On Intuitionistic Fuzzy Topological Vector Space

Mohammed Jassim Mohammed
Ghufran Adeel Ataa
Department of Mathematics,
College of Education for pure science /University of The-Qar
www.ghufranalmusawi21@gmail.com
www.dr.mohammed965@yahoo.com

Abstract.

In this paper, we introduce some type of neighborhood of intuitionistic fuzzy topological space, and prove some results about it, also we define intuitionistic fuzzy topological vector space, intuitionistic fuzzy locally convex, and we prove some result about it.

Keywords: intuitionistic fuzzy topology, some type of neighborhood, intuitionistic fuzzy locally convex.

الملخص:

في هذا البحث نحن قدمنا بعض انواع من الجوارات في الفضاء التبلوجي الضبابي الحدسي, و برهنا بعض النتائج حول ذلك , كذلك عرفنا الفضاء التبولوجي المتجه الضبابي الحدسي , المجموعه المحدبه محليا الحدسية الضبابيه, وبرهنا بعض النتائج حول ذلك.

Introduction

The concept of fuzzy topological vector space was introduced rationally by Katsaras in 1981 [8]. According to the standardized terminology in [6], it should be called *I*-topological vector space, where $I = [0, 1]$. It is known that locally convex spaces play a quite important role in theory of classical topological vector spaces. So, it is also important tointroduce and to study locally convex *I*-topological vector spaces in the research of *I*-topological vector spaces.. In 1984,Katsaras [9] first gave the definition of locally convex *I*-topological vector spaces by means of *W*-neighborhood base [12]. Almost at the same time,Wu and Li [13,14] gave the other definition of locally convex *I*-topological vector spaces by using *Q*-neighborhood base [10] and investigated some properties of such spaces. It goes without saying that to make the relation between these two definitions clear is very necessary for the research of *I*-topological vector spaces.

In 2006, Hui Zhang,and Jin-xuan[15]studied the relation between two definition of locally convex*I*-topological vector spaces. Our results show that the locally convex *I*-topological vector spaces in the sense ofWu and Li[13,14] are certainly that in the sense of Katsaras [9]. The converse is false. That is, all locally convex *I*-topological vector spaces in the sense ofWu and Li [13,14] form a special subclass of that in the sense of Katsaras[9].

In 1983the idea of "intuitionistic fuzzy set" was first published by T.Atanassov [2] and many researchers have followed the same author and his colleagues.
In 1995, Coker D. [4] constructed the basic concept, so called "intuitionistic fuzzy points " and related objects such as "qusi-coincidence.
In 1997, Coker D. [5] gave the basic definition of "intuitionistic fuzzy topological space ".
In this paper, they studied the definition of intuitionistic fuzzy set, intuitionistic fuzzy topological space and we define convex intuitionistic fuzzy set, study some type of intuitionistic fuzzy neighborhood, prove some result about it, we write the definition of topological vector space, we write two definition of locally convex in intuitionistic fuzzy topological vector space, first definition of locally convex intuitionistic fuzzy topological vector space by means of intuitionistic fuzzy W- neighborhood base, second definition of locally convex intuitionistic fuzzy topological vector space by means of intuitionistic fuzzy Q- neighborhood base and we prove the relation between two definitions and we prove some properties about it.

1.Intuitionistic fuzzy topology

We will gave some definitions of intuitionistic fuzzy set, intuitionistic fuzzy point intuitionistic fuzzy topology. We will gave some preliminaries ,needed in this work. X will denoted a nonempty set; $I = [0,1]$, the closed unit interval of real line;I_0=(0,1]; I_1=[0,1); $IF(X)$the set of all intuitionistic fuzzy sets of X. By $(\underline{\alpha}, \underline{\beta})$we denote the constant intuitionistic fuzzy sets of X.

<u>Definition (1.1) [16]</u>

Let X be a non-empty set and let I be the closed interval [0,1] of the real line. A fuzzy set μ in X is characterized by membership function $\mu: X \longrightarrow I$, which associates with each point $x \in X$ its grade or degree of membership $\mu(x) \in I$.

<u>Definition (1.2) [2]</u>

Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A is an object having the form:

A=$\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$, where the functions $\mu_A: X \longrightarrow I$ and $v_A: X \longrightarrow I$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + v_A(x) \le 1$ for each $x \in X$.
Furthermore, we call :
$\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, $x \in X$, the intuitionistic index or hesitancy degree of x in A. It is obvious that $0 \le \pi(x) \le 1$, for each $\in X$.
Note: Every fuzzy set A on a set X is obviously an IFS having the form :
$\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle, x \in X\}$

Definition: (1.3)[5]

$\tilde{0} = \{\langle x, 0,1\rangle, x \in X\}$

$\tilde{1} = \{\langle x, 1,0\rangle, x \in X\}$

are the intuitionistic fuzzy sets corresponding to empty set and the entire universe respectively

Theorem(1.4) [2,11]

Let X be aset and an intuitionistic fuzzy sets A and B be in the forms: $A=\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$, $B=\{\langle x, \mu_B(x), v_B(x)\rangle, x \in X\}$, then :

1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$.

2)$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

3) $A^c = \{\langle x, v_A(x), \mu_A(x)\rangle, x \in X\}$.

4)$A \cap B=\{\langle x, \{\mu_A(x) \wedge \mu_B(x)\}, \{v_A(x) \vee v_B(x)\}\rangle, x \in X\} = \{\langle x, min\{\mu_A(x), \mu_B(x)\}, max\{v_A(x), v_B(x)\}\rangle, x \in X\}$.

5)$A \cup B = \{\langle x, \{\mu_A(x) \vee \mu_B(x)\}, \{v_A(x) \wedge v_B(x)\}, x \in X\rangle\} = \{\langle x, max\{\mu_A(x), \mu_B(x)\}, min\{v_A(x), v_B(x)\}, x \in X\rangle\}$

6)If $A \subseteq B \implies B^c \subseteq A^c$.

Definition (1.5) [2]

Let $\{A_i, i \in J\}$ be an arbitrary family on intuitionistic fuzzy setsin X, then :

1) $\cap A_i = \left\{\langle x, \wedge_i \mu_{A_i}(x), \vee_i v_{A_i}(x)\rangle, x \in X\right\}$.

2) $\cup A_i = \left\{\langle x, \vee_i \mu_{A_i}(x), \wedge_i v_{A_i}(x)\rangle, x \in X\right\}$.

Definition (1.6) [4]

Let X be a nonempty set. An intuitionistic fuzzy point, denoted by $x_{(\alpha,\beta)}$ is an intuitionistic fuzzy set Let $A = \{< x, \mu_A(x), v_A(x) >: x \in X\}$, such that

$$\mu_A(y) = \begin{cases} \alpha & if\ x = y \\ 0 & otherwise \end{cases}$$

$$v_A(y) = \begin{cases} \beta & if\ x = y \\ 1 & otherwise \end{cases}$$

Where $x \in X$ is a fixed point, and constants $\alpha, \beta \in I$, satisfy $\alpha + \beta \leq 1$.The set of all intuitionistic fuzzy points $x_{(\alpha,\beta)}$ is denoted by $P_t(IF(X))$and we say that $x_{(\alpha,\beta)} \in A$ if and only if $\alpha \leq \mu_A(x)$ and $\beta \geq v_A(x)$ for each $x \in X$.

Definition (1.7)

The constant intuitionistic fuzzy set in X is an intuitionistic fuzzy set has membership and non membership define as the following

$A = \{< x, \mu_A(x), v_A(x) >, x \in X\}$ where

$\mu_A(x) = \alpha$ and $v_A(x) = \beta$, $\forall x \in X$ Where $\alpha \in I_0, \beta \in I_1$

We denoted by $(\underline{\alpha}, \underline{\beta})$, clearly $(\underline{0}, \underline{1}) = \tilde{0}$ and $(\underline{1}, \underline{0}) = \check{1}$

Definition (1.8) [4]

Let $A = \{< x, \mu_A(x), v_A(x) >: x \in X\}$and Let$B = \{< x, \mu_B(x), v_B(x) >: x \in X\}$be two intuitionistic fuzzy sets in X. A is said to be quasi-coincident with B (written $A\ q\ B$) if there exists an element $x\ in\ X$ such that$\mu_A(x) + \mu_B(x) > 1 and v_A(x) + v_B(x) < 1$.Otherwise$A$ not consincident $A\overline{q}B$. And we say that $x_{(\alpha,\beta)}$ quasi-coincident with A, denoted $x_{(\alpha,\beta)}qA$ if$x_{(\alpha,\beta)} \in P_t(IF(X))\ and\ A \in IF(X)$. then $\mu_A(x) + \alpha > 1 and v_A(x) + \beta < 1$.otherwise$x_{(\alpha,\beta)}$not consincident$B x_{(\alpha,\beta)}\overline{q}B$.

Theorem(1.9)[4]

If A and B are intuitionistic fuzzy sets in X, then :

(1) $A\ \overline{q}\ B$ if and only if $A \subseteq B^c$

(2) AqB if and only if $A \not\subseteq B^c$.

Lemma(1.10)

For $A, A_i, B \in IF(X)\ and\ x_{(\alpha,\beta)} \in P_t(IF(X))$, we have, $A \subseteq B$ if and only if for $x_{(\alpha,\beta)} \in A$ then $x_{(\alpha,\beta)} \in B$.

Proof

($\Rightarrow$) Let $x_{(\alpha,\beta)} \in A$, then $\forall y \in X$, $\alpha \le \mu_A(y)$ and $\beta \ge v_A(y)$.

Since $\subseteq B$, we have $\mu_A(y) \le \mu_B(y)$ and $v_A(y) \ge v_B(y)$.

Thus $\alpha \le \mu_B(y)$ and $\beta \ge v_B(y)$. Hence $x_{(\alpha,\beta)} \in B$.

($\Leftarrow$) Let $x_{(\alpha,\beta)} \in A$, then $x_{(\alpha,\beta)} \in B$

This mean $\alpha \le \mu_A(x), \beta \ge v_A(x) \Longrightarrow \alpha \le \mu_B(x), \beta \ge v_B(x)$

Thus $\mu_A(x) \le \mu_B(x)$ and $v_A(x) \ge v_B(x)$. $\forall x \in X$

Hence $A \subseteq B$ ■

Definition (1.11)[5]

1)Let $A=\{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$ be an intuitionistic fuzzy set in X, the image of A under the function $f: X \longrightarrow Y$ which is denote it by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$f(A)=\{\langle y, f(\mu_A)(y), (1-f(1-v_A))(y)\rangle : y \in Y\}$ where $f(\mu_A)(y)=\begin{cases} sup_{x\in f^{-1}(y)}\mu_A(x) & , \text{ if } f^{-1}(y) \neq \emptyset \\ 0 & , \ otherwise \end{cases}$

and $(1\text{-}f(1\text{-}v_A))(y)=\begin{cases} inf_{x\in f^{-1}(y)} v_A(x) & , \ if\ f^{-1}(y) \neq \emptyset \\ 1 & , \ otherwise \end{cases}$

2)Let $B=\{\langle y, \mu_B(y), v_B(y)\rangle, y \in Y\}$ be an intuitionistic fuzzy set in Y, the preimage of B under f which is denote it by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by :
$f^{-1}(B)=\{\langle x, f^{-1}(\mu_B)(x), f^{-1}(v_B)(x)\rangle : x \in X\}$, where

$f^{-1}(\mu_B)(x) = \mu_B(f(x))$ and $f^{-1}(v_B)(x) = v_B\big(f(x)\big)$.

Definition (1.12)[5]

An intuitionistic fuzzy topology (IFT for short) on a non-empty set X is a family τ of an intuitionistic fuzzy set in X whichsatisfies the following conditions:

(a) $\tilde{0}, \tilde{1} \in \tau$;

(b) If $A, B \in \tau$, then $A \cap B \in \tau$, (closed under intersection);

(c) If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$, (closed under arbitrary union).

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space,($IFTS$ for short. Every member of $\tau$is called an intuitionistic fuzzy open fuzzy set ($IFOS$), and The complement A^c of an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy closed set ($IFCS$ for short).

Definition(1.13) [5]

Let (X, τ) be an intuitionistic fuzzy topological space and
$A = \{\langle x, \mu_A(x), v_A(x)\rangle, x \in X\}$ be an intuitionistic fuzzy set in X then, an intuitionistic fuzzy interior and intuitionistic fuzzyclosure of A are defined by.

$$int(A) = A^{\circ} = \cup \{ G : G\ is\ an\ IFOS\ in\ X\ and\ G \subseteq A\}$$

$$cl(A) = A' = \cap \{K: K is\ an\ IFCS\ in\ X\ and\ A \subseteq K\}$$

Definition(1.14) [5]

Let $(X,\tau), and\ (Y,\delta)$ are two intuitionistic fuzzy topological spaces and let $f: X \to Y$be a function. Then f is said to be an intuitionistic fuzzy continuous (IF continuous for short) if the preimage of each intuitionistic fuzzy set in δ is an intuitionistic fuzzy set in τ.

Definition (1.15)

Let τ be an intuitionistic fuzzy topological space on , the pair (X,τ) is called an intuitionistic fuzzy topological vector space ($IFTVS$ for short), if the following two operations of IFS on X

Addition

$+: X \times X \to X,\qquad (x,y) = x + y$ and

$$\cdot : F \times X \to X, \quad (k,x) = kx$$

Are intuitionistic fuzzy continuous, when F has the usual intuitionistic fuzzy topology, ($\Im_U$ generated by the usual topology U on F) and $X \times X$ and $F \times X$ the corresponding product intuitionistic fuzzy topologies.

Definition(1.16)

Let be an A, B intuitionistic fuzzy setin a vector space X over $F$and $\lambda \in F$ then we define $A + B$ and λA is an intuitionistic fuzzy set in X define by

$(A+B)(z) = \{< z, \vee_{x+y=z}\{\mu_A(x) \wedge \mu_B(y)\},\ \wedge_{x+y=z}\{\nu_A(x) \vee \nu_B(y)\} >\}$ and

$\lambda A = \{< x, \lambda\mu_A(x), \lambda\, \nu_A(x) >\}$ where

$$\lambda\mu_A(x) = \begin{cases} \mu_A(\frac{1}{\lambda}x) & if\ \lambda \neq 0, \quad \forall x \in X \\ 0 & if\ \lambda = 0, \quad x \neq 0 \\ \sup_{y \in X} \mu_A(y) & if\ \lambda = 0, \quad x = 0 \end{cases}$$

And

$$\lambda\nu_A(x) = \begin{cases} \nu_A(\frac{1}{\lambda}x) & if\ \lambda \neq 0, \quad \forall x \in X \\ 1 & if\ \lambda = 0, \quad x \neq 0 \\ \inf_{y \in X} \nu_A(y) & if\ \lambda = 0, \quad x = 0 \end{cases}$$

Definition(1.17)

Let X be a vector space over F, An Intuitionistic fuzzyset A in X is called **convex** intuitionistic fuzzy set if and only if,

$$\mu_A(\lambda x + (1-\lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$$

$v_A(\lambda x + (1-\lambda)y) \leq \max\{v_A(x), v_A(y)\}$,

for each $\lambda \in F$ with $0 \leq \lambda \leq 1$.

2.Some type of neighborhood

Definition(2.1)

Let (X,τ) be an intuitionistic fuzzy topological space and $x \in X$, an IFsubset A on X is called an
Intuitionistic fuzzy $\boldsymbol{W}$**- neighborhood** of x if there exist $G \in \tau$ such that
$\mu_G(t) \leq \mu_A(t)$ and $\nu_G(t) \geq \nu_A(t)$, $\forall t \in X$ and $\mu_G(x) = \mu_A(x) > 0$ and $\nu_G(x) = \nu_A(x) \leq 1$.
The family consisting of all Intuitionistic fuzzy $\boldsymbol{W}$**- neighborhood** of x is called the system of an Intuitionistic fuzzy $\boldsymbol{W}$**- neighborhood** of x and is denoted by $N_{IFW}(x)$.

Definition(2.2)

Let (X,τ) be an Intuitionistic fuzzytopological space and $x \in X$, A family $\mathfrak{B}_X$ of an Intuitionistic fuzzy $\boldsymbol{W}$ **−neighborhood** of x is called an Intuitionistic fuzzy $\boldsymbol{W}$**-neighborhood base** of x if for each $A \in N_{IFW}(x)$ and $\alpha \in [0, \mu_A(x))$, $\beta \in (\nu_A(x), 1]$ there exist $B \in \mathfrak{B}_X$, such that $\mu_B(t) \leq \mu_A(t)$ and $\nu_B(t) \geq \nu_A(t)$, $t \in X$ and $\mu_B(x) > \alpha$, $\nu_B(x) < \beta$.

Remark (2.3)

Every intuitionistic fuzzy open set is an intuitionistic fuzzy W- neighborhood .

Definition (2.4)

Let (X,τ) be an intuitionistic fuzzy topological space and $x_{(\alpha,\beta)} \in p_t(IF(X))$ an intuitionistic fuzzy subset A of X is called an **intuitionistic fuzzy** $\boldsymbol{R}$**-neighborhood** of $x_{(\alpha,\beta)}$ if there exist $G \in \tau$ such that $x_{(\alpha,\beta)} \notin G^c$ and $A \subseteq G^c$.

Definition (2.5)

Let (X,τ) be an Intuitionistic fuzzy topological space and $x_{(\alpha,\beta)} \in pt(IF(X))$ an Intuitionistic fuzzy subset A of X is called an Intuitionistic fuzzy $\boldsymbol{Q}$**-neighborhood** of $x_{(\alpha,\beta)}$ if there exist $G \in \tau$ such that $x_{(\alpha,\beta)} q G \subseteq A$.
The family consisting of all Intuitionistic fuzzy $\boldsymbol{Q}$**-neighborhood** of $x_{(\alpha,\beta)}$ is called the system of an Intuitionistic fuzzy $\boldsymbol{Q}$**-neighborhood** of $x_{(\alpha,\beta)}$ and is denoted by $N_{IFQ}(x_{(\alpha,\beta)})$.

Definition(2.6)

Let (X,τ) be an Intuitionistic fuzzy topological space and $x_{(\alpha,\beta)} \in pt(IF(X))$ a family $\Im_{x_{(\alpha,\beta)}}$ of an Intuitionistic fuzzy $\boldsymbol{Q}$**-neighborhood** of $x_{(\alpha,\beta)}$ is called an Intuitionistic fuzzy $\boldsymbol{Q}$**-neighborhood base** of $x_{(\alpha,\beta)}$ if for each $A \in N_{IFQ}(x_{(\alpha,\beta)})$ there exist $B \in \Im_{x_{(\alpha,\beta)}}$ such that $\mu_B(x) \leq \mu_A(x)$ and $\nu_B(x) \geq \nu_A(t), x \in X$.

Definition(2.7)

Let (X,τ) be an intuitionistic fuzzy topological vector space, an intuitionistic fuzzy set A on X is called intuitionistic fuzzy neighborhood of $0_{(\alpha,\beta)}$ if there exist $B \in \tau$ such that $0_{(\alpha,\beta)}qB \subseteq A$.

Lemma (2.8)

Let (X,τ) be an intuitionistic fuzzy topological vector space and $A \in (IF(X))$. If A is an intuitionistic fuzzy convex, $Q -$ neighborhood of $0_{(\alpha,\beta)}$, then $int(A)$ is also intuitionistic fuzzy convex, $Q -$ neighborhood of $0_{(\alpha,\beta)}$, where $int(A)$ denotes the interior of A.

Proof

Since A is an intuitionistic fuzzy $Q -$ neighborhood of $0_{(\alpha,\beta)}$

$\Rightarrow$ there exists $G \in \tau$ such that $0_{(\alpha,\beta)}qG \subseteq A$

Since G is an intuitionistic fuzzy open set

So, $G = int(G)$ and since $G \subseteq A$

$\Rightarrow int(G) \subseteq int(A)$ it follows that $0_{(\alpha,\beta)}qG = int(G) \subseteq int(A)$, hence $0_{(\alpha,\beta)}qG \subseteq int(A)$ then $int(A)$ is an intuitionistic fuzzy $Q -$ neighborhood of $0_{(\alpha,\beta)}$.

If A is a convex intuitionistic fuzzy set, then its interior $int(A)$is also intuitionistic fuzzy set, then we get A is an intuitionistic fuzzy convex , Q- neighborhood of $0_{(\alpha,\beta)}$ ■

Theorem (2.9)

Let A be an intuitionistic fuzzy set on X then A is an intuitionistic fuzzy Q – neighborhood of $x_{(\alpha,\beta)}$ if and only if A^c is an intuitionistic fuzzy R – neighborhood of $x_{(\alpha,\beta)}$.

Proof

Suppose that A is an intuitionistic fuzzy Q – neighborhood of $x_{(\alpha,\beta)}$

$\Rightarrow \exists\ G \in \tau$ such that $x_{(\alpha,\beta)} qG \subseteq A$

$\Rightarrow \exists x \in X\ such\ that\ \ \mu_G(x) > 1-\alpha \wedge \nu_G(x) < 1-\beta$ and

$$\mu_G(t) \leq \mu_A(t) \wedge \nu_G(t) \geq \nu_A(t),\ \forall t \in X$$

Since $x_{(\alpha,\beta)} qG \Rightarrow x_{(\alpha,\beta)} \notin G^c \ldots (1)$ [by Theorem 1.9]

Now

Since $G \subseteq A$

$\Rightarrow A^c \subseteq G^c$ . .. (2) [by Theorem 1.4]

From (1) and (2) we get $\exists G \in \tau$ such that $x_{(\alpha,\beta)} \notin G^c$ and $A^c \subseteq G^c$

$\Rightarrow A^c$ is an intuitionistic fuzzy R – neighborhood of $x_{(\alpha,\beta)}$

<u>The converse</u>

Suppose that A^c be an intuitionistic fuzzy R – neighborhood of $x_{(\alpha,\beta)}$

$\Rightarrow \exists G \in \tau$ such that $x_{(\alpha,\beta)} \notin G^c$ and $A^c \subseteq G^c$

$\Rightarrow \alpha > \nu_G(x) \vee \beta < \mu_G(x)$ and $\nu_A(t) \leq \nu_G(t) \wedge \mu_A(t) \geq \mu_G(t) \forall t \in X$

Since $x_{(\alpha,\beta)} \notin G^c \Rightarrow x_{(\alpha,\beta)} qG \ldots (3)$ [by Theorem 1.9]

since $A^c \subseteq G^c$

$\Rightarrow G \subseteq A \ldots (4)$ [by Theorem 1.4]

Then from (3) and (3) we get $\exists G \in \tau$ such that $x_{(\alpha,\beta)} qG \subseteq A$

Then A is an intuitionistic fuzzy Q – neighborhood of $x_{(\alpha,\beta)}$ ■

<u>Theorem (2.10)</u>

Let A be an intuitionistic fuzzy neighborhood of $\tilde{0}$ if and only if A be an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)}$, $\forall \alpha \in I_0, \beta \in I_1$.

Proof

Suppose that A be an intuitionistic fuzzy neighborhood of $\tilde{0}(i.e\ \ \tilde{0} = 0_{(1,0)})$

$\Rightarrow \exists G \in \tau$ such that $\tilde{0} qG \subseteq A$

$\Rightarrow 0 \leq \mu_G(0) \wedge \nu_G(0) \leq 1$ and

$$\mu_G(t) \leq \mu_A(t) \wedge \nu_G(t) \geq \nu_A(t),\ \forall t \in X$$

Since $G \in \tau$ and $\mu_G(0) \geq 0$ and $\alpha \in (0,1]$

$\Rightarrow \mu_G(0) \geq 1 - \alpha$ . . . (1)

Also

Since $\nu_{\mathrm{G}}(0) \leq 1$ and $\beta \in [0,1)$

$\Rightarrow \nu_{\mathrm{G}}(0) \leq 1 - \beta$ . . .(2)

From (1) and (2) we get $0_{(\alpha,\beta)} q G \subseteq A$

$\Rightarrow A$ be an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)}$

<u>The converse</u>

suppose that A be an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)} \Rightarrow \exists G \in \tau$ such that $0_{(\alpha,\beta)} q G \subseteq A$

$\Rightarrow \mu_G(0) > 1 - \alpha \wedge \nu_{\mathrm{G}}(0) < 1 - \beta$ and

$$\nu_{\mathrm{G}}(t) \geq \nu_{\mathrm{A}}(t) \wedge \mu_G(t) \leq \mu_A(t)\ ,\ \forall t \in X$$

Since $G \in \tau \wedge \mu_G(0) > 1 - \alpha\ ,\ \alpha \in (0,1]$

$$\Rightarrow \mu_G(0) \geq 0$$

Also

Since

$$\nu_{\mathrm{G}}(0) < 1 - \beta,\ \beta \in [0,1)$$

$$\Rightarrow \nu_{\mathrm{G}}(0) \leq 1$$

And $G \subseteq A$

$\Rightarrow A$ be an intuitionistic fuzzy neighborhood of $\tilde{0}$ ■

<u>Corollary (2.11)</u>

Let A be an intuitionistic fuzzy neighborhood of $\tilde{0}$ if and only if A^c be an intuitionistic fuzzy R- neighborhood of $0_{(\alpha,\beta)}$, $\forall \alpha \in I_0, \beta \in I_1$.

Proof

Let A be an intuitionistic fuzzy neighborhood of $\tilde{0}$

$\Leftrightarrow A$ be an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)}, \forall \alpha \in I_0, \beta \in I_1$

[by Theorem 2.10]

$\Leftrightarrow A^c$ be an intuitionistic fuzzy R- neighborhood of $0_{(\alpha,\beta)}, \forall \alpha \in I_0, \beta \in I_1$

[by Theorem 2.9] ■

<u>Theorem (2.12)</u>

Let (X, τ) be an intuitionistic fuzzy topological vector space and let A be an intuitionistic fuzzy set on X, $x \in X$ then A is an intuitionistic fuzzy W-neighborhood of x if and only if A is an intuitionistic fuzzy Q- neighborhood of $x_{(\alpha,\beta)}$, for each $\alpha \in (1-\mu_U(x), 1]$, $\beta \in [0, 1-\nu_U(x))$.

Proof

Suppose that A is an intuitionistic fuzzy W- neighborhood of x

there exist $G \in \tau$ such that

$\mu_G(t) \leq \mu_A(t) \ldots (1)$ and $\nu_G(t) \geq \nu_A(t) \ldots (2)$, $\forall t \in X$

also $\mu_G(x) = \mu_A(x) > 0$ and $\nu_G(x) = \nu_A(x) < 1$

Hence

for each $\alpha \in (1-\mu_A(x), 1]$, we have $\mu_G(x) = \mu_A(x) > 1-\alpha$

$\Rightarrow \mu_G(x) + \alpha > 1 \ldots (3)$

also, for each $\beta \in [0, 1-\nu_A(x))$

$$\nu_G(x) = \nu_A(x) < 1-\beta$$

$\Rightarrow \nu_G(x) + \beta < 1 \ldots (4)$

From (3)and (4) we get $x_{(\alpha,\beta)} qG \ldots (5)$

And from (1) and (2) we get $G \subseteq A \ldots (6)$

Then from (5) and (6) we have

$$x_{(\alpha,\beta)} qG \subseteq A$$

$\Rightarrow A$ is an intuitionistic fuzzy Q- neighborhood of $x_{(\alpha,\beta)}$.

<u>The converse</u>

Suppose that A is an intuitionistic fuzzy Q - neighborhood of $x_{(\alpha,\beta)}$ for each $\alpha \in (1-\mu_A(x), 1]$, $\beta \in [0, 1-\nu_A(x))$

$\Rightarrow \exists\, G_{(\alpha,\beta)} \in \tau$ such that $x_{(\alpha,\beta)} q\, G_{(\alpha,\beta)} \subseteq A i.e\ \mu_{G_{(\alpha,\beta)}}(x) + \alpha > 1$ and $\nu_{G_{(\alpha,\beta)}}(x) + \beta < 1$

And

$\mu_{G_{(\alpha,\beta)}}(t) \leq \mu_A(t)$ and $\nu_{G_{(\alpha,\beta)}}(t) \geq \nu_A(t)$, $\forall t \in X$

Put

$G = \cup\{G_{(\alpha,\beta)}: \alpha \in (1-\mu_A(x), 1], \ \beta \in [0, 1-\nu_A(x))\}$.

Since $G_{(\alpha,\beta)} \in \tau$ and τ topology on intuitionistic fuzzy set X

$$\Rightarrow \cup G_{(\alpha,\beta)} \in \tau \Rightarrow G \in \tau$$

And since $G_{(\alpha,\beta)} \subseteq A$

$$\Rightarrow \mu_{G_{(\alpha,\beta)}}(t) \leq \mu_A(t)$$

$$\Rightarrow \sup\left\{\mu_{G_{(\alpha,\beta)}}, \ \alpha \in (1-\mu_A(x), 1], \beta \in [0, 1-\nu_A(x))\right\} \leq \mu_A(t)$$

Also $\nu_{G_{(\alpha,\beta)}}(t) \geq \nu_A(t)$

$$\Rightarrow \inf\left\{\nu_{G_{(\alpha,\beta)}}, \ \alpha \in (1-\mu_A(x), 1], \beta \in [0, 1-\nu_A(x))\right\} \geq \nu_A(t)$$

So,

$\mu_G(t) \leq \mu_A(t)$ and $\nu_G(t) \geq \nu_A(t)$, $\forall t \in X \Rightarrow G \subseteq A$

Note that

$$\mu_G(x) = \left\{\sup \mu_{G_{(\alpha,\beta)}}(x), \alpha \in (1-\mu_A(x), 1], \beta \in [0, 1-\nu_A(x))\right\}$$

$$\geq \sup\{1-\alpha, \alpha \in (1-\mu_A(x), 1]\} = \mu_A(x)$$

Also

$$\nu_G(x) = \inf\left\{\nu_{G_{(\alpha,\beta)}}, \ \alpha \in (1-\mu_A(x), 1], \beta \in [0, 1-\nu_A(x))\right\}$$

$$\leq \inf\{1-\beta, \beta \in [0, 1-\nu_A(x))\} = \nu_A(x)$$

So

$\mu_G(x) = \mu_A(x) > 0$ and

$$\nu_G(x) = \nu_A(x) < 1$$

Hence A is an intuitionistic fuzzy W- neighborhood of x ∎

<u>Theorem (2.13)</u>

Let (X, τ) be an intuitionistic fuzzy topological vector space and let $\mathcal{B} \in \big(IF\ \ (X)\big)$, then is an intuitionistic fuzzy W- neighborhood base of x if for each $A \in \mathcal{B}$, $\mu_A(x) > 0$ and $\nu_A(x) < 1$ and for each $\alpha \in I_0, \beta \in I_1$

$\mathfrak{T}_{(\alpha,\beta)} = \{A : A \in \mathcal{B},\ \ \mu_A(x) > 1 - \alpha\ , \nu_A(x) < 1 - \beta\}$ is an intuitionistic fuzzy Q- neighborhood base of $x_{(\alpha,\beta)}$.

Proof

Suppose that is an intuitionistic fuzzy W- neighborhood base of x

And $A \in \mathfrak{T}_{(\alpha,\beta)} \Rightarrow A \in \mathcal{B}$ and

$\mu_A(x) > 1 - \alpha\ \wedge\ \nu_A(x) < 1 - \beta$ . . . (1)

Since $A \in \mathcal{B}$ and an intuitionistic fuzzy W- neighborhood base of x

$\Rightarrow \mathcal{B}$ is a family of an intuitionistic fuzzy W- neighborhood of x

$\Rightarrow A$ is an intuitionistic fuzzy W- neighborhood of x

Then $\exists G \in \tau$ such that $G \subseteq A$ and

$\mu_G(x) = \mu_A(x) > 0\ \wedge\ \nu_G(x) = \nu_A(x) < 1$ . . . (2)

Since $\mu_A(x) > 1 - \alpha$ and $\mu_G(x) = \mu_A(x) > 0$

$\Rightarrow \mu_G(x) > 1 - \alpha$ . . .(3)

Also

Since $\nu_A(x) < 1 - \beta$ and $\nu_G(x) = \nu_A(x) < 1$

$$\Rightarrow \nu_G(x) < 1 - \beta$$

From (1) and (2) we get

$\mu_G(x) > 1 - \alpha\ \wedge\ \nu_G(x) < 1 - \beta$ . . .(4)

From (3) and (4) we get $x_{(\alpha,\beta)} q\ G$ and $G \subseteq A$

$$\Rightarrow x_{(\alpha,\beta)} qG\ \ \subseteq U$$

$\Rightarrow A$ is an intuitionistic fuzzy Q- neighborhood of $x_{(\alpha,\beta)}$

$\Rightarrow \mathfrak{T}_{(\alpha,\beta)}$ is a family of an intuitionistic fuzzy Q- neighborhood of $x_{(\alpha,\beta)}$

Let $B \in N_{IFQ}\ \big(x_{(\alpha,\beta)}\big)$

$\Rightarrow \exists\ H \in \tau$ such that $x_{(\alpha,\beta)} q H \subseteq B$

Since $H \in \tau \Rightarrow H$ is an intuitionistic fuzzy $W-$ neighborhood of x (by Remark 2.3)

Since is an intuitionistic fuzzy W- neighborhood base of x

$\exists\ C \in \mathcal{B}$ such that $C \subseteq H$ and $\mu_C(x) > 1-\alpha \wedge \nu_C(x) < 1-\beta$

$\Rightarrow C \in \mathfrak{T}_{(\alpha,\beta)}$ and $C \subseteq H \subseteq B$

by definition of an intuitionistic fuzzy Q- neighborhood base of $x_{(\alpha,\beta)}$

$\mathfrak{T}_{(\alpha,\beta)}$ is an intuitionistic fuzzy Q- neighborhood base of $x_{(\alpha,\beta)}$

<u>The converse</u>

Suppose that $\mathfrak{T}_{(\alpha,\beta)}$ is an intuitionistic fuzzy Q- neighborhood base of $x_{(\alpha,\beta)}$

and $A \in \mathcal{B}$

$\Rightarrow \mu_A(x) > 0 \wedge \nu_A(x) < 1$ by the condition of this theorem

$$\alpha \in (0,1], \qquad \beta \in [0,1)$$

Let $\alpha \in (1-\mu_A(x), 1]$, $\beta \in [1-\nu_A(x), 1)$

From $\alpha \in (1-\mu_A(x), 1]$

We have $\mu_A(x) > 1-\alpha \ldots (1)$

And from $\beta \in [1-\nu_A(x), 1)$

We have $\nu_A(x) < 1-\beta \ldots (2)$

From (1) and (2) we get

$$A \in \mathfrak{T}_{(\alpha,\beta)}, \quad \forall \alpha \in (1-\mu_A(x), 1], \qquad \beta \in [0, 1-\nu_A(x))$$

$\mathfrak{T}_{(\alpha,\beta)}$ is an intuitionistic fuzzy Q- neighborhood base of $x_{(\alpha,\beta)}$

$\Rightarrow \mathfrak{T}_{(\alpha,\beta)}$ is a family of an an intuitionistic fuzzy Q- neighborhood of $x_{(\alpha,\beta)}$

$\Rightarrow A$ is an intuitionistic fuzzy W- neighborhood of x [Theorem 2.12]

Now

let $A \in N_{IFW}(x)$, and $\forall \lambda \in [0, \mu_A(x))$, $\sigma \in (\nu_A(x), 1]$

Since $A \in N_{IFW}(x)$

$\Rightarrow A$ is an intuitionistic fuzzy W- neighborhood of x

$\Rightarrow A$ is an intuitionistic fuzzy Q- neighborhood of $x_{(1-\lambda,1-\sigma)}$ [Theorem2.12]

Thus $\exists\ B \in U_{(1-\lambda,1-\sigma)}$

$\Rightarrow B \in \mathcal{B}$ such that $\mu_{\mathrm{B}}(x) > 1-(1-\lambda)$

$\Rightarrow \mu_{\mathrm{B}}(x) > \lambda$ and

$$\nu_{\mathrm{B}}(x) < 1-(1-\sigma)$$

$$\Rightarrow \nu_{\mathrm{B}}(x) < \sigma$$

Such that $B \subseteq A$

Hence is an intuitionistic fuzzy W- neighborhood base of x ■

Theorem (2.14)

Let (X,τ) be an intuitionistic fuzzy topological vector space and let $U_{(\alpha,\beta)}$ be an intuitionistic fuzzy Q- neighborhood base of $0_{(\alpha,\beta)}$ in X, for each $\alpha \in I_0, \beta \in I_1$, then it has the following properties.

1) If $A \in U_{(\alpha,\beta)}$ then there exist $\alpha_0 \in (0,\alpha), \beta_0 \in (\beta,1)$ such that for each $m \in [\alpha_0,1], n \in [0,\beta_0]$ there exists $B \in U_{(\alpha,\beta)}$ with $B \subseteq A$.
2) If $A, B \in U_{(\alpha,\beta)}$ then there exist $C \in U_{(\alpha,\beta)}$ such that $C \subseteq A \cap B$.

Proof

1) Let $A \in U_{(\alpha,\beta)}$ $\Rightarrow A$ is an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)}$

$\Rightarrow \exists G \in \tau$ such that $0_{(\alpha,\beta)} q G \subseteq A$

$\Rightarrow$ $\mu_G(0) > 1-\alpha \ \wedge\ \nu_{\mathrm{G}}(0) < 1-\beta$ and

$$\mu_G(x) \le \mu_A(x) \ \wedge\ \nu_{\mathrm{G}}(x) \ge \nu_{\mathrm{A}}(x),\ \forall x \in X$$

Since $0 \in X$

$$\Rightarrow \mu_G(0) \le \mu_A(0) \ \wedge\ \nu_{\mathrm{G}}(0) \ge \nu_{\mathrm{A}}(0)$$

Taking $\alpha_0 \in (0,\alpha), \beta_0 \in (\beta,1)$ with $\mu_G(0) > 1-\alpha_0 > 1-\alpha \ \wedge\ \nu_{\mathrm{G}}(0) < 1-\beta_0 < 1-\beta$

For any $m \in [\alpha_0,1], n \in [0,\beta_0]$

We have $0_{(m,n)} q G \subseteq A$

Hence A is an intuitionistic fuzzy Q- neighborhood of $0_{(m,n)}$

Note that $U_{(m,n)}$ is an intuitionistic fuzzy Q- neighborhood base of $0_{(m,n)}$

Thus $\exists B \in U_{(m,n)}$ such that $B \subseteq A$.

2) Let $A, B \in U_{(\alpha,\beta)}$

$\Rightarrow A, B$ an intuitionistic fuzzy Q- neighborhood of $0_{(\alpha,\beta)}$

$\Rightarrow \exists G, H \in \tau$ such that $0_{(\alpha,\beta)} qG \subseteq A,\ 0_{(\alpha,\beta)} qH \subseteq B$

$\Rightarrow \mu_G(0) > 1-\alpha \wedge \nu_{\mathrm{G}}(0) < 1-\beta$ and

$$\mu_H(0) > 1-\alpha \wedge \nu_{\mathrm{H}}(0) < 1-\beta$$

And

$\mu_G(x) \le \mu_A(x) \wedge \nu_{\mathrm{G}}(x) \ge \nu_{\mathrm{A}}(x),\ \forall x \in X$ and

$$\mu_H(x) \le \mu_B(x) \wedge \nu_{\mathrm{H}}(x) \ge \nu_{\mathrm{B}}(x),\ \forall x \in X$$

Now

Since $, H \in \tau$, (τ topology on X)

$$\Rightarrow G \cap H \in \tau$$

Since $\{< \min\{\mu_G(x), \mu_H(x)\}, \max\{\nu_{\mathrm{G}}(x), \nu_{\mathrm{H}}(x)\} >\} = G \cap H \in \tau$

Let $N = G \cap H \in \tau \Rightarrow N \in \tau$

Since $\mu_G(0) > 1-\alpha \wedge \mu_H(0) > 1-\alpha$

$\Rightarrow \mu_H(0)\} > 1-\alpha$ and

since $\min\{\mu_G(0), \mu_H(0)\} = \mu_N(0) \Rightarrow \mu_N(0) > 1-\alpha$ . . . (1) and

Since $\nu_{\mathrm{G}}(0) < 1-\beta \wedge \nu_{\mathrm{H}}(0) < 1-\beta$

$\Rightarrow \max\{\nu_{\mathrm{G}}(0), \nu_{\mathrm{H}}(0)\} < 1-\beta$ and

Since $\max\{\nu_{\mathrm{G}}(0), \nu_{\mathrm{H}}(0)\} = \nu_{\mathrm{N}}(0) \Rightarrow \nu_{\mathrm{N}}(0) < 1-\beta$ . . . (2)

From (1) and (2) we get $0_{(\alpha,\beta)} qN$ . . . (*)

Now $\mu_G(x) \le \mu_A(x) \wedge \mu_H(x) \le \mu_B(x),\ \forall x \in X$

$$\Rightarrow \mu_N(x) = \min\{\mu_G(x), \mu_H(x)\} \le \min\{\mu_A(x), \mu_B(x)\} = \mu_C(x)$$

$\Rightarrow \mu_N(x) \le \mu_C(x)$ . . . (3)

Also

$$\nu_{G}(x) \geq \nu_{A}(x) \ \wedge \ \nu_{H}(x) \geq \nu_{B}(x), \ \forall x \in X$$

$$\Rightarrow \nu_{N}(x) = \max\{\nu_{G}(x), \nu_{H}(x)\} \geq \max\{\nu_{A}(x), \nu_{B}(x)\} = \nu_{C}(x)$$

$\Rightarrow \nu_{N}(x) \geq \nu_{C}(x) \ldots (4)$

From (3) and (4) we get $N \subseteq C \ldots (**)$

From (*) and (**) we get $0_{(\alpha,\beta)} q N \subseteq C$

$i.e \ \exists N \in \tau$ such that $0_{(\alpha,\beta)} q N \subseteq C$

When

$N = \{\min\{\mu_{G}(x), \mu_{H}(x)\}, \max\{\nu_{G}(x), \nu_{H}(x)\}\} = G \cap H$ and

$$C = \{\min\{\mu_{A}(x), \mu_{B}(x)\}, \max\{\nu_{A}(x), \nu_{B}(x)\}\} = A \cap B$$

Note that

$$\mu_{C}(x) = \min\{\mu_{A}(x), \mu_{B}(x)\}$$

$\Rightarrow \mu_{C}(x) \leq \min\{\mu_{A}(x), \mu_{B}(x)\} \ldots (5)$ also

$$\nu_{C}(x) = \max\{\nu_{A}(x), \nu_{B}(x)\}$$

$\Rightarrow \nu_{C}(x) \geq \max\{\nu_{A}(x), \nu_{B}(x)\} \ldots (6)$

From (5) and (6) we get

$$C \subseteq A \cap B \blacksquare$$

3. Intuitionistic fuzzy locally convex

In this section, we define an intuitionistic fuzzy locally convex about intuitionistic fuzzy topological vector space and we prove some result about it.

Definition (3.1)

An intuitionistic fuzzy topological vector space (X, τ) is called intuitionistic fuzzy locally convex if it has an intuitionistic fuzzy W- neighborhood base of $\tilde{0}$ consisting of convex intuitionistic fuzzy set.

Definition (3.2)

An intuitionistic fuzzy topological vector space (X, τ) is called intuitionistic fuzzy locally convex if there exist a family U of convex intuitionistic fuzzy subset on X such that for each $\alpha \in I_0, \beta \in I_1$

$U_{(\alpha,\beta)} = \{A \cap (\underline{r}, \underline{s}) | A \in U, r \in (1-\alpha, 1], \ s \in [0, 1-\beta)\}$ is an intuitionistic fuzzy Q- neighborhood base of $0_{(\alpha,\beta)}$.

Theorem(3.3)

An intuitionistic fuzzy topological vector space (X,τ) is an intuitionistic fuzzy locally convex in the sense of (3.1) if and only if it has an intuitionistic fuzzy Q-neighborhood base of $0_{(\alpha,\beta)}$ consisting of convex intuitionistic fuzzy sets for each $\alpha \in I_0, \beta \in I_1$.

Proof

suppose that (X,τ) is a locally convex intuitionistic fuzzy topological vector space

Then by definition(3.2) of intuitionistic fuzzy locally convex

(X,τ) it has an intuitionistic fuzzy W- neighborhood base of $\tilde{0}$ consisting of convex intuitionistic fuzzy set

let $U_{(\alpha,\beta)} = \{A: A \in \mathcal{B},\ \ \mu_A(x) > 1-\alpha\ , \nu_A(x) < 1-\beta\}$ then by Theorem 2.13

$U_{(\alpha,\beta)}$is an intuitionistic fuzzy Q-neighborhood base of$0_{(\alpha,\beta)}$ for each $\alpha \in I_0,\ \beta \in I_1$

And since an intuitionistic fuzzy W- neighborhood base of $\tilde{0}$ consisting of convex intuitionistic fuzzy set

$\Rightarrow \forall A \in \mathcal{B}$ is a convex intuitionistic fuzzy set and

Since each element of $U_{(\alpha,\beta)}$ belongs to

Then $\forall A \in U_{(\alpha,\beta)}$ is a convex intuitionistic fuzzy set

Then we have $U_{(\alpha,\beta)}$ is an intuitionistic fuzzy Q- neighborhood base of $0_{(\alpha,\beta)}$ consisting a convex intuitionistic fuzzy set for each $\alpha \in I_0,\ \beta \in I_1$

<u>The converse</u>

Assume that for each $\alpha \in I_0,\ \beta \in I_1, \mathcal{B}_{(\alpha,\beta)}$ is an intuitionistic fuzzy Q-neighborhood base of$0_{(\alpha,\beta)}$ consisting a convex intuitionistic fuzzy sets.

ByLemma2.8, we assume that every member of $\mathcal{B}_{(\alpha,\beta)}$ is a convex intuitionistic fuzzy set (intuitionistic fuzzy open set)

Put

$\mathcal{B} = \cup_{\substack{\alpha \in I_0 \\ \beta \in I_1,}} \mathcal{B}_{(\alpha,\beta)}$ and

$$U_{(\alpha,\beta)} = \{B: B \in \mathcal{B},\ \mu_B 0 > 1-\alpha, \qquad \nu_B(0) < 1-\beta\}$$

Obviously , $\mu_B(0) > 0,\ \ \nu_B(0) \leq 1$, for each $B \in \mathcal{B}$

Notice that $\mathcal{B}_{(\alpha,\beta)} \subseteq \mathcal{U}_{(\alpha,\beta)}$ and

Every member of $\mathcal{U}_{(\alpha,\beta)}$ is a convex intuitionistic fuzzy set (intuitionistic fuzzy open set, $\mathcal{Q}$- neighborhood of $0_{(\alpha,\beta)}$)

Hence $\mathcal{U}_{(\alpha,\beta)}$ is a convex intuitionistic fuzzy set

(intuitionistic fuzzy open set, $\mathcal{Q}$- neighborhood base of $0_{(\alpha,\beta)}$ for each $\alpha \in I_0, \beta \in I_1$)

So by Theorem 2.13 , we know that is an intuitionistic fuzzy $\mathcal{Q}$- neighborhood base of consisting of convex intuitionistic fuzzy set

There for (X, τ) is a locally convex intuitionistic fuzzy topological vector space in the since of Definition 3.1 ■

Theorem (3.4)

An intuitionistic fuzzy topological vector space(X, τ) has an intuitionistic fuzzy $\mathcal{Q}-$ neighborhood base of $0_{(\alpha,\beta)}$ consisting of absolutely convex intuitionistic fuzzy sets for each $\alpha \in (0,1]$, $\beta \in [0,1)$, then intuitionistic fuzzy locally convex in the sense of Definition 3.1.

Proof

Suppose that (X, τ)intuitionistic fuzzy topological vector space has an intuitionistic fuzzy $\mathcal{Q}-$ neighborhood base of $0_{(\alpha,\beta)}$ consisting of absolutely convex intuitionistic fuzzy set, $\alpha \in (0,1]$, $\beta \in [0,1)$

$\Rightarrow$ (X, τ)intuitionistic fuzzy topological vector space has an intuitionistic fuzzy $\mathcal{Q}-$ neighborhood base of $0_{(\alpha,\beta)}$ consisting of convex intuitionistic fuzzy set

By Theorem 2.13 we have (X, τ) locally convex intuitionistic fuzzy topological vector space ■

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