Reliability Function of (1 Strength and 4 Stresses) for Rayleigh distribution

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ABSTRACT

In this paper, a reliability function was found for one of the stress-robustness models where the model consists of one component that has robustness expressed by the random variable X and is subjected to four stresses expressed by random variables $Y_1,\ Y_2, Y_3\ and\ Y_4$ that follow the Rayleigh distribution. three methods of estimation (maximum likelihood method, least squares method, and weighted least squares method), were used to estimate the parameters into the reliability function of the model. A Monte Carlo simulation was also performed to compare the results of the estimation methods using the mean squared error criterion. The comparison showed that ML is the best estimator of the reliability function.

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1. Introduction

Interest in the term stress-strength in industrial systems increased significantly before the second half of the twentieth century. When dealing with the term stress-strength, there is a relationship between two random variables, one represents stress, the other represents strength and tries to find the probability that one will overcome the other. The development that has taken place in the world in various branches, as scientific, medical and construction fields, has led to extension and complexity in stress and strength models, and meanwhile, the dependency is basis for measuring the performance of manufacturing models work ended time, so it has a major effect on refining the performance of this systems and increases their efficiency. Reliability is defined as the lifetime of a component where the component remains in a working state as long as it can resist the stresses to which it is subjected, expressed by the random variable Y with its strength, and expressed by the random variable X, where $R = P(Y < X)$ and stops working (fails) if the stresses exceed the strength of the component $X < Y$.

Many papers included strength-stress, Haddad and Batah [1] studied the reliability of the strength-stress model when the factors follow the (Rayl. – Par) distribution. Khaleel and Karam [2,3] derived a special reliability model from the Cascade models (2+1), where the model contains two basic components and an excess component with an active standby state. Khaleel [4] studied a special model of reliability (3+1), which contains three basic components and a component with an active standby state. Khaleel [5] derived a model of a single component that has strength and is subjected to several stresses when the stress and strength factors follow the Lomax distribution. Salman and Hamad [6] studied the estimation of the reliability function by several different estimation methods when the stress and durability factors trace the Lomax distribution.

This paper aims to find a reliability function for a model consisting of one component, where this component is subjected to four stresses, and these stresses have their own strength, assuming that the stress and strength factors follow the Rayleigh distribution and are independent, as well as estimating the reliability function in three estimation methods "maximum likelihood", "least square" and "weighted least square" and working Monte Carlo simulation to compare the estimation results.

2. Mathematical Model

It is known that the life of a component is determined in stress - strength models according to its strength X, through which it can resist the stress Y to which the component is exposed, where

$(X > Y)$, but if $(Y > X)$, the component fails and does not continue to work .The mathematical formula for the reliability of a one-component model can be expressed as follows:

$$\mathcal{R} = \mathrm{pr}(\mathrm{Y} < \mathrm{X}) = \int_{-\infty}^{\infty} \mathrm{f}(x)\mathrm{F}_{\mathcal{Y}}(x)\mathrm{dx} \quad \ldots(1)$$

As for the case when the component is subjected to four stresses Y_1, Y_2, Y_3 and Y_4 and resists these stresses with one strength X, the reliability of this model can be expressed:

$$\mathcal{R} \int_{-\infty}^{\infty} \mathrm{pr}(\mathrm{Y}_1 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_2 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_3 < \mathrm{X})\mathrm{pr}(\mathrm{Y}_4 < \mathrm{X})\mathrm{f}_{x(x)}\,\mathrm{dx}$$

Assume that the random variables of stress and durability are independent and indexical, so the mathematical formula for the reliability of the model is as follows:

$$\mathcal{R} = pr(Max(Y_1, Y_2, Y_3, Y_4) < X)$$

$$= \int_0^{\infty} \int_0^{\mathcal{Y}_1} \int_0^{\mathcal{Y}_2} \int_0^{\mathcal{Y}_3} \int_0^{\mathcal{Y}_4} f(\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \mathcal{Y}_4, x)\, d\mathcal{Y}_4 d\mathcal{Y}_3 d\mathcal{Y}_2 d\mathcal{Y}_1 dx \quad \ldots(3)$$

Then

$$\mathcal{R} = \int_0^{\infty} \int_0^{\mathcal{Y}_1} \int_0^{\mathcal{Y}_2} \int_0^{\mathcal{Y}_3} \int_0^{\mathcal{Y}_4} f(\mathcal{Y}_1)f(\mathcal{Y}_2)f(\mathcal{Y}_3)f(\mathcal{Y}_4)\, f(x) d\mathcal{Y}_4 d\mathcal{Y}_3 d\mathcal{Y}_2 d\mathcal{Y}_1 dx$$

$$\mathcal{R} = \int_0^{\infty} F_{1\mathcal{Y}_1}(x)F_{2\mathcal{Y}_2}F_{3\mathcal{Y}_3}(x)F_{4\mathcal{Y}_4}(x)f(x)dx \quad \ldots(4)$$

Assuming that the random variables follow a Raleigh distribution where $X \sim R\ (2, \delta)$ and $Y_r \sim R(2, \delta_r)$;r=1,2,3,4 then the pdf and CDF are:

$$f(x) = \tfrac{2}{\delta} x\, e^{-\frac{x^2}{\delta}} \quad \ldots(5)$$

$$F(x) = 1 - e^{-\frac{x^2}{\delta}} \quad \ldots(6)$$

And

$$F_r(\mathcal{Y}_\mathrm{r}) = 1 - e^{-\frac{\mathcal{Y}_r^2}{\delta_r}} \quad ;\mathrm{r}=1,2,3,4 \quad \ldots(7)$$

Equations 5, 6 and 7 will be used in Equation 4 as follows:

$$\mathcal{R} = \int_0^{\infty} \left[[1 - \mathrm{e}^{-\frac{x^2}{\delta_1}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_2}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_3}}]\, [1 - \mathrm{e}^{-\frac{x^2}{\delta_4}}] \right] \tfrac{2}{\delta} x\, \mathrm{e}^{-\frac{x^2}{\delta}} dx$$

$$= \int_0^{\infty} \Big[1 - e^{-\frac{x^2}{\delta_1}} - e^{-\frac{x^2}{\delta_2}} - e^{-\frac{x^2}{\delta_3}} - e^{-\frac{x^2}{\delta_4}} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)x^2}$$

$$+ e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2} + e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2} + e^{-\left(\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} - e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2} - e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}$$

$$- e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} - e^{-\left(\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} + e^{-\left(\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2} \Big] \tfrac{2}{\delta} x\, e^{-\frac{x^2}{\delta}} dx$$

$$\mathcal{R}=\int_0^\infty \tfrac{2}{\delta}x\,e^{-\frac{x^2}{\delta}}dx-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}\right)x^2}dx-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}\right)x^2}dx-\int_0^\infty \tfrac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_3}\right)x^2}dx$$

$$-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)x^2}dx+\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)x^2}dx+\int_x^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)x^2}dx$$

$$+\int_0^\infty \tfrac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2}dx+\int_0^\infty \tfrac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

$$-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)x^2}dx-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)x^2}dx-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

$$-\int_0^\infty \tfrac{2}{\delta}x\,e^{-\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx+\int_0^\infty \tfrac{2}{\delta}xe^{-\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)x^2}dx$$

Then

$$\mathcal{R}=1-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_3}\right)}\right]-\left[\frac{1}{\beta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}\right)}\right]$$

$$+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}\right)}\right]$$

$$-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]-\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]+\left[\frac{1}{\delta}\frac{1}{\left(\frac{1}{\delta}+\frac{1}{\delta_1}+\frac{1}{\delta_2}+\frac{1}{\delta_3}+\frac{1}{\delta_4}\right)}\right]$$

Finally, the reliability model is:

$$\mathcal{R}=1-\left[\frac{\delta_1}{(\delta+\delta_1)}\right]-\left[\frac{\delta_2}{(\delta+\delta_2)}\right]-\left[\frac{\delta_3}{(\delta+\delta_3)}\right]-\left[\frac{\delta_4}{(\delta+\delta_4)}\right]+\left[\frac{\delta_1\delta_2}{(\delta\delta_1+\delta\delta_2+\delta_1\delta_2)}\right]+\left[\frac{\delta_1\delta_3}{(\delta\delta_1+\delta\delta_3+\delta_1\delta_3)}\right]$$

$$+\left[\frac{\delta_1\delta_4}{(\delta\delta_1+\delta\delta_4+\delta_1\delta_4)}\right]+\left[\frac{\delta_2\delta_3}{(\delta\delta_2+\delta\delta_3+\delta_2\delta_3)}\right]+\left[\frac{\delta_2\delta_4}{(\delta\delta_2+\delta\delta_4+\delta_2\delta_4)}\right]+\left[\frac{\delta_3\delta_4}{(\delta\delta_3+\delta\delta_4+\delta_3\delta_4)}\right]$$

$$-\left[\frac{\delta_1\delta_2\delta_3}{(\delta\delta_1\delta_2+\delta\delta_1\delta_3+\delta\delta_2\delta_3+\delta_1\delta_2\delta_3)}\right]-\left[\frac{\delta_1\delta_2\delta_4}{(\delta\delta_1\delta_2+\delta\delta_1\delta_4+\delta\delta_2\delta_4+\delta_1\delta_2\delta_4)}\right]-\left[\frac{\delta_1\delta_3\delta_4}{(\delta\delta_1\delta_3+\delta\delta_1\delta_4+\delta\delta_3\delta_4+\delta_1\delta_3\delta_4)}\right]$$

$$-\left[\frac{\delta_2\delta_3\delta_4}{(\delta\delta_2\delta_3+\delta\delta_2\delta_4+\delta\delta_3\delta_4+\delta_2\delta_3\delta_4)}\right]+\left[\frac{\delta_1\delta_2\delta_3\delta_4}{(\delta\delta_1\delta_2\delta_3+\delta\delta_1\delta_2\delta_4+\delta\delta_1\delta_3\delta_4+\delta\delta_2\delta_3\delta_4+\delta_1\delta_2\delta_3\delta_4)}\right] \quad \ldots(8)$$

3. Estimation

Three different estimation methods are used to estimate the parameters $\delta, \delta_1, \delta_2, \delta_3$ *and* δ_4, then the reliability function will be estimated.

3.1 Maximum likelihood function (ML):

To estimate the parameter δ by the ML method, we can start by using equation (5) as follows[7]:

$$\mathrm{L}(x_1,x_2,\ldots,x_\mathrm{n};2,\delta)=\left(\tfrac{2}{\delta}\right)^{\mathrm{n}}\prod_{\mathrm{i}=1}^{\mathrm{n}}x_\mathrm{i}\,\mathrm{e}^{-\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}}x_\mathrm{i}^2}{\delta}} \quad \ldots(9)$$

The logarithm is taken for equation 9:

$$\ln L=n\ln 2-n\ln\delta+\sum_{i=1}^{n}\ln x_i-\frac{\sum_{i=1}^{n}x_i^2}{\delta} \quad \ldots(10)$$

Equation (10) is partially derived for the parameter δ:

$$\frac{\partial\ln L}{\partial\delta}=-\frac{n}{\delta}+\frac{\sum_{i=1}^{n}x_i^2}{\delta^2}$$

Then get as $\hat{\delta}_{(ML)}$:

$$\hat{\delta}_{(ML)} = \frac{\sum_{i=1}^{n} {x_i}^2}{n} \quad ...(11)$$

With the same previous steps, $\hat{\delta}_{1(ML)}$, $\hat{\delta}_{2(ML)}$, $\hat{\delta}_{3(ML)}$, $\hat{\delta}_{4(ML)}$ are obtained:

$$\hat{\delta}_{1(ML)} = \frac{\sum_{j_1=1}^{n_1} y_{j1}^2}{n_1} \quad ...(12)$$

$$\hat{\delta}_{2(ML)} = \frac{\sum_{j_2=1}^{n_2} y_{j2}^2}{n_2} \quad ...(13)$$

$$\hat{\delta}_{3(ML)} = \frac{\sum_{j_3=1}^{n_3} y_{j3}^2}{n_3} \quad ...(14)$$

$$\hat{\delta}_{4(ML)} = \frac{\sum_{j_4=1}^{n_4} y_{j4}^2}{n_4} \quad ...(15)$$

3.2 Least Square Method (LS) :

Using the least squares method [8], it is possible to estimate the parameters by the minimization of the following equation:

$$\mathrm{S} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \left[\mathrm{F}(\mathrm{X}_{(\mathrm{i})}) - \mathrm{E}\left(\mathrm{F}(\mathrm{X}_{(\mathrm{i})})\right)\right]^2 \quad ...(16)$$

Where $E\left(F(X_{(i)})\right) = P_i$; $P_i = \frac{i}{n+1}$; $i = 1,2, \dots, n$

$$P_i = 1 - e^{-\frac{x_{(i)}^2}{\delta}}$$

$$(1 - P_i) = e^{-\frac{x_{(i)}^2}{\delta}}$$

Then

$$\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta} = 0 \quad ...(17)$$

By substation equation (17) is used in equation (16):

$$S = \sum_{i=1}^{n} \left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right]^2 \quad ...(18)$$

Equation (18) is derived for the parameter δ:

$$\frac{\partial S}{\partial \delta} = \sum_{i=1}^{n} 2 \left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right] \left(\frac{-x_{(i)}^2}{\delta^2}\right)$$

$$\sum_{i=1}^{n} 2 \left[\ln(1 - P_i) + \frac{x_{(i)}^2}{\delta}\right] \left(\frac{-x_{(i)}^2}{\delta^2}\right) = 0 \quad ...(19)$$

Then $\hat{\delta}_{(LS)}$ is obtained:

$$\hat{\delta}_{(LS)} = \frac{-\sum_{i=1}^{n} x_{(i)}^4}{\sum_{i=1}^{n} x_{(i)}^2 \ln(1-P_i)} \quad ....(20)$$

In the same way, $\hat{\delta}_{1(LS)}$, $\hat{\delta}_{2(LS)}$, $\hat{\delta}_{3(LS)}$ *and* $\hat{\delta}_{4(LS)}$ are obtained:

$$\hat{\delta}_{1(LS)} = \frac{-\sum_{j_1=1}^{n_1} y_{1(j_1)}^4}{\sum_{j_1=1}^{n_1} y_{1(j_1)}^2 \ln\left(1-P_{j_1}\right)} \quad \ldots(21)$$

$$\hat{\delta}_{2(LS)} = \frac{-\sum_{j_2=1}^{n_2} y_{2(j_2)}^4}{\sum_{j_2=1}^{n_2} y_{2(j_2)}^2 \ln\left(1-P_{j_2}\right)} \quad \ldots(22)$$

$$\hat{\delta}_{3(LS)} = \frac{-\sum_{j_3=1}^{n_3} y_{3(j_3)}^4}{\sum_{j_3=1}^{n_3} y_{3(j_3)}^2 \ln\left(1-P_{j_3}\right)} \quad \ldots(23)$$

$$\hat{\delta}_{4(LS)} = \frac{-\sum_{j_4=1}^{n_4} y_{4(j_4)}^4}{\sum_{j_4=1}^{n_4} y_{4(j_4)}^2 \ln\left(1-P_{j_4}\right)} \quad \ldots(24)$$

3.3 Weighted Least Square Method (WLS) :

Using equations (17) and (18) to estimate of parameter δ by the weighted least squares method as follows [9]:

$$\sum_{i=1}^{n} w_i \left[ln(1-P_i) + \frac{x_{(i)}^2}{\delta}\right]^2 = 0 \quad \ldots(25)$$

Where $w_i = \frac{1}{Var[F(x_{(i)})]}$

By deriving (25) with respect to δ:

$$\sum_{i=1}^{n} w_i x_{(i)}^2 \ln(1-P_i) + \frac{1}{\delta}\sum_{i=1}^{n} w_i x_{(i)}^4 = 0 \quad \ldots(26)$$

So, $\hat{\delta}_{(WLS)}$ is:

$$\hat{\delta}_{(WLS)} = \frac{-\sum_{i=1}^{n} w_i x_{(i)}^4}{\sum_{i=1}^{n} w_i x_{(i)}^2 \ln(1-P_i)} \quad \ldots(27)$$

In similar way, $\hat{\delta}_{1(WLS)}$, $\widehat{\delta}_{2(WLS)}$, $\widehat{\delta}_{3(WLS)}$ *and* $\hat{\delta}_{4(WLS)}$ are:

$$\hat{\delta}_{1(WLS)} = \frac{-\sum_{j_1=1}^{n_1} \mathrm{w}_{j_1} y_{1(j_1)}^4}{\sum_{j_1=1}^{n_1} \mathrm{w}_{j_1} y_{1(j_1)}^2 \ln\left(1-P_{j_1}\right)} \quad \ldots(28)$$

$$\hat{\delta}_{2(WLS)} = \frac{-\sum_{j_2=1}^{n_2} \mathrm{w}_{j_2} y_{2(j_2)}^4}{\sum_{j_2=1}^{n_2} \mathrm{w}_{j_2} y_{2(j_2)}^2 \ln\left(1-P_{j_2}\right)} \quad \ldots(29)$$

$$\hat{\delta}_{3(WLS)} = \frac{-\sum_{j_3=1}^{n_3} \mathrm{w}_{j_3} y_{3(j_3)}^4}{\sum_{j_3=1}^{n_3} \mathrm{w}_{j_3} y_{3(j_3)}^2 \ln\left(1-P_{j_3}\right)} \quad \ldots(30)$$

$$\hat{\delta}_{4(WLS)} = \frac{-\sum_{j_4=1}^{n_4} \mathrm{w}_{j_4} y_{4(j_4)}^4}{\sum_{j_4=1}^{n_4} \mathrm{w}_{j_4} y_{4(j_4)}^2 \ln\left(1-P_{j_4}\right)} \quad \ldots(31)$$

4. Simulation

A Monte Carlo simulation is performed to compare the results of different methods of estimation using MSE. It is also made to compare estimation methods for R using different sample sizes[10].

4.1 The Algorithm:

The MATLAB program is used in the simulation as shown in the steps below**:**

1. The random samples $x_1, x_2, \dots, x_n; y_{11}, y_{12}, \dots, y_{1n_1}; y_{21}, y_{22}, \dots y_{2n_2}; y_{31}, y_{32}, \dots y_{3n_3}$ and $y_{41}, y_{42}, \dots y_{4n_4}$ of different sizes $(n, n_1, n_2, n_3, n_4) = (20{,}20{,}20{,}20{,}20), (40{,}40{,}40{,}40{,}40)$, $(50{,}50{,}50{,}50{,}50), (70{,}70{,}70{,}70{,}70) and (90{,}90{,}90{,}90{,}90)$ are generated.

2. Let the values of the parameters $\delta, \delta_1, \delta_2, \delta_3, \delta_4$ be the reliability of the five experiments as shown below:

Table 1: The parameter values and reliability

Excrement	$\boldsymbol{\delta}$	$\boldsymbol{\delta_1}$	$\boldsymbol{\delta_2}$	$\boldsymbol{\delta_3}$	$\boldsymbol{\delta_4}$	R
1	0.7	0.7	0.7	0.7	0.7	**0.2000**
2	1.6	0.5	0.9	0.4	0.2	**0.5337**
3	0.9	1.3	1.1	1.1	1.4	**0.1294**
4	1.2	1.1	0.8	0.5	1.2	**0.2996**
5	2.6	0.3	0.6	0.3	0.5	**0.7163**

3. Parameters $(\delta, \delta_1, \delta_2, \delta_3, \delta_4)$ were estimated in equations (11), (12), (13), (14), (15), (20), (21), (22), (23), (24), (27),(28), (29), (30),(31).

4. Calculate the mean with the formula:

$$Mean = \frac{\sum_{i=1}^{L} \hat{\mathcal{R}}_i}{L}$$

5. By using the mean squares error, the results of the estimation methods are compared, the mathematical formula of which is:

$$\text{MSE}(\hat{\mathcal{R}}) = \frac{1}{L} \sum_{i=1}^{L} (\hat{\mathcal{R}}_i - \mathcal{R})^2$$

4.2 Results

The tables below represent the results obtained from the simulation procedure:

Table 2: Simulation results of experiment 1

S.S	Criterion	**MLE**	**LSE**	**WLSE**	**Best**
(20, 20, 20, 20, 20)	Mean	0.1662	0.1621	0.1569	MLE
	MSE	0.0123	0.0131	0.0138	
(40, 40, 40, 40, 40)	Mean	0.1640	0.1603	0.1527	
	MSE	0.0107	0.0114	0.0125	
(50, 50, 50, 50, 50)	Mean	0.1612	0.1587	0.1458	
	MSE	0.0086	0.0092	0.0109	
(70, 70, 70, 70, 70)	Mean	0.1585	0.1565	0.1376	
	MSE	0.0077	0.0081	0.0105	
(90, 90, 90, 90, 90)	Mean	0.1585	0.1565	0.1355	
	MSE	0.0074	0.0077	0.0104	

Table 3: Simulation results of experiment 2

S.S	Criterion	MLE	LSE	WLSE	Best
$(20, 20, 20, 20, 20)$	Mean	0.4729	0.4674	0.4613	MLE
	MSE	0.0241	0.0265	0.0291	
$(40, 40, 40, 40, 40)$	Mean	0.4734	0.4676	0.4577	
	MSE	0.0211	0.0233	0.0270	
$(50, 50, 50, 50, 50)$	Mean	0.4765	0.4730	0.4577	
	MSE	0.0172	0.0186	0.0240	
$(70, 70, 70, 70, 70)$	Mean	0.4770	0.4745	0.4526	
	MSE	0.0159	0.0168	0.0240	
$(90, 90, 90, 90, 90)$	Mean	0.4803	0.4780	0.4547	
	MSE	0.0148	0.0156	0.0230	

Table 4: Simulation results of experiment 3

S.S	Criterion	MLE	LSE	WLSE	Best
$(20, 20, 20, 20, 20)$	Mean	0.1103	0.1074	0.1036	MLE
	MSE	0.0074	0.0077	0.0080	
$(40, 40, 40, 40, 40)$	Mean	0.1076	0.1052	0.0994	
	MSE	0.0059	0.0062	0.0066	
$(50, 50, 50, 50, 50)$	Mean	0.1043	0.1028	0.0932	
	MSE	0.0045	0.0048	0.0056	
$(70, 70, 70, 70, 70)$	Mean	0.1027	0.1012	0.0869	
	MSE	0.0040	0.0042	0.0053	
$(90, 90, 90, 90, 90)$	Mean	0.1039	0.1029	0.0874	
	MSE	0.0039	0.0040	0.0051	

Table 5: Simulation results of experiment 4

S.S	Criterion	MLE	LSE	WLSE	Best
$(20, 20, 20, 20, 20)$	Mean	0.2412	0.2338	0.2249	
	MSE	0.0230	0.0242	0.0256	MLE
$(40, 40, 40, 40, 40)$	Mean	0.2370	0.2312	0.2182	
	MSE	0.0213	0.0224	0.0246	
$(50, 50, 50, 50, 50)$	Mean	0.2381	0.2333	0.2107	
	MSE	0.0188	0.0197	0.0233	
$(70, 70, 70, 70, 70)$	Mean	0.2373	0.2341	0.2024	
	MSE	0.0175	0.0181	0.0231	
$(90, 90, 90, 90, 90)$	Mean	0.2386	0.2357	0.2011	
	MSE	0.0172	0.0178	0.0232	

Table 6: Simulation results of experiment 5

S.S	Criterion	MLE	LSE	WLSE	Best
$(\mathbf{20,20,20,20,20})$	Mean	0.6060	0.5962	0.5842	
	MSE	0.0470	0.0508	0.0557	MLE
$(\mathbf{40,40,40,40,40})$	Mean	0.6048	0.5963	0.5784	
	MSE	0.0471	0.0503	0.0576	
$(\mathbf{50,50,50,50,50})$	Mean	0.6081	0.6018	0.5714	
	MSE	0.0438	0.0461	0.0578	
$(\mathbf{70,70,70,70,70})$	Mean	0.6085	0.6038	0.5605	
	MSE	0.0426	0.0443	0.0611	
$(\mathbf{90,90,90,90,90})$	Mean	0.6079	0.6039	0.5569	
	MSE	0.0412	0.0425	0.0600	

4. Conclusions

The conclusions were reached as follows:

- The reliability value of the model increases by increasing the value of the parameter δ and the reliability value decreases by increasing the values of the parameters $\delta_{\mathbf{1}}, \delta_{\mathbf{2}}, \delta_{\mathbf{3}}, \delta_{\mathbf{4}}$, this is clear from Table 1 when comparing Experiment 2 with experiment 3, as well as when comparing experiment 4 with experiment 5.
- The performance of the MLE estimator was the best for estimating $\mathcal{R}$.

Table 7: Best estimator of model reliability

Values of parameters and sample size	Best
Sizes of Sample $(n, n_1, n_2, n_3, n_4) = (20,20,20,20,20), (40,40,40,40,40), (50,50,50,50,50), (70,70,70,70,70) and (90,90,90,90,90)$ and values of parameter are $(\delta, \delta_1, \delta_2, \delta_3, \delta_4) = (0.7,0.7,0.7,0.7,0.7), (1.6,0.5,0.9,0.4,0.2), (0.9,1.3,1.1,1.1,1.4), (1.2,1.1,0.8,0.5,1.2)$ *and* $(2.6,0.3,0.6,0.3,0.5)$,	MLE

- There is a convergence between the performance of MLE and LSE estimators.

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دالة المعولية لـ (1 متانة و4 ضغوط) لتوزيع رالي

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المستخلص

في هذه البحث، تم ايجاد دالة المعولية لأحد نماذج المتانة والإجهاد حيث يتكون النموذج من مكونة واحدة لها متانة يعبر عنها المتغير العشوائي X ويخضع لأربعة اجهادات يعبر عنها بالمتغيرات العشوائية Y_1 و Y_2 و Y_3 و Y_4 بافتراض أنها تتبع توزيع رالي. تم استخدام ثلاث طرق للتقدير (طريقة الإمكان الاعظم وطريقة المربعات الصغرى وطريقة المربعات الصغرى الموزونة) لتقدير المعلمات في دالة المعولية للنموذج. كما تم إجراء محاكاة مونت كارلو أيضا لمقارنة نتائج طرق التقدير باستخدام معيار متوسط مربع الخطأ. أظهرت المقارنة أن مقدر الإمكان الاعظم هو الأفضل لتقدير دالة المعولية.