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Some weak hereditary properties

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ABSTRACT: called f-normal space, which we studied and identified some of its properties as well as relationships with other sets, and we obtained some results that show the relationship between sets using theories obtained using the set from Style (f-open)

Keywords: Normal space and Og-normal space and f-normal space and ff- normal space and f-fg-normal space

1. INTRODUCTION

In this chapter we are going to study other features for normal space:

Og-normal space and f-normal space and ff-normal space and f-fg-normal space.

As we know before in general way that said about a topological feature is hereditary, if and only if achieved for each subspace from a space had done. And said about a topological feature is weak hereditary if and only if achieved for each close subspace from a space had done [1].

Now in particular, we asked the following question:-

Let X be a topological space, and possesses any of the normal traits above and Y was subset from X, does sub space Y have the same feature that X had ?

That is what we are going to justify throughout our study for features of the subset which clarified for each kinds of normal space as state above.

A the beginning, we mention the following theorem which justify if X was normal space and Y was closed subset if X then subspace Y is normal space.

2. PRELIMINARIES

Definition 2.1

1. Assume that X is a topological space and $A \subseteq X$. The letter $\overline{A}$ denotes the closure of A is defined by :- $\overline{A} = \cap\{F \subseteq X; F$ is closed set and $A \subseteq F\}$

2. Let X is a topological space and $A \subseteq X$. The letter A° denotes the interior of A is defined by:- $A^\circ = \cup\{G \subseteq X : G \textit{ is open set and } G \subseteq A\}$.

3. A subset A of a topological space X, is called semi-open (s-open set if there exists an open set O such that $O \subseteq A \subseteq \overline{O}$.

4. A subset A of a topological space X is called semi-closed (s-closed set if there exists a closed set O such that $O^\circ \subseteq A \subseteq O$

5. A subset A of a topological space X is said to be feebly open set if there exists an open set U in X, such that $U \subseteq A \subseteq \overline{U}^s$, and the complement of feebly open set is called feebly closed set.

6. Let X a topological space, X is said to be f-normal space for each two disjoint closed set A and B in X there exists are two disjoint f-open set , U and V in X such that $A \subseteq U$, $B \subseteq V$

Theorem 2.2 [2]

Let X be normal space and Y closed subset in X, then subspace Y is normal space.

Remark 2.3

Look at [3] which justify in the example if X is normal space and Y is sub set in X, is not necessary sub space Y normal space. Which can be said here the description of normality is not genetic description, in another word its weak genetic description.

Remark 2.4

If $X$is og-normal space and Y is subset from X, then subspace Y is not necessary og-normal space, as showed that in the following example, if Y is g-closed subset in X then subspace Y be og-normal space.

Example 2.5

Let $X = \{a, b, c, d\}$ and

$T_x = \{x, \emptyset, (a, c\}, (a, d\}, (a, c, d\}, (a\}\}$ a topological space in X, and let

$Y = (a, c, d\} \subset X$

And $T_Y = \{\emptyset, Y, (a, c\}, (a, d\}, \{a\}$ a topological space in Y

To proof X is og-normal space g-closed set in $X = (X, \emptyset, (b\}, (a, b\}, (b, c\}, (b, d\}, (a, b, c\}, (a, b, d\}, a, c, d\}, \{b, c, d\}\}$.

Let $A = X$ and $B = \emptyset \Longrightarrow A \cap B = \emptyset$

Let $U = X$ *and* $V = \emptyset \Longrightarrow U \cap V = \emptyset$

Thus $A \subseteq U$ *and* $B \subseteq V$

Hence X is og-normal space

To proof Y is og-normal space

g-closed set in $Y = \{\emptyset, Y, (d\}, (c\}, (c, d\}\}$

Let $A = \{d\}$ and $B = (c\} \Longrightarrow A \cap B = \emptyset$ (g-closed set in Y)

There is not exists two disjoint open set counting {d} and {c}, in T_Y.

Hence Y is not og-normal space

And now we are introducing the following lemma which we need it to proof the coming theorem.

Lemma 2.6

Let X be a topological space, if $A \subseteq Y \subseteq X$ was and A g-closed set in Y was, and Y g-closed set in X, then A is going to be g-closed set in X

Proof : Look [4].

We are going to introduce the following theorem which justify if X og-normal space was, and Y g-closed subset was in X then subspace Y is going to be og-normal space:-

Theorem 2.7

Let X be og-normal space and Y is g-closed subset in X, then subspace Y is og-normal space.

Proof:

Let X is og-normal space,

And let A *and* B are two g-closed set in Y such that $A \cap B = \emptyset$

Hence, by lemma (3.2.5)

A *and* B a two disjoint g-closed set in X

Since X is og-normal space

Hence, there exists two disjoint open set U *and* V in X

Such that $A \subseteq U$ *and* $B \subseteq V$

Let $U_1 = Y \cap U$ and $V_1 = Y \cap V$

Hence U_1 *and* V_1 are two disjoint open set in Y.(relative topology).

Such that $A \subseteq U_1$ and $B \subseteq V_1$

Hence, a subspace Y is og-normal space.

Throughout the theorem (3.2.6) we can get the following corollary:-

Corollary 2.8

Let X be og-normal space and Y is closed subset in X then subspace Y is og-normal space.

Remark 2.9

Let X be f-normal space and Y was subset in X, then subspace is not necessary f-normal space, as justify in the following example (3.2.9). and if Y was closed and open set at the same time (clopen) in X, then subspace Y be f-normal space. We will clarify that in a later theorem

Example 2.10

Let $X = \{a, b, c, d\}$ and
$T_x = \{x, \emptyset, (a\}, (b\}, (a, b\}, (a, b, d\}, (a, b, c\}\}$ a topological space in X.
See (2.4)
Hence X is f-normal space,
Now teak $Y = \{a, c, d\} \subset X$
$T_Y = \{Y, \emptyset, (a\}, (a, d\}, (a, c\}\}$ a topological space in Y
To proof Y is f-normal space.
f-open set in Y=$\{Y, \emptyset, (a\}, (a, c\}, (a, d\}\}$
now clearly A *and* B are two closed set disjoint
let $A = \{c\}$ and $B = \{d\}$ there is not exists two disjoint f-open set counting {c} and {d}, in T_Y.
Hence, subspace Y is not f-normal space.

The following theorem justify if X is f-normal space and Y is subset closed and open set (clopen) at the same time in X then subspace Y is f-normal space.

Theorem 2.11

Let X be f-normal space and Y is closed and open subset at the same time in X, then subspace Y is f-normal space according to that subspace Y is f-normal space.

Let X is f-normal space And let A and B are two closed set in Y such that $A \cap B = \emptyset$
Hence, A and B are two disjoint closed set in X, by [2].
Since X is f-normal space
Thus there exists two disjoint f-open set U and V in X
Such that $A \subseteq U$ and $B \subseteq V$
And let $U_1 = Y \cap U$ and $V_1 = Y \cap V$
Thus, by proposition (1.1.12).
U_1 and V_1 are two disjoint f-open set in X.
Such that $A \subseteq U_1$ and $B \subseteq V_1$
According to that subspace Y is f-normal space.

Remark 2.12

ff-normal space has the same feature as f-normal space, if Y is was any subset from X then subspace Y does not ff-normal space. If Y was closed and open subset at the same time in X, then subspace Y is ff-normal space.

That we going to justify in a later theorem.

Example 2.13

From example(2.19) paragraph (2)
Clearly, X is ff-normal space
Now, teak $Y = (a, c, d\} \subset X$
$T_Y = \{Y, \emptyset, (a\}, (a, d\}, (a, c\}\}$ a topological space in Y f-open set in $Y = \{Y, \emptyset, (a\}, (a, d\}, (a, c\}\}$ f-closed set in $Y = \{\emptyset, Y, (c, d\}, (c\}, (d\}\}$

Now notice that $A = (c\}$ and $B = \{d\}$ are two disjoint f-closed set in Y there is not exists two disjoint f-open set counting {c} and {d}, both of them in a row

According to that subspace Y is not ff-normal space.

Now, we are introducing the following lemma which we need it to proof in a later theorem.

Lemma 2.14

Let X be a topological space, if $A \subseteq Y \subseteq X$ and A was f-closed set in Y and Y is closed and open set at the same time in X then A is g-closed set in X.

Proof:

Assume that Y is closed and open(clopen) subset at the same time in X,
Let A f-closed set in Y
To proof A is f-closed set in X.
Since A is f-closed set in Y
Hence, there exist B closed set in Y
Such that $\left(B^{\circ}\right)_Y \subseteq A \subseteq B$
Since $B^{\circ} = \left(B^{\circ}\right)_Y \cap Y^{\circ}$ by [5].
Therefore $B^{\circ} = \left(B^{\circ}\right)_Y \cap Y$ (Y is open set in X),
Thus $B^{\circ} = \left(B^{\circ}\right)_Y$
Since B is closed set in Y and Y is closed set in X,
Hence B is closed set in X. By [2]

Then $B^\circ \subseteq A \subseteq B$

Hence A is f-closed set in X........

The following theorem justify that if X was ff-normal space and Y was open and close subset at the same time in X then subspace Y be ff-normal space.

Theorem 2.15

Let X ff-normal space and Y was open and close subset at the same time in X then subspace Y is ff-normal space.

Proof:

Assume that X is ff-normal space

Let it be A *and* B are two f-closed set in Y. such that $A \cap B = \emptyset$

Hence, by lemma (2.14).

A *and* B are two disjoint f-closed set in X

Since X is ff-normal space

Hence there exists are two disjoint f-open set U *and* V in X

Such that $A \subseteq U$ *and* $B \subseteq V$

Let $U_1 = Y \cap U$ and $V_1 = Y \cap V$

Thus, by proposition (1.1.12)

U_1 *and* V_1 are two disjoint f-open set in Y

Such that $A \subseteq U_1$ *and* $B \subseteq V_1$

Then, subspace Y is ff-normal space ....

Remark 2.16

Let X be f-fg-normal space and Y is subset from X then subspace Y is not necessary be f-fg-normal space at state in coming example (2.17).

But if Y is fg-open and close set at the same time in X then subspace Y is f-fg-normal space. And we will justify that in a later theorem.

Example 2.17

From example (2.39) paragraph (2).

Clear X is f-fg-normal space

Now teak $Y = \{a, c, d\} \subset X$

It is also clear that Y subspace is not f-fg-normal space.

Because $\{c\}$ and $\{d\}$ are two disjoint fg-closed set,

But, is not there exists are two disjoint f-open set counting {c} and {d}, both of them in a row. And now we are introducing lemma which we need it to prove the proof of the coming theorem.

Lemma 2.18

Let X be a topological space, and let $B \subseteq Y \subseteq X$ such that fg-closed and open set at the same time in X then B is fg-closed set in Y if and only if B is fg-closed set in X.

Proof:

See [6] .

We are introducing the following theorem which justify if X is fg-normal space and Y is fg-closed and open set in X then sub space Y is f-fg-normal space .

Theorem 2.19

let X be f-fg-normal space and Y was fg-closed and open set at the same time in X, then subspace Y is f-fg-normal space.

Proof:

Assume that X is f-fg-normal space

Let A *and* B are two fg-closed set in Y,

Such that $A \cap B = \emptyset$

Thus, by lemma (2.18)

A *and* B are two disjoint fg-closed set in X

Since X is f-fg-normal space

Hence there exists are two disjoint f-open set U *and* V in X.

Such that $A \subseteq U$ *and* $B \subseteq V$

Let $U_1 = Y \cap U$ and $V_1 = Y \cap V$

Thus by proposition (1.1.12)

U_1 *and* V_1 are two disjoint f-open set in Y

Such that $A \subseteq U_1$ *and* $B \subseteq V_1$

Then subspace Y is f-fg-normal space

Through theorem (2.19), the following results can be obtained

3. CONCLUSION

1. Let X be f-fg-normal space and Y is f-closed and open subset at the same time then subspace Y is f-fg-normal space.
2. Let X be f-fg-normal space and Y is closed and open subset at the same time in X then subspace Y is f-fg-normal space.

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