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New Types of Continuous Function and Open Function

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ABSTRACT: In this paper, we continue to study the properties of the relation with some type of open sets, and we introduce α-continuous function, semi-continuous function, α^*-continuous function, and α^{**}-continuous function are studied and some of their characteristics are discussed. In this work, we need to introduce the concepts of function, especially the inverse function to find all continuous function, so we want to prove some examples, theorems, and observations of our subject with the help of new concepts for the alpha-open sets of sums to make it easier for us to find a relationship between these formulas as well as the converse relationship has been studied and explained with illustration many examples. Hence, reaching to get a relationship (continuous, α-continuous, semi α-continuous) function at new condition.

Keywords: (semi-continuous, α-continuous, semi α-continuous, α * - continuous, α ** -continuous) function

1. INTRODUCTION

Open sets and closed sets play a key role in constructing topological space, which is why scientists and researchers in the field of mathematics paid great attention to them, and used new patterns as synonyms for open and closed sets. Our studies focus on continuous functions of the alpha type.it is known that the continuity of functions is evident from the concept of open and closed function s according to the following criterion. Let the function be defined from the topological space to another one, $f : (X, \tau_x) \to (Y, \tau_y)$. Then the function is continuous if and only if the invers image of each set is open or closed in the second space is also open or closed in the first space.

2. PRELIMINARIES

Definition 2.1. [1]

If f :$(X, T_X)\to (Y, T_Y)$ be two topological space. Then f is named α-**continuous** function if and only if, for each A *is open* set *in Y.* Thus f^{-1} $(A) is \alpha\text{-}open$ Set in (X, T_X).

Definition 2.2. [2]

If f : $(X, T_X) \to$ (Y, T_Y) be two topological space, thus f is termed *semi*-**continuous** function. When $A is open set$ $in Y$, $f^{-1}(A)$ in(Y,τ_x) thus $f^{-1}(A) is semi$-**open** set in (X, τ_x). Such that $(f^{-1}(A) \subseteq Cl\ Int\ f^{-1}(A))$

Theorem 2.3. [2]

Each $contiuous function$ is *semi-contiuous* function.

Proof:

Let $f$be *contiuous*, there exists $f^{-1}(A)$ open in (Y,τ_Y)
Therefore $f^{-1}(A)$is open in(X, τ_x). since (**every open set is *semi*-open set**)
Then f is *semi*-continuous.
The opposite of the previous theorem does not have to be true and the following Example illustrates this.

Example 2.4 .

If $X = (0, 2, 4, 6\}$, $T_x = \{\varnothing, (4\}, (0, 2\}, (0, 2, 4\}$. Define in$(X, T_x$)space,
And let $Y = (1, 3, 5\}$, $T_y = (\varnothing, (5\}, (3\}, (1, 3\}, (5, 3\}, Y\}$. Define in Y space,
Then $f : X \to Y$; $f(x_1) = f(x_2) = 3$, $f(x_3) = 5$, $f(x_4) = 1$, ,
Clearly ; open sets of space Y : $(\varnothing, \{5\}, (3\}, (1, 3\}, (5, 3\}, Y\}$.
And also *semi*-open sets of space X :$SO(X, T_x)$
$\{\varnothing, (4\}, (0, 2\}, (0, 2, 4\}, (0, 2, 6\}, (4, 6\}, X\}$.
That is perfect f is *semi*-continuous,
But f is not continuous because ;
$f^{-1}((1, 5\}) = \{4, 6\} \notin T_x$**.**

Remark 2.5.

Let $f : (X, T_x) \to (Y, T_y)$;
Continuous $\to$**α-continuous** $\to$*semi* **α-continues**. But the convers is not Right in general.

Theorem 2.6. [3]

If (X , T_X) too (Y , T_Y) are a *topological spaces* and if $f : X \to Y$ be *α-continuous* function, then f is *semi-continuous*.

Proof :

Since f is α-continuous function. Thus $f^{-1}(A) \subseteq Int\ Cl\ Int f^{-1}(A)$.
By (proposition **let $f : X \to Y$ is α -continuous if and only i f each open Set A of Y** $f^{-1}(A) \subseteq Int\ Cl\ Int\ f^{-1}(A)$ **.)**
Obviously $Int\ Cl\ Int\ f^{-1}(A) \subseteq Cl\ Int\ f^{-1}(A)$,
So $f^{-1}(A) \subseteq Cl\ Int\ f^{-1}(A)$. hence f is *semi*-continuous.
(**By, let (X, T_X) and (Y, T_Y) be two** *topological Space*,
Thus $f : X \to Y$ is semi-*continuous* **if every** *open set Ain Y*,
$f^{-1}(A) \subseteq cl\ Int\ f^{-1}(A)$**).**
The convers of theorem **(2.1.6.)** is not certainly true in general. To get this, We offer the previous counter example is given.

Example 2.7.

If $X = (7, 8, 9\}$, $T_x = (\varnothing , (7\}, (8\}, (7, 8\}, X\}$,
$T_y = (\varnothing , (7\}, (8, 9\}, X\}$,Therefore the self-function, since $X = Y$.
$f : (X, T_x) \to (X, T_y)$ be *semi*-continuous,
However no α-continuous.
Every continuous *function is α − continuou*s function, so *it is* semi α- *continuous*, On the other hand the convers is false in universal.

Example 2.8.

If $X = (0, 1, 3, 5\}$, and $T_x = (\varnothing, (0\}, X\}$, let $y = (2, 4, 6\}$, $T_y = (\varnothing, (2\}, Y\}$.
The α-open sets define on space X are ;

$$T^{\alpha}_{(x)} = T_x \cup ((0, 1\}, (0, 3\}, (0, 5\}, (0, 1, 3\}, (0, 1, 5\}, (0, 3, 5\}\},$$

The α-open sets define on space Y are ; $T^{\alpha}_{y} = (\varnothing, (2\}, (2, 4\}, (2, 6\}, Y\}$,
If $f : X \to Y$, define are $f(x_1) = f(x_2) = 2$, $f(x_3) = 4$, $f(x_4) = 6$.
Since f is *α-continuous* function, however it is not *continuous* function.
Because $\{2\}$ is open in space Y, as $f^{-1}((2\}) = (0, 1\}$,
But $\{0, 1\}$is not open in Space X.
To fined semi α-continuous ; $S\alpha O(X) = T^{\alpha}_{x}$, and $S\alpha O(Y) = T^{\alpha}_{y}$
Since f **is** *semi* **α-continuous** function, but it is **not continuous**, $\{2\}$is open,
However $f^{-1}((2\}) = (0, 1\}$, is not open in T_x.
Each α-continuous function is *semiα*-continuous, however convers is not True In General.

Example 2.9.

Give $X = (4, 6, 8\}$, $T_x = (\varnothing, (4\}, (6\}, (4, 6\}, X\}$,
Then the α-*open sets in spaceX* ; $T_x^\alpha = T_x$,
As well as the*semi* α-*open sets in spaceX*, $S\alpha O(X) = T^\alpha_{(x)} \cup ((6, 8\}, (4, 8\}\}$.
Thus function define by *identity* ; $f(x_1) = 4$, $f(x_2) = f(x_3) = 6$,
Therefore function is *semi*α-continuous , but it is not α-continuous.
because, $\{6\}$is **open** set, then $f^{-1}((w\}) = (w, e\} \notin T_x^\alpha$.
To find a *semi*α-continuous ; $S\alpha O(X) = T_x^\alpha \cup \{(w, e\}, (q, e\}\}$,
Then f **is** *semi* α**- continuous**, however it is **not** α**-continuous**.

3. CONTINUITY AND FUNCTION RELATIONSHIP

Theorem 3.1. [4]

let f : (X, τ_x) → (Y, τ_y) and g : (Y, τ_y) → (Z, τ_z), are equally continuous function then the composition $gof : (X, \tau_x) \to (Z, \tau_z)$ is continuous.

Proof :

If $M \in \tau_Z$, then $g^{-1}(M) \in \tau_y$, (by g is continuous). Such that $g^{-1}(M) \subseteq Y$
Therefore $f^{-1}(\ g^{-1}(M)) \in \tau_x$. (by f be continuous)
And $(f^{-1} o g^{-1})(M) \in \tau_X$,
Thus $(gof)^{-1}(M) \in \tau_x$, (by $(gof)^{-1} = f^{-1} o g^{-1}$)
Then gof is composition.(continuous)

Remark 3.2. [4]

The composition of finite number of continuous function is continuous.
Explain : the composition of four or seven or fifty continuous function is continuous (if f, g, h, k are continuous, so $kohogof$ is continuous. . .).
If f:X→Y, g:Y→Z,*are* α-*continuous*, and the arrangement function,
gof **is not necessary** α**-continuous.**

Example 3.3.

Let $X = (0, 3, 5, 7\}$, $T_x = (\varnothing, (5\}, (0, 5\}, (0, 3, 5\}, X\}$.
And let $Y = (2, 4, 6\}$, $T_y = (\varnothing, (6\}, Y\}$. If α-open sets in space X ;
$T^\alpha_{(x)} = T_x \cup \{(3, 5\}, (5, 7\}, (5, 3, 7\}, \{0, 5, 7\}\}$. .And the α-open sets in space Y ;
$T^\alpha_{(y)} = T_y \cup ((2, 6\}, (4, 6\}\}$ .So f:X→Y ; $(x_1) = f(x_2) = 2$, $f(x_3) = f(x_4) = 4$.
And g:Y→X ;g (y_1)=g(y_2)= 5, g (y_3)= 0. Therefore f, g are α-continuous.
$gof : X \to X$, $gof(0) = gof(3) = 5$, $gof(5) = gof(7) = 0$.The gof,
is not α- continuous because; $\{5\}$is *open set* of *spaceX* ,
$(gof)^{-1}(5) = (0, 3\}$, but $\{0,3\}$be not α-*open* of *space* X.

Definition 3.4. [5]

If f: X→ Y. Then f is named α^***-continuous**,
for Each N is α-*openset* of Y**,** thus f^{-1} (N)be α-*open set* of X.

Theorem 3.5. [1]

A function $f : X \to Y$ Therefore the following statement are equivalent
a) f*issemi*α-*continuous*.
b) f*issemi*α-*continuous* at each point $x \in X$.

Proof:

a) ⟹ b)
let $f : X \longrightarrow Y$ is a *semi*α-continuous.
And $x \in X$, N is an *open set* of Y having $f(x)$.
Then $x \in f^{-1}(N)$. also f is *semi*α-continuous.
So $M = f^{-1}(N)$ is *semi*α-*open set* in X holding (x).
therefore $f(M) \subset N$.
b)⟹ a)
if f : X⟶ Y is a *semi*α-continous for all point in X.
And N *open set* in Y. Let $x \in f^{-1}(N)$.
Then N is *open set inY* containing $f(x)$.
By (b), at hand is *semi*α-*open setM* of X containing x.
Since $f(x) \in f(M) \subseteq N$. Therefor M $\subseteq f^{-1}$ (N).

Hence f^{-1} (N)= ∪{M :x∈ f^(-1) (N) }.
Then f^{-1} (N) is *semiα-open in X*.

Remark 3.6.
The notions of continuity and α^*-continuity are independent,.

Example 3.7.
If $X = (\ 1, 2, 3, 4\ \}$, $T_X\ = (\ \varnothing\ , (1\ \}, (2\ , 3\ \}\ , (1, 2, 3\ \}, X\ \}$, $T^{\alpha}_{(X)} = T_X$.
And $Y = (\ 5,\ 6\ 7\}$, $T_Y = \{\ \varnothing\ ,\ (5\},\ Y\}$,
$T^{\alpha}_{(Y)} =\ T_Y \cup \{(5, 6\},\ (5, 7\}\}$.
If f :X→Y by f (x_1)= 5 , f (x_2)= 6 , f (x_3)=f $(x_4$)= 7.
Then f is **continuous,** However it is not α^***-continuous**.
Since $(\ 5, 6\ \} \in\ \tau^{\alpha}_{(Y)}$, but $f^{-1}\ (\ 5,\ 6\ \} = (1,\ 2\ \}\ \notin\ T^{\alpha}_{(X)}$.
Hence f is **continuous**. And f is **not** α^***-continuous** function.

Example 3.8.
If $X = (1, 2, 3, 4\}$, $T_X = \{\varnothing,\ (1\},\ X\}$,
$T^{\alpha}_x = T_x \cup ((1, 2\}, (1, 3\}, (1, 4\}, (1, 2, 3\}, (1, 2, 4\},\ (1, 3, 4\}\}$.
$Y = \{5, 6, 7\}$, $T_y = \{\varnothing,\ (5\},\ Y\ \}$,
$T^{\alpha}_y =\ T_Y \cup \{\ (5, 6\},\ (5, 7\}\ \}$,
If $f : X\ \rightarrow\ Y$, with f (x_1)= f (x_2)= 5 , f (x_3)= 6 , f (x_4)= 7
Hence f is α^*-continuous, but it's not continuous, Because{5}is open set in Y,
However f^{-1} ((5 }) = { 1, 2 } is not open in X.
As a result f is α^***-continuous**, however f is **not continuous** .

Proposition 3.9. [3], [1]
1. A function $f\ : (X, T_x\ \rightarrow (Y, T_y)$ is an open, continuous and bijective, then f is $\alpha^* - continuous$
2. A meaning $f\ : (X,\ T_x \rightarrow \left(Y, T_y\right)$ are α^*−continuous iff, $f\ : (X\ , T^{\alpha}_x) \rightarrow (\ Y\ ,\ T^{\alpha}_y)$ are continuous

Proof :
Let $E \in\ T^{\alpha}_x$, to prove f^{-1}(E)∈ T^{α}_x, Then $f^{-1}\ (E) \subseteq Int\ Cl\ Int\ f^{-1}$(E)
If $x \in\ f^{-1}$(E)⟹ f (x) ∈(E). and f(x)∈ $Int\ Cl\ Int\ E$ (since $E \in T^{\alpha}_y$) .
And so, there occurs $N open\ set$ of Y. Since $f(x) \in N \subseteq Cl\ Int\ E$.
And $x \in f^{-1}(N) \subseteq f^{-1}(cl\ Int\ E)$, then $f^{-1}(cl\ Int\ E) \subseteq cl\left(f^{-1}(IntE)\right)$.
(then $f^{-1}$is continuous, which is same to f is open and bijective)
Thus $x \in f^{-1}(N) \subseteq Cl\left(f^{-1}(Int\ E)\right)$.
Since $x \in f^{-1}(N) \subseteq Cl(f^{-1}IntE)) \subseteq Cl(Int(f^{-1}(E)))$, ($f$ **is continuous**)
Therefore $x \in f^{-1}$(N)⊆ $Cl(Int f^{-1}$(N),
But $f^{-1}(N)$ is open set in X, (f **is continuous**)
Thus $x \in Int\ Cl$ (Int$\left(f^{-1}(N)\right)$, ,As a result $f^{-1}(N) \subseteq Int\ Cl$ Int$\left(f^{-1}(N)\right)$,
Then f^{-1}(N)∈ T^{α}_x. therefore f is α^*-continuous function.
To prove (2) is obviously.

Remark 3.10. [1]
The concepts of continuity and *semiα* -continuity are independent,
Example.

Example 3.11.
If $X = (0, 2, 4, 6\}$, $T_x = (\varnothing, (0\}, (0, 4\}, (2, 4, 6\}, X\}$. Thus $T^{\alpha}_x = T_X$,
Let $y = (7, 8, 9\}$, $T_y = (\varnothing, (7\}, Y\}$, $T^{\alpha}_y = T_y \cup ((7, 8\}, (7, 9\}\}$.
Define f:X→Y, by $f(x_1) = 7$, $f(x_2) = 8$, $f(x_3) = f(x_4) = 9$.
It is simply seen, f be continuous, then be no *semiα**-continous, then
$(7, 8\} \in S\alpha O(Y)$, but $f^{-1}((7, 8\}) = (0, 2\} \notin S\alpha O(X)$.
Therefore f is continuous however it is not *semiα**-continuous.

Example 3.12.
Let us equip that the sets X and Y of the above example with topologies,

$$T_x = (\varnothing, (0\}, X\},\ T^{\alpha}_x = T_x \cup ((0, 2\}, (0, 4\}, (0, 6\}, (0, 2, 4\}, (0, 2, 6\}, (0, 4, 6\}\}$$

$S\alpha O$ (X)= T^{α}_x, T_y ={∅,{7},Y}, $T^{\alpha}_y = T_y \cup ((7, 8\}, (7, 9\}\}$, $S\alpha O(Y) = T^{\alpha}_y$,
Then describe f:X→Y, by $f(x_1) = 7$, $f(x_2) = 8$, $f(x_3) = f(x_4) = 9$.
It is simply told that f is *semiα**-continuous, but it is not continuous,

Because $\{7\}$is *open* of Y. then $f^{-1}((7\}) = \{0, 2\}$be *open* of X.
Therefore f is *semiα**-continuous, however it is not continuous.

Definition 3.13.. [17]

If $f : X \to Y$ is a *function*, thus f *is* termed α^{**}-*continuous* if and only if, For each N α-*open set* of Y , thus $f^{-1}(N)$ be *open set* of X .

Example 3.14.

If $X = (5, 3, 1, 0\}$, $T_X = (\varnothing, (5, 1\}, (5, 3, 1\}, X\}$,
$T_x^{\alpha} = T_X \cup \{5, 3, 0\}$. With f *is* Identity function.
$f(x_1) = f(x_2) = 3$, $f(x_3) = f(x_4) = 1$. 1. Thus $f$be α-*open* set in Y,
Because $\{5,3,1\}$ is *op* $f(x_1) = f(x_2) = 3$, $f(x_3) = f(x_4) = 1$then
of X, $f^{-1}((5, 3, 1\}) = X$ an open in X.
Hence f is α-open and open function. So f is α^{**}-continuous.

4. CONCLUSION

For topological space, through our study between the relations, continuous, alpha-continuous, and semi-alpha-continuous. we get a direct representation of their abbreviation ; the relationship continuous→alpha-continuous→semi-alpha continous. And prove ; $f : (X, T_x) \to \left(Y, T_y\right)$ *are* alpha star-continuous $\Longleftrightarrow f : (X, T_x^{\alpha}) \to (Y, T_y^{\alpha})$ are continuous.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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