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Calculate Energy Levels, Energy Ratios and Electric Quadrupole Transition Probability B(E2), of the Even-Even Yb-164 Isotopes Using IBM-1

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ABSTRACT

The nuclear structure for even-even nuclei $^{164}_{70}Yb_{94}$ estimated by using interacting boson model (IBM-1). Also this program was used to determine energy levels of g- state, energy ratios and electric quadrupole transitions probability B(E2) which have been calculated. Depending on the follow up of energy ground bands (g, β, and γ) . The dynamical symmetry and energy spectrum of selected isotope are determined by B(E2). The electric quadrupole transition probability of the (2_g+-0_g+) transitions for Yb (A=164) isotopes with the dynamical symmetry SU(3)- SU(5) from rotational SU(3) to vibrational SU(5). All obtained results of the study were compared with experimental facts and acceptable agreement obtained.

KEYWORDS: Nuclear Structure, Energy Ratios, Dynamical Symmetry, Electric Quadrupole Transition Probability B(E2), Boson.

الخلاصة

التركيب النووي للنواة الزوجية –زوجية (A=164) Yb تم تحديده باستخدام نموذج البوزونات المتفاعلة الاول IBM-1. كما ان هذا البرنامج ا ستخدم لتحديد مستويات الطاقة ونسب الطاقة واحتمالية الانتقالات الكهربائية لرباعي القطب الكهربائي B(E2). والتي تم حسابها بالاعتماد على متابعة حزم الطاقة الارضية (g, β, γ). يتم تحديد التماثل الديناميكي وطيف الطاقة للنظير المحدد بواسطة B(E2). احتمالية الانتقال الرباعي للقطب الكهربائي للانتقالات (2_g^+-0_g^+) للنظير (A=164) Yb ذو التناظر الديناميكي SU(5)-SU(3) وهو الانتقال من الحالة الدورانية (3) SU الى الحالة الاهتزازية (5) SU. تمت مقارنة جميع النتائج التي تم الحصول عليها من الدراسة مع الحقائق التجريبية وتم الحصول على توافق مقبول.

INTRODUCTION

An algebraic interacting boson model (IBM) proposed by Arima and Iachello [1]. This program studied the properties of the collective status of the heavy and medium mass nuclei. And achieve to describing the spectra of nuclei by assuming that an even-even nucleus consists of an inert core plus some valence particles (i.e. particles outside the major closed shells at 2, 8, 20, 28, 50, 82, 126 and 184) this magic numbers of nuclear shell structure. The valence particles tend to pair together to form bosons with angular momentum 0 and 2 [2].

The proton (neutron) bosons with angular momentum L = 0 were symbolizes it sπ (sν) denominate s-bosons, while proton (neutron) bosons with angular momentum L = 2 were denoted by dπ (dν) were denominate d-bosons [3].

According to this version (IBM-1), the bosons of s (L=0) and d (L=2) have six components (sub states) and therefore define a six dimensional space, this produce to characterize in terms of unification group in six ambit U(6) [4, 5].that yields three dynamical chains: the SU(5), SU(3) and O(6).

MATERIALS AND METHOD

Hamiltonian operator can be written as under the program of IBM-1 in terms of creation and annihilation operators on this formula [6, 7]:

$$\hat{H} = \varepsilon \hat{n}_d + a_o(\hat{P}^{\dagger}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad (1)$$

Six independent free parameters (ε, a0, a1, a2, a3, a4) are fitted to the data corresponding to the

dynamical symmetry according to the operators, and the operators $\hat{n}_d, \hat{P}, \hat{L}, \hat{Q}, \hat{T}_3, \hat{T}_4$ can be written as follows [6, 7]:

$$\hat{n}_d = \left(\hat{d}^\dagger.\hat{\tilde{d}}\right)$$
$$\hat{P} = \frac{1}{2}\left(\hat{\tilde{d}}.\hat{\tilde{d}}\right) - \frac{1}{2}\left(\hat{\tilde{s}}.\hat{\tilde{s}}\right)$$
$$\hat{I} = \sqrt{10}\left(\hat{d}^+.\hat{\tilde{d}}\right)\left[\left(\hat{d}^\dagger.\hat{\tilde{d}}\right)\right]^{(1)}$$
$$\hat{Q} = \left[\hat{d}^+ \times \hat{\tilde{s}} + \hat{s}^\dagger \times \hat{\tilde{d}}\right]^{(2)} - \qquad (2)$$
$$\qquad \chi\left[\left(\hat{d}^\dagger.\hat{\tilde{d}}\right)\right]^{(2)}$$
$$\hat{T}_3 = \left[\left(\hat{d}^\dagger \times \hat{\tilde{d}}\right)\right]^{(3)}$$
$$\hat{T}_4 = \left[\left(\hat{d}^\dagger \times \hat{\tilde{d}}\right)\right]^{(4)}$$

Where

$\hat{n}_d = d - $ Boson operator
$\hat{P} = $ pairing operator
$\hat{I} = $ angular momentum operator
$\hat{Q} = $ quadrupole operator
$\chi = -\sqrt{7}/2$ for deformed nuclei
$\hat{T}_3 = $ octapole operator
$\hat{T}_4 = $ hexadeapole operator
and N is the total number of Bosons defined as[3]:

$$N = n_s + n_d \qquad (3)$$

So that,
$$\varepsilon_s\left(\hat{s}^\dagger.\hat{\tilde{s}}\right) = \varepsilon_s\hat{\tilde{n}}_s = \varepsilon_s\left(\hat{N} - \hat{\tilde{n}}_d\right) = \varepsilon_s\hat{N} - \varepsilon_s\hat{\tilde{n}}_d$$
calculate of the energy levels for Yb-164 To applied the equivalent equation of the Hamiltonian operators and to illustrate the composition of the energy levels energy bands spectrum and other related parameters.

SU(3)-SU(5) transitional dynamical symmetry

One of the most important goals of this research is to show the structure of deformed nuclei Yb-164. The nucleus of this class has transitional properties contain the two groups, SU(3) and SU(5). General formula of Hamiltonian operator for SU(3)- SU(5) region will represented by [8, 9]:

$$\hat{H} = \in \hat{n}_d + a_1\hat{I}.\hat{I} + a_2\hat{Q}.\hat{Q} \qquad (4)$$

Properties of this variety depend on the ratio of ($\in/a_2$), when the ratio is great it tends to be specific limiting SU(5) and while this ratio is small, it tends to be SU(3) . Also, the B(E2) values are influenced by the ratio ($\in/a_2$). Where $\in$=Boson's energy and a_2 = Phenomenological Parameter used in IBM-1 to define the Hamiltonian function. On the other hand, the values of the electric quadrupole transition B(E2) are influenced by information ratios. (Branching Ratios) R enable to be appointed by [7]:

$$R = \frac{B(E2; 2_2^+ \to 0_1^+)}{B(E2; 2_2^+ \to 2_1^+)} \qquad (5)$$

Where:
R= 0   in SU(5) region
R=7/10   in SU(3) region

RESULTS AND DISCUSSION

Table 1 listed theoretical and experimental values of energy transition levels of nuclei Yb-164 understudy according to energy bands (g- band, β-bands and γ-bands).

The calculated values of energy levels for even-even isotope were compared with the available experimental values. The existing results give an approval with the experimental data [10].

One of the most important concepts in nuclear structure is the concept of symmetry, which must be defined accurately because the nucleus form has an intrinsic relationship to determine nuclear properties such as energy levels and probability reduced electromagnetic transmission, surface voltage function, inertial torque and rotational power. (N) the total number of bosons is an important factor to identify the dynamical symmetry of Yb-164. The total number of bosons N=12 (6-protons-bosons and 6-neutrons-bosons).

Figure 1 This figure compared between the theoretical and experimental calculations for nucleus Yb-164, which have 6-protons-bosons. Energy levels of even-even isotope have been classified in accordance with the three bands (g, β and γ –bands). The SU(5) limit occurs because, the β1 -band became before γ1. The layout of these bands is very important because of it is used to calculate the behavior of selected nucleus and moment of inertia values for all bands.

Table 1. Comparison of theoretical and practical values of energy levels data for the isotope Yb (A=164).

Isotopes	I^π	Energy Level (MeV)		Spin Sequence	Transition Energy (MeV)	
		Exp.[10]	IBM-1 (pw)		Exp.[10]	IBM-1 (pw)
$^{164}_{70}Yb_{94}$	0^+_1	0.0000	0.0000	--------	--------	--------
	2^+_1	0.1233	0.12214	$2^+_1 - 0^+_1$	0.1233	0.1224
	4^+_1	0.3856	0.4070	$4^+_1 - 2^+_1$	0.2622	0.2846
	6^+_1	0.7597	0.8546	$6^+_1 - 4^+_1$	0.3740	0.4476
	2^+_2	0.8639	0.8840	$2^+_2 - 2^+_1$	0.7405	0.7619
				$2^+_2 - 0^+_1$	0.8639	0.8840
	0^+_2	0.9756	0.7594	$0^+_2 - 2^+_1$	0.8522	0.6372
	3^+_1	1.0038	1.0345	$3^+_1 - 4^+_1$	0.6182	0.6275
				$3^+_1 - 2^+_1$	0.8805	0.6275
	2^+_3	1.0738	0.9105	$2^+_3 - 4^+_1$	0.6878	0.5035
				$2^+_3 - 2^+_1$	0.9502	0.7884
				$2^+_3 - 0^+_1$	1.0738	0.9105
	4^+_2	1.1444	1.1739	$4^+_2 - 4^+_1$	0.7588	0.7669
	8^+_1	1.2235	1.4646	$8^+_1 - 6^+_1$	0.4632	0.5806
	4^+_3	1.3232	1.2077	$4^+_3 - 4^+_1$	0.9376	0.7937
				$4^+_3 - 2^+_1$	1.1999	1.0785
	10^+_1	2.0233	2.2368	$10^+_1 - 8^+_1$	0.7995	0.7722

Table 2 includes the energy ratios of corresponding limits, these values were one of the basis states for nuclear structure [11].

Table 3 illustrates theoretical and experimental values of energy ratios calculated as it's like in $E(4^+_1)/E(2^+_1)$, $E(6^+_1)/E(2^+_1)$ and $E(8^+_1)/E(2^+_1)$, it appears that Yb-164 nucleus lies within symmetry SU(5)-SU(3). The nuclei in this district $N \geq 90$ are malformed with the elevated energy ratios of$E(4^+_1)/E(2^+_1)$ $E(6^+_1)/E(2^+_1)$, and $E(8^+_1)/E(2^+_1)$.

Table 2 shows the practical energy ratios that can be adopted. In addition, table 2 can be used to compare theoretical ratios with them. In Table 2 and table 3 are shown that the isotope tends to be symmetrical SU(5)-SU(3).

These values indicated the nucleus has transitional symmetry from vibrational excitations up to completely rotational excitations [11, 12].

The nuclei in this region dominance of γ – soft and moreover to vibrational nuclei, i.e. the characteristic character of these nuclei in this area is format of SU(3) – O(6) – SU(5).
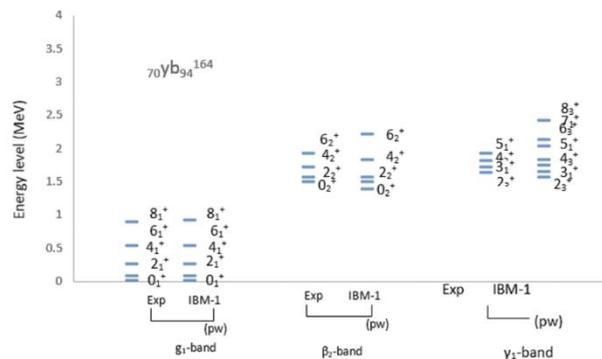
Figure 1. The calculated IBM (pw) compared with experimental energy levels states) g, β, and γ (in isotope $^{164}_{70}Yb_{94}$ of the dynamical symmetry SU(3) and SU(5).

Table 2. The energy ratios of corresponding limits [10].

Limit	$E(4^+_1)/E(2^+_1)$	$E(6^+_1)/E(2^+_1)$	$E(8^+_1)/E(2^+_1)$
SU(5)	2.3	3	4
O(6)	2.5	4.5	7
SU(3)	3.33	7	12

Table 3. Comparison of theoretical and experimental values of energy ratios for selected even-even isotopes Yb (A=164).

Isotopes	$E(4_1^+)/E(2_1^+)$		$E(6_1^+)/E(2_1^+)$		$E(8_1^+)/E(2_1^+)$	
	Exp.[10]	IBM-1 (pw)	Exp.[10]	IBM-1 (pw)	Exp.[10]	IBM-1 (pw)
$^{164}_{70}Yb_{94}$	3.1273	3.3333	6.1613	6.9991	9.9229	11.9950

Table (4) shows practical and theoretical values of energy levels. B(E2) measure of Yb (A=164) the appropriate values of electric quadrupole transition probabilities for SU(5) and SU(3) dynamical symmetry can be written [7].

$$SU(5) \ B(E2; 2_1^+ - 0_1^+) = \alpha_2^2 N$$
$$= \alpha_2^2 \frac{1}{5} N(2N + 3) \quad (6)$$

where α_2 is the role of effective boson charge. Figure 2 shows the difference between the practical and theoretical values of transitions through the given values of experimental transition $(2_1^+ - 0_1^+)$, we enable to calculate the value of the α_2^2 parameter for the isotope, where α_2^2 indicates square of the effective charge.

B(E2) for the transitions $(2_1^+ - 0_1^+)$ in the present work is calculated to find the deformation degree of the nucleus corresponding to the limit of the dynamical symmetry [12, 13].
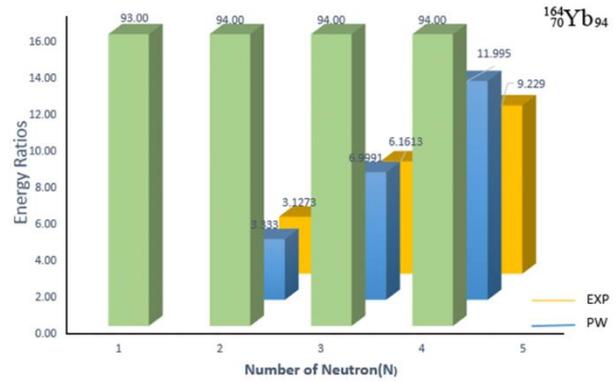
Figure 2. Comparison between of practical and theoretical values of energy levels data for selected even-even isotopes $^{164}_{70}Yb_{94}$.

Table 4. Calculated and experimental energy levels their transitions and B (E2) values of Yb (A=164).

Isotopes	I^π	Energy Level (MeV)		I_i - I_f	B(E2)(eb)^2	
		Exp.[10]	IBM1 (pw) Calculated		Exp.[14]	IBM1 (pw) Calculated
$^{164}_{70}Yb_{94}$	2_g^+	0.123	0.1221	$2_g^+ - 0_g^+$	0.918	0.9154
	2_β^+	1.070	0.8840	$2_\beta^+ - 0_g^+$	--------	1.141

CONCLUSIONS

In the present work, the interacting boson model (IBM-1) was used in the calculations of the energy levels, energy ratios, electric quadrupole transitions probability B(E2), It is through the results we obtained show that the isotope yb-164 has transitional properties (SU (3) - (SU (5). According to the values we obtained for energy levels, energy ratios and the probability of quadrupole transmission B(E2),

Our results confirmed good agreement with the available experimental data.

and their energy ratios from the first 4^+ levels, R = E(4^+)/E(2^+).

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