Intuitionistic Fuzzy Ideals Topological Spaces

Prof. Mohammed Jassim Mohammed

Wedad Salman Mohammad

College of Educatior for Pure Sciences

Department of Mathematics ,Thi-Qar University , Thi-Qar , Iraq

Abstract

In this paper we introduce the definition of intuitionistic fuzzy Remotedneighbourhoods and prove some results about it . Also the concept of limit points and cluster points of intuitionistic fuzzy ideal are also introduced and prove some results about them .

Keywords: intuitionistic fuzzy ideal ,intuitionistic fuzzy topological space , intuitionistic fuzzy point .

Introduction

The concept of fuzzy set and fuzzy operations was first introduced by Zadeh [15]. According fuzzy topological spaces introduced by Chang [4]. Several researches were study the generalizations of the notion of fuzzy sets and fuzzy topology [4,23]. The idea of intuitionistic fuzzy set was first published by Atanassov [11,12,13].subsequently , Coker and Saadati [5,21] defined the notion of intuitionistic fuzzy topology and studied the basic concept of intuitionistic fuzzy point [5,6,21]. The notion of ideal topological spaces studied by Kuratowski [14] and Vaidyanathaswamy [22] , and many authors did a lot of works on ideal topological space [7,18,19,20,24]. The concept of fuzzy ideal topological spaces ,initiated by Sarker [8] . M. H. Ghanim , A. A. Nouh and G. A. Gameel[17] they gave several characterizations for Remotedneighbourhood ,limit point and cluster point in fuzzy ideal topological space . Also G. A. Gameel [9] introduced the Remotedneighbourhood in fuzzy topological space .A. A. Salama [1] generalize the concept of fuzzy ideal topological spaces into intuitionistic fuzzy ideal topological spaces and prove some results and defined intuitionistic fuzzy ideal set for a set which is considered as a generalization of fuzzy ideal studied in [2,3,8,16,17] and introduced the notion of intuitionistic fuzzy local function corresponding to intuitionistic fuzzy topological space .This paper consists four sections , In section one we study some definitions which needed in other sections , In section twowe introduce the definition of intuitionistic fuzzy ideal with some properties , section three include the definition ofRemoted in intuitionistic fuzzy and prove some results about it and the concept limit points and cluster points of intuitionistic fuzzy ideal introduced in section four .

1. Preliminaries

Let X is anon empty fixed set , An intuitionistic fuzzy set A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of member ship (namely $\mu_A(x)$) and the degree of non-member ship (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively , and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$ [1,4,11,12]. Let X be anon empty set an intuitionistic fuzzy point ,denoted by $x_{(\alpha,\beta)}$ is an intuitionistic fuzzy set A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, suchthat

$$\mu_A(y) = \begin{cases} \alpha if x = y \\ 0 \quad e.w \end{cases}$$

$$\nu_A(y) = \begin{cases} \beta if x = y \\ 1 \quad e.w \end{cases}$$

Where $x \in X$is a fixed point , and constants $\alpha, \beta \in I$, satisfy $\alpha + \beta \leq 1$.

The set of all intuitionistic fuzzy points $x_{(\alpha,\beta)}$ is denoted by $IFP(X)$. An intuitionistic fuzzy topology on a non empty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms : (i) $0_\sim, 1_\sim \in \tau$, (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$, (iii) $\bigcup G_i \in \tau$ for any arbitrary family $\{ G_i : i \in J \} \subseteq \tau$,in this case the pair (X, τ) is called an intuitionistic fuzzy topological space [6,20] and any intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X (IFOS for short) . The complement A^c of IFOS A in intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy closed set (IFCSfor short)[2,5]. The closure [8] of an intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ of X denoted by $cl(A)$, is the intersection of all intuitionistic fuzzy closed sets which contains A .The interior [8] of an intuitionistic fuzzy set A of X denoted by $int(A)$ is the union of all intuitionistic fuzzy open sets of X contained in A . Let A= $\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X \}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X \}$ be two intuitionistic fuzzy sets in X. A is said to be quasi-coincident with B (written A q B) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$ [5] , otherwise A not quasi-coincident with B and denoted by $A\tilde{q}B$. If $x_{(\alpha,\beta)} \in IFP(X)$ and A$\in$ IFS .We say that $x_{(\alpha,\beta)}$ quasi-coincident with A, denoted $x_{(\alpha,\beta)}qA$ if and only if $\alpha > \nu_A(x)$ or $\beta < \mu_A(x)$other wise $x_{(\alpha,\beta)}$ not quasi-coincident with A and denoted by $x_{(\alpha,\beta)}\tilde{q}A$[5]. Let X is non-empty set and L a non-empty family of intuitionistic fuzzy sets.We will call L is an intuitionistic fuzzy ideal on X if : (i)$A \in L$ and $B \subseteq A$ implies $B \in L$ [heredity], (ii)$A \in L$ and $B \in L$ implies $A \vee B \in L$[finite additivity]. The triple (X, τ, L) denotes an intuitionistic fuzzy ideal topological space where L intuitionistic fuzzy ideal and τ intuitionistic fuzzy topology .The local function [1] for an intuitionistic fuzzy set A of X with respect to τ and L denoted by $A^*(L, \tau)$(briefly $A^*(L)$)in an intuitionistic fuzzy ideal topological space (X, τ, L) is the union of all intuitionistic fuzzy point $x_{(\alpha,\beta)}$ such that if $U \in \mathcal{N}_{x(\alpha,\beta)}$ and $A^*(L, \tau) = \{ x_{(\alpha,\beta)} \in X : A \wedge U \notin L$ for every $U \in \mathcal{N}_{x(\alpha,\beta)} \}$ .The intuitionistic fuzzy clousure operator of an intuitionistic fuzzy set A denoted by $cl^*(A)$ in (X,τ,L) defined as $cl^*(A) = A^* \vee A$ for any IFS A of X . In an intuitionistic fuzzy ideal topological space (X,τ,L) , the collection $\tau^*(L)$ means an extension of IFTS finer than τ via IFL which constructed by considering the class $\mathcal{B}(L,\tau) = \{A - B : A \in \tau, B \in L\}$ as a base . such that $\tau^*(L)$ be IFT generated by cl^* .

2. Intuitionistic Fuzzy Ideal

Definition2.1:Let A be an intuitionistic fuzzy set in X then the (α,β)-level set and defined by $A_{(\alpha,\beta)}$=$\{x \in X : \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$ where $\alpha \in I_0, \beta \in I_1$.

Theorem2.2:Let A and B be two intuitionistic fuzzy sets then for any $\alpha \in I_0, \beta \in I_1$ we get :

1. $(A \cap B)_{(\alpha,\beta)} = A_{(\alpha,\beta)} \cap B_{(\alpha,\beta)}$.
2. $(A \cup B)_{(\alpha,\beta)} = A_{(\alpha,\beta)} \cup B_{(\alpha,\beta)}$such that $\alpha + \beta = 1$.

Definition2.3[1] :

Let X is a nonempty set and L a nonempty family of intuitionistic fuzzy sets . We will call L is an intuitionistic fuzzy ideal (IFL for short) on X if :

1) $A \in L$ and $B \subseteq A \implies B \in L$.(heredity)
2) $A \in L$ and $B \in L \implies A \vee B \in L$.(finite additivity)

Let (X,τ) be a topological space and L be an intuitionistic fuzzy ideal on X then (X,τ,L) is said to be intuitionistic fuzzy ideal topological space .

An intuitionistic fuzzy ideal L is called a δ- Intuitionistic fuzzy ideal if $\{A_j\}_{j\in N} \leq L$ implies $\bigvee_{j\in J} A_j \in L$ [countable] .

The smallest and largest intuitionistic fuzzy ideals on a nonempty set X are $\{0_\smile\}$ and intuitionistic fuzzy sets on X . Also ,$F.L_f$, $F.L_c$are denoting the intuitionistic fuzzy ideal set of fuzzy subsets having finite and countable support of X respectively .

Example2.4:Let A= $\langle x, 0.2, 0.6\rangle$, $B = \langle x, 0.5, 0.3\rangle$, $C = \langle x, 0.3, 0.4\rangle$,then the family L={ $0_\smile$,A,B,C} of intuitionistic fuzzy sets is an intuitionistic fuzzy ideal on X .

Definition 2.5[1]: Let A is a nonempty intuitionistic fuzzy set in X , then $\{B \in IFS : B \subseteq A\}$ is an IFL on X .This is called the principal intuitionistic fuzzy ideal of and denoted by IFL$\langle A\rangle$ and A is called base L .

Remark 2.6[1]:

i. If $1_{\sim} = \{\langle x, 1,0\rangle : x \in X\} \notin L$,then L is called intuitionistic fuzzy proper ideal .
ii. If $1_{\sim} \in L$, then L is called intuitionistic fuzzy improper ideal .
iii. $0_{\sim} = \{\langle x, 0,1\rangle : x \in X\} \in L$, $\forall$ L .

Definition 2.7[1]:LetL_1 and L_2 are two IFLs on X .Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2if $L_1 \leq L_2$. If also $L_1 \neq L_2$ .Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2 .
Two IFLs said to be comparable if one is finer than the other .The set of all IFLs on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in IFSs .
Salama in [1] state some proposition with out prove in this section we prove this proposition

Proposition2.8[1]: Let $\{L_j : j \in J\}$ be any non empty family of intuitionistic fuzzy ideal sets on a set X .Then $\bigcup_{j\in J} L_j$ and $\bigcap_{j\in J} L_j$ are IFL on X where $\bigcup_{j\in J} L_j=\langle \vee \mu_{L_j} , \wedge \nu_{L_j}\rangle$ such that

$\vee \mu_{L_j}(x)$ =sup $\{\mu_{A_i}(x) : i \in J , x \in X\}$

$\wedge \nu_{L_j}(x)$ =inf $\{\nu_{A_i}(x) : i \in J , x \in X\}$

and where$\bigcap_{j\in J} L_j=\langle \wedge \mu_{L_j} ,\vee \nu_{L_j}\rangle$ such that

$\wedge \mu_{L_j}(x)$ =inf $\{\mu_{A_i}(x) : i \in J , x \in X\}$

$\vee \nu_{L_j}(x)$=sup $\{\nu_{A_i}(x) : i\in J , x \in X\}$

Proposition2.9[1]: An intuitionistic fuzzy set A in intuitionistic fuzzy ideal L on X is a base of L if and only if every member of L contained in A .

Proposition2.10:Let $f :(X,\tau_1) \longrightarrow (Y,\tau_2)$ be a function and L , J are two intuitionistic fuzzy ideals on X and Y respectively Then

1. f(L)={f(A) : A $\in$ L } is an intuitionistic fuzzy ideal .

2. If f is injection .Then $f^{-1}(J)$ is an intuitionistic fuzzy ideal .

Proposition2.11[5]: Let A , B be IFSs and $x_{(\alpha,\beta)}$ an IFP in X .Then

1. $A\tilde{q}B^c \Leftrightarrow A \leq B$,
2. $AqB \Leftrightarrow A \nleq B^c$,
3. $x_{(\alpha,\beta)} \in A \Leftrightarrow x_{(\alpha,\beta)}\tilde{q}A^c$,
4. $x_{(\alpha,\beta)}qA \Leftrightarrow x_{(\alpha,\beta)} \notin A^c$.

Definition2.13: The set of all lower semi –continuous function from (X,τ) into the closed unit interval equipped with the usual topological constitutes, an intuitionistic fuzzy topology on X . It is called the induced intuitionistic fuzzy topology by (X,τ) and is denoted by $(X,\omega(\tau))$.

Definition 2.14: Let (X,τ) be an intuitionistic fuzzy topological space and $\alpha \in I_0$, $\beta \in I_1$ then the family $\{A_{(\alpha,\beta)} : A \in \tau\}$, constitutes a subbase for the so called initial topology associated with (X,τ) , and is denoted by $(X,\ell(\tau))$.

Definition2.15: An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong strongly to an intuitionistic fuzzy set $A = \langle x,\mu_A,\gamma_A\rangle$ in X , denoted by $x_{(\alpha,\beta)} \in^* A$ if $\alpha < \mu_A(x)$ and $\beta > \gamma_A(x)$.

Definition2.16[5]: AnIFP$x_{(\alpha,\beta)}$ is said to be belongs to an intuitionistic fuzzy set A in X , denoted by $x_{(\alpha,\beta)} \in A$ if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.

3. <u>Remotedneighbourhoods</u>

Definition3.1: Let (X,τ) be an intuitionistic fuzzy topological space and $x_{(\alpha,\beta)} \in IFP(X)$ then :

i. An intuitionistic fuzzy set A such that $x_{(\alpha,\beta)} \in A$ is said to be aneighbourhood of $x_{(\alpha,\beta)}$ if $\exists\, G \in \tau , x_{(\alpha,\beta)} \in G \subseteq A$. Let $\mathcal{N}x_{(\alpha,\beta)}$ be the intuitionistic fuzzy filter generated by all the neighbourhoods of $x_{(\alpha,\beta)}$ i.e $\mathcal{N}x_{(\alpha,\beta)} = \{A \in IFS : \exists G \in \tau , x_{(\alpha,\beta)} \in G \subseteq A\}$ each intuitionistic fuzzy set belonging $\mathcal{N}x_{(\alpha,\beta)}$ is said to be a nbd. of $x_{(\alpha,\beta)}$.

ii. An intuitionistic fuzzy set A such that $x_{(\alpha,\beta)} \in^* A$ is said to be a$*$-neighbourhood of $x_{(\alpha,\beta)}$ .The symbol $\mathcal{N}^*x_{(\alpha,\beta)}$ denoted the intuitionisticfilter generated by all the $*$-neighbourhood of $x_{(\alpha,\beta)}$.

i.e

$$\mathcal{N}^*x_{(\alpha,\beta)} = \{A \in IFS : \exists G \in \tau , x_{(\alpha,\beta)} \in^* G \subseteq A\}$$

Each intuitionistic fuzzy set belong to $\mathcal{N}^*x_{(\alpha,\beta)}$ is said to be $*$-neighbourhood of $x_{(\alpha,\beta)}$.

Definition 3.2:The intuitionistic fuzzy filter generated by all the Q-neighbourhood of an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is defined by

$$\mathcal{N}^{Q}x_{(\alpha,\beta)} = \{A \in IFS : \exists G \in \tau , x_{(\alpha,\beta)} q G \subseteq A \}$$

Each intuitionistic fuzzy set belonging to $\mathcal{N}^{Q}$ is said to be a Q-neighbourhood of $x_{(\alpha,\beta)}$.

Definition 3.3:An intuitionistic fuzzy set A is said to be an R-neighbourhood of an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ if there exist closed set $G \subseteq \tau^{c}$ we have $x_{(\alpha,\beta)} \notin G$ and $A \subseteq G$. The collection of all R-neighbourhoods of an intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is denoted by $R_{x_{(\alpha,\beta)}}$

$$R_{x_{(\alpha,\beta)}} = \left\{ A \in IFS : \exists G \in \tau^{c} , x_{(\alpha,\beta)} \notin G , A \subseteq G \right\}$$

Theorem 3.4 :Let(X, τ) be an IFTS , $x_{(\alpha,\beta)} \in IFP(X)$ and $A \in IFS$. Then $A \in \tau^{c}$ if and only if $A \in R_{x(\alpha,\beta)}$ $\forall$ $x_{(\alpha,\beta)} \notin A$.

Proof

$(\Longrightarrow)$Let $A \in \tau^{c}$ and $x_{(\alpha,\beta)} \notin A$, then $\exists A \in \tau^{c}$ s.t $x_{(\alpha,\beta)} \notin A$, $A \subseteq A$

Hence $A \in R_{x(\alpha,\beta)}$

$(\Longleftarrow)$let$A \in R_{x(\alpha,\beta)}$, $\forall$ $x_{(\alpha,\beta)} \notin A$, then $\exists B \in \tau^{c}$ such that

$x_{(\alpha,\beta)} \notin B$, $B \supseteq A$. Thus $\exists G = B^{c} \in \tau$ such that

$x_{(\beta,\alpha)} \in^{*} G$, $G = B^{c} \subseteq A^{c}$by proposition (1.1.10)

Then $A^{c} \in \tau$

Therefore $A \in \tau^{c}$ ■ .

Theorem 3.5:Let(X, τ) be an intuitionistic fuzzy topological space , $x_{(\alpha,\beta)} \in IFP(X)$ and $A, B \in IFS$. Then :

i. $0_{\sim} \in R_{x(\alpha,\beta)}$.
ii. If $A, B \in R_{x(\alpha,\beta)}$, then $A \cup B \in R_{x(\alpha,\beta)}$.
iii. If $A_{i} \in R_{x(\alpha,\beta)}$, $i \in I$, then $\bigcap_{i \in I} A_{i} \in R_{x(\alpha,\beta)}$.

Proof

(i) Since $\exists\, 0_{\sim} \in \tau^{c}$ such that $x_{(\alpha,\beta)} \notin 0_{\sim}$, $0_{\sim} \supseteq 0_{\sim}$.
Then $0_{\sim} \in R_{x(\alpha,\beta)}$

(ii) Let $A, B \in R_{x(\alpha,\beta)}$, then $\exists H_{1}, H_{2} \in \tau^{c}$ such that

$x_{(\alpha,\beta)} \notin H_1$ and $A \subseteq H_1$ and $x_{(\alpha,\beta)} \notin H_2$ and $B \subseteq H_2$ for some $x \in X$

Then $x_{(\alpha,\beta)} \notin \max\{H_1, H_2\}$.

So $x_{(\alpha,\beta)} \notin (H_1 \cup H_2)$ and $H_1 \cup H_2 \supseteq A \cup B$, $H_1 \cup H_2 \in \tau^c$[because τ^ctopology]

Then $x_{(\alpha,\beta)} \notin H_1 \cup H_2$, $H_1 \cup H_2 \supseteq A \cup B$

Hence $A \cup B \in R_{x(\alpha,\beta)}$.

(iii) Let $A_i \in R_{x(\alpha,\beta)}$ $i \in I$, then $\exists\, B_i \in \tau^c$ such that

$$x_{(\alpha,\beta)} \notin B_i \ , \ A_i \subseteq B_i$$

$$\Rightarrow \bigcap A_i \subseteq \bigcap B_i$$

Since $B_i \in \tau^c \Rightarrow \bigcap B_i \in \tau^c$

Since $x_{(\alpha,\beta)} \notin B_i \Rightarrow x_{(\alpha,\beta)} \notin \bigcap B_i$ then $x_{(\alpha,\beta)} \notin \bigcap A_i\ i \in I$

Hence $\bigcap_{i\in I} A_i \in R_{x(\alpha,\beta)}$ ■ .

Theorem 3.6 :Let X be a non empty set and $x_{(\alpha,\beta)} \in IFP(X)$,let $R_{x(\alpha,\beta)} \subseteq IFS$ satisfies the properties in theorem (3.5) .

$\tau^c(R) = \left\{ A \in IFS : \ \forall\, x_{(\alpha,\beta)} \notin A \ , A \in R_{x(\alpha,\beta)} \right\} \cup \{1_\smile\}$.

Then $\tau(R) = \{ G \in IFS : G^c \in \tau^c(R)\}$ is an intuitionistic fuzzy topological space on X .

Proof

(i)*since* $0_\smile, 1_\smile \in \tau^c(R)$, then $1_\smile, 0_\smile \in \tau(R)$.

(ii)Let $A, B \in \tau(R) \Rightarrow A^c, B^c \in \tau^c(R)$

Then $x_{(\alpha,\beta)} \notin A^c$ and $x_{(\alpha,\beta)} \notin B^c$

Since $A^c, B^c \in \tau^c(R)$, then $A^c \in R_{x(\alpha,\beta)}$ and $B^c \in R_{x(\alpha,\beta)}$

Thus by condition (ii) in theorem (1.3.5)

Then $A^c \cup B^c \in R_{x(\alpha,\beta)}$. Hence $A^c \cup B^c \in \tau^c(R)$

$$\Rightarrow A \cap B \in \tau(R)$$

(ii) Let $\{ A_j : j \in J \} \subseteq \tau(R)$

Then $\{ A^c{}_j : j \in J \} \subseteq \tau^c(R)$

Let $x_{(\alpha,\beta)} \notin \bigcap_{j\in J} A^c{}_j$, then $\exists\, j_\circ \in J$ such that $x_{(\alpha,\beta)} \notin A_{j_\circ}$

Since $A^c{}_{j_\circ} \in \tau^c(R)$, then $A^c{}_{j_\circ} \in R_{x(\alpha,\beta)}$. But $\bigcap_{j\in J} A^c{}_j \subseteq A^c{}_{j_\circ}$

$$\bigcap_{j\in J} A^c{}_j \in R_{x(\alpha,\beta)} \quad \Rightarrow \quad \bigcap_{j\in J} A^c{}_j \in \tau^c(R)$$

Hence $\bigcup_{j\in J} A_j \in \tau(R)$ ■ .

Proposition 3.7:Let (X,τ) be an intuitionistic fuzzy topological space and $A,B \in$ IFS and $\alpha \in I_0$, $\beta \in I_1$ $s.t$ $\alpha+\beta=1$Then :

i. $A \in \mathcal{N}^{Q}x_{(\alpha,\beta)}$ if and only if $A^c \in Rx_{(\alpha,\beta)}$.

ii. $A \in Rx_{(\alpha,\beta)}$ if and only if $A^c \in \mathcal{N}^{*}x_{(\beta,\alpha)}$ if $\mu_A(x)+\nu_A(x)=1$ for any intuionistic fuzzy set .

iii. If $A \subseteq B$ and $C \in R_A$then $\in R_B$.

Proof

(i)($\Longrightarrow$) suppose that $A \in \mathcal{N}^{Q}x_{(\alpha,\beta)}$

Then $\exists\, G \in \tau$ such that $x_{(\alpha,\beta)}qG \subseteq A$

Since $x_{(\alpha,\beta)}qG$

By proposition (2.11) in (4)

We get $x_{(\alpha,\beta)} \notin G^c$

Since $G \subseteq A$

Therefore $A^c \subseteq G^c$, let $G^c = F$

Thus $\exists\, F \in \tau^c$ such that $x_{(\alpha,\beta)} \notin F$ and $A^c \subseteq F$

Hence$A^c \in Rx_{(\alpha,\beta)}$.

($\Longleftarrow$)suppose that $A^c \in Rx_{(\alpha,\beta)}$

$\Longrightarrow$ $\exists\, G \in \tau^c$such that $x_{(\alpha,\beta)} \notin G$ and $A^c \subseteq G$

Since $x_{(\alpha,\beta)} \notin G$

By proposition (2.11)$\Longrightarrow$ $x_{(\alpha,\beta)}qG^c$ ……(1)

Since $A^c \subseteq G$ $\Longrightarrow G^c \subseteq A$ ……..(2)

$\Longrightarrow$ $\exists\, G^c \in \tau$such that $x_{(\alpha,\beta)}qG^c \subseteq A$

Then A is an intuitionistic fuzzy Q-nbd. of$x_{(\alpha,\beta)}$

Thus $A \in \mathcal{N}^{Q}x_{(\alpha,\beta)}$

(ii)($\Longrightarrow$)suppose that $A \in Rx_{(\alpha,\beta)}$

$\Longrightarrow$ $\exists\, G \in \tau^c$, $x_{(\alpha,\beta)} \notin GandA \subseteq G$

Since $x_{(\alpha,\beta)} \notin G$

Then $x_{(\alpha,\beta)}qG^c$

Therefore $\exists\, x \in X$, $\alpha > \mu_G(x)$ or $\beta < \nu_G(x)$

Since $\alpha+\beta=1$ and $\mu_A(x)+\nu_A(x)=1$

Therefore $\beta < \nu_G(x)$ and $\alpha > \mu_G(x)$

Then $x_{(\beta,\alpha)} \in G^c$, let$G^c = H$ such that $H \in \tau$

Then $x_{(\beta,\alpha)} \in H$

Then $x_{(\beta,\alpha)} \in^{*} G^c = H$ ……..(1)

Since $A \subseteq G$

Then $G^c \subseteq A^c$

Therefore $H \subseteq A^C$ ……… (2)

$$A^c \in \mathcal{N}^* x_{(\beta,\alpha)}$$

($\Leftarrow$)suppose that $A^c \in \mathcal{N}^* x_{(\beta,\alpha)}$

$$\Longrightarrow \exists F \in \tau \ , \quad x_{(\beta,\alpha)} \in^* F \subseteq A^c$$

$$\Longrightarrow \beta < \mu_F(x) \wedge \alpha > \nu_F(x)$$

$$\Longrightarrow 1-\beta \geq 1-\mu_F(x) \vee 1-\alpha \leq 1-\nu_F(x)$$

Since $\mu_F(x)+\nu_F(x)=1$ and $\alpha+\beta=1$

Then $\alpha \geq \nu_F(x) \ \wedge \ \beta \leq \mu_F(x)$

Therefore $x_{(\alpha,\beta)} \notin F^c$ ……… (1) such that $F^c \in \tau^c$

Since $F \subseteq A^c \quad \Longrightarrow A \subseteq F^c$ ……… (2)

Therefore $A \in Rx_{(\alpha,\beta)}$.

(iii) suppose that $A \subseteq B$ and $D \in R_A$ such that$A \subseteq B$

$R_A = \{H \in IFS : \exists G \in \tau^c$such that $A \nsubseteq G$ and $H \subseteq G \}$

$\Longrightarrow \exists G \in \tau^c$such that $A \nsubseteq G$ and $D \subseteq G$

Since $A \nsubseteq G$

$$\Longrightarrow B \nsubseteq G$$

So $\exists G \in \tau^c$ such that $B \nsubseteq G$ and $D \subseteq G$

$\Longrightarrow D \in R_B$ ■ .

Definition 3.8:Let (X,τ)be a topological space a set A is said to be an R-neighabourhood of a point x if there exist a set $G \in \tau^c$ we have $x \notin G$ and $A \subseteq G$ .The collection of all R-neighabourhoods of a point x is denoted by R_x.

$R_x = \{ A \subseteq X : \exists G \in \tau^c , x \notin G$ and $A \subseteq G \}$.

Theorem 3.9: For each $\alpha \in (0,1]$, $\beta \in [0,1)$, $A \in IFS$ and $x_{(\alpha,\beta)} \in IFP(X)$, we have :

i. $A_{(\alpha,\beta)} \in R_x in\big(X, \ell(\tau)\big)$ if and only if $A \in Rx_{(\alpha,\beta)}$ in (X,τ) .

ii. $A_{(\alpha,\beta)} \in R_x$ in (X,τ) if and only $A \in Rx_{(\alpha,\beta)}$ in $\left(X, w_{(\alpha,\beta)}(\tau)\right)$.

Proof /

(i)($\Longrightarrow$)suppose that $A_{(\alpha,\beta)} \in R_x$ in $(X, \ell(\tau))$ such that

Sub base $\sigma = \{A_{(\alpha,\beta)} : A \in \tau\}$

$\Longrightarrow\ \exists\, G_{(\alpha,\beta)} \in (\ell(\tau))^c \ni x \notin G_{(\alpha,\beta)}$ and $A_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$ such that $G \in \tau$, $A \in \tau$

Then $x_{(\alpha,\beta)} \notin G$ ... ... (1)

Since $A_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$ such that $A \subseteq G$ ... ... (2)

$\Longrightarrow\ A \in Rx_{(\alpha,\beta)}$ in (X,τ) .

($\Longleftarrow$) suppose that $A \in Rx_{(\alpha,\beta)}$ in (X,τ)

$\Longrightarrow\ \exists\, G \in \tau^c \ni x_{(\alpha,\beta)} \notin G$ and $A \subseteq G$

Since $x_{(\alpha,\beta)} \notin G$

Thus $x \notin G_{(\alpha,\beta)}$ ... ... (1)

Since $A \subseteq G$

Then $A_{(\alpha,\beta)} \subseteq B_{(\alpha,\beta)}$ ... ... (2)

Hence $A_{(\alpha,\beta)} \in R_x$ in $(X, \ell(\tau))$.

(ii)($\Longrightarrow$)suppose that $A_{(\alpha,\beta)} \in R_x$ in (X,τ)

$\Longrightarrow\ \exists\, G_{(\alpha,\beta)} \in \tau^c \ni$,$x \notin G_{(\alpha,\beta)}$ and $A_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$

Then $A \subseteq G$

Since $A_{(\alpha,\beta)} \in \tau$ then $A \in w_{(\alpha,\beta)}(\tau)$and sinceG$_{(\alpha,\beta)} \in \tau^c$ then $G \in (w_{(\alpha,\beta)}(\tau))^c$

$\exists\, G \in (w_{(\alpha,\beta)}(\tau))^c$and we get $x_{(\alpha,\beta)} \notin G$ ... ... (1)

Since $A_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$so ,$A \subseteq G$ ... ... (2)

Hence $A \in Rx_{(\alpha,\beta)}$ in $(X, w_{(\alpha,\beta)}(\tau))$.

($\Longleftarrow$) suppose that $A \in Rx_{(\alpha,\beta)}$ in $(X, w_{(\alpha,\beta)}(\tau))$

Therefore

$\exists\, G \in (\square_{(\square,\square)}(\square))^{\square}$such that $x_{(\alpha,\beta)} \notin G$ and $A \subseteq G$

Since $G \in (\square_{(\square,\square)}(\square))^{\square}$ then $G_{(\alpha,\beta)} \in \tau^c$

$A \subseteq G$ therefore $A_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$ and since $x_{(\alpha,\beta)} \notin G$

Thus $x \notin G_{(\alpha,\beta)}$

Hence $A_{(\alpha,\beta)} \in R_x$ in $(X,\tau)\square$.

4. Limit points and cluster points of intuitionistic fuzzy ideal

Definition 4.1: Let (X,τ) be an intuitionistic fuzzy topological space and L be an intuitionistic fuzzy ideal in X and $x_{(\alpha,\beta)}$ intuitionistic fuzzy point then :

1. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be limit point of L (written as $L \longrightarrow x_{(\alpha,\beta)}$) if $\forall A \in R_{x_{(\alpha,\beta)}}, \exists G \in L, A \subseteq G$ or equivalently $Rx_{(\alpha,\beta)} \subseteq L$. In this case we say that L convergence to $x_{(\alpha,\beta)}$.
2. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be an accumulation (or a cluster , or adherent point of L)(written as $L \propto x_{(\square,\square)}$) if $\forall A \in R_{x_{(\alpha,\beta)}}, \forall G \in L$ we *have* $A \cup G \neq 1_{\smile}$.

The union of all limit points (respectively all cluster points of L will be denoted by Lim(L) (respectively Adh(L)) i.e

$$\mathrm{Lim}(L) = \cup \{ x_{(\alpha,\beta)} : L \longrightarrow x_{(\alpha,\beta)} \}$$

$$\mathrm{Adh}(L) = \{ x_{(\alpha,\beta)} : L \propto x_{(\square,\square)} \} .$$

Proposition 4.2: Let L be an intuitionistic fuzzy ideal of an intuitionistic fuzzy topological space (X,τ) and $x_{(\alpha,\beta)} \in IFP(X)$ Then :

i. $L \longrightarrow x_{(\alpha,\beta)}$ iff $x_{(\alpha,\beta)} \in \mathrm{Lim}(L)$.
ii. $L \propto x_{(\square,\square)}$ iff $x_{(\alpha,\beta)} \in \mathrm{Adh}(L)$.

Proof

(i)($\Longrightarrow$)suppose that $L \longrightarrow x_{(\alpha,\beta)}$

Since $\mathrm{Lim}(L) = \cup \{ x_{(\alpha,\beta)} : L \longrightarrow x_{(\alpha,\beta)} \}$

From definition we get $x_{(\alpha,\beta)} \in \mathrm{Lim}(L)$.

($\Longleftarrow$)suppose that $x_{(\alpha,\beta)} \in \mathrm{Lim}(L)$,

And $G \in R_{x_{(\alpha,\beta)}}$, since $x_{(\alpha,\beta)} \notin G$ so $\mathrm{Lim}(L) \nsubseteq G$.

Then $\exists y_{(\alpha_\circ,\beta_\circ)} \in IFP(X) \ni L \longrightarrow y_{(\square_\circ,\square_\circ)}$ and $y_{(\alpha_\circ,\beta_\circ)} \notin G$,

So $G \in R_{y_{(\alpha_\circ,\beta_\circ)}}$ and $R_{y_{(\alpha_\circ,\beta_\circ)}} \subseteq L$ then $R_{x_{(\alpha,\beta)}} \subseteq L$,

Therefore $L \longrightarrow x_{(\alpha,\beta)}$.

(ii)($\Longrightarrow$)suppose that $L \propto x_{(\alpha,\beta)}$

i.e $x_{(\alpha,\beta)}$ a cluster point of L

From definition we get $x_{(\alpha,\beta)} \in \cup \{x_{(\alpha,\beta)} : L \propto x_{(\alpha,\beta)}\}$

Hence $x_{(\alpha,\beta)} \in Adh(L)$.

($\Leftarrow$)suppose that $x_{(\alpha,\beta)} \in Adh(L)$

Let $G \in R_{x_{(\alpha,\beta)}}$, since $x_{(\alpha,\beta)} \notin G$ then $Adh(L) \nsubseteq G$.

$\exists\, y_{(\alpha_\circ,\beta_\circ)} \in IFP(X)$ such that $L \propto y_{(\alpha_\circ,\beta_\circ)}$ and $y_{(\alpha_\circ,\beta_\circ)} \notin G$.

So ,$\forall A \in R_{y_{(\alpha_\circ,\beta_\circ)}}$, $\forall H \in L$ we get $A \cup H \neq 1_\sim$

Since $y_{(\alpha_\circ,\beta_\circ)} \notin G$, then $G \in Ry_{(\alpha_\circ,\beta_\circ)}$ and $G \cup H \neq 1_\sim$

Thus $\forall G \in R_{x_{(\alpha,\beta)}}$, $\forall H \in L \Longrightarrow G \cup H \neq 1_\sim$

Therefore $L \propto x_{(\alpha,\beta)}$ □.

Theorem4.3:Let L be a maximal intuitionistic fuzzy ideal in an intuitionistic fuzzy topological space (X,τ) . Then $Lim(L) = Adh(L)$.

Proof

Let $x_{(\alpha,\beta)} \in Lim(L)$ by proposition(4.2) we get $L \longrightarrow x_{(\alpha,\beta)}$ so $R_{x_{(\alpha,\beta)}} \subseteq L$

Let $A \in R_{x_{(\alpha,\beta)}}$ and $B \in L$

Since $R_{x_{(\alpha,\beta)}} \subseteq L \Rightarrow A \in L$

Since L intuitionistic fuzzy ideal then $A \vee B \in L$

Since L is a maximal intuitionistic fuzzy ideal $\Rightarrow 1_\sim \notin L$, so $A \cup B \neq 1_\sim$

Therefore for any $A \in Rx_{(\alpha,\beta)}$, $\forall B \in L$

therefore $A \cup B \neq 1_\sim$

then□ ∝ □(□,□)

By proposition(4.2) $x_{(\alpha,\beta)} \in Adh(L)$,

Hence $Lim(L) \subseteq Adh(L)$.

Let $x_{(\alpha,\beta)} \in Adh(L)$ by proposition(4.2) L ∝ x(□,□)

Let $L^* = \{ A \in IFS : \exists H \in L \text{ and } B \in R_{x_{(\alpha,\beta)}} \text{ with } A \subseteq H \cup B \}$

We prove that L^* is an intuitionistic fuzzy ideal

1) Let $A \in L^* \ni \exists H \in L$ and $C \in R_{x_{(\alpha,\beta)}} \Longrightarrow A \subseteq H \cup C$

And let $B \in L^* \ni \exists F \in L$ and $D \in R_{x_{(\alpha,\beta)}} \Rightarrow B \subseteq F \cup D$

$$\Longrightarrow A \cup B \subseteq (H \cup F) \cup (C \cup D)$$

$$\Longrightarrow A \cup B \in L^*$$

2) Let $A \in L^*$, $B \subseteq A$

Therefore $\exists$ $H \in L$ and $C \in R_{x_{(\alpha,\beta)}}$ and $A \subseteq H \cup C$

Since $B \subseteq A$ then $B \subseteq H \cup C$

So $B \in L^*$, and L^* is an intuitionistic fuzzy ideal

Clearly $R_{x_{(\alpha,\beta)}} \subseteq L^*$, and□ ⊆ □*.

since L is a maximal intuitionistic fuzzy ideal , then $L = L^*$

Thus$R_{x_{(\alpha,\beta)}} \subseteq L$ and $L \longrightarrow x_{(\alpha,\beta)}$

By theorem we get $x_{(\alpha,\beta)} \in Lim(L)$ then $Adh(L) \subseteq Lim(L)$

Hence $Adh(L) = Lim(L)$□ .

Theorem 4.4:Let L is intuitionistic fuzzy ideal . Then the family $L_{(\alpha,\beta)}(L) = \{ A_{(\alpha,\beta)} : A \in L \}$ is an ideal on X where $\in (0,1]$, $\beta \in [0,1)$□ + □ = *1*.

Proof

1. Let $A, B \in L_{(\alpha,\beta)}(L)$, $\exists$ $G, H \in L$ such tha
$$A = G_{(\alpha,\beta)} \quad , \quad B = H_{(\alpha,\beta)} \quad , G, H \in L \ , \ \alpha + \beta = 1$$
Since L is an intuitionistic fuzzy ideal then$G \cup H \in L$
Thus $(G \cup H)_{(\alpha,\beta)} \in L_{(\alpha,\beta)}(L)$
Since $(G \cup H)_{(\alpha,\beta)} = G_{(\alpha,\beta)} \cup H_{(\alpha,\beta)} \in L_{(\alpha,\beta)}(L)$ therefore
$$A \cup B = G_{(\alpha,\beta)} \cup H_{(\alpha,\beta)} = (G \cup H)_{(\alpha,\beta)} \in L_{(\alpha,\beta)}(L)$$
Then $A \cup B \in L_{(\alpha,\beta)}(L)$
2. Let $A \in L_{(\alpha,\beta)}(L)$ and $B \subseteq A$
Since $A \in L_{(\alpha,\beta)}(L)$ then $\exists$ $G \in L$ such that $A = G_{(\alpha,\beta)}$.
So $\forall \alpha \in (0,1], \beta \in [0,1)$, $\exists H \in IFS$ $\ni B = H_{(\alpha,\beta)}$
Since $B \subseteq A = G_{(\alpha,\beta)} \Rightarrow B \subseteq G_{(\alpha,\beta)}$ such that $G \in L$
So $H_{(\alpha,\beta)} \subseteq G_{(\alpha,\beta)}$
Then $H \subseteq G$, since $G \in L$ and L is an intuitionistic fuzzy ideal
Thus $H \in L$, therefore $B \in L_{(\alpha,\beta)}(L)$
Then $L_{(\alpha,\beta)}(L)$ is an intuitionistic fuzzy ideal .

Proposition 4.5:Let (X, τ) be an intuitionistic fuzzy topological space andL_1, L_2 be intuitionistic fuzzy ideals in X such that $L_1 \subseteq L_2$then :

i. If $L_2 \propto x_{(\alpha,\beta)}$ implies that $L_1 \propto x_{(\alpha,\beta)}$.

ii. If $L_1 \longrightarrow x_{(\alpha,\beta)}$ implies that $L_2 \longrightarrow x_{(\alpha,\beta)}$.

Proof

(□)Suppose that $L_2 \propto x_{(\alpha,\beta)}$, and let $G \in Rx_{(\alpha,\beta)}$, $A \in L_1$

Since $L_1 \subseteq L_2$, then $A \in L_2$

Since $L_2 \propto x_{(\alpha,\beta)}$

Then we have $A \cup G \neq 1_{\smile}$.

Hence$L_1 \propto x_{(\alpha,\beta)}$ □ .

(□□)Suppose that $L_1 \longrightarrow x_{(\alpha,\beta)}$ so $Rx_{(\alpha,\beta)} \subseteq L_1$

Since $L_1 \subseteq L_2$

$$Rx_{(\alpha,\beta)} \subseteq L_2$$

Hence$L_2 \longrightarrow x_{(\alpha,\beta)}$ □ .

Theorem 4.6:Let (X,τ) be an intuitionistic fuzzy space ,$x_{(\alpha,\beta)} \in IFP(X)$, $A \in IFS$ and L be an intuitionistic fuzzy ideal in X . Then $L \longrightarrow x_{(\alpha,\beta)}$ (respectively □ ∝ □$_{(□,□)}$ in (X,τ)) iff $L_{(\alpha,\beta)}(L) \longrightarrow x$ (respectively $L_{(\alpha,\beta)}(L) \propto_x$in $\left(X, \ell_{(\alpha,\beta)}(\tau)\right)$ $\forall \alpha \in (0,1], \beta \in [0,1)$.

Proof

$L \longrightarrow x_{(\alpha,\beta)}in(X,\tau)$ and let $F \in R_x$ in $(X, \ell_{(\alpha,\beta)}(\tau))$

Then $F = G_{(\alpha,\beta)}$ s.t $G \in \tau$, by theorem (3.9) $G \in Rx_{(\alpha,\beta)}$ in (X,τ)

Since $L \longrightarrow x_{(\alpha,\beta)}$

Then $\exists A \in L$ such that $G \subseteq A$ so $G_{(\alpha,\beta)} \subseteq A_{(\alpha,\beta)} \in L_{(\alpha,\beta)}(L)$

Therefore $L_{(\alpha,\beta)}(L) \longrightarrow x$ in $(X, \ell_{(\alpha,\beta)}(\tau))$.

Conversely let $\ell_{(\alpha,\beta)}(L) \longrightarrow x$ in $(X, \ell_{(\alpha,\beta)}(L))$ and let $G \in Rx_{(\alpha,\beta)}$

By theorem (3.9) $G_{(\alpha,\beta)} \in R_x$in $(X, \ell_{(\alpha,\beta)}(\tau))$, then $\exists F \in \ell_{(\alpha,\beta)}(L)$

$G_{(\alpha,\beta)} \subseteq F$, so $G_{(\alpha,\beta)} \subseteq A_{(\alpha,\beta)}$ for some $A \in L \implies G \subseteq A$

Thus $L \longrightarrow x_{(\alpha,\beta)}$in (X,τ) .

Let $L \propto_{(□,□)}$in (X,τ) ,and$F \in R_x$ in $(X, \ell_{(\alpha,\beta)}(\tau))$ Let $H \in \ell_{(\alpha,\beta)}(L)$

Then $\exists G \in$ □$^{□}$, $A \in L$ where $H = A_{(\alpha,\beta)}$, $F = G_{(\alpha,\beta)} \in R_x$ by theorem (3.9)

$G \in Rx_{(\alpha,\beta)}$.since $L \propto x_{(□,□)}$then $A \cup G \neq 1_{\smile}$.

Thus $A_{(\alpha,\beta)} \cup G_{(\alpha,\beta)} \neq X$, $\forall \alpha \in (0,1] , \beta \in [0,1)$, $F \cup H \neq X$.

Hence $\ell_{(\alpha,\beta)}(L) \propto x$in $(X, \ell_{(\alpha,\beta)}(L))$.

Conversely , Let $\ell_{(\alpha,\beta)}(L) \propto_x$ in $\left(X, \ell_{(\alpha,\beta)}(\tau)\right)$
and$G \in Rx_{(\alpha,\beta)}$ in $(X,\tau), S \in L$.
by theorem (3.9) , $G_{(\alpha,\beta)} \in R_x$ in $\left(X, \ell_{(\alpha,\beta)}(L)\right)$.
Then $\exists\ F \in \ell_{(\alpha,\beta)}(\tau)$, $H \in \ell_{(\alpha,\beta)}(L)$ such that$F = G_{(\alpha,\beta)}$, $H = S_{(\alpha,\beta)}$,
Since $\ell_{(\alpha,\beta)}(L) \propto x$ in $\left(X, \ell_{(\alpha,\beta)}(\tau)\right)$,
Then $G_{(\alpha,\beta)} \cup S_{(\alpha,\beta)} \neq X\ \ \forall\ \alpha \in (0,1]$, $\beta \in [0,1)$ so$G \cup S \neq 1_{\sim}$.
Thus $L \propto x_{(\alpha,\beta)}$ in (X,τ)□ .

Theorem 4.7:Let L is an ideal , then the family $w_{(\alpha,\beta)}(L) = \{ A :\ A_{(\alpha,\beta)} \in J \}$ is an intuitionistic fuzzy ideal on X and □ + □ = *1*.
Proof
1. Let $A, B \in w_{(\alpha,\beta)}(L)$, then $A_{(\alpha,\beta)}$, $B_{(\alpha,\beta)} \in L$
   Since L is ideal then$A_{(\alpha,\beta)} \cup B_{(\alpha,\beta)} \in L$, since $(A \cup B)_{(\alpha,\beta)} = A_{(\alpha,\beta)} \cup B_{(\alpha,\beta)}$
   So ,$(A \cup B)_{(\alpha,\beta)} \in L$, Hence $A \cup B \in w_{(\alpha,\beta)}(L)$.

2. Let $A \in w_{(\alpha,\beta)}(L)$, $B \subseteq A$
   Then $A_{(\alpha,\beta)} \in L$ and since $B \subseteq A$
   Therefore $B_{(\alpha,\beta)} \subseteq A_{(\alpha,\beta)}$, so sinceL is an ideal
   Then $B_{(\alpha,\beta)} \in L$, Thus $B \in w_{(\alpha,\beta)}(L)$
   Hence $w_{(\alpha,\beta)}(L)$ is an intuitionistic fuzzy ideal □ .

Theorem 4.8:Let (X,τ) be a topological space and L be ideal in X ,$L \longrightarrow x$ (respectively $L \propto x$) in (X,τ) iff $w_{(\alpha,\beta)}(L) \longrightarrow x_{(\alpha,\beta)}$ (respectively $w_{(\alpha,\beta)}L \propto x_{(\alpha,\beta)}$ in $(X, w_{(\alpha,\beta)}(\tau))$.
Proof
Let $L \longrightarrow x$ in (X,τ) and let $A \in Rx_{(\alpha,\beta)}$in $(X, w_{(\alpha,\beta)}(\tau))$
by theorem (3.9) $A_{(\alpha,\beta)} \in R_x$ in (X,τ) . Since $L \longrightarrow x$in (X,τ),
then ,$\exists\ B \in L$ such that $A_{(\alpha,\beta)} \subseteq B$, so $A_{(\alpha,\beta)} \subseteq H_{(\alpha,\beta)}$, $\forall\ \alpha \in I_0$, $\beta \in I_1$ for some $H \in w_{(\alpha,\beta)}(L)$. Hence $A \subseteq H$ .Thus $w_{(\alpha,\beta)}(L) \longrightarrow x_{(\alpha,\beta)}$in $(X, w_{(\alpha,\beta)}(\tau))$.

Conversely , let $w_{(\alpha,\beta)}(L) \longrightarrow x_{(\alpha,\beta)}$ in $\left(X, w_{(\alpha,\beta)}(\tau)\right)$ and let $B \in R_x$ in (X,τ) . Then $\exists\ G \in w_{(\alpha,\beta)}(\tau)$where $B = G_{(\alpha,\beta)}$ .by theorem (3.9) we get $G \in Rx_{(\alpha,\beta)}$ in $\left(X, w_{(\alpha,\beta)}(\tau)\right)$, since

$w_{(\alpha,\beta)}(L) \longrightarrow x_{(\alpha,\beta)}$, then $\exists\, S \in w_{(\alpha,\beta)}(L)$ such that $G \subseteq S$, so $G_{(\alpha,\beta)} \subseteq S_{(\alpha,\beta)}$, $\forall\, \alpha \in (0,1]$, $\beta \in [0,1)$ i.e $B \subseteq S_{(\alpha,\beta)}$, $S_{(\alpha,\beta)} \in L$

$\Longrightarrow L \longrightarrow x$ in (X,τ) .

Let $L \propto x$ in (X,τ) and let $A \in Rx_{(\alpha,\beta)}$, $B \in w_{(\alpha,\beta)}(L)$.by theorem (3.9) $A_{(\alpha,\beta)} \in R_x$ in (X,τ) .

Then $\exists\, F \in \tau$, $H \in L$ such that $F = A_{(\alpha,\beta)}$, $H = B_{(\alpha,\beta)}$,

sinceL $\propto x$ in (X,τ)so $B_{(\alpha,\beta)} \cup A_{(\alpha,\beta)} \neq X$, $\forall\, \alpha \in I_0$, $\beta \in I_1$, so $F \cup H \neq 1_{\sim}$.

Hence $w_{(\alpha,\beta)}(L) \longrightarrow x_{(\alpha,\beta)}$in $(X, w_{(\alpha,\beta)}(\tau))$.

Conversely let $w_{(\alpha,\beta)}(L) \propto x_{(\alpha,\beta)}$ in $(X, w_{(\alpha,\beta)}(\tau))$ and $F \in R_x$ in (X,τ) , $H \in L$, then $\exists\, G \in w_{(\alpha,\beta)}(\tau)$, $B \in w_{(\alpha,\beta)}(L)$

Where $F = G_{(\alpha,\beta)}$, $H = B_{(\alpha,\beta)}$ by theorem $G \in Rx_{(\alpha,\beta)}$.

sincew$_{(\alpha,\beta)}(L) \propto x_{(\alpha,\beta)}$

Since $G \cup B \neq 1_{\sim}$so ,$G_{(\alpha,\beta)} \cup B_{(\alpha,\beta)} \neq X$, $\forall\, \alpha \in I_0$, $\beta \in I_1$

Then $L \propto x$ in (X,τ)□ .

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