074448	2.14	0.623203	1.13797
^{74}Se	34	3PF	-0.09	4.488	0.5933	0.082787	2.141	0.315388	2.07989

Table 2- Calculated occupation numbers of 2s, 1f, and 2p orbitals of the considered nuclei together with $\langle r^2\rangle_{cal}^{1/2}$ and $\langle r^2\rangle_{exp}^{1/2}$

Nuclei	Occupation No. of 2*s* $(2-\alpha_1)$	Occupation No. of 1*f* $(Z-20-\alpha_2)$	Occupation No. of 2*p* $(\alpha_1+\alpha_2)$	$\langle r^2\rangle_{cal}^{1/2}$ *(fm)*	$\langle r^2\rangle_{exp}^{1/2}$ *(fm)* [9,10]
^{70}Zn	1.321877	9.06898	1.60914	4.044	4.044
^{72}Ge	1.376796	10.86203	1.76117	4.05	4.05
^{74}Se	1.684612	11.92011	2.39527	4.07	4.07

Figure (1) demonstrates the CDD's for ^{70}Zn, ^{72}Ge and ^{74}Se nuclei which have been calculated using Eq. (2) and denoted in this figure as dashed and solid curves with $\alpha_1=\alpha_2=0$ and $\alpha_1\neq\alpha_2\neq 0$, respectively. For comparison, the experimental data [11, 15] have been also presented in Figure (1) and denoted by the filled circle along with calculated CDD curves. This figure shows a very clear discrepancy between the dashed curve and the experimental data especially at the central region. Taking the effect of the occupation number of higher orbits α_1 and α_2 into consideration leads to reduce this discrepancy and significantly at the central region and subsequently improving the result by bringing the distributions of solid curves closer to the experimental data. Figure (2) shows the $n(k)$ (fm^3) versus k (fm^{-1}) for ^{70}Zn, ^{72}Ge and ^{74}Se nuclei calculated with shell model using single particle harmonic oscillator wave function in momentum space. The experimental and theoretical $n(k)$ obtained by CDFM, using experimental and theoretical CDD in Eq. (14), have been also presented in this figure and denoted by the filled circle symbols and solid curves, respectively. It is clearly seen that the calculated $n(k)$ distributions using shell model (dashed curves) has a steep slope behavior, which are in disagreement with the studies [1,6, 16-18]. This disagreement refer to the fact that the short-range correlation dose not included in to the ground state wave function of the shell model calculations which is responsible for the behavior of $n(k)$ in the high momentum [17, 18]. The calculated $n(k)$ obtained by CDFM for the interested nuclei are much closer to the experimental data than the shell model calculation. The CDFM corrected the steep behavior of the $n(k)$ curves to a long tail manner for k $\geq$2 fm^{-1} region. The property of long-tail manner obtained by CDFM is connected to the presence of high densities $\rho_x(r)$ in the decomposition of Eq. (12), though their fluctuation function $|f(x)|^2$ are small. The dependence of F(q) on the momentum transfer q (in fm^{-1}) for considered nuclei is shown in Figure 3. As there is no data available for ^{70}Zn, ^{72}Ge and ^{74}Se nuclei we have compared the calculated form factors (solid curves) of these nuclei obtained in the framework of CDFM using the theoretical weight function of Eq. (22), with those obtained by the Fourier transform of the 2PF and 3PF (filled circle symbols) [11,15]. This figure shows that the calculated form factors are in very good agreement with those fitted to the experimental data.

Conclusions

It is concluded that considering the effect of higher shells through introducing the parameters α_1 and α_2 removes the discrepancy between the CDD of the simple shell model and those of the experimental results at short distance. The calculated PMD in the coherent density fluctuation model (using theoretical and experimental weight function) are mainly characterized by the long tail behaviour at high momentum k, since this feature agrees with predictions of other theoretical and experimental studies.

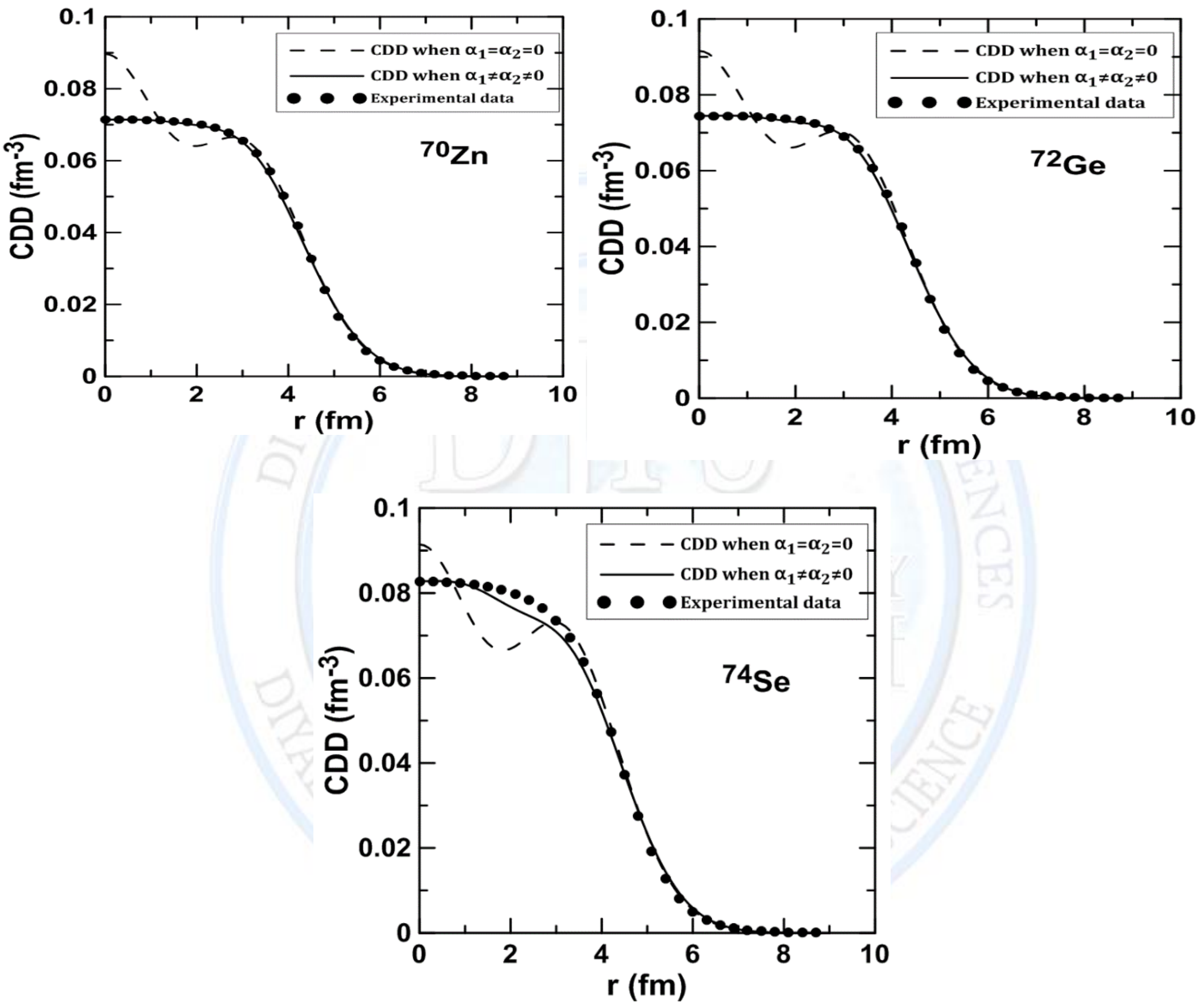

Figure 1: The CDD for ^{70}Zn, ^{72}Ge and ^{74}Se nuclei. The dashed and solid curves are the calculated CDD of Eq. (2) when $\alpha_1 = \alpha_2 = 0$ and $\alpha_1 \neq \alpha_2 \neq 0$, respectively. The filled circle symbols are the fitted to the measured data [11,15].

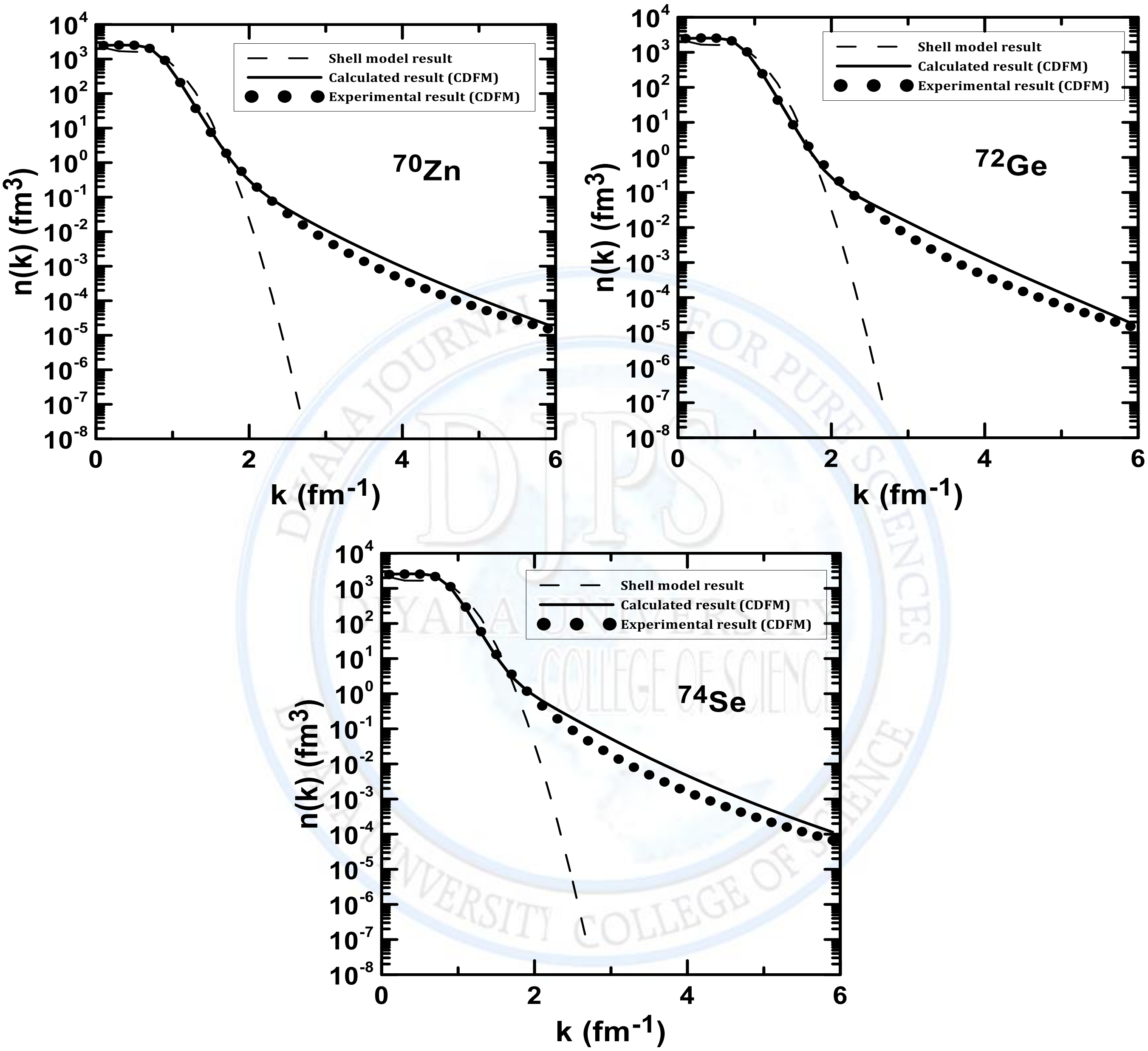

Figure 2: The PMD for ^{70}Zn, ^{72}Ge and ^{74}Se nuclei. The dashed curves are the results of Eq. (6) obtained by the shell model calculation using the single-particle harmonic oscillator wave functions in momentum space. The solid curves and filled circle symbols are the calculated $n(k)$ expressed by the CDFM of Eq. (14) using the theoretical CDD of Eq. (2) and the measured data of ref. [11,15], respectively.

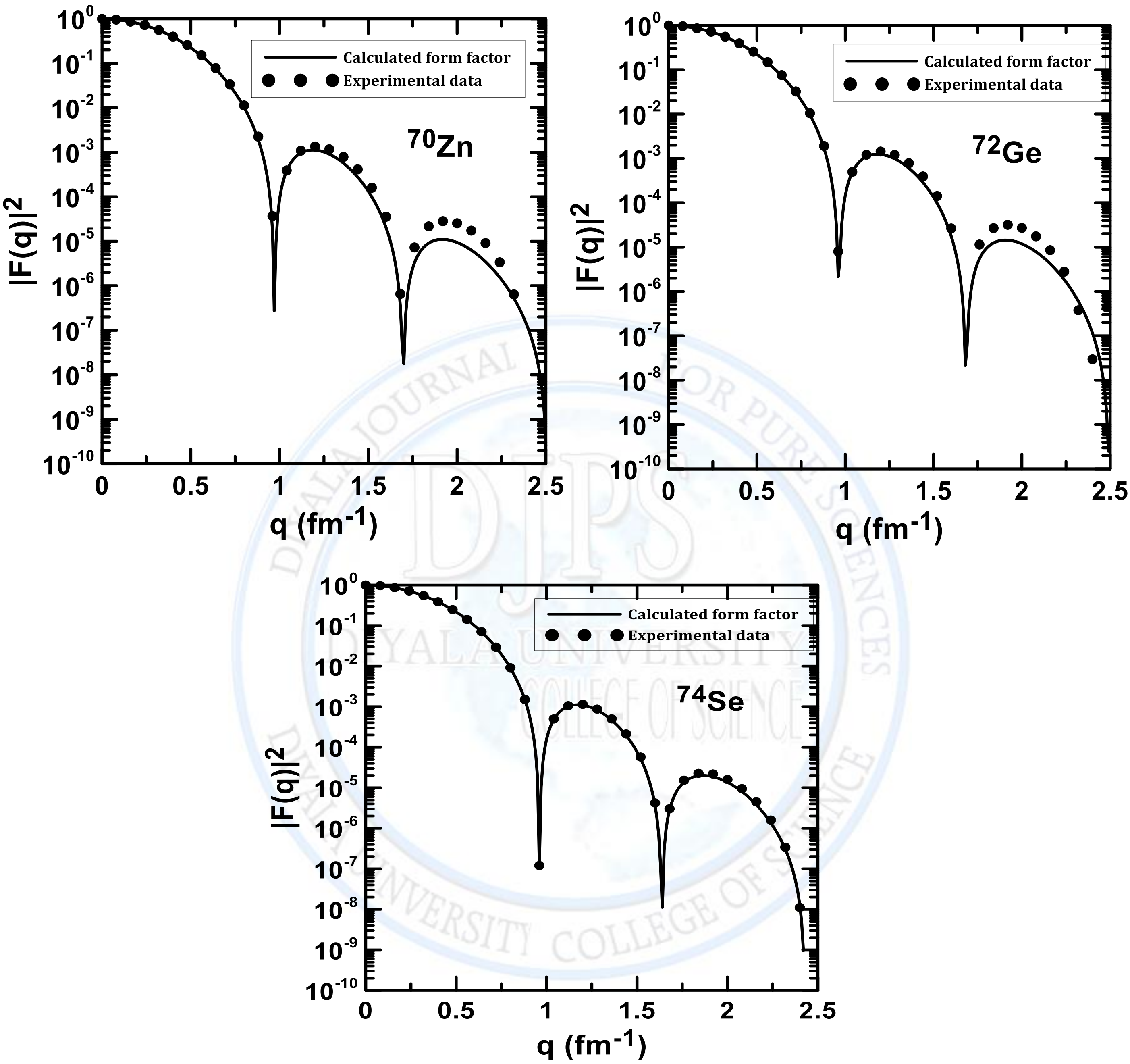

Figure 3: Elastic form factors for ^{70}Zn,^{72}Ge and ^{74}Se nuclei. The solid curves are the form factors calculated using Eq. (16) are the form factors obtained by the Fourier transform of the 2PF (^{70}Zn,^{72}Ge) and 3PF (^{74}Se) [11,15].

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