Laguerre- Galerkin's method for solving second kind- fuzzy Fredholm integral equation

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Abstract

This paper is concerned with introducing Laguerre Galerkin's method for solving second kind Fuzzy Fredholm integral equations. Some specific numerical examples are discussed to demonstrate the validity and applicability of the proposed method. The obtained numerical results are comparing with the analytical known solution.

Keyword: Fuzzy Fredholm, Laguerre polynomials, Galerkin's method.

الملخص:

يهتم هذا البحث بتقديم طريقة لاكير- كالركن لحل معادلات فريدهولم الضبابيه التكاملية من النوع الثاني .تمت مناقشة بعض الأمثلة المحدده لشرح صحة وتطابق النتائج العددية للطريقة المقترحة. النتائج العددية التي تم الحصول عليها تم مقارنتها مع الحلول التحليلية المعطاة.

1. Introduction

The subject fuzzy have been associated by differential and integral equation by many scientists and researchers due to its importance in applications, which based on differential or integral equations, such as fuzzy control, approximate reasoning. The difficulty of finding the analytical solution for equation of fuzzy type in many cases, phased by using numerical method as a great choice in solution.

Numerical method used by Altaie.S.A.(2012) to solve fuzzy Fredholm integral equation of second kind using Bernstein polynomials to approximate the unknown fuzzy function. Also Khora.S.M.(2010) used approximate method based homotopy analysis method for linear Fredholm integral equations, while Abbasbandy.S.(2007) employed numerical Rung-Kutta method to solve n-order fuzzy differential equations. Finally, Allahviranloo.T.(2007) compute the numerical solution of fuzzy differential equations by using Predictor-Corrector method. In this paper numerical method based on Galerkin's method proposed to solve second kind of linear fuzzy Fredholm integral equation by approximating the solution in Laguerre Polynomials.

2. Basic Concepts

In this section, the most basic notations and definitions used in fuzzy

Definition (2.1) :[Abbasbandy.S.(2007), Khora.S.M.(2010)]

A fuzzy number is a fuzzy set $\tilde{y} : R \to I = [0,1]$

Which represented by an ordered pair of functions $\tilde{y} = (\underline{y}(r), \overline{y}(r))$ $, 0 \le r \le 1$ which satisfy the following properties:

1- $\underline{y}(r)$ is a bounded non decreasing left continuous function on [0,1].

2- $\overline{y}(r)$ is a bounded non creasing left continuous function on [0,1].

3- $\underline{y}(r) \le \overline{y}(r)$ $\quad 0 \le r \le 1$

Remark (2.1) :[Altaie.S.A.(2012)]

If $\tilde{y} = (\underline{y}(r), \overline{y}(r))$, and $\tilde{x} = (\underline{x}(r), \overline{x}(r))$ any fuzzy numbers and $k \in \Re$,the addition and scalar multiplication by k is

$$\tilde{x} + \tilde{y} = [\underline{x}(r) + \underline{y}(r), \overline{x}(r) + \overline{y}(r)]$$

$$k\tilde{y} = [k\underline{y}(r), k\overline{y}(r)] \qquad k \ge 0$$

and

$$k\tilde{y} = [k\overline{y}(r), k\underline{y}(r)], \qquad k < 0$$

The set of all fuzzy numbers is denoted by E .

Definition(2.2):[Reza Ezzati . (2008)]

Any fuzzy system of the form $A\tilde{x} = B\tilde{x} + \tilde{y}$ where $A = (a_{ij}), B = (b_{ij})$ $1 \le i \le m, 1 \le j \le n$ are crisp $m \times n$ matrices, and $m \le n$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ,..., \tilde{x}_n)^T$ and $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_m)^T$ are fuzzy number vectors.

Remark (2.2):[Altaie.S.A.(2012), Reza Ezzati . (2008)]

Consider the fuzzy system $A\tilde{x} = B\tilde{x} + \tilde{y}$ and transform its $n \times n$ coefficient matrices A, and B into $2n \times 2n$ crisp linear system,

$$SX = TX + Y \Rightarrow (S - T)X = Y \Rightarrow X = (S - T)^{-1} Y \qquad \ldots(1)$$

where the coefficients matrix $S = (S_{ij})$ and $T = (t_{ij})$, $1 \le i, j \le 2n$ the element S_{ij} *and* t_{ij} , are determined as follows:

$$\text{1) If } \begin{cases} a_{ij} \ge 0 \Rightarrow S_{ij} = a_{ij} \quad and \quad S_i + n_j + n = a_{ij} \\ or \\ b_{ij} \ge 0 \Rightarrow t_{ij} = b_{ij} \quad and \quad t_i + n_j + n = b_{ij} \end{cases} \qquad \ldots(2)$$

$$\text{2) If } \begin{cases} a_{ij} < 0 \Rightarrow S_{ij} + n = -a_{ij} \quad and \quad S_i + n_j = -a_{ij} \\ or \\ b_{ij} < 0 \Rightarrow t_{ij} + n = -b_{ij} \quad and \quad t_i + n_j = -b_{ij} \end{cases} \qquad \ldots(3)$$

For any element S_{ij} *and* t_{ij} which has no assigned value from the coefficient matrices A, and B is set to be zero, while the variable vector are:

$$X = [\underline{x_1} \quad \underline{x_2} \quad \underline{x_3} \ldots \underline{x_n} - \overline{x_1} \quad - \overline{x_2} \quad - \overline{x_3} \cdots - \overline{x_n}]^T$$
$$and$$
$$Y = [\underline{y_1} \quad \underline{y_2} \quad \underline{y_3} \cdots \underline{y_n} - \overline{y_1} \quad - \overline{y_2} \quad - \overline{y_3} \cdots - \overline{y_n}]^T$$

3. Laguerre polynomials and its properties :[**Krasikov.I. (2010),Poularikas A.D.(1999)]**

Laguerre polynomial has a wide range of applications in many areas including in permutation statistics. In addition, these polynomials and their properties are powerful mathematical techniques for solving some problems in pure and applied methods. The general form of these polynomials is:-

$$L_n(x) = \sum_{k=0}^{n} \frac{(-1)^k n! x^k}{(k!)^2 (n-k)!} \quad 0 \le x \le \infty \qquad \ldots(4)$$

The most important properties of Laguerre polynomials are the recurrence relation and orthogonal property:-

$$L_{n+1}(x)=\frac{(2n+1)-x}{n+1}L_n(x)-\frac{n}{n+1}L_{n-1}(x) \quad n\geq 1 \qquad \ldots (5)$$

where

$$L_0(x)=1 \ , L_1(x)=1-x \quad .$$

The Laguerre polynomials are orthogonal with respect to the weight function $w(x)=e^{-x}$ on the interval $[0,\infty)$ so,

$$\int_0^\infty e^{-x}L_n(x)L_m(x)dx=0 \qquad n\neq m$$

<u>4. Second kind fuzzy Fredholm integral equations (2-FFIE):</u> [Omid Fard.S. (2011), Reza Ezzati . (2008)]

In this section, the equation of Second Kind Fuzzy Fredholm integral equation will be studied and discussed, as follows:

$$\tilde{y}(x,r)=\tilde{f}(x,r)+\lambda\int_a^b k(x,t)\tilde{y}(t,r)dt \qquad \ldots (6)$$

where $\lambda>0$ and $k(x,t)$ is an arbitrary function called the kernel over the interval $a\leq x,t\leq b$,

$\tilde{f}(x,r)=[\underline{f}(x,r),\overline{f}(x,r)]$ is fuzzy function on [a,b],and

$\tilde{y}(x,r)=[\underline{y}(x,r),\overline{y}(x,r)]$ where $0\leq r\leq 1$

Equation (6) can be written using the above equivalently as

$$\underline{y}(x,r)=\underline{f}(x,r)+\lambda\int_a^b k(x,t)\underline{y}(t,r)dt \qquad \ldots$$
(7)

and

$$\overline{y}(x,r)=\overline{f}(x,r)+\int_a^b k(x,t)\overline{y}(t,r)dt \qquad \ldots (8)$$

<u>5. Main approximate function:</u>

In this part of the paper we spot lit on approximate the unknown function $\tilde{y}(x,r)$ in Equation (6) by using Lagurre polynomials as follows:

$$\tilde{y}(x,r) \cong \sum_{i=0}^{n} \tilde{c}_i L_i(x) \quad \dots (9)$$

where $\tilde{C}_n = (\underline{c_0}, \underline{c_1}, \dots, \underline{c_n}, -\overline{c_0}, -\overline{c_1}, \dots, -\overline{c_n})$ the fuzzy coefficient.

By substituting Equation (9) into Equation (6) we have

$$\sum_{i=0}^{n} \tilde{c}_i \tilde{L}_i(x) = \tilde{f}(x,r) + \lambda \int_a^b k(x,t) \sum_{i=0}^{n} \tilde{c}_i L_i(t) dt \quad \dots (10)$$

So the problem know is how to find the approximate values of fuzzy coefficient $\tilde{c}_i, i = 0,1,\dots,n$

6. Galrkin's method for solving 2nd-kind fuzzy Fredholm integral equation (2-FFIE):

Consider the Equation $\sum_{i=0}^{n} \tilde{c}_i \tilde{L}_i(x) = \tilde{f}(x,r) + \lambda \int_a^b k(x,t) \sum_{i=0}^{n} \tilde{c}_i L_i(t) dt$

Let $R(x,r) = \sum_{i=0}^{n} \tilde{c}_i L_i(x) - \tilde{f}(x,r) + \lambda \int_a^b k(x,t) \sum_{i=0}^{n} \tilde{c}_i L_i(t) dt \quad \dots (11)$

be the fuzzy residue function ,and by put

$$L_j(x) = \frac{\partial}{\partial \tilde{c}_j} \sum_{j=0}^{n} \tilde{c}_j L_j(x)$$

So

$$I(\tilde{c}_i, r) = \int_a^b R(x,r).L_j(x) dx \quad \dots (12)$$

by substituting $R(x,r)$ into Equation (12), we have

$$I(\tilde{c}_i, r) = \int_a^b (\sum_{i=0}^{n} \tilde{c}_i L_i(x) - \tilde{f}(x,r) - \lambda \int_a^b k(x,t) \sum_{i=0}^{n} \tilde{c}_i L_i(t) dt) L_j(x) dx \quad \dots (13)$$

Put $I(\tilde{c}_i, r) = 0$ we get

$$\sum_{i=0}^{n} \tilde{c}_i \int_a^b L_i(x) L_j(x) dx = \int_a^b \tilde{f}(x,r) L_j(x) dx + \lambda \sum_{i=0}^{n} \tilde{c}_i \int_a^b (\int_a^b k(x,t) L_i(t) dt) L_j(x) dx) \quad \dots (14)$$

If $M_i(x,r) = \int_a^b k(x,t) L_i(t) dt$ then Equation (14) will be

$$\sum_{i=0}^{n}\tilde{c}_i\int_a^b L_i(x)L_j(x)dx=\int_a^b \tilde{f}(x,r)L_j(x)dx+\lambda\sum_{i=0}^{n}\tilde{c}_i\int_a^b M_i(x,r)L_j(x)dx \qquad \ldots(15)$$

Equation (15) can be written in matrix form as follows;

$$A\tilde{C}=\tilde{F}+B\tilde{C} \qquad \ldots(16)$$

where $A=\begin{bmatrix} a_{00} & a_{01}\cdots & a_{0j}\cdots & a_{0n} \\ a_{10} & a_{11}\cdots & a_{1j}\cdots & a_{1n} \\ \vdots & \vdots & \vdots\ \ddots & \vdots \\ a_{i0} & a_{i1}\cdots & a_{ij}\cdots & a_{in} \\ \vdots & \vdots & \vdots\ \ddots & \vdots \\ a_{n0} & a_{n1} & a_{n2}\cdots & a_{nn} \end{bmatrix}$

and $a_{ij}=\int_a^b L_I(x)L_j(x)dx$

$$B=\begin{bmatrix} b_{00} & b_{01}\cdots & b_{0j}\cdots & b_{0n} \\ b_{10} & b_{11}\cdots & b_{1j}\cdots & b_{1n} \\ \vdots & \vdots & \vdots\ \ddots & \vdots \\ b_{i0} & b_{i1}\cdots & b_{ij}\cdots & b_{in} \\ \vdots & \vdots & \vdots\ \ddots & \vdots \\ b_{n0} & b_{n1} & b_{n2}\cdots & b_{nn} \end{bmatrix}$$

$$b_{ij}=\int_a^b M_i(x)L_j(x)dx$$

$$\tilde{F}=[\underline{f_0},\underline{f_1},\cdots,\underline{f_j},\cdots,\underline{f_n},-\overline{f_0},-\overline{f_1},\cdots,-\overline{f_j},\cdots,-\overline{f_n}]^T$$

$$\underline{f_j}=\int_a^b \underline{f}(x,r)L_j(x)dx \quad \text{and} \quad \overline{f_j}=\int_a^b \overline{f}(x,r)L_j(x)dx$$

Now, transform the $n\times n$ coefficient matrices A and B into $2n\times 2n$ crisp matrices S and T respectively as mention in remark (2.2) we have the system :

$$S\tilde{C}=T\tilde{C}+F$$

So,

$$(S-T)\tilde{C}=F$$

To find the fuzzy coefficients $\tilde{C}_i$ the following system:

$$\tilde{C} = (S - T)^{-1} F$$

will be solved using matlab program.

Finally, the approximate fuzzy solution $\tilde{y}(x,c) = [\underline{y}(x,c), \overline{y}(x,c)]$ will be given as follows:

$$\underline{y}(x,r) = \sum_{i=0}^{n} \underline{c_i} L_i(x) \quad \text{and} \quad \overline{y}(x,r) = \sum_{i=0}^{n} \overline{c_i} L_i(x)$$

7. Numerical examples:

In this section, we show how the proposed method will be implementing in order to obtain the solution of second kind fuzzy Fredholm Integral equations.

Example (7.1): [Altaie,S.A.(2012)]

Consider the 2- kind Fuzzy Fredholm integral equation where $\lambda = 1 \quad and \quad a = 0, b = 1$

$$\tilde{y}(x,r) = \tilde{f}(x,r) + \int_0^1 k(x,t)\tilde{y}(t,r)dt$$

where

$$k(x,t) = x + t \qquad 0 \le x,t \le 1$$

$$\underline{f}(x,r) = r(\frac{1}{2}x - \frac{1}{3})$$
$$\overline{f}(x,r) = (2-r)(\frac{1}{2}x - \frac{1}{3})$$

where $\underline{y}(x,r) = rx$ the exact is lower solution, and $\overline{y}(x,r) = (2-r)x$ is the exact upper solution.

Assume that $\tilde{y}(x,r) = \sum_{i=0}^{3} \tilde{c}_i L_i(x)$ … (16)

So,

$$\tilde{y}(x,r) = \tilde{c}_0 L_0(x) + \tilde{c}_1 L_1(x) + \tilde{c}_2 L_2(x) + \tilde{c}_3 L_3(x)$$
$$= \tilde{c}_0 + \tilde{c}_1(1-x) + \tilde{c}_2(\frac{1}{2}(x^2 - 4x + 2)) + \tilde{c}_3(1 - 3x + \frac{3}{2}x^2 - \frac{1}{6}x^3) \qquad \ldots(17)$$

By substituting Equation (16) into the [2- FFIE] to calculate the residue function as follows:

$$R(x,r) = \sum_{i=0}^{3} \tilde{c}_i L_i(x) - \tilde{f}(x,r) + \int_0^1 k(x,t) \sum_{i=0}^{3} \tilde{c}_i L_i(t)dt$$

also

$$\frac{\partial \sum_{j=0}^{3} \tilde{c}_j L_j(x)}{\partial c_j} = L_j(x) \qquad j = 0,1,2,3$$

So

$$R(x,r) = \tilde{c}_0 L_0 + \tilde{c}_1 L_1(x) + \tilde{c}_2 L_2(x) + \tilde{c}_3 L_3(x) - (\tilde{f}(x,r) - \int_0^1 (x,t)(\tilde{c}_0 L_0(t) + \tilde{c}_1 L_1(t) + \tilde{c}_2 L_2(t) + \tilde{c}_2 L_3(t)dt$$

$$R(x,r) = c_0 + \tilde{c}_1(1-x) + \tilde{c}_2(\frac{1}{2}(x^2 - 4x + 2)) + \tilde{c}_3(1 - 3x + \frac{3}{2}x^2 - \frac{1}{6}x^3)$$
$$- \tilde{f}(x,r) - \int_0^1 (x,t)(\tilde{c}_0 + \tilde{c}_1(1-t) + \tilde{c}_2(\frac{1}{2}t^2 - 2t + 1) + \tilde{c}_3(1 - 3t + \frac{3}{2}t^2 - \frac{1}{6}t^3)$$

Let

$I(\tilde{c}_i, r) = \int_0^1 R(x,r) L_j(x)dx$ since $I(\tilde{c}_i, r) = 0$ and substituting $R(x,r)$, we have

$$\int_0^1 (\tilde{c}_0 + \tilde{c}_1(1-x) + \tilde{c}_2(\frac{1}{2}x^2 - 2x + 1) + \tilde{c}_3(1 - 3x + \frac{3}{2}x^2 - \frac{1}{6}x^3) L_j(x)dx$$

$$= \int_0^1 (\int_0^1 \tilde{c}_0 + \tilde{c}_1(1-t) + \tilde{c}_2(\frac{1}{2}t^2 - 2t + 1) + \tilde{c}_3(1 - 3t + \frac{3}{2}t^2 - \frac{1}{6}t^3)dt) L_j(x)dx + \int_0^1 \tilde{f}(x,r) L_j(x)dx \qquad j = 0,1,2,3$$

Rewriting this system in matrix form we get

$$A\tilde{C} = B\tilde{C} + F$$

where

$$A = \begin{bmatrix} 1 & 0.5 & 0.1667 & -0.0417 \\ 0.5 & 0.3333 & 0.2083 & 0.1167 \\ 0.1667 & 0.2083 & 0.2167 & 0.2028 \\ -0.0417 & 0.1167 & 0.2028 & 0.2373 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0.4167 & 0.0417 & -0.1792 \\ 0.4167 & 0.1667 & 0.0069 & -0.0861 \\ 0.0917 & 0.0069 & -0.0139 & -0.0247 \\ -0.1792 & -0.0861 & -0.0247 & 0.0132 \end{bmatrix}$$

By transforming the above 4×4 matrices A and B into 8×8 matrices S, T that considered in remark (2.2), we get

$$S=\begin{bmatrix} 1 & 0.5 & 0.1667 & 0 & 0 & 0 & 0 & 0.0417 \\ 0.5 & 0.3333 & 0.2083 & 0.1167 & 0 & 0 & 0 & 0 \\ 0.1667 & 0.2083 & 0.2167 & 0.2028 & 0 & 0 & 0 & 0 \\ 0 & 0.1167 & 0.2028 & 0.2373 & 0.0417 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0417 & 1 & 0.5 & 0.1667 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.3333 & 0.2083 & 0.1167 \\ 0 & 0 & 0 & 0 & 0.1667 & 0.2083 & 0.2167 & 0.2083 \\ 0.0417 & 0 & 0 & 0 & 0 & 0.1167 & 0.2083 & 0.2373 \end{bmatrix}$$

$$H=\begin{bmatrix} -1/12r \\ -1/12r \\ -11/144r \\ -47/720 \\ 1/6-1/12r \\ 1/6-1/12r \\ 11/72-11/144r \\ 47/360-47/720r \end{bmatrix} \quad \text{Then} \quad \tilde{C}=\begin{bmatrix} \underline{c}_0 \\ \underline{c}_1 \\ \underline{c}_2 \\ \underline{c}_3 \\ -\overline{c}_0 \\ -\overline{c}_1 \\ -\overline{c}_2 \\ -\overline{c}_3 \end{bmatrix}=\begin{bmatrix} 0.7067r+0.2933 \\ -1.1528r+0.1528 \\ 0.2301r-0.2301 \\ 0.1166r-0.1166 \\ 0.7067r-1.7067 \\ -1.1528r+2.1528 \\ 0.2301r-0.2301 \\ 0.1166r-0.1166 \end{bmatrix}$$

By substituting the fuzzy coefficient in approximate solution, we get

$$\underline{y}(x,r)=\underline{c}_0+\underline{c}_1(1-x)+\underline{c}_2(\frac{1}{2}x^2-2x+1)+\underline{c}_3(1-3x+\frac{3}{2}x^2-\frac{1}{6}x^3)$$

$$\underline{y}(x,r)=0.7067r+0.2933+(-1.1528r+0.1528)(1-x)+(0.2301r-0.2301)(\frac{1}{2}x^2-2x+1)$$
$$+(0.1166r-0.1166)(1-3x+\frac{3}{2}x^2-\frac{1}{6}x^3)$$

$$\overline{y}(x,r)=\overline{c}_0L_0+\overline{c}_1L_1+\overline{c}_2L_2+\overline{c}_3L_3$$

$$\overline{y}(x,r)=(-0.7067r+1.7067)+(1.1528r-2.1528)(1-x)+(-0.2301r+0.2301)(\frac{1}{2}x^2-2x+1)$$
$$+(-0.1166r+0.1166)(1-3x+\frac{3}{2}x^2-\frac{1}{6}x^3)$$

The comparison between the exact and approximate solution for values of r are shows in table(1).

r	$\underline{y}(1,r)$ **exact**	$\underline{y}(1,r)$ **approximate**	$\overline{y}(1,r)$ **exact**	$\overline{y}(1,r)$ **approximate**
0.0	**0.000000**	**0.000000**	**2.000000**	**2.000000**
0.1	**0.100000**	**0.100000**	**1.900000**	**1.900000**
0.2	**0.200000**	**0.200000**	**1.800000**	**1.800000**
0.3	**0.300000**	**0.300000**	**1.700000**	**1.700000**
0.4	**0.400000**	**0.400000**	**1.600000**	**1.600000**
0.5	**0.500000**	**0.500000**	**1.500000**	**1.500000**
0.6	**0.600000**	**0.600000**	**1.400000**	**1.400000**
0.7	**0.700000**	**0.700000**	**1.300000**	**1.300000**
0.8	**0.800000**	**0.800000**	**1.200000**	**1.200000**
0.9	**0.900000**	**0.900000**	**1.100000**	**1.100000**
1.0	**1.000000**	**1.000000**	**1.000000**	**1.000000**

Table (1): Comparison between the exact solution and the approximate solution of example (7.1)

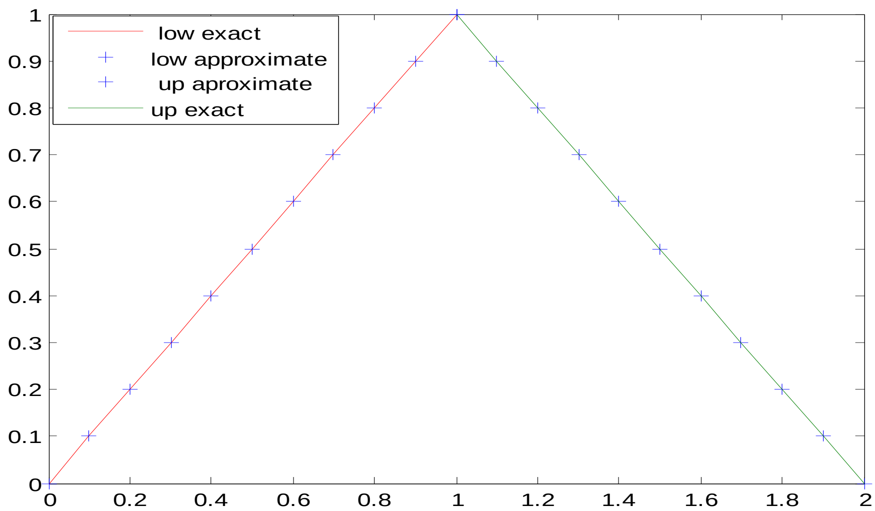

Figure (1): Exact and approximate solution for example (7.1)

<u>Example (7.2):</u>[Altaie,S.A.(2012)]

Consider the following Fuzzy Fredholm Integral equation of the second kind where $a = 0$ and $b = 2\pi$

$$\tilde{y}(x,r)=\tilde{f}(x,r)+\int_0^{2\pi} k(x,t)\tilde{y}(t,r)dt$$

where $k(x,t)=0.1\sin(t)\sin(\frac{1}{2}x)$ $\quad 0\le x,t\le 2\pi$

$$\underline{f}(x,r)=[\frac{13}{15}(r^2+r)+\frac{2}{15}(4-r^3-r)]\sin(\frac{x}{2}),\ \bar{f}(x,r)=[\frac{2}{15}(r^2+r)+\frac{13}{15}(4-r^3-r)]\sin(\frac{x}{2})$$

Where the lower solution $\underline{y}(x,r)=(r^2+r)\sin\frac{1}{2}x$,

The upper exact solution $\bar{y}(x,r)=(4-r-r^3)\sin\frac{1}{2}x$

Assume that $\tilde{y}(x,r)=\sum_{i=0}^{8}\tilde{c}_i L_i(x)$,where $\tilde{y}(x,r)=(\underline{y}(x,r),\bar{y}(x,r))$ and $\tilde{c}_i=(\underline{c_i},\overline{c_i})$

The same steps in example (7.1) are listing the approximate results of example (7.2) in table (2) for $0\le r\le 1$.

r	$\underline{y}(pi,r)$ **Lower approximate**	$\underline{y}(pi,r)$ **lower exact**	$\bar{y}(pi,r)$ **upper exact**	$\bar{y}(pi,r)$ **upper approximate**
0.0	**0.0000043**	**0.00**	**4.000**	**4.0001145**
0.1	**0.1100041**	**0.11**	**3.899**	**3.8990049**
0.2	**0.2400032**	**0.24**	**3.792**	**3.7920056**
0.3	**0.3900064**	**0.39**	**3.673**	**3.6730028**
0.4	**0.5600029**	**0.56**	**3.536**	**3.5360012**
0.5	**0.7500055**	**0.75**	**3.375**	**3.3750007**
0.6	**0.9600013**	**0.96**	**3.184**	**3.1840051**
0.7	**1.1900036**	**1.19**	**2.957**	**2.9570083**
0.8	**1.4400054**	**1.44**	**2.688**	**2.6880019**
0.9	**1.7100071**	**1.71**	**2.371**	**2.3710028**
1.0	**2.0000437**	**2.00**	**2.000**	**2.0000329**

Table (2): Comparison between the exact solution and the approximate solution for example (7.2)

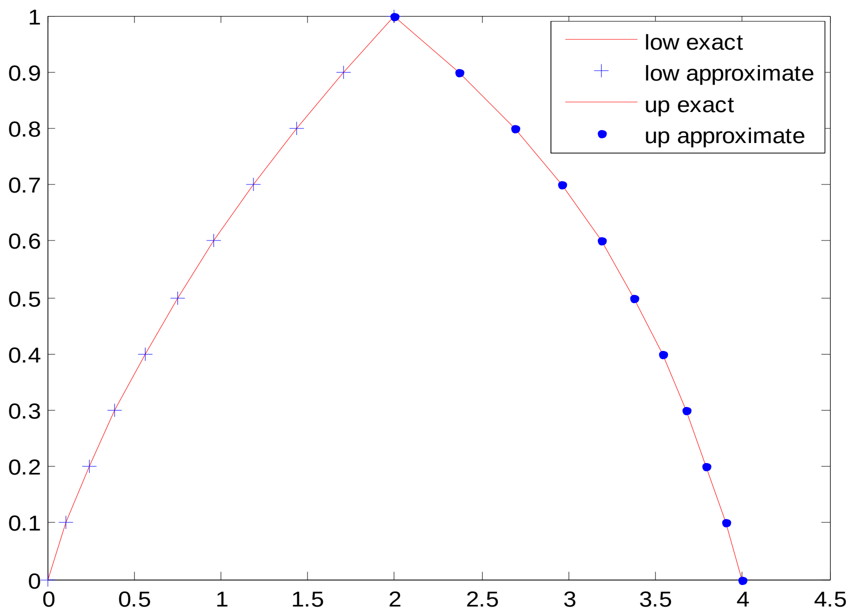

Figure (2): Exact solution and approximate solution of example (7.2)

Conclusions:

In this work, we illustrated a numerical method for solving fuzzy fredholm integral equations of the second kind, using Galerkin's method.

The unknown fuzzy function was approximated by Lagurre polynomials. Comparison of the numerical solutions and the exact solution shows that the proposed method is more efficient and practical for solving these kinds of equations.

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