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Concepts Of Bi-supra Topological Space Via graph Theory

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ABSTRACT: Definition of bi-supra topological space via graph theory was introduced in this study. We also studied some concepts related of bi supra-topological space via graph theory. "At least many theorems were proofed as a characterization and some examples introduced to explain the subject".

Keywords: subgraph, closure, interior, boundary, exterior

1. INTRODUCTION

Toplogy is a branch of pure mathematics [1]. In 2019 Gufran Ali [2] we introduced concept bi-supra topological space. In A graph G is defined as a non-empty set of elements called "vertices" and we symbolize it sometimes by with the family of unordered pairs of vertices set and each element of is called "edge" and we symbolize it sometimes by [3]. In 2020 Aiad and Atef [4, 5] and Abdu [6] and Mahdi [7] the link between graph theory and topological space as the definition topological graph. In this paper new definition bi-supra topological graph with concept of bi-supra topological by graphing concept.

2. PRELIMINARIES

Definition 2.1 [2]: suppose X be a non-blank set. "Let $\mathcal{ST}$ be the set of all semi open subset of X (for short $\mathcal{S}o\ x$ [8] and Let $\mathcal{PT}$ be the set of all pre-open subset of X (for short $\mathcal{P}$o(x)) [9], then we say that (X,$\mathcal{ST}$, $\mathcal{PT}$) is a bi-supra topological space". after both of (X, $\mathcal{ST}$) and (X, $\mathcal{PT}$) *are supra topological space*".
Definition 2.2: "A subset A of a topological space (X,τ) is called.

a) a pre-open set if [9] A int(cl(A)) and a pre-closed set ifcl(int(A)) ⊆ A;

b) a semi-open set [8] if A cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A"

Definition 2.3 [3]: "Let $G\,(V, E)$ be a graph, we call H is a subgraph from

G If $V\ (H) \subseteq V\ (G)$, $E\ (H) \subseteq E\ (G)$, in this case we would write $H \subseteq$. The spanning subgraph from a graph G is a subgraph acquired by edge deletions only. A deduced subgraph of a graph G is a subgraph acquired by vertex deletions along with the incident edges".

Definition 2.4 [4]: "Let $G(V\ ,\ E)$ be a graph, $v \in\ V\,(G)$ then we define the post stage vR is the set of all vertices which is not neighborhood of v. S_G is the collection of (vR) is called subbasis of graph. $B_G\ =\ \bigcap_{i=1}^{n} S_{G_i}$is called bases of graph. Then the union of B_G is form a topology on G and $(V\,(G),\ \ \tau_G)$ is called topological graph".

Definition 2.5 [4]: "Let $G(V\ ,\ \ E)$ be a graph, H be a subgraph from G.

Then the graph closure of $V(H)$ has the shape":

$$Cl_G(V(H)) = V(H) \cup (v \in V(G) : vR \cap V(H) \neq \varnothing\}.$$

Definition 2.6 [4]:"Let $G = (V, E)$ be a graph, H be a subgraph from G". Then the graph internal of $V(H)$ had the shape: $Int_G(V(H)) = \{v \in V(G) : vR \subseteq V(H)\}$.

Definition 2.7 [10]: "Let $G(V, E)$ be a graph that contains a topological graph $(V(G), \tau_G)$, H be a subgraph of G is called open subgraph if $Int_G(V(H)) = V(H)$. It is called closed subgraph if its complement is open subgraph".

Definition 2.8 [10]:"Let $G(V, E)$ be a graph which contain a topological graph $(V(G), \tau_G)$, H be a subgraph of G was named semi-open subgraph if $V(H) \subseteq Cl_G(Int_G(V(H)))$." "The family of all semi-open subgraph from G will be denoted by $SO(V(G))$. The complement of a semi-open subgraph is called a semi-closed subgraph and the family of all semi- closed subgraph from G will be denoted by $SF(V(G))$".

Definition 2.10 [10]: "Let $G(V, E)$ be a graph that contains a topological graph $(V(G), \tau_G)$, H be a subgraph from $G$is called preopen subgraph if $V(H) \subseteq Int_G(Cl_G(V(H)))$. The family of all preopen subgraph from G will be denoted by $PO(V(G))$. The complement of a preopen subgraph is called pre-closed subgraph and the family of all pre-closed subgraph from G will be denoted" by $PF(V(G))$.

3. CONSTRUCT OF BI-SUPRA TOPOLOGICAL VIA GRAPH

Definition 3.1: suppose $G(V, E)$ be a graph. Let ST_G be the set of all open subgraph subset of G and let PT_Gbe the set of all open subgraph subset of G. Then we say that $(V(G), ST_G, PT_G)$ is a bi-supra topological graph. When each of $(V(G), ST_G)$ and $(V(G), PT_G)$ are supra a topological graph.

Example 3.2: We construct a topological space for G then

$v_1R = (v_3\}, v_2R = (v_3, v_4\}, v_3R = (v_1, v_2\}, v_4R = \{v_2\}$.

Then a topology subbase was

$S_G = \{(v_3\}, (v_3, v_4\}, (v_1, v_2\}, \{v_2\}\}$.

The base is

$\beta_G = \{V(G), \emptyset, (v_2\}, (v_3\}, (v_3, v_4\}, (v_1, v_2\}\}$.

Hence, the topological graph on G is

$$\tau_G = \{V(G), \emptyset, (v_2\}, (v_3\}, (v_2, v_3\}, (v_1, v_2, v_3\}, (v_1, v_2\}\}$$

$, (v_2, v_3, v_4\}, (v_3, v_4\}\}$.

"$\tau_G^c = \{\emptyset, V(G), (v_1, v_3, v_4\}, (v_1, v_2, v_4\}, (v_1, v_4\}, (v_1\}, (v_4\}, (v_1, v_2\},$
$(v_3, v_4\}\}$.

$$ST_G = \{V(G), \varnothing, (v_2\}, (v_3\}, (v_1, v_2\}, (v_1, v_3\}, (v_2, v_3\}, (v_2, v_4\}, (v_3, v_4\},$$

$(v_1, v_2, v_3\}, (v_1, v_2, v_4\}, (v_1, v_3, v_4\}, \{v_2, v_3, v_4\}$.

$$PT_G = \{V(G), \varnothing, \{v_1\}, \{v_2\}, (v_3\}, (v_4\}, (v_1, v_2\}, (v_1, v_3\}, (v_1, v_3\}, (v_2, v_3\},$$

$(v_2, v_4\}, (v_3, v_4\}, (v_1, v_2, v_3\}, (v_1, v_2, v_4\}, (v_1, v_3, v_4\}, \{v_2, v_3, v_4\}$".

Hence $(V(G), ST_G, PT_G)$ is bi-supra topological graph.

Definition 3.3: Let $(V(G), ST_G, PT_G)$ be a bi-supra topological graph and suppose $V(H)$ be a subgraph of $V(G)$. So $V(H)$ was supposed to be:

1- "(ST_G, PT_G)-supra open subgraph if $V(H) = V(K) \cup V(L)$ *where* $V(K) \in ST_G$ *and* $V(L) \in PT_G$. The complement of (ST_G, PT_G)-supra open subgraph is called (ST_G, PT_G)-supra closed subgraph.

2-$(ST_G, PT_G)^*$- supra open subgraph if $V(H) = V(K) \cup V(L)$ *where* $V(K) \in ST_G$ *and*$V(L) \in PT_G$ such that $V(L) \notin ST_G$ *or* $V(K) \in PT_G$, $V(L) \in ST_G$ *such that* $V(K) \notin PT_G$.

The complement of $(ST_G, PT_G)^*$-supra open subgraph is called $(ST_G, PT_G)^*$-supra closed subgraph.

3- bi- supra open subgraph if $V(G) = V(K)$ where, $V(K) \in \tau_G$. The complement of bi-supra open subgraph is called bi-closed subgraph".

Proposition 3.4 :

1- "Every bi-supra open subgraph is (ST_G, PT_G)-supra open subgraph and every bi-supra closed subgraph is (ST_G, PT_G)-supra closed subgraph but the convers is not true.

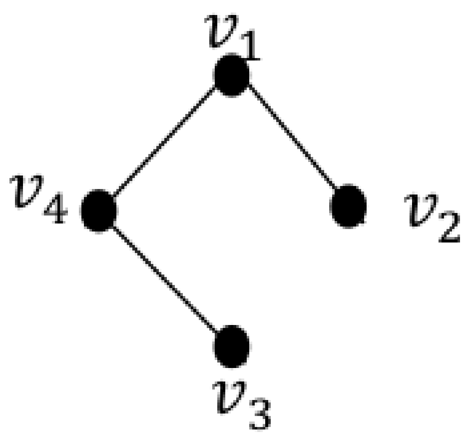

FIGURE 1. Simple graph

2- Every $(ST_G,\ PT)^*$-supra open subgraph is $(ST_G,\ PT_G)$-supra open graph and Every $(ST_G,\ PT)^*$-supra closed subgraph is $(ST_G,\ PT_G)$-supra closed graph but the converse is not true.

3- The $(ST_G,\ PT_G)^*$-supra open subgraph, bi-supra open subgraph are independent and The $(ST_G,\ PT_G)^*$-supra closed subgraph, bi-supra closed subgraph are independent".

Remark 3.5: The set of all $(ST_G,\ PT_G)$[res. $(ST_G,\ PT_G)^*$, bi]-supra open subgraph and $(ST_G,\ PT_G)$[res. $(ST_G,\ PT_G)^*$, bi]-supra closed subgraph was require not essentially form a topological graph it was a supra topological graph.

Example 3.6: From Example 3.2,

"$\tau_G = \{V\ (G)\ ,\ \varnothing\ ,(v_2\}\ ,(v_3\}\ ,(v_2, v_3\}\ ,(v_1\ ,v_2, v_3\}\ ,(v_1\ ,v_2\}\}$

$$ST_G = \{V(G), \varnothing, (v_2\},\ (v_3\}, (v_1, v_2\}, (v_1, v_3\}, (v_2, v_3\}, (v_2, v_4\}, (v_3, v_4\},$$

$(v_1, v_2, v_3\}, (v_1, v_2, v_4\}, (v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$.

$$PT_G = \{V(G), \varnothing, \{v_1\}, \{v_2\}, (v_3\},\ (v_4\}, (v_1, v_2\}, (v_1, v_3\}, (v_1, v_4\},\ (v_2, v_3\},$$

$(v_2, v_4\},\ (v_3, v_4\}, (v_1, v_2, v_3\}, (v_1, v_2, v_4\}, (v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$. "$(ST_G,\ PT_G)$-open supra subgraph".

$= \{V(G), \varnothing, \{v_1\}, \{v_2\}, (v_3\},\ (v_4\}, (v_1, v_2\},$

$$(v_1, v_3\}, (v_1, v_3\},\ (v_2, v_3\}, (v_2, v_4\},\ (v_3, v_4\}, (v_1, v_2, v_3\}, (v_1, v_2, v_4\},$$

$(v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$. $(ST_G,\ PT_G)$-closed supra subgraph $= \{V(G), \varnothing, \{v_1\}, \{v_2\}, (v_3\},\ (v_4\},$

$$(v_1, v_2\}, (v_1, v_3\}, (v_1, v_4\},\ (v_2, v_3\}, (v_2, v_4\},\ (v_3, v_4\}, (v_1, v_2, v_3\},$$

$(v_1, v_2, v_4\}, (v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}$. $(ST_G,\ PT_G)^*$-supra open subgraph= $\{V(G), \varnothing,\ \{v_1\}, \{v_4\},\ (v_1, v_4\}\}$ $(ST_G,\ PT_G)^*$-supra closed subgraph $= \{V(G), \varnothing,\ \{v_2, v_3, v_4\}, \{v_1, v_2, v_3\},\ (v_2, v_3\}\}$.

Bi-supra open subgraph $= \{V\ (G)\ ,\ \emptyset\ ,(v_2\}\ ,(v_3\}\ ,(v_2, v_3\}\ ,(v_1\ ,v_2, v_3\}\ ,$
$(v_1\ ,v_2\}\}$.

Bi-supra closed subgraph $= \{V\ (G)\ ,\ \emptyset\ ,(v_1, v_3, v_4\}\ ,(v_1, v_2, v_4\}\ ,(v_1, v_4\}\ ,$
$(v_4\}\ ,(v_3, v_4\}\}$".

4. SOME CONCEPTS OF BI-SUPRA TOPOLOGICAL GRAPH

Definition 4.1: "suppose $G(V\ ,\ \ E)$ be a graph, $V(H)$ be a subgraph from $V(G)$. "

Then the graph closure of bi-supra topological graph $(V(G), ST_G, PT_G)$ have the shape:

"$bi - Cl_G(V(H)) = \ \cap\{V(K) : V(H) \subseteq V(K),\ V(K)\ is\ bi - supra$
$closed\ subgraph\}$."

Theorem 4.2: "suppose $G(V\ ,\ \ E)$ be a graph that contain bi-supra topological graph $(V(G),\ ST_G,\ PT_G)$." If H , W are subgraphs from G; then:

(1) $V(H)\ \subseteq\ bi-Cl_G(V(H))$.

(2) If $H\ \subseteq\ W$, then $bi-Cl_G(V(H))\ \subseteq\ bi-Cl_G(V(W))$.

(3)" $bi-Cl_G(V(H)\ \cup\ V(W))\ =\ bi-Cl_G(V(H)\ \cup\ bi-Cl_G(V(W))$."

(4)" $bi-Cl_G(V(H)\ \cap\ V(W))\ \subseteq\ bi-Cl_G(V(H))\ \cap\ bi-(V(W))$."

Proof : 1- Suppose that $v\ \in\ V(H)$, by definition 4.2.Then $V(H) \in bi-Cl_G(V(H))$. Therefore, $V(H) \subseteq bi-Cl_G(V(H))$

(2) From (1 , $V\ (H) \subseteq bi-Cl_G(V(H)$, $\ V(W) \subseteq bi-Cl_G(V(W)$

Since, $H \subseteq W$, then $V(H) \subseteq V(W)$. Therefore, $bi-Cl_G(V(H))\ \subseteq\ bi-Cl_G(V(W))$.

(4) From (1 , $V(H) \subseteq bi-Cl_G(V(H)$, $V(W) \subseteq bi-Cl_G(V(W)$ Since

$V(H)\ \cap\ V(W)\ \subseteq\ V(H),\ \ V(H)\ \cap\ V(W)\ \subseteq\ V(W)$. Then

"$bi-Cl_G(V(H)\ \cap\ V(W))\ \subseteq\ bi-Cl_G(V(H))$,

$bi-Cl_G(V(H)\ \cap\ V(W))\ \subseteq\ bi-Cl_G(V(W))$. Therefore,

$bi-Cl_G(V(H)\ \cap\ V(W))\ \subseteq\ bi-Cl_G(V(H)\ \cap\ bi-Cl_G(V(W))$."

Definition 4.3: Let $G(V\ ,\ \ E)$ be a graph, H be a subgraph from G.

So the graph internal of bi-supra topological graph $(V(G), ST_G, PT_G)$ had the shape:

"$bi-Int_G(V(H)) =\ \cup\{V(L) : V(L) \subseteq V(H),\ V(L)\ is\ bi-supra$
$open\ subgraph\}$."

Theorem 4.4: Suppose $G(V\ ,\ \ E)$ be a graph that contain bi-supra
topological graph $(V(G),\ ST_G,\ PT_G)$. If H , W are subgraphs from G; then:

(1) If $H\ \subseteq\ G$, then $bi-Int_G(V(H))\ \subseteq\ V(H)$.

(2) If $H\ \subseteq\ W$, then $bi-Int_G(V(H))\ \subseteq\ bi-Int_G(V(W))$.

(3) $bi-Int_G(V(H)\ \cap V(W)) = bi-Int_G(V(H))\ \cap\ bi-Int_G(V(W))$.

(4) $bi-Int_G(V(H)) \cup bi-Int_G(V(W)) \subseteq bi-Int_G(V(H)\ \cup\ V(W))$.

Proof: (4) Suppose that, $V(H)\ , V(W)\ \subseteq\ V(G)$, since
$V(H)\ \subseteq\ V(H)\ \cup V(W), V\ (W)\ \subseteq\ V\ (H)\ \cup\ V\ (W)$. Then

$$bi-Int_G(V(H))\ \subseteq\ bi-Int_G(V(H)\ \cup\ V(W)),$$

$bi-Int_G(V(W))\ \subseteq\ bi-Int_G(V(H)\ \cup\ V(W))$. Therefore,
$bi-Int_G(V(H))\ \cup\ bi-Int_G(V(W))\ \subseteq\ bi-Int_G(V(H)\ \cup\ V(W))$.

Example 4.5: From Example 3.6. Let $V(H) = \{v_1, v_3\}$.

Bi-supra open subgraph=" $\{V\ (G)\ ,\ \ ,(v_2\}\ ,(v_3\}\ ,(v_2, v_3\}\ ,(v_1\ , v_2, v_3\}\ ,$
$(v_1\ , v_2\}\}$."

Bi-supra closed subgraph=" $\{V\ (G)\ ,\ \ ,(v_1, v_3, v_4\}\ ,(v_1, v_2, v_4\}\ ,(v_1, v_4\}\ ,$
$(v_4\}\ ,(v_3, v_4\}\}$."

Then $bi-Cl_G(V(H)) = \{v_1, v_3, v_4\}$ and
$bi-Int_G(V(H)) = \{v_3\}$.

Remark 4.6: Suppose $G(V,\ \ E)$ be a graph, H is a subgraph from G.
Then, $bi-Int_G(V(H)) \subseteq V(H) \subseteq bi-Cl_G(V(H))$

Proposition 4.7: "Let $G(V\ ,\ \ E)$ be a graph that contains bi-supra
topological graph $(V(G),\ ST_G,\ PT_G)$". If H be a subgraph from G, so:

(1) $bi-Cl_G(V(G)\ -\ V(H))\ =\ V(G)\ -\ bi-Int_G(V(H))$.

(2) $bi-Int_G(V(G)\ -\ V(H))\ =\ V(G)\ -\ bi-Cl_G(V(H))$.

Proof:

(1) Suppose that $v\ \in\ V(G)\ -\ V(H), then\ v\ \in\ V(G),\ v\ \notin\ V(H)$ By
Theorem 4.4, $v\ \notin\ bi-Int_G(V(H))$.

So, $v\ \in\ V(G)\ -\ bi-Int_G(V(H))$.

Conversely,

Assume that $v\ \in\ V(G)\ -\ bi-Int_G(V(H)),\ v\ \in\ V(G)$ and by definition 4.3, $v\ \notin\ bi-Int_G(V(H))$, Then, $v\ \in\ V(G),\ \ V(K) \subseteq\ V(H)$, for every $v\ \in\ V(G)$. So $v\ \in\ V(G),\ \ v\ \notin\ V(H)$. This means $v\ \in\ V(G)\ -\ V(H)$, so $V(G)\ -\ V(H)\ \subseteq\ bi-Cl_G(V(G)\ -\ V(H))$. Therefore, $bi-Cl_G(V(G)\ -\ V(H))\ =\ V(G)\ - bi-\ Int_G(V(H))$.

(2) Assume that $v\ \in\ bi-Int_G(V(G)\ -\ V(H))$. Then by definition
4.3, for every $v\ \in\ V(G)$ such that $V(L\ \subseteq\ V(G)\ -\ V(H)$ Then $V(L) \subseteq\ V(G),\ V(L) \nsubseteq\ V(H)$. This means $v\ \in\ bi-Int_G(V(G))\ ,\ V(L)\ \cap\ V(H)\ =\ \emptyset$. Then, $v\ \in\ V(G), v\ \notin\ bi-Cl_G(V(H))$.

Therefore, $v\ \in\ V(G)\ -\ bi-Cl_G(V(H))$.

Definition 4.8: "Let $G\,(V,\ \ E)$ be a graph, H be a subgraph from G.
Then the graph exterior of bi-supra topological graph has the shape":
$bi-Ext_G(V(H)) = bi-Int_G(C\ (V(H)))$ or

$$bi-Ext_G(V(H)) = \ C(bi-Cl_G(V(H)))$$

Theorem 4.9: "Let $G(V\ ,\ \ E)$ be a graph that contains bi-supra topological graph $(V(G),\ ST_G,\ PT_G)$. If $H\ ,W$ are subgraphs from G; then":
(1) $bi-Ext_G(V(H))\ \cap\ V\,(H)\ =\ \emptyset$.
(2) If $H\ \subseteq\ W$, then $bi-Ext_G(V\,(W))\ \subseteq\ bi-Ext_G(V\,(H))$.
(3) $bi-Ext_G(V\,(H) \cup V\,(W)) \subseteq bi-Ext_G(V\,(H)) \cap bi-Ext_G(V\,(W))$.

Definition 4.10: Suppose $G\,(V,\ \ E)$ be a graph, H be a sub-graph from G.
So, the graph boundary of bi-supra topological graph has the shape:
$bi-B_G(V(H)) = \ bi-Cl_G(V(H)) - bi-Int_G(V(H))$.

Theorem 4.11: Suppose $G(V\ ,\ \ E)$ be a graph that contain bi-supra topological graph $(V(G),\ ST_G,\ PT_G)$. If H be a sub-graph from G so
(1) $bi-B_G(V(H))\ \cap\ bi-Int_G(V(H))\ =\ \emptyset$.
(2) $bi-B_G(V(H))\ \cap\ bi-Ext_G(V(H))\ =\ \emptyset$.
(3) $bi-Int_G(V(H))\ \cap\ bi-Ext_G(V(H))\ =\ \emptyset$.
(4) $bi-Int_G(V(H))\ \cup\ bi-Ext_G(V(H))\ \cup\ bi-B_G(V(H))\ =\ V(G)$.
(5) $Cl_G(V\,(H)\ =\ Int_G(V\,(H))\ \cup\ B_G(V\,(H))$.

Proof:
(1) By definition 4.10, it‘s clear.
(2) Assume that $V\,(H)\ \subseteq\ V\,(G)\,,\ \ bi-B_G(V(H)) \cap\ bi-Ext_G(V(H))$, by theorem 4.9
$\rightarrow bi-B_G(C(V(H)))\ \cap\ bi-Int_G(C(V\,(H))) =\ \emptyset$,
$= bi-B_G(G\ -\ V\,(H))\ \cap\ bi-Int_G(C(G\ -\ V\ (H)))\ =\ $.
(3) Assume that $V\,(H)\ \subseteq\ V\,(G)\,,\ \ bi-Int_G(V(H\ \cap\ bi-Ext_G(V(H)$
$= bi-Int_G(V(H)) \cap bi-Int_G(C(V(H))) \subseteq V(H) \cap bi-Int_G(V(H))$,

$$= bi-Int_G(V(H))\ \cap\ (C(bi-Cl_G(V(H)))$$

$= bi-Int_G(V(H))\ \cap\ (G\ -\ bi-Cl_G(V(H)))$, by distributing intersection,

$$\rightarrow (bi-Int_G(V(H))\ \cap\ G)\ -\ (bi-Int_G(V(H))\ \cap\ bi-Cl_G(V(H))$$

$= bi-Int_G(V(H))\ -\ bi-Ext_G(V(H))\ =\ \emptyset$.
(4) Assume that $V\,(H)\ \subseteq\ V\,(G)$,

$$bi-Int_G(V(H))\ \cup\ bi-Ext_G(V(H))\ \cup\ bi-B_G(V(H))$$

$= bi-Cl_G(V\,(H))\ \cup\ C(bi-Cl_G(V\,(H)))\ =\ G$.
(5) Suppose $v\ \in\ V\,(H)\,$,

$$bi-B_G(V(H))\ \cup\ bi-Int_G(V(H)) =\ (bi-Cl_G(V(H)) -$$

$bi-Int_G(V(H)))\ \cup bi-Int_G(V\,(H))$. Therefore,
$bi-Cl_G(V\,(H)\ =\ bi-Int_G(V\,(H))\ \cup\ bi-B_G(V\,(H))$.

Example 4.12: From example 3.6, let $V\,(H) = \{v_1, v_3\}$. Then
$bi-Cl_G(V(H)) = \{v_1, v_3, v_4\}$ and
$bi-Int_G(V(H)) = \{v_3\}$.
$bi-Ext_G(V(H)) = \{v_2\}$.
$bi-B_G(V\,(H)) = \{v_1, v_4\}$.

Definition 4.13: Suppose $G(V\ ,\ \ E)$ be a graph, H be a sub-graph from
G, and $(V(G), ST_G, PT_G)$ bi-supra topological graph is called:
1- $(ST_G,\ PT_G)^*Cl_G(V(H)) = \cap\{V\,(K) : V\,(H) \subseteq V\,(K)\,,\ V\,(K)\ \ is$
$(ST_G,\ PT_G)^* - closed\ \ subgraph\}$.
2- $(ST_G,\ PT_G)^*Int_G(V(H)) = \cup\{V\,(L) : V\,(L) \subseteq V\,(H)\,,\ V\,(L)\ \ is$
$(ST_G,\ PT_G)^* - open\ \ subgraph\}$.

Example 4.14: From example 3.6, let $V(H) = \{v_1, v_3\}$. Then
$(ST_G,\ PT_G)^* Cl_G(V(H)) = \{v_1, v_2, v_3\}$.
$(ST_G,\ PT_G)^* Int_G(V(H)) = \{v_1\}$.

Proposition 4.15: Suppose $G(V,\ E)$ be a graph, H is a sub-graph from G.
Then, $(ST_G,\ PT_G)^* Int_G(V(H)) \subseteq V(H) \subseteq (ST_G,\ PT_G)^* Cl_G(V(H))$

5. CONCLUSION

The main result in this paper is to explain the relations between bi-supra Topological Space and topological graph theory which illustrated by many proposition as 3.4 and some examples 3.6 and another theorem by 4.2,4.4 and 4.9 by these theorems can study more of subject in graph theory.

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