Charge density Distributions and Elastic Electron Scattering from ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei using the occupation numbers of the states

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Abstract

The charge density distribution (CDD) and elastic electron scattering form factors, F(q), for some 1*f*-2*p* shell nuclei, such as ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei have been evaluated using the wave functions of the harmonic oscillator and occupation numbers of states. It found that considering the effect of higher shells through introducing additional parameters namely δ_1 and δ_2 lead to astonishing accordance between the calculated and the observed results of the CDD and elastic form factors F(q).

Keywords: Elastic electron scattering, Charge density distributions, Occupation numbers.

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Charge Density Distributions and Elastic Electron Scattering from ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei using the occupation numbers of the states

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توزيعات كثافة الشحنة والاستطارة الالكترونية المرنة من النوى ^{58}Ni، ^{64}Zn ، ^{70}Ge و ^{76}Se باستخدام حالات اعداد الملئ

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الخلاصة

تم حساب توزيعات كثافة الشحنة (CDD) وعوامل التشكل للاستطارة الالكترونية المرنة لبعض النوى الواقعة ضمن القشرة النووية $1f$-$2p$ مثل النوى ^{58}Ni، ^{64}Zn، ^{70}Ge و ^{76}Se وذلك باستخدام الدوال الموجية لجهد المتذبذب التوافقي واعداد الملئ للمستويات النووية. لقد وجد ان ادخال اعلومات اضافية (δ_1 و δ_2)، والتي تمثل اعداد الملئ للمستويات العليا يؤدي الى توافق رائع بين النتائج النظرية والنتائج العملية لتوزيع كثافة الشحنة النووية وعوامل التشكل للاستطارة الالكترونية المرنة.

الكلمات المفتاحية: الاستطارة الالكترونية المرنة ، توزيعات كثافة الشحنة، اعداد الملئ .

Introduction

The charge density distribution (CDD) and form factors (a factor depends on the charge, current and magnetization distributions in the target nucleus) are fundamental properties of the nucleus and can be obtained experimentally from electron- nucleus scattering. The electron-nucleus interaction is considered by the first Born approximation as an exchange of a virtual photon. In this case the initial and final particles are considered free and can be represented by plane waves [1]. The electron scattering from the nucleus at high energy gives important information about the nuclear structure. Information obtained from the high energy electron scattering by the nuclei depends on the magnitude of the de Broglie wave length that is associated with the electron which is compared with the range of the nuclear forces. When the energy of the incident electron is in the range of 100 MeV and more, the de Broglie wave length will be in the range of the spatial extension of the target nucleus. Thus with this energy, the electron represents a best probe to study the nuclear structure [2,3]. Al-Rahmani and Mahdi have calculated the charge density distribution and elastic form factors for some 2*s*-1*d* shell nuclei using the plan-wave Born approximation and illustrated that the inclusion additional parameters in the calculations improves the results and makes them in remarkable accordance with the measured results [4].

The aim of the present work is to extend the calculations of Al-Rahmani and Mahdi to higher shells (such as the 1*f*-2*p* shell nuclei) and to derive an analytical form for the charge density distribution and elastic form factors F(q).

Theory

The charge density distribution (CDD) can be determined in terms of the harmonic oscillator radial wave function ($R_{n\ell}$) as [4,5]:

$$\rho_{ch}(r) = \frac{1}{4\pi}\sum_{n\ell} 2(2\ell+1)|R_{n\ell}|^2 \tag{1}$$

According to the simple shell model, the 1*f*-2*p* shell nuclei are assumed as an inert core of filled 1*s*, 1*p*, 1*d*, and 2*s* while the 1*f* orbit is occupied by $(Z-20)$ protons, where Z is the atomic number of nuclei. Using this assumption the CDD of 1*f*-2*p* shell nuclei can be written as:

$$\rho_{ch}(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}\, b^3}\left[5 + 4\left(\frac{r}{b}\right)^4 + (Z-20)\frac{8}{105}\left(\frac{r}{b}\right)^6\right] \tag{2}$$

Where b is the harmonic oscillator size parameter.

The normalization condition of the ρ_{ch} given in eq (1) is [4,5]

$$Z = 4\pi\int_0^\infty \rho_{ch}(r)\, r^2\, dr \tag{3}$$

and the mean square radius (MSR) is [4,5]

$$\langle r^2\rangle = \frac{4\pi}{Z}\int_0^\infty \rho_{ch}(r)\, r^4\, dr \tag{4}$$

The calculated results obtained by Eq. (2) have a poor agreement with experimental data. In the present work, it is assumed that there is a core of filled $1s$, $1p$ and $1d$ orbitals and the proton occupation numbers in $2s, 1f$ and $2p$ orbitals are equal to, respectively, $(2-\delta_1)$, $(Z-20-\delta_2)$ and $(\delta_1+\delta_2)$ and not to 2, $(Z-20)$ and 0 as in the simple shell model, where the parameters δ_1 and δ_2 are the occupation number of higher shells. Using this assumption, with the help of Eq. (1), the charge density distribution can be written as:

$$\rho_{ch}(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}\, b^3}\Big[5-\frac{3}{2}\delta_1+\Big(\frac{11}{3}\delta_1+\frac{5}{3}\delta_2\Big)\Big(\frac{r}{b}\Big)^2+\Big(4-2\delta_1-\frac{4}{3}\,\delta_2\Big)\Big(\frac{r}{b}\Big)^4 + \Big(\frac{4}{21}\delta_2+\frac{8}{105}(Z-20)+\frac{4}{15}\delta_1\Big)\Big(\frac{r}{b}\Big)^6\Big] \qquad (5)$$

and the corresponding MSR is

$$\langle r^2\rangle = \frac{b^2}{Z}\left[\frac{9Z-60}{2}+\delta_1\right] \qquad (6)$$

The central CDD, $\rho_{ch}(r=0)$ is obtained from Eq. (5) as

$$\rho_{ch}(0) = \frac{1}{\pi^{3/2}\, b^3}\left[5-\frac{3}{2}\delta_1\right] \qquad (7)$$

then δ_1 is obtained from Eq. (7) as

$$\delta_1 = \frac{2}{3}\left[5-\rho_{ch}(0)\pi^{3/2}\, b^3\right] \qquad (8)$$

The elastic electron scattering form factors is determined by the ground state charge density distribution (CDD) [4,5], *i.e.*

$$F(q) = \frac{4\pi}{Z}\int_0^\infty \rho_{ch}(r) j_0(qr) r^2 dr \qquad (9)$$

where

$$j_0(qr) = \frac{\sin(qr)}{qr} \qquad (10)$$

is the zeroth order spherical Bessel function, q is the momentum transfer from the incident electron to the target nucleus and the $\rho_{ch}(r)$ is the CDD of the ground state.

An analytical form for elastic electron scattering form factor, F(q), can be obtained by introducing the form of the CDD of eq. (5) into eq. (9), and performing the integration, i.e.,

$$F(q) = \frac{1}{Z}\left\{Z + \left[\frac{(10-Z)}{2} - \frac{\delta_1}{6}\right](qb)^2 + \left[\frac{\delta_1}{20} + \frac{\delta_2}{24} + \frac{(Z-15)}{20}\right](qb)^4 - \left[\frac{\delta_1}{240} + \frac{\delta_2}{336} + \frac{(Z-20)}{840}\right](qb)^6\right\} e^{-q^2b^2/4} \qquad (11)$$

Inclusion the corrections of the center of mass $F_{cm}(q) = exp(b^2q^2/4A)$ [6] and the finite nucleon size $F_{fs}(q) = exp(-0.43q^2/4)$ [6] in the calculations needs multiplying the form factor of Eq. (11) by these corrections.

In this work, the calculated CDD for nuclei under study are compared with the fitted to the experimental data of two parameter Fermi model (2PF) and three parameter Fermi model (3PF), which are extracted from the analysis of elastic electron-nuclei scattering experiments, and are given by [7]

$$\rho_{ch}(r) = \rho_0 / \left(1 + exp\left((r-c)/z\right)\right)$$

(12)

$$\rho_{ch}(r) = \rho_0(1 + wr^2/c^2) / \left(1 + exp\left((r-c)/z\right)\right)$$

Results and Discussion

The calculated CDD's for some even- *A* of 1*f*-2*p* shell nuclei, such as ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei have been obtained using the analytical form of eq. (5) and compared with the fitted to the experimental data of two parameter Fermi (2PF) model and three parameter Fermi (3PF) model.

Table (1) demonstrates the values of the parameters (c and z) and (w, c and z) used to extract, respectively, 2PF and 3PF CDD's together with central charge densities $\rho_{exp}(0)$ for ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei. Table (2) exhibits the size parameter b which gives the experimental *rms* radii and the calculated parameters of δ_1 and δ_2 for nuclei under study. The occupation numbers for 2*s*, 1*f*, and 2*p* shells and *rms* radii which are calculated in the present work are given in table (3).

Table 1- Parameters of CDD for consider nuclei together with $\rho_{exp}(0)$.

Nuclei	*Parameters of the experimental CDD* [7,8]				$\rho_{exp}(0)$ fm^{-3}
	model	*w*	*c (fm)*	*z (fm)*	[7, 8]
^{58}Ni	3PF	-0.1308	4.3092	0.5169	0.0816325
^{64}Zn	2PF	----	4.285	0.584	0.0768541
^{70}Ge	2PF	----	4.43	0.5807	0.0751088
^{76}Se	3PF	-0.039	4.512	0.6189	0.0770452

Table 2- Calculated size parameters *b* utilized in Eq. (5) of the present study along with the calculated δ_1 and δ_2

Nuclei	Z	*b* (*fm*)	δ_1	δ_2
^{58}Ni	28	2.04	0.75908	0.635
^{64}Zn	30	2.116	0.62869	0.851
^{70}Ge	32	2.143	0.58763	1.349
^{76}Se	34	2.18	0.36842	1.950

Table 3- Calculated occupation numbers of 2*s*, 1*f*, and 2*p* shells together with the calculated and experimental *rms*.

Nuclei	Occupation No. of $2s$ $(2-\delta_1)$	Occupation No. of $1f$ (Z-20-δ_2)	Occupation No. of $2p$ $(\delta_1+\delta_2)$	$\langle r^2\rangle_{cal}^{1/2}$	$\langle r^2\rangle_{exp}^{1/2}$ [7, 8]
^{58}Ni	1.240	7.365	1.394	3.792	3.772
^{64}Zn	1.371	9.149	1.479	3.97	3.965
^{70}Ge	1.412	10.651	1.936	4.055	4.055
^{76}Se	1.631	12.050	2.318	4.152	4.152

Figure 1 illustrates the calculated CDD's for ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei. The blue and red curves are the calculated results using eq. (5) with $\delta_1=\delta_2=0$ and $\delta_1\neq\delta_2\neq 0$, respectively whereas the filled circle symbols correspond to the measured data [7, 8]. This figure illustrates that the blue curves are in disagreement with the measured data, especially for small r. Inclusion of the parameters δ_1 and δ_2 (i.e., considering the higher orbitals) in the calculation leads to astonishing accordance with the measured data as demonstrated by the red curves. As it is evident from figure 1 that the red curves of ^{64}Zn and ^{76}Se deviates slightly from the measured data of 2PF and 3PF, respectively, especially at the region $(1.5 < r < 2.8)$ fm. In general, considering the effect higher shells in Eq.(5) improves strongly the calculated CDD of ^{64}Zn and ^{76}Se nuclei, but these higher shells are not enough for resolving completely the problem of slight deviation. This deviation may be attributed to the necessity of considering other higher shell, such as $1g$ shell. However, this deviation doesn't affect generally the very well agreement with the experimental data throughout the whole range of r. The elastic electron scattering form factors from considered spin-zero nucleus are calculated in terms of the ground state CDD using the plane wave Born-Approximation (PWBA), where the form factor is a Fourier transform of CDD as given in eq (9). The form factors of ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei have been calculated using eq (11) and depicted in figure 2 as a function of the momentum transfer. In this figure the filled circle symbols and red curves are the experimental data and the calculated results, respectively.

The form factors of ^{58}Ni and ^{64}Zn nuclei are presented in figures 2(a) and 2(b), respectively. These figures show that the experimental form factors of ^{58}Ni [9] and ^{64}Zn [10] nuclei are in a very good agreement with those of calculated result up to $q \approx 0.9\ fm^{-1}$, while for higher q the calculated results for these nuclei underestimate the experimental data. It is evident from figure 2(a) that the diffraction minima and maxima of ^{58}Ni nucleus are reproduced in the correct places.

Figures 2(c) and 2(d) illustrate the form factor of ^{70}Ge and ^{76}Se nuclei, respectively. These figures give an indication that the observed data [8] of ^{70}Ge and ^{76}Se nuclei are in very good agreement with the calculated results for all momentum transfer values. It is evident from these figures that the observed first and second diffraction minima are quite well described by the calculated results.

Summary & Conclusions

Analytical expressions for the ground state CDD and elastic electron scattering from factors for some 1*f*-2*p* shell nuclei, namely ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei has been derived from a method that based on used the wave function of the harmonic oscillator and occupation numbers of states. From the calculated charge density distributions and form factors, it is concluded that:

1- Including the effect of higher shells through introducing the parameters (δ_1 and δ_2) removes the discrepancy between the CDD of the simple shell model and those of the experimental results at short distance, which means, higher configuration are important to the included.

2- The theoretical forms for the CDD and elastic electron scattering from factors give us a satisfactory description with those of experimental data for the 1*f*-2*p* shell nuclei under study.

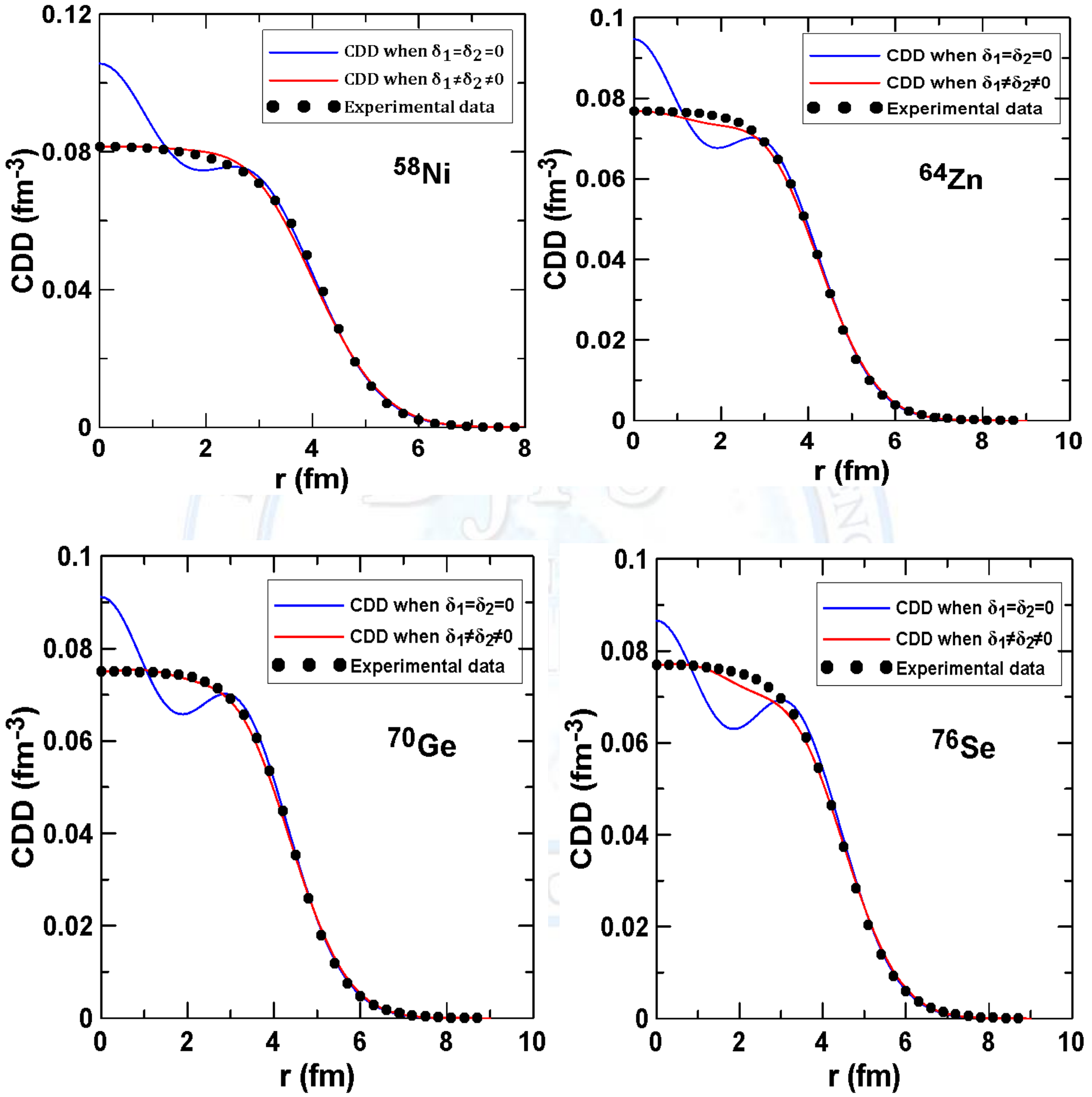

Figure 1: The CDD for ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei. The blue and red lines are the evaluated CDD of Eq. (5) when $\delta_1 = \delta_2 = 0$ and $\delta_1 \neq \delta_2 \neq 0$, respectively. The filled circle symbols are the measured data of refs. [7, 8].

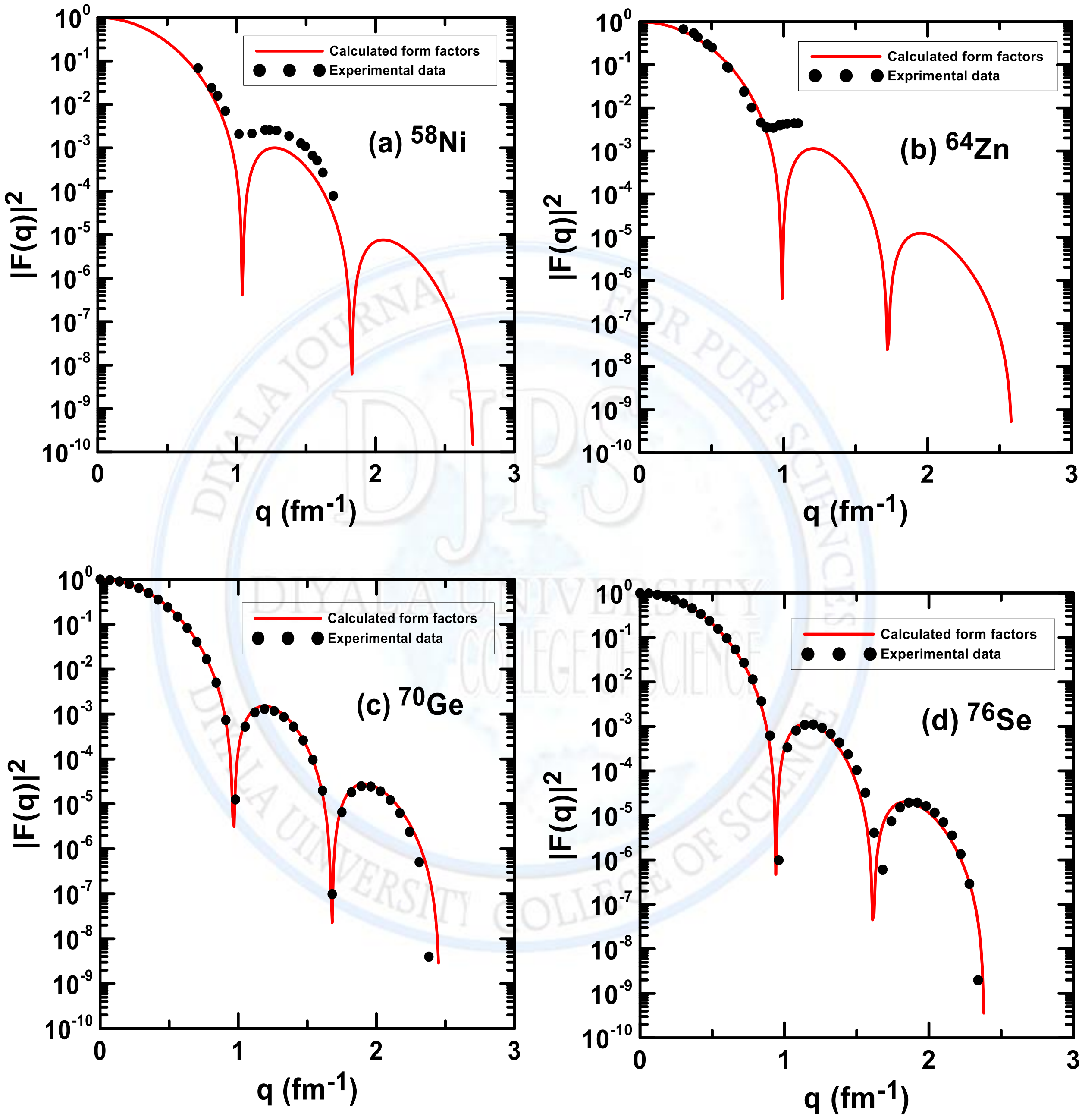

Figure 2: The elastic form factors for ^{58}Ni, ^{64}Zn, ^{70}Ge and ^{76}Se nuclei. The filled circle symbols and red curves are the experimental data and the calculated results, respectively.

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