The Closed Form Solution of the Inverse *Kinematics of a 6-DOF Robot*

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Abstract:-

The kinematics of manipulator is a central problem in the automatic control of robot manipulators. Theoretical background of the analysis of the MA-2000 educational robot arm kinematics is presented in this paper. The revolute robot consists of six rotary joints (6-DOF) with base, shoulder, elbow, wrist pitch, wrist yaw and wrist roll. The kinematics problem is defined as transformation from the cartesian space to the joint space. The Denavit-Harbenterg (D-H) model of representation is used to model robot links and joints in this study. Both forward and inverse kinematics solutions for this robot are presented. Four sets of exact solution for the vector of the joint angles $\{\theta_i\}$ pertaining to the inverse kinematics problem of a MA-2000 robot with two different kinds of gripper configurations. An effective method is suggested to decrease multiple solutions in inverse kinematics.

Keyword: *Forward kinematics robot, inverse kinematics robot, closed form solution*

1.Introduction:

A serial link manipulator consists of a sequence of mechanical links connected together by actuated joints. Such a structure forms a kinematics chain and may be analyzed by methods developed by Denavite and Hartenberg (D-H)[1]. The results of this analysis are the matrix equations expressing manipulator end-effector Cartesian position and orientation in terms of the joint coordinates. These equations may be obtained for any manipulator independent of the number of links or degree of freedom.

The problem of the inverse kinematics for robot manipulators has always been a challenging problem in their design. Essentially, the problem is to find the vector of the joint angles, say for an n-axis revolute manipulator, given the position and orientation of the

end- effector or the gripper [2].
Piper and Roth[3], were the first to derive a set of solutions for a number of special robot manipulators. They also devoted a portion of their work to the iterative type algorithms to numerically solved the inverse kinematics problem. Lumelsky [4] presented iterative solutions for the inverse kinematics problem for one type of a robot manipulator. Paul and Mayer [5] presented the basis for all advanced manipulator control which is relationship between the Cartesian coordinates of the end-effector and manipulator joint coordinate. Dieh, Z. [6] proposed a closed form solution algorithm for solving the inverse position and orientation problem for the NASHI-750 robot by imposing a castrating condition to the problem and projection the tool frame on subspace of the robot. Duffy [7] further found some special geometric manipulators, which can solve the same problem analytically. Since then, many methods have been presented to solve the IK problem.

2.Coordinate frame:

A serial link manipulator consists of a sequence of links connected together by actuated joints. For n-degrees of freedom manipulator, there will be n-links and n-joints. The base of the manipulator is link "0" and is not considered one of the six links. Link "1" is connected to the base link by joint "1". There is not joint at the end of the final link. The only significance of links is that they maintain a fixed relationship between the manipulator joints at each end of the link. Any link can be characterized by two dimensions. The common normal distance "a_i" and" α_i "the angle between the axes in a plane perpendicular to a_i .It is customary to call a_i "the length" and α_i "the twist" of the link (see Fig.-1). Generally, two links are connected at each joint axis.
The axis will have two normal connected to it, one for each link. The relative position of two such connected links is given by d_i "the distance between the normal along the joint n-axis and θ_i the angle between the normal measured in a plane normal to the axis".[8]

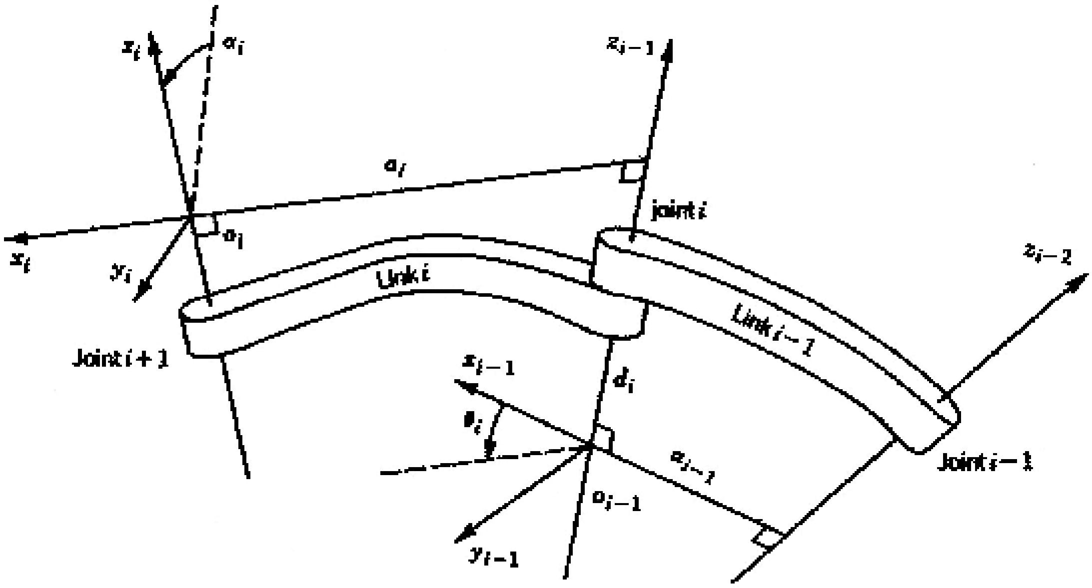


Fig. -1: Link parameter d, a, α and θ

The coordinates for the generalized link coordinate system are chosen as follows:

1. Numbering:

Number the joints from 1 to n starting with the base and ending with tool Yaw, Pitch and Roll in that order.

2- Base frame:

Assign a right-handed orthonormal coordinate frame O_0 to the robot base, making sure that $\hat{Z}_0$ aligns with the axis of joint 1. Set i=1

3- $\hat{Z}_i$ Axis

Align $\hat{Z}_i$ with the axis of joint i+1

4- O_i Origin:

If $\hat{Z}_i \cap \hat{Z}_{i-1}$ then

Locate O_i at the point of intersection.

Else

Locate O_i at the intersection of $\hat{Z}_i$ with a common normal between $\hat{Z}_i$ and $\hat{Z}_{i-1}$.

End if

5- $\hat{X}_i$ Axis:

If $\hat{Z}_i // \hat{Z}_{i-1}$ then

  Point $\hat{X}_i$ away from $\hat{Z}_{i-1}$

Else

  Establish $\hat{X}_i$ perpendicular on both $\hat{Z}_i$ and $\hat{Z}_{i-1}$.

End if

6- $\hat{Y}_i$ Axis:

Establish $\hat{Y}_i$ to from a right handed orthonormal coordinate frame.

7- Decision:

Set i=i+1

If $i < n$ then

  Go to step 3

Else

  Go to step 8

End if

8- Tool frame:

- Locate O_n at the tool tip.
- Align $\hat{Z}_n$ with the approach vector $\hat{a}$.
- Align $\hat{Y}_n$ with the sliding vector $\hat{s}$.
- Align $\hat{X}_n$ with the normal vector $\hat{n}$.

Stop

constitute a minimally $d_i, \alpha_i, \theta_i, a_i$ The four parameters ( ) sufficient set to determine the kinematics configuration of each link of the robotic arm. d_i, α_i, a_i

Note that for a plane revolute joint, generally ( ) are all constant while θ_i varies as link i rotate about the axis of joint i. on the other hand, for a prismatic joint (α_i, θ_i, a_i) are constant while Thus, d_i varies as link i slides along the axis of joint i. depend α_i and a_i for both cases are generally constant and on the design of the robot. Once the Denavit-Harbenterg (D-H) coordinate system for each link is established a homogenous transformation matrix can be easily developed relating the i-th coordinate frame to the (i-1)th coordinate frame. The following homogenous transformation matrices:

1- joint variable q_i:
set i=1

$$q_i = \zeta_i \theta_i + (1-\zeta_i) d_i$$

.....(1)

$$\zeta_i = \begin{cases} 1 & \text{joint i revolute} \\ 0 & \text{joint i prismatic} \end{cases}$$

2- HTM's of two adjacent frames:
From the HTM's between two adjacent link frames by substituting the link parameters from the link parameter table into:

$$^{i-1}T_i(q) = \begin{bmatrix} C_{\theta i} & -S_{\theta i}C_{\theta i} & S_{\theta i}S_{\alpha i} & a_i C_{\theta i} \\ S_{\theta i} & C_{\theta i}C_{\theta i} & -C_{\theta i}C_{\theta i} & a_i S_{\theta i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

.....(2)

3- Decision:
Set i=i+1
If i $\leq$ n then
Go to step 1
Else
Got to step 4
End if

4- Manipulator matrices:

The wrist matrix: $^{Base}T_{Wrist}(q) = {}^0T_3(q_3) = {}^0T_1(q_1)\,{}^1T_2(q_2)\,{}^2T_3(q_3)$

The hand matrix: $^{Wrist}T_{Tool}(q) = {}^3T_n(q) = {}^3T_4(q_4)\,{}^4T_5(q_5)\,.........\,{}^{n-1}T_n(q_n)$

The arm matrix: $^{Base}T_{Tool}(q) = {}^{Base}T_{Wrist}(q)\;{}^{Wrist}T_{Tool}(q\;) = {}^0T_n(q_n)$

...(3)

Stop

Kinematics equations:

The task of a robot usually specified in Cartesian space coordinate, ^0X_T (Cartesian position and orientation of the tool frame T relative to a fixed frame 0), the controllers require joint space coordinate θ, to control the motion of the robot. Therefore, coordinate transformation from one space to another is important. Such transformations are often referred to as forward (direct) and inverse kinematics. The forward kinematic problem transforms on one side, joint space coordinate θ, into task space coordinate, ${}^{0}X_{T}$, via nonlinear transforms, f, determine by the homogenous transformation matrices ${}^{0}T_{T}$,such that [9].

$$ {}^{0}X_{T} = f(\theta) \qquad ....(4)$$

$$ {}^{0}T_{T} = \begin{bmatrix} {}^{0}R_{T} & {}^{0}P_{T} \\ 0_{3}^{T} & 1 \end{bmatrix} \qquad ….(5)$$

Where 0_{0}^{T} is zero vector, ${}^{0}T_{T}$ is homogenous transformation matrix, ${}^{0}R_{T}$ and ${}^{0}P_{T}$ are the rotation matrix and position vector of frame T relative to frame 0, respectively.

The description of the end-effector, link coordinate frame 6, with respect to link coordinate frame n-1 is given by U_n:

$$ U_{n} = A_{n}\, A_{n+1} ....A_{6} \qquad …(6)$$

The position and orientation of the end-effector respect to base, known as T_6 is given by U_1:

$$ T_{6} = U_{1} = A_{1} * A_{2} * A_{3} * A_{4} * A_{5} * A_{6} = \begin{bmatrix} {}^{0}R_{6} & {}^{0}P_{6} \\ 0_{3}^{T} & 1 \end{bmatrix} \qquad …..(7)$$

For the first version of MA-2000 educational robot, we consider the gripper axis of motion to be co directional with the axis of wrist roll (Fig.-2). For the second version of this same robot we consider the gripper axis of motion to be perpendicular to the axis of wrist roll (Fig.-3).

A. First case:

Consider (Fig.-2) we can construct the following table for the joint parameter (Table- 1).

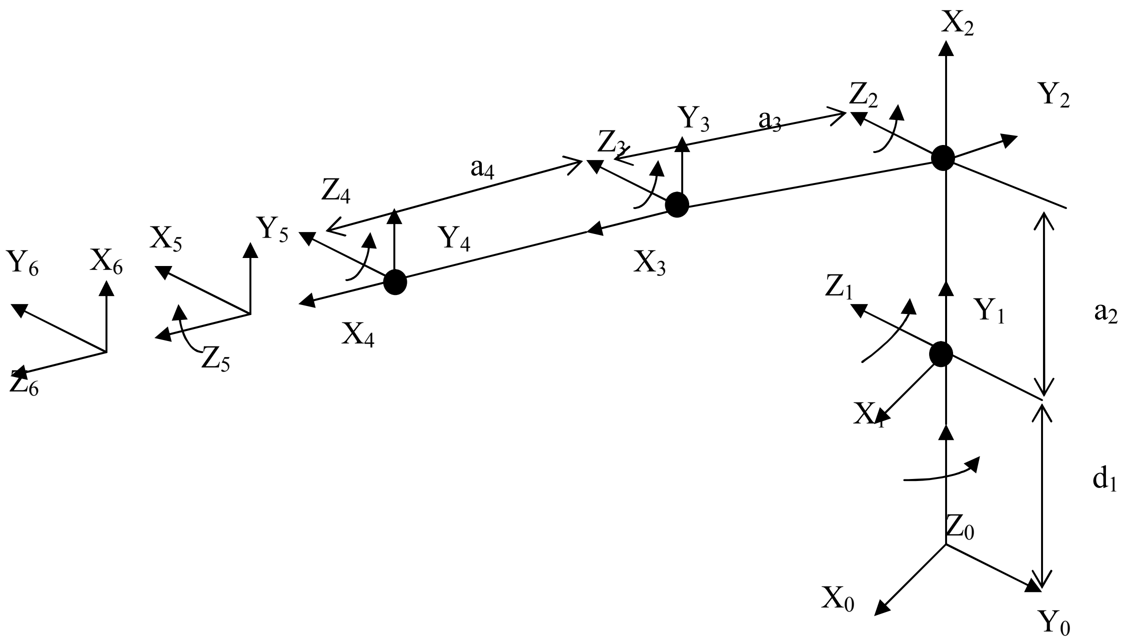


Fig-2: Link frame assignment for MA-2000 Robot with first version of gripper.

Joint	θ_i	α_i	a_i	d_i
1	θ_1	90	0	d_1
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	90	a_4	0
5	θ_5	90	0	0
6	θ_6	0	0	d_6

Table-1 : joint parameter for 6-DOF robot with first version

$$^{0}T_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots(8)$$

$$
{}^{1}T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
….(9)

$$
{}^{2}T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
….(10)

$$
{}^{3}T_4 = \begin{bmatrix} C_4 & 0 & S_4 & a_4C_4 \\ S_4 & 0 & -C_4 & a_4S_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
…(11)

$$
{}^{4}T_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
…(12)

$$
{}^{5}T_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$
…(13)

Multiplying the matrices T_1 through T_6 and setting the result equal to ${}^{0}T_6$ yields the following twelve equations for the determination of the vector of the joint angle.

$$n_x = C_1(C_{234}C_5C_6 - S_{234}S_6) - S_1S_5S_6$$
....(14)

$$n_Y = S_1(C_{234}C_5C_6 + S_{234}S_6) + S_1S_5S_6$$
…(15)

$$n_Z = -S_{234}C_5C_6 - C_{234}S_6$$
…(16)

$$o_x = -C_1(C_{234}C_5C_6 + S_{234}S_6) - S_1S_5S_6$$
…(17)

$$o_x = -S_1(C_{234}C_5C_6 + S_{234}S_6) + S_1S_5S_6$$

…(18)

$$o_Z = S_{234}C_5C_6 - C_{234}S_6$$

….(19)

$$a_x = C_1C_{234}S_5 - S_1C_5$$

…(20)

$$a_Y = S_1C_{234}S_5 + C_1C_5$$

…(21)

$$a_Z = S_{234}S_5$$

…(22)

$$p_x = C_1(d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}) - S_1C_5d_6$$

...(23)

$$p_y = S_1(d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}) + S_1C_5d_6$$

..(24)

$$p_Z = d_1 + d_6S_{234}S_5 - a_3S_{23} - a_2S_2 - a_4S_{234}$$

..(25)

Multiplying Eq.(24) by C_1 and Eq.(23) by S_1 and substituting the resulting equations, as the following is obtained:

$$C_1\ {}^0p_{y6} - S_1\ {}^0p_{x6} = d_6C_5$$

…(26)

Multiplying Eq.(20) by S_1 and Eq.(21) by C_1 and substituting the resulting equations, as the following is obtained:

$$C_5 = C_1a_y - S_1a_x$$

…(27)

From Eqs.(26 and 27) ,we can obtain:

$$C_1\ {}^0p_{y6} - S_1\ {}^0p_{x6} = d_6C_1a_y - d_6a_xS_1$$

.. (28)

Hence,

$$\theta_1 = a\tan 2((p_y - d_6a_y),(p_x - d_6a_x)$$

…(29)

From eq(27), hence,

$$S_5 = \pm\sqrt{1 - C_5^2}$$

…(30)

$$\theta_5 = a\tan 2(S_5, C_5)$$

…(31)

Now multiplying Eq.(20) by C_1 and Eq.(21) by S_1 and adding the resulting equations yields:

$$S_5C_{234} = -(a_xC_1 + a_yS_1)$$

…(32)

From Eq.(22) and Eq.(32) find that:

$\theta_{234} = a\tan 2(-a_z, (a_x C_1 + a_y S_1))$ for $\theta_5 > 0$

…(33)

and,

$\theta_{234} = \theta_{234} + \pi$ for $\theta_5 < 0$

…(34)

Multiplying Eq.(23) by C_1 and Eq.(24) by S_1 and adding the resulting equations yields:

$P_1 = C_1 p_{x6} + S_1 p_{y6} + d_6 S_5 C_{234} - a_4 C_{234}$

Where,

$P_1 = a_2 C_2 + a_3 C_{23}$

…(35)

Rearranging eq.(25) yields:

$P_2 = -p_{z6} - d_1 + d_6 S_5 S_{234} - a_4 S_{234}$

Where,

$P_2 = a_3 S_{23} - a_2 S_2$

…(36)

Thus, from eqs.(35) and (36) we find:

$a_3 C_{23} = P_1 - a_2 C_2$

…(37)

$a_3 S_{23} = P_2 - a_2 S_2$

…(38)

Hence from Eqs(37) and (38) we have:

$a_3^2 = (P_1 - a_2 C_2)^2 + (P_2 - a_2 S_2)^2$

…(39)

Which yield the following:

$$P_1 C_2 + P_2 S_2 = \frac{P_1^2 + P_2^2 + a_2^2 - a_3^2}{2a_2} = N$$

….(40)

Then it clearly follows that:

$\theta_2 = a\tan 2(P_2, P_1) + a\tan 2(\sqrt{P_1^2 + P_2^2 - N}, N)$

…(41)

Consequently from (37) and (38):

$\theta_{23} = a\tan 2(P_2 - a_2 S_2, P_1 - a_2 C_2)$

….(42)

$\theta_3 = \theta_{23} - \theta_2$

….(43)

and finally,

$\theta_4 = \theta_{234} - \theta_{23}$

….(44)

Multiplying Eq.(15) by C_1 and Eq.(14) by S_1 and adding the resulting equations yields:

$S_5C_6 = n_yC_1 - n_xS_1$

..(45)

Multiplying Eq.(17) by S_1 and Eq.(18) by S_1 and adding the resulting equations yields:

$S_5S_6 = (o_xS_1 - o_yC_1)$

…(46)

Hence,

$\theta_6 = a\tan 2((o_xS_1 - o_yC_1),(n_yC_1 - n_xS_1))$ for $\theta_5 > 0$

... (47)

$\theta_6 = \theta_6 + \pi$ for $\theta_5 < 0$

... (48)

These are the complete solutions for the inverse kinematics $\theta_5 \neq 0$ problem of a robot when (nondegenerate).

one of $\theta_5 = 0, \pm n\pi$ If S_5=0( ) the arm geometry becomes essentially a degenerate kind because θ_{234} becomes $\theta_{234} = k$ arbitrary based on Eq.(22). For the sake of calculation let us assume that

$\theta_1 = a\tan 2((p_y - d_6a_y),(p_x - d_6a_x)$

… (49)

$\theta_2 = a\tan 2(P_2, P_1) + a\tan 2(\sqrt{P_1^2 + P_2^2 - N}, N)$

… (50)

$\theta_{23} = a\tan 2(P_2 - a_2S_2, P_1 - a_2C_2)$

…(51)

$\theta_3 = \theta_{23} - \theta_2$

…(52)

$\theta_4 = k - \theta_{23}$

… (53)

$\theta_5 = 0$

… (54)

From Eqs.(16) and (19) and further yield:

$C_6 = n_zS_{234}C_5 - o_zC_{234}$

... (55)

$S_6 = o_zS_{234}C_5 - n_zC_{234}$

…(56)

$\theta_6 = a\tan 2(S_6, C_6)$

… (57)

These are complete solutions for one of the degenerate case of a robot with a gripper of the first case.

B. Second case:
Consider (Fig.-3) we can construct the following table for the joint parameter (Table-2).

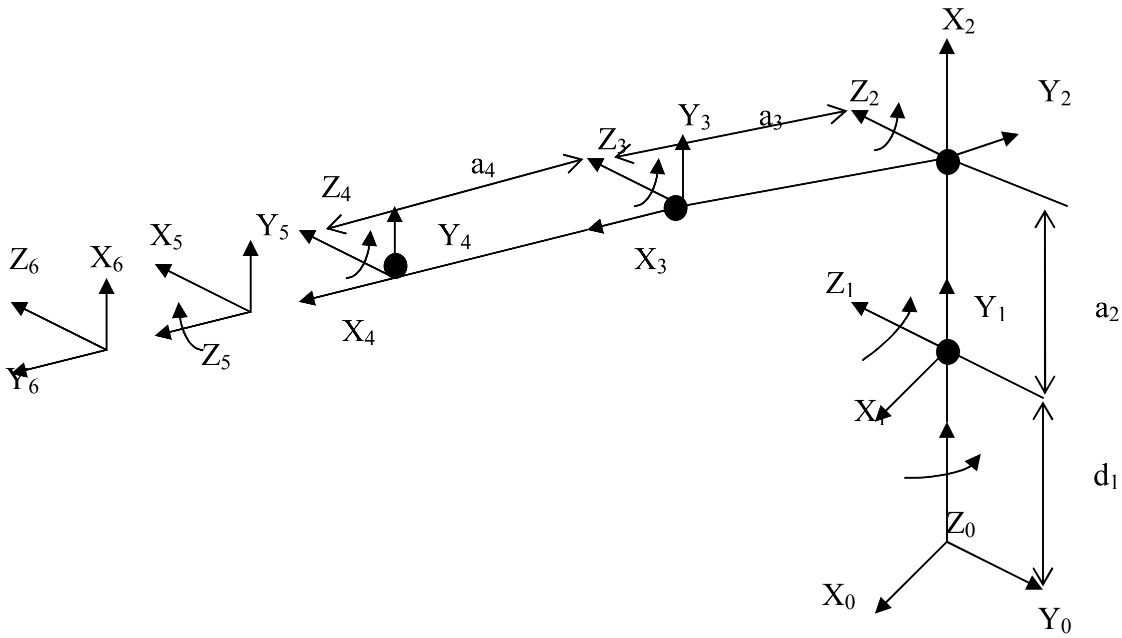


Fig-3: Link frame assignment for MA-2000 Robot with second version of gripper.

Joint	θ_i	α_i	a_i	d_i
1	θ_1	90	0	d_1
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	90	a_4	0
5	θ_5	90	0	0
6	θ_6	90	0	d_6

Table-2: joint parameter for 6-DOF robot with second version

Now bases on above table the homogenous transformation matrices will be the same as before expect for T_6 which becomes:

$$^{5}T_{6} = \begin{bmatrix} C_6 & 0 & S_6 & 0 \\ S_6 & 0 & -C_6 & 0 \\ 0 & 1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

… (59)

The homogenous transformations of the end-effector to the base can be multiplying the above six matrices and setting the results equal to $^{0}T_{6}$:

$$n_x = C_1(C_{234}C_5C_6 - S_{234}S_6) - S_1S_5S_6$$

....(60)

$$n_Y = S_1(C_{234}C_5C_6 - S_{234}S_6) + S_1S_5S_6$$

…(61)

$$n_Z = -S_{234}C_5C_6 - C_{234}S_6$$

…(62)

$$o_x = C_1C_{234}S_5 + S_1C_5$$

…(63)

$$o_y = S_1C_{234}S_5 - C_1C_5$$

…(64)

$$o_Z = S_{234}S_5$$

….(65)

$$a_x = -C_1(C_{234}C_5C_6 + S_{234}S_6) + S_1S_5S_6$$

…(66)

$$a_x = -C_1(C_{234}C_5C_6 + S_{234}S_6) - C_1S_5S_6$$

…(67)

$$a_Z = S_{234}C_5S_6 - C_{234}C_6$$

….(68)

$$p_x = C_1(d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}) - S_1C_5d_6$$

...(69)

$$p_y = S_1(d_6S_5C_{234} - a_4C_{234} - a_2C_2 - a_3C_{23}) + S_1C_5d_6$$

...(70)

$$p_Z = d_1 + d_6S_{234}S_5 - a_3S_{23} - a_2S_2 - a_4S_{234}$$

...(71)

Note that the structures of these equations are similar to the structure of previous (Eqs.(14) through (25)) except that the roles of o-vector and the roles of a-vector are exactly replaced respectively by.

$$\theta_1 = a\tan 2((p_y - d_6 a_y),(p_x - d_6 a_x)$$

… (72)

$$\theta_2 = a\tan 2(P_2, P_1) + a\tan 2(\sqrt{P_1^2 + P_2^2 - N}, N)$$

...(72)

$$\theta_{23} = a\tan 2(P_2 - a_2 S_2, P_1 - a_2 C_2)$$

…(73)

$$\theta_3 = \theta_{23} - \theta_2$$

…(74)

Multiplying eq.(63) by S_1 and eq.(64) by C_1,

$$C_5 = S_1 o_x - C_1 o_y$$

$$S_5 = \pm\sqrt{1 - C_5^2}$$

$$\theta_5 = a\tan 2(S_5, C_5)$$

…(75)

Now multiplying eq.(63)by C_1 and eq.(64)by S_1 and adding the resulting equations yield:

$$S_5 C_{234} = o_x C_1 + o_y S_1$$

…(76)

From eq.(65) and eq.(76) find that:

$$\theta_{234} = a\tan 2(o_z,(o_x C_1 + o_y S_1))$$ for $\theta_5 > 0$

…(77)

And,

$$\theta_{234} = \theta_{234} + \pi$$ for $\theta_5 < 0$

…(78)

$$\theta_4 = \theta_{234} - \theta_{23}$$

...(79)

Multiplying eq.(60) by C_1 and eq.(61) by S_1 and adding the resulting equations yield:

$$S_5 C_6 = n_y C_1 - n_x S_1$$

...(80)

Multiplying eq.(66) by S_1 and eq.(67) by C_1 and adding the resulting equations yield:

$$S_5 S_6 = a_x S_1 - a_y C_1$$

...(81)

Hence,

$$\theta_6 = a\tan 2(S_6, C_6)$$ for $\theta_5 > 0$

…(82)

And,

$$\theta_6 = \theta_6 + \pi$$ for $\theta_5 < 0$

…(83)

These are complete solutions for inverse kinematics problem of a degenerate case $\theta_5 = 0, \pm n\pi$ robot for $\theta_5 \neq 0$. For the we find that: when ( ) and assume ($\theta_{234} = k$),

$$\theta_1 = a\tan 2((p_y - d_6 a_y),(p_x - d_6 a_x) \quad ..(84)$$

$$\theta_2 = a\tan 2(P_2, P_1) + a\tan 2(\sqrt{P_1^2 + P_2^2 - N}, N) \quad ..(85)$$

$$\theta_{23} = a\tan 2(P_2 - a_2 S_2, P_1 - a_2 C_2) \quad ..(86)$$

$$\theta_3 = \theta_{23} - \theta_2 \quad ..(87)$$

$$\theta_4 = k - \theta_{23} \quad ..(88)$$

$$\theta_5 = 0 \quad ..(89)$$

From eqs.(62) and (68) and further yield:

$$C_6 = -n_z S_{234} C_5 - a_z C_{234} \quad .. (90)$$

$$S_6 = a_z S_{234} C_5 - n_z C_{234} \quad ..(91)$$

$$\theta_6 = a\tan 2(S_6, C_6) \quad ..(92)$$

These are complete solutions for one of the degenerate case of a robot with a gripper of the second case.

Results and discussion:

An analysis technique was introduced to reduce the multiple solutions in inverse kinematics. Forward and inverse kinematics solution are generated and implemented by program (Fig.4). For the numerical confirmation of the exact solutions the following procedure has been adopted. First the values of the structure of a robot a_2, a_3, a_4, d_1, d_6 *and* α_i are specified (Table-3). Then a set of θ_i's are chosen to calculate position and orientation of the end effector by forward kinematics. The values of the attitude matrix are then employed to calculate the joint angles by inverse kinematics. The inverse kinematics problem is solved by closed form solution because the kinematic equations in general have multiple solutions; having closed form solution allows one to develop rules for choosing a particular solution among several. The results obtained for the components of the vector of the joint angles are compared with the chosen values of θ_i's. (Table4 and 5). They are in fairly excellent agreement with the chosen values for θ_i's.

Joint	α_i	a_i	d_i
1	90	0	15
2	0	15	0
3	0	15	0
4	90	10	0
5	90	0	0
6	0	0	15

Table-3: D-H parameters of MA-2000 Robot arm

Joint	θ_i Chosen	θ_i Calculated
1	25	25
2	45	44.99998
3	30	30.001
4	40	39.998
5	20	19.999
6	30	30.0003

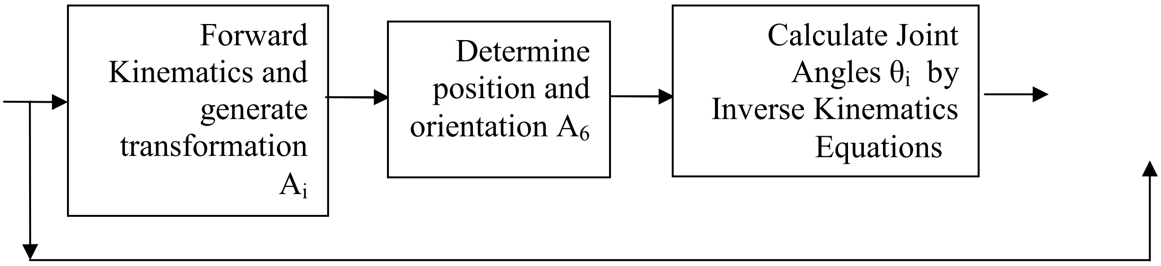
Table-4: the results for the non degenerate case1

Joint	θ_i Chosen	θ_i Calculated
1	25	25
2	45	45.002
3	30	29.997
4	40	40
5	0	0
6	30	29.897

Table-5: the results for the degenerate case1

Joint Angles θ_i

Compare joint angles



Fig(4): Block diagram for the numerical confirmation of the exact solution for MA-2000 Robot

Conclusions:

Two different gripper configurations were used to find exact solution of the joint angles. Two sets pertaining to the degenerate cases were nonunique and the other two sets pertaining to the nondegenerate cases were unique for each gripper configurations.
The numerical calculation by forward kinematics showed that the calculated exact solutions were in prefect agreement with the chosen vector of the joint angle for each gripper configuration.

References:

1. Jing-Shan Zhao, Kai Zhou, Zhi-Jing Feng, A theory of degrees of freedom for mechanisms, Mechanism and Machine Theory 39 (6) (2004) 621–643.

2. Coiffet, P., Chirouze, M.: An Introduction to Robot Technology. Kogan Page, London, 1983.

3. Piper,D.L. and Roth,B.” the kinematics of manipulator under computer control” proc.2nd Int.cong.on the theory of mach.and mech. Vol.2,1969.

4. Lumelsky,V.J.”Iterative procedure for inverse coordinate transformation for one class of robots”. Report general electric company Feb.1983.

5.Paul P.R. and Mayer G.”kinematics control equations for simple manipulator” IEEE Trans. On systems Man. And Cyberntics VolSNC-11 No.6 June 1995.

6.Dieh Z.” Simulation and off-programming of a 5-DOF welding robot” M.Sc. Thesis, Baghdad University Dept. of Mech.Eng.1998.

7. Duffy, J., 1980. Analysis of Mechanism and Robot Manipulator. Wiley, NY, USA.

8. Craid J.J “Introduction to robotics: mechanics and control” Ad ison-Wesely.1989.

9.Baki Koyuncu “Application of 6 axes robot arm by using inverse kinematics equations” Journal of computer engineering Vol.1,No.1,2007.