Open, closed and continuous function in bi-pre-supra topological space

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Abstract
In this paper we construction a new space called bi-pre-supra topological space . Many concepts ($\mathcal{T},\mathcal{PT}$)-open set ,($\mathcal{T},\mathcal{P}^{*}\mathcal{T}$)-open set , bi-open set) were introduced . At last through this paper we introduced a new class of functions (open , closed and continuous) in bi-pre-supra topological space . We study and investigate some properties and characterization of above concepts .

Keywords: ($\mathcal{T},\mathcal{PT}$)-open function ,($\mathcal{T},\mathcal{P}^{*}\mathcal{T}$)-open function , bi-open function ($\mathcal{T},\mathcal{PT}$)-closed function ,($\mathcal{T},\mathcal{P}^{*}\mathcal{T}$)- closed function , bi- closed function , ($\mathcal{T},\mathcal{PT}$)-continuous function, ($\mathcal{T},\mathcal{P}^{*}\mathcal{T}$)-continuous function , bi-continuous function .

الملخص: في هذا البحث قدمنا نوع جديد من الفضاءات اطلق عليه (bi-pre-supra topological space) حيث تم التعرف على مجموعة مفاهيم في هذا الفضاء مثل المجموعه المفتوحه المزدوجه كما تم تقديم نوع جديد من الدوال (مفتوحه , مغلقه ومستمره) في هذا الفضاء الجديد كما تم دراسة بعض الخواص والصفات للمفاهيم السابقة .

1-Introduction

In 1963 Kelley J. C. [5] was first introduced the concept of bi-topological spaces , where X is a non-empty set and $\mathcal{T}_1$, $\mathcal{T}_2$ are topologies on X . In 1982 Almashhor [1] introduced the concept of pre-open sets in topological space . By using this concept , several authors' [4], [6], [7] defined and studies stronger or weaker types of topological concept .

In this paper , we introduced the concepts of bi-pre-supra topological space , via ($\mathcal{T},\mathcal{PT}$)-open set ,($\mathcal{T},\mathcal{P}^{*}\mathcal{T}$)-open set and bi-open set in bi-pre-supra topological space , and we study their basic properties and relationships with other concepts of sets. At last through this paper we introduced a new class of functions (open , closed and continuous) in bi-pre-supra topological space . We study and investigate some properties and characterization of above concepts .

2-Preliminaries

Definition 2.1 [1] A subset A of a space (X, $\mathcal{T}$) is called pre-open, if A$\subseteq$ int (cl(A)) . The complement of pre-open set is said to be pre-closed .

Definition 2.2 [2] A subfamily $\mathcal{T}$of a family of subset of X is said to be a supra topology on X if:

1) X,$\emptyset \in \mathcal{T}$
2) If $A_i \in \mathcal{T}$ for all i$\in$I then $\cup A_i \in \mathcal{T}$

(X, $\mathcal{T}$) is called a supra topological space . The element of $\mathcal{T}$ are called supra open set in (X, $\mathcal{T}$) and complement of a supra open set is called a supra closed set .

Definition 2.3 [7] Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a bi-topological space , and let G be a subset of X . Then G is said to be (i,j)-open set if G=A∪B where A∈ $\mathcal{T}_1$ and B∈ $\mathcal{T}_2$. The complement of (i,j)-open set is called (i,j)-closed set .

Remark 2.4 [7] Notice that (i,j)-open set need not necessarily form a topology .

Definition 2.4 [3] A subset A of a bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ is called (i,j)-neighborhood of a point x in X if there exists an (i,j)-open set G such that x∈G⊆A. And denoted (i,j)-nbd .

Definition 2.5 [3] Let A be a subset of bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. A point x in X is said to (i,j)-limit point of A if for each (i, j)-open set G containing x such that A∩(G \{x})≠ ∅ .The set of all (i,j)-limit point of A is called (i,j)-derived set of A and denoted by (i,j)-d(A) .

Definition 2.6 [7] Let A be a subset of bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then the (i,j)-closure of G denoted by (i,j)-cl(A) , is defined by ⋂{ F : A ⊆ F and F is (i,j)-closed set }

.**Definition 2.7** [7] Let A be a subset of bi-topological space $(X, \mathcal{T}_1, \mathcal{T}_2)$. Then the (i,j)-interior of A denoted by (i,j)-int(A) , is defined by ∪{ G : G ⊆ A and F is (i,j)-open set }

Definition 2.8 [8] A function f:$(X, \mathcal{T}_X) \rightarrow (Y, \mathcal{T}_Y)$ is called open function if the image of every open set is open .

Definition 2.9 [8] A function f:$(X, \mathcal{T}_X) \rightarrow (Y, \mathcal{T}_Y)$ is called closed function if the image of every closed set is closed .

Definition 2.10 [8]

A function f:$(X, \mathcal{T}_X) \rightarrow (Y, \mathcal{T}_Y)$ is called continuous function if the inverse image of any $\mathcal{T}_Y$-open set G is $\mathcal{T}_X$-open set .

3-Bi-pre-supra topological spaces

Definition 3.1 Let X be a non-empty set , let $\mathcal{T}$ be a topology on X and let $\mathcal{PT}$ is the set of all pre-open subset of X (for short Po(X)), then We say that $(X, \mathcal{T}, \mathcal{PT})$ is a bi-pre-supra topological space .

Now the deference between bi-topological space [Kelly] and bi-pre-supra topological space $\mathcal{PT}$ is supra topology not topology .

Example 3.2 Let X ={1,2,3,4}

$\mathcal{T}$ = {∅, X , {1} , {2,3} , {1,2,3}}

$\mathcal{P}$oX = $\mathcal{PT}$ = {∅, X, {1} , {2,3} , {1,2,3} , {2} , {3} , {1,2} ,{1,3} ,{1,2,4} ,{1,3,4}}

(X , $\mathcal{T}$, $\mathcal{PT}$) is bi-pre-supra topological space

Definition 3.3 Let (X , $\mathcal{T}$, $\mathcal{PT}$) be a bi-pre-supra topological space, and let G be a subset of X . Then

i) G is said to be $(\mathcal{T}, \mathcal{PT})$-open set if G=A∪B where A ∈ $\mathcal{T}$ and B ∈ $\mathcal{PT}$.
ii) The complement of $(\mathcal{T}, \mathcal{PT})$-open set is called $(\mathcal{T}, \mathcal{PT})$-closed set.
iii) G is said to be $(\mathcal{T}, \mathcal{PT})$*-open set if G=A∪B where A ∈ $\mathcal{T}$, B ∈ $\mathcal{PT}$ and B∉ $\mathcal{T}$.
iv) The complement of $(\mathcal{T}, \mathcal{PT})$*-open set is called $(\mathcal{T}, \mathcal{PT})$*-closed set.

v) G is said to be bi-open set if G = A where $A \in \mathcal{T}$ and $A \in \mathcal{PT}$.
vi) The complement of bi-open set is called bi-closed set.

Proposition 3.4

1) Every bi-open set is $(\mathcal{T},\mathcal{PT})$-open set and every bi-closed set is $(\mathcal{T},\mathcal{PT})$-closed set but the converse is not true .
2) Every $(\mathcal{T},\mathcal{PT})^*$-open set is $(\mathcal{T},\mathcal{PT})$-open set and every $(\mathcal{T},\mathcal{PT})^*$-closed set is $(\mathcal{T},\mathcal{PT})$-closed set but the converse is not true .

Example 3.5

Let X ={1,2,3,4}
$\mathcal{T}$ = {∅ , X , {2} , {1,3} , {1,2,3}}
$\mathcal{T}^c$ = {∅, X , {1,3,4} , {2,4} , {4}}
$\mathcal{PT}$ = {∅ , X , {2} , {1,3} , {1,2,3} , {1} , {3} , {1,2} , {2,3} ,{1,2,4} ,{2,3,4}}
$(\mathcal{T},\mathcal{PT})$-open sets = {∅ , X , {2} , {1,2,3} , {2,3} , {1,2} , {1,3} , {1,2,4} , {2,3,4}}
$(\mathcal{T},\mathcal{PT})$-closed sets = {∅ , X , {1,3,4} , {4} , {1,4} , {3,4} , {2,4} , {3} , {1}}
$(\mathcal{T},\mathcal{PT})^*$-open set= {∅ , X , {1,2} , {2,3} , {1,2,4} , {2,3,4} ,{1,3} {1,2,3}}
$(\mathcal{T},\mathcal{PT})^*$-closed sets = {∅ , X , {3,4} , {1,4} , {3} , {1} , {2,4} , {4}}
bi-open sets = {∅ , X , {2} , {1,3} , {1,2,3}}
bi-closed sets = {∅, X , {1,3,4} , {2,4} , {4}}

Definition 3.6 Let (X , $\mathcal{T}$, $\mathcal{PT}$) be a bi-pre-supra topological space, and let A be a subset of X . Then

i. The $(\mathcal{T},\mathcal{PT})$-closure of G denoted by $(\mathcal{T},\mathcal{PT})$-cl(A) , is defined by ⋂{ F : A ⊆ F and F is $(\mathcal{T},\mathcal{PT})$-closed set }
ii. The $(\mathcal{T},\mathcal{PT})^*$-closure of A denoted by $(\mathcal{T},\mathcal{PT})^*$-cl(A) , is defined by ⋂{ F : A ⊆ F and F is $(\mathcal{T},\mathcal{PT})^*$-closed set }
iii. The bi- closure of A denoted by bi-cl(A) , is defined by
⋂{ F : A ⊆ F and F is bi-closed set }

Example 3.7 Let X = {1,2,3,4}
$\mathcal{T}$ = {∅ , X , {4} , {1,3} , {1,3,4}}
$\mathcal{T}^c$ = {∅ , X , {1,2,3} , {2,4} , {2}}
$\mathcal{PT}$ = {∅ , X , {4} , {1,3} , {1,3,4} , {1} , {3} , {1,4} , {3,4} , {1,2,4} , {2,3,4}}
$(\mathcal{T},\mathcal{PT})$-open sets = {∅ , X , {4} , {1,3,4} , {1,4} , {3,4} , {1,2,4} , {2,3,4} , {1,3}}
$(\mathcal{T},\mathcal{PT})$-closed sets = {∅ , X , {1,2,3} , {2} , {2,3} , {1,2} , {3} , {1} , {2,4}}
$(\mathcal{T},\mathcal{PT})^*$-open sets={∅ , X , {1,4} , {3,4} , {1,2,4} , {2,3,4} , {1,3} , {1,3,4}}
$(\mathcal{T},\mathcal{PT})^*$-closed sets={∅ , X , {2,3} , {1,2} , {3} , {1} , {2,4} , {2}}
bi-open sets = {∅ , X , {4} , {1,3} , {1,3,4}}
bi-closed sets = {∅ , X , {1,2,3} , {2,4} , {2}}
Take G = {1,2} , H={1,2,3}
$(\mathcal{T},\mathcal{PT})$-cl(G) = {1,2}
bi-cl(G) = {1,2,3}
$(\mathcal{T},\mathcal{PT})$-cl(H) = {1,2,3}
$(\mathcal{T},\mathcal{PT})^*$-cl(H)=X

Definition 3.8 Let (X , $\mathcal{T}$, $\mathcal{PT}$) be a bi-pre-supra topological space, and let A be a subset of X . Then :

(i) The $(\mathcal{T},\mathcal{PT})$-interior of A denoted by $(\mathcal{T},\mathcal{PT})$-int(A) , is defined by ∪{ F : F ⊆ A and F is $(\mathcal{T},\mathcal{PT})$-open set }

(ii) The $(\mathcal{T},\mathcal{PT})$*-interior of A denoted by $(\mathcal{T},\mathcal{PT})$*-int(A) , is defined by ∪{ F : F ⊆ A and F is $(\mathcal{T},\mathcal{PT})$*-open set }

(iii) The bi-interior of A denoted by bi-int(A), is defined by ∪{ F : F ⊆ A and F is bi-open set }

Example 3.9 Let X = {1,2,3,4}
$\mathcal{T}$ = {∅ , X , {1} , {2,4} , {1,2,4}}
$\mathcal{T}^c$ = {∅ , X , {2,3,4} , {1,3} , {3}}
$\mathcal{PT}$ = {∅ , X , {1} , {2,4} , {1,2,4} , {2} , {4} , {1,2} , {1,4} , {1,2,3} , {1,3,4}}
$(\mathcal{T},\mathcal{PT})$-open sets = {∅ , X , {1} , {1,2,4} , {1,2} , {1,4} , {1,2,3} , {1,3,4} , {2,4}}
$(\mathcal{T},\mathcal{PT})$*-open sets= {∅ , X , {1,2,4} , {1,2} , {1,4} , {1,2,3} , {1,3,4} , {2,4}}
bi-open sets = {∅ , X , {1} , {2,4} , {1,2,4}}
Take G = {1,2,3}
$(\mathcal{T},\mathcal{PT})$-int(G) = {1,2,3}
$(\mathcal{T},\mathcal{PT})$*-int(G)={1,2,3}
bi-int(G) = {1}

4-Open and closed function in bi-pre-supra topological space

In this section we introduce a new class of open and closed function in bi-pre-supra topological space .

Definition 4.1 A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is called

1- bi-$(\mathcal{T},\mathcal{PT})$-open function if the image of every $(\mathcal{T}_X,\mathcal{PT}_X)$-open set is $(\mathcal{T}_Y,\mathcal{P}-\mathcal{T}_Y)$-open .
2- bi-$(\mathcal{T},\mathcal{PT})$*-open function if the image of every $(\mathcal{T}_X,\mathcal{PT}_X)$*-open set is $(\mathcal{T}_Y,\mathcal{PT}_Y)$*-open .
3- bi-open function if the image of every bi-open set is bi-open

Definition 4.2 A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is called

1- bi-$(\mathcal{T},\mathcal{PT})$-closed function if the image of every $(\mathcal{T}_X,\mathcal{PT}_X)$-closed set is $(\mathcal{T}_Y,\mathcal{PT}_Y)$-closed .
2- bi-$(\mathcal{T},\mathcal{PT})$*-closed function if the image of every $(\mathcal{T}_X,\mathcal{PT}_X)$*-closed set is $(\mathcal{T}_Y,\mathcal{PT}_Y)$*-closed .
3- bi-closed function if the image of every bi- closed set is bi- closed

Example 4.3

X={1,2,3,4}
$\mathcal{T}_X$ = {∅ , X , {1} , {2} , {1,2}}
$\mathcal{T}_X^c$={∅,X,{2,3,4} ,{1,3,4} ,{3,4}}
$\mathcal{PT}_X$ = {∅ , X , {1} , {2} , {1,2} , {1,2,3} , {1,2,4}}
$(\mathcal{T}_X,\mathcal{PT}_X)$-open sets={∅,X ,{1} ,{2} ,{1,2} ,{1,2,3} ,{1,2,4}}
$(\mathcal{T}_X,\mathcal{PT}_X)$-closed sets={∅,X ,{2,3,4} ,{1,3,4} ,{3,4} ,{4} ,{3}}
$(\mathcal{T}_X,\mathcal{PT}_X)$*-open sets={∅,X ,{1,2,3} ,{1,2,4}}
$(\mathcal{T}_X,\mathcal{PT}_X)$*-closed sets={∅,X ,{4} ,{3}}

Y={a,b,c,d}
$\mathcal{T}_Y$ = {∅ , Y , {a} , {b} , {a,b} , {a,b,c}}
$\mathcal{T}_Y^c$={∅,Y ,{b,c,d} ,{a,c,d} ,{c,d} ,{d}}
$\mathcal{PT}_Y$ = {∅ , Y , {a} , {b} , {a,b} , {a,b,c} , {a,b,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-open sets={∅,Y ,{a} ,{a,b} ,{a,b,c} ,{a,b,d} ,{b}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-closed sets={∅,Y ,{b,c,d} ,{c,d} ,{d} ,{c} ,{a,c,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-open sets={∅,Y {a,b,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-closed sets={∅,Y {c}}
Let f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) defined by
f(1)=a f(2)=b f(3)=c f(4)=d . Then all types of function in def.[4.1],[4.2] are holding .

Diagram 4.4

The following diagram is valid

Example 4.5

X={1,2,3,4}
$\mathcal{T}_X$={∅,X,{1}}
$\mathcal{T}_X^c$={∅,X,{2,3,4}}
$\mathcal{PT}_X$={∅,X,{1} ,{1,2},{1,2,3},{1,3} ,{1,4} ,{1,2,4} ,{1,3,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)-open sets = { ∅ , X , {1} , {1,2} , {1,2,3} , {1,3} , {1,4} , {1,2,4} , {1,3,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)-closed sets={∅,X,{2,3,4} ,{3,4} ,{4} ,{2,4} ,{3} ,{2}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)*-open sets={∅,X,{1,3},{1,4} ,{1,2,3} ,{1,2,4} ,{1,3,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)*-closed sets={∅,X,{2,4} ,{2,3} ,{4} ,{3} ,{2}}
Y={a,b,c,d}
$\mathcal{T}_Y$={∅,X,{a},{a,c,d}}
$\mathcal{T}_Y^c$={∅,Y,{b,c,d} ,{b}}
$\mathcal{PT}_Y$={∅, Y, {a}, {a,b}, {a,c},{a,d}, {a,b,c} ,{a,b,d}, {a,c,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-open sets = {∅, Y, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d} ,{a,c,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-closed sets = {∅, Y ,{b,c,d} ,{c,d} ,{b,d} ,{b,c} ,{d} ,{c} ,{b}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-open sets = {∅,Y ,{a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-closed sets = {∅,Y ,{b,d}, {b,c}, {d}, {c}, {b}}
Let f:(Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) → (X, $\mathcal{T}_X$, $\mathcal{PT}_X$)defined by
f(a)=1 f(b)=2 f(c)=3 f(d)=4 . Then f is bi-($\mathcal{T}$,$\mathcal{PT}$)-open (closed) function not bi-open (closed) function

Example 4.6

X={1,2,3,4}
$\mathcal{T}_X$={∅,X,{1} ,{2} ,{1,2}}
$\mathcal{T}_X^c$={∅,X,{2,3,4} ,{1,3,4} ,{3,4}}

$\mathcal{PT}_X$={∅,X,{1},{2},{1,2},{4},{1,2,3},{1,2,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)-open sets={∅,X,{1},{1,2},{2},{1,2,3},{1,2,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)-closed sets={∅,X,{2,3,4} ,{ 3,4} ,{1,3,4} ,{4} ,{3}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)*-open sets={∅,X,{ 1,2,3},{1,2,4}}
($\mathcal{T}_X$, $\mathcal{PT}_X$)*-closed sets={∅,X,{4} ,{3} ,{4}}
Y={a,b,c,d}
$\mathcal{T}_Y$={∅,X,{a},{b},{a,b},{a,b,d}}
$\mathcal{T}_Y^c$={∅,Y,{b,c,d},{a,c,d},{c,d},{c}}
$\mathcal{PT}_Y$={∅, Y, {a},{b}, {a,b},{a,b,c} ,{a,b,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-open sets ={∅, Y, {a},{b}, {a,b},{a,b,c} ,{a,b,d}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)-closed sets = {∅, Y ,{b,c,d} ,{a,c,d} ,{c,d} ,{d} ,{c}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-open sets = {∅,Y ,{a,b,c}}
($\mathcal{T}_Y$, $\mathcal{PT}_Y$)*-closed sets = {∅,Y ,{d}}
Let f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) defined by
f(1)=a f(2)=b f(3)=c f(4)=d . Then f is bi-($\mathcal{T},\mathcal{PT}$)-open (closed) function not bi-$\mathcal{T},\mathcal{P}^* - \mathcal{T} -$ open (closed) function

Theorem 4.7

A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is bi-($\mathcal{T},\mathcal{PT}$)-open iff f(($\mathcal{T},\mathcal{PT}$)-int(A))⊆ ($\mathcal{T},\mathcal{PT}$)-int(f(A))for all A⊆X

Proof:

Let f bi-($\mathcal{T},\mathcal{PT}$)-open function and A⊆X
Since ($\mathcal{T},\mathcal{PT}$)-int(A) is($\mathcal{T},\mathcal{PT}$)-open set and f is bi-($\mathcal{T},\mathcal{PT}$)-open function then f(($\mathcal{T},\mathcal{PT}$)-int(A))is($\mathcal{T},\mathcal{PT}$)-open set subset of Y
Since ($\mathcal{T},\mathcal{PT}$)-int(A)⊆A then :
f(($\mathcal{T},\mathcal{PT}$)-int(A))⊆ ($\mathcal{T},\mathcal{PT}$)-int(f(A))

Conversely:

Suppose that the condition is true and A is ($\mathcal{T},\mathcal{PT}$)-open set subset of X
Now f(A)=f(($\mathcal{T},\mathcal{PT}$)-int(A))⊆ ($\mathcal{T},\mathcal{PT}$)-int(f(A))
i.e f(A)=($\mathcal{T},\mathcal{PT}$)-int(f(A))
then f(A) is ($\mathcal{T},\mathcal{PT}$)-open

Theorem 4.8

A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is bi-($\mathcal{T},\mathcal{PT}$)-closed iff ($\mathcal{T},\mathcal{PT}$)-cl(f(A))⊆ f(($\mathcal{T},\mathcal{PT}$)-cl(A))for all A⊆X .

Proof:

Let f bi-($\mathcal{T},\mathcal{PT}$)-closed function and A⊆X
Since ($\mathcal{T},\mathcal{PT}$)-cl(A) is ($\mathcal{T},\mathcal{PT}$)-closed set and f is bi-($\mathcal{T},\mathcal{PT}$)-closed function then
f(($\mathcal{T},\mathcal{PT}$)-cl(A))= ($\mathcal{T},\mathcal{PT}$)-cl(f(($\mathcal{T},\mathcal{PT}$)-cl (A))
But A⊆ ($\mathcal{T},\mathcal{PT}$)-cl(A)
This f(A)⊆f(($\mathcal{T},\mathcal{PT}$)-cl(A))
⟹ ($\mathcal{T},\mathcal{PT}$)-cl(f(A))⊆ ($\mathcal{T},\mathcal{PT}$)-cl(f(($\mathcal{T},\mathcal{PT}$)-cl (A))
⟹ ($\mathcal{T},\mathcal{PT}$)-cl(f(A))⊆ f(($\mathcal{T},\mathcal{PT}$)-cl(A))

Conversely:

If the condition is true and $A \subseteq X$ closed set

Then $(\mathcal{T},\mathcal{PT})$-cl(f(A)) $\subseteq$ f($(\mathcal{T},\mathcal{PT})$-cl(A))=f(A)

i.e $(\mathcal{T},\mathcal{PT})$-cl(f(A))= f(A)

Then f(A) is $(\mathcal{T},\mathcal{PT})$-closed set subset of Y .

5- Continuous function in bi-pre-supra topological space

In this section we introduce a new class of continuous function in bi-pre-supra topological space .

Definition 5.1 A function $f:(X, \mathcal{T}_X, \mathcal{PT}_X) \to (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ is called

1- bi-$(\mathcal{T},\mathcal{PT})$-continuous function if the inverse image of any $(\mathcal{T}_Y, \mathcal{PT}_Y)$-open set G is($\mathcal{T}_X, \mathcal{PT}_X$)-open set .

2- bi-$(\mathcal{T},\mathcal{PT})^*$-continuous function if the inverse image of any $(\mathcal{T}_Y, \mathcal{PT}_Y)^*$-open set G is($\mathcal{T}_X, \mathcal{PT}_X$)*-open set .

3- bi-continuous function if the inverse image of any bi-open set is bi-open .

Example 5.2 X={1,2,3,4}

$\mathcal{T}_X$ = {∅ , X , {1} , {2} , {1,2}}

$\mathcal{PT}_X$ = {∅ , X , {1} , {2} , {1,2} , {1,2,3} , {1,2,4}}

$(\mathcal{T}_X, \mathcal{PT}_X)$-open sets={∅,X ,{1} ,{2} ,{1,2} ,{1,2,3} ,{1,2,4}}

$(\mathcal{T}_X, \mathcal{PT}_X)^*$-open sets={∅,X ,{1,2,3} ,{1,2,4}}

Y={a,b,c,d}

$\mathcal{T}_Y$ = {∅ , Y , {a} , {b} , {a,b} , {a,b,c}}

$\mathcal{PT}_Y$ = {∅ , Y , {a} , {b} , {a,b} , {a,b,c} , {a,b,d}}

$(\mathcal{T}_Y, \mathcal{PT}_Y)$-opensets={∅,Y ,{a} ,{a,b} ,{a,b,c} ,{a,b,d} ,{b}}

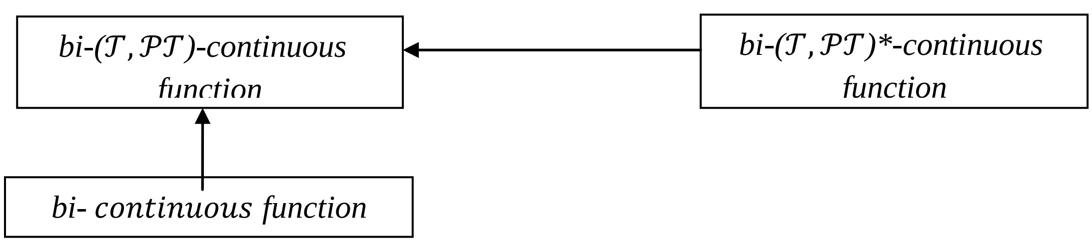
$(\mathcal{T}_Y, \mathcal{PT}_Y)^*$-opensets={∅,Y {a,b,d}}

Let $f:(X, \mathcal{T}_X, \mathcal{PT}_X) \to (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ defined by

f(1)=a f(2)=b f(3)=c f(4)=d . Then all types of function in def.[5.1] are holding

Diagram 5.3

The following diagram is valid



Example 5.4

X={1,2,3,4}

$\mathcal{T}_X$={∅,X,{1}}

$\mathcal{PT}_X$={∅,X,{1} ,{1,2},{1,2,3},{1,3} ,{1,4} ,{1,2,4} ,{1,3,4}}

$(\mathcal{T}_X, \mathcal{PT}_X)$-open sets={∅ ,X ,{1} ,{1,2} ,{1,2,3} ,{1,3} ,{1,4} ,{1,2,4} ,{1,3,4}}

$(\mathcal{T}_X, \mathcal{PT}_X)^*$-open sets={∅,X,{1,3},{1,4} ,{1,2,3} ,{1,2,4} ,{1,3,4}}

Y={a,b,c,d}

$\mathcal{T}_Y$={∅,X,{a},{a,c,d}}

$\mathcal{PT}_Y$={∅, Y, {a}, {a,b}, {a,c},{a,d}, {a,b,c} ,{a,b,d}, {a,c,d}}

$(\mathcal{T}_Y, \mathcal{PT}_Y)$-open sets = {∅, Y, {a}, {a,b}, {a,c}, {a,d}, {a,b,c}, {a,b,d} ,{a,c,d}}
$(\mathcal{T}_Y, \mathcal{PT}_Y)$*-open sets = {∅,Y ,{a,c}, {a,d}, {a,b,c}, {a,b,d}, {a,c,d}}
Let f:$(Y, \mathcal{T}_Y, \mathcal{PT}_Y) \rightarrow (X, \mathcal{T}_X, \mathcal{PT}_X)$defined by
f(a)=1 f(b)=2 f(c)=3 f(d)=4 . Then f is bi-$(\mathcal{T}, \mathcal{PT})$-continuous function not bi-continuous function

Example 5.5

X={1,2,3,4}
$\mathcal{T}_X$={∅,X,{1} ,{2} ,{1,2}}
$\mathcal{PT}_X$={∅,X,{1},{2},{1,2},{4},{1,2,3},{1,2,4}}
$(\mathcal{T}_X, \mathcal{PT}_X)$-open sets={∅,X,{1},{1,2},{2},{1,2,3},{1,2,4}}
$(\mathcal{T}_X, \mathcal{PT}_X)$*-open sets={∅,X,{ 1,2,3},{1,2,4}}
Y={a,b,c,d}
$\mathcal{T}_Y$={∅,X,{a},{b},{a,b},{a,b,d}}
$\mathcal{PT}_Y$={∅, Y, {a},{b}, {a,b},{a,b,c} ,{a,b,d}}
$(\mathcal{T}_Y, \mathcal{PT}_Y)$-open sets ={∅, Y, {a},{b}, {a,b},{a,b,c} ,{a,b,d}}
$(\mathcal{T}_Y, \mathcal{PT}_Y)$*-open sets = {∅,Y ,{a,b,c}}
Let f:$(X, \mathcal{T}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ defined by
f(1)=a f(2)=b f(3)=c f(4)=d . Then f is bi-$(\mathcal{T}, \mathcal{PT})$-continuous function not bi-$(\mathcal{T}, \mathcal{PT})$*-continuous function

Theorem 5.6 Let the function f:$(X, \mathcal{T}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ and g:$(Y, \mathcal{T}_Y, \mathcal{PT}_Y) \rightarrow (Z, \mathcal{T}_Z, \mathcal{PT}_Z)$ be bi-$(\mathcal{T}, \mathcal{PT})$-continuous. Then the composition function gof:$(X, \mathcal{T}_X, \mathcal{PT}_X) \rightarrow (Z, \mathcal{T}_Z, \mathcal{PT}_Z)$ is also bi-$(\mathcal{T}, \mathcal{PT})$-continuous .

Proof:

Let G be an $(\mathcal{T}, \mathcal{PT})$-open subset of Z .
Then $g^{-1}(G)$ is $(\mathcal{T}, \mathcal{PT})$-open in Y since g is continuous .
But f is also bi-$(\mathcal{T}, \mathcal{PT})$-continuous , so $f^{-1}[g^{-1}(G)]$ is $(\mathcal{T}, \mathcal{PT})$-open in X .
Now $(gof)^{-1}(G)= f^{-1}[g^{-1}(G)]$
Thus $(gof)^{-1}(G)$ is $(\mathcal{T}, \mathcal{PT})$-open in X for every $(\mathcal{T}, \mathcal{PT})$-open subset G of Z
gof is continuous .

Theorem 5.7

A function f:$(X, \mathcal{T}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ is bi-$(\mathcal{T}, \mathcal{PT})$-continuous iff the inverse image of every $(\mathcal{T}, \mathcal{PT})$-closed subset of Y is a $(\mathcal{T}, \mathcal{PT})$-closed subset of X .

Proof:

Suppose f:$(X, \mathcal{T}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ is bi-$(\mathcal{T}, \mathcal{PT})$-continuous . and let F be a $(\mathcal{T}, \mathcal{PT})$-closed subset of Y .
Then F^c is $(\mathcal{T}, \mathcal{PT})$-open , and so $f^{-1}(F^c)$ is $(\mathcal{T}, \mathcal{PT})$-open in X .
But $f^{-1}(F^c)= [f^{-1}(F)]^c$
Therefore $f^{-1}(F)$ is $(\mathcal{T}, \mathcal{PT})$-closed .

Conversely:

Assume F is $(\mathcal{T}, \mathcal{PT})$-closed in Y implies $f^{-1}(F)$ is $(\mathcal{T}, \mathcal{PT})$-closed in X.
Let G be an $(\mathcal{T}, \mathcal{PT})$-open subset of Y.
Then $G^c$is $(\mathcal{T}, \mathcal{PT})$-closed in Y , and so $f^{-1}(G^c)= [f^{-1}(G)]^c$ is $(\mathcal{T}, \mathcal{PT})$-closed in X .
Accordingly , $f^{-1}(G)$ is $(\mathcal{T}, \mathcal{PT})$-open and therefore f is bi-$(\mathcal{T}, \mathcal{PT})$-continuous .

Theorem 5.8

A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous iff for every subset G⊆X, f(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)).

Proof:

Suppose f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous

Now f(G)⊆ ($\mathcal{T}$,$\mathcal{PT}$)- (f(G)) , so

G ⊆ f^{-1}(f(G))⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)))

But ($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)) is ($\mathcal{T}$,$\mathcal{PT}$)-closed .

And so f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G))) is also ($\mathcal{T}$,$\mathcal{PT}$)-closed .

Hence G⊆($\mathcal{T}$,$\mathcal{PT}$)-cl(G)⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)))

And therefore f(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G))

($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G))=f(f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)))

Conversely:

Assume f(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl(f(G)) for any G⊆X , and let F be a ($\mathcal{T}$,$\mathcal{PT}$)-closed subset of Y .

Set G=f^{-1}(F) , i.e ($\mathcal{T}$,$\mathcal{PT}$)-cl(G)=G .

Now

f(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))=f(($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(F)))⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl(f(f^{-1}(F))) = ($\mathcal{T}$,$\mathcal{PT}$)-cl(F)=F

Hence ($\mathcal{T}$,$\mathcal{PT}$)-cl(G)⊆ f^{-1}(f(($\mathcal{T}$,$\mathcal{PT}$)-cl(G)))⊆ f^{-1}(F)=G

But G⊆($\mathcal{T}$,$\mathcal{PT}$)-cl(G)

So ($\mathcal{T}$,$\mathcal{PT}$)-cl(G)=G and f is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous function .

Theorem 5.9

A function f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→(Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous iff for every subset G⊆Y , ($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(G))⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G)).

Proof:

let f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$) →(Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) be bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous function . To prove that ($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(G))⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G)) for every subset G⊆X .

Since G⊆($\mathcal{T}$,$\mathcal{PT}$)-cl(G), Then

($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(G))⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))) …..(1)

($\mathcal{T}$,$\mathcal{PT}$)-cl(G) is ($\mathcal{T}$,$\mathcal{PT}$)-closed in Y, f is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous function

Implies ($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G)) is ($\mathcal{T}$,$\mathcal{PT}$)-closed in X .

Implies ($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))=f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))..(2)

From (1) and (2) we get ($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(G))⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G))

Conversely:

Suppose that f:(X, $\mathcal{T}_X$, $\mathcal{PT}_X$)→ (Y, $\mathcal{T}_Y$, $\mathcal{PT}_Y$) is a function such that ($\mathcal{T}$,$\mathcal{PT}$)-cl(f^{-1}(G))⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(G)) for every subset G⊆X .

To prove that f is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous function

Let F ⊆Y be an arbitrary ($\mathcal{T}$,$\mathcal{PT}$)-closed set then ($\mathcal{T}$,$\mathcal{PT}$)-cl(F)=F

By hypothesis

($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(F)) ⊆ f^{-1}(($\mathcal{T}$,$\mathcal{PT}$)-cl(F)) =f^{-1}(F) ….(3)

But f^{-1}(F)⊆ ($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(F)) for every F ….(4)

From (3) and (4) we get f^{-1}(F)=($\mathcal{T}$,$\mathcal{PT}$)-cl (f^{-1}(F))

Then f^{-1}(F) is ($\mathcal{T}$,$\mathcal{PT}$)-closed .

So by theorem 3.2.17

f is bi-($\mathcal{T}$,$\mathcal{PT}$)-continuous .

Theorem 5.10

A function $f:(X, \mathcal{T}_X, \mathcal{PT}_X)\to(Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ is bi-$(\mathcal{T},\mathcal{PT})$-continuous iff for every subset $G\subseteq X$, $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq (\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$.

Proof:

let $f:(X, \mathcal{T}_X, \mathcal{PT}_X)\to(Y, \mathcal{T}_Y, \mathcal{PT}_Y)$ be bi-$(\mathcal{T},\mathcal{PT})$-continuous function . To prove that $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq(\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$ for every subset $G\subseteq Y$.

$G\subseteq Y$ implies $(\mathcal{T},\mathcal{PT})\text{-int}(G)\subseteq (Y, \mathcal{T}_Y, \mathcal{PT}_Y)$

Implies $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq(X, \mathcal{T}_X, \mathcal{PT}_X)$ since f is bi-$(\mathcal{T},\mathcal{PT})$-continuous function

Implies$(\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G)))=f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))$..(1)

$(\mathcal{T},\mathcal{PT})\text{-int}(G)\subseteq G$ implies $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq f^{-1}(G)$

Implies $(\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G)))=f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G)) \subseteq (\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$

Implies $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq (\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$

Conversely:

Suppose that $f^{-1}((\mathcal{T},\mathcal{PT})\text{-int}(G))\subseteq (\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$..(2)

To prove that f bi-$(\mathcal{T},\mathcal{PT})$-continuous function .

Let G be an $(\mathcal{T},\mathcal{PT})$-open subset of Y and hence $(\mathcal{T},\mathcal{PT})\text{-int}(G)=G$.

If we show $f^{-1}(G)$ is $(\mathcal{T},\mathcal{PT})$-open in X , the result will follow.

$f^{-1}(G)=f^{-1}((\mathcal{T},\mathcal{PT})\text{-int }(G))\subseteq (\mathcal{T},\mathcal{PT})\text{- int}(f^{-1}(G))$ [by(2)]

Then $f^{-1}(G)\subseteq (\mathcal{T},\mathcal{PT})\text{- int}(f^{-1}(G))$ …..(3)

But $(\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))\subseteq f^{-1}(G)$ is always true …..(4)

From (3) and (4) we get that $f^{-1}(G)=(\mathcal{T},\mathcal{PT})\text{-int}(f^{-1}(G))$

So that $f^{-1}(G)$ is $(\mathcal{T},\mathcal{PT})$-open .

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