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ADAPTIVE FUZZY CONTROL CONCEPTS AND SURVEY

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Abstract: In this paper an adaptive fuzzy control concepts and survey are introduced. Starting with the global adaptive control towered the adaptive fuzzy control, the required concepts are explained. Some of the adaptive fuzzy control subjects are viewed as sequential steps with simplifying their views to enable the reader to get a fast and global idea with some details if it is necessary. Most of the stability considerations in the corresponding references are proved by using the lyapunov criteria, where the derivation is a mathematical concept with long steps. Therefore, it is mentioned without details, and for more information, the corresponding reference must be studied. It can be seen from this topic, that the main role of the fuzzy system in adaptive control is the system identification, controller construction and output predictor. The adaptive fuzzy control survey is presented at the end, so the reader can go along with the topics after he reviewed the necessary concepts.

Keywords: *fuzzy adaptive control, nonlinear system, feedback linearization, back stepping, adaptive control direction, Nussbaum gain function.*

1. Introduction

Because of its nature to deal with the uncertainty behaviors, the fuzzy system has a wide area of application in adaptive control system. It has important characteristics; the fuzzy control system construction and uncertain system identification as it represents a global approximation [1]. These two characteristics represent basic requirements of the adaptive control system. Really, the fuzzy system and neural network represent a type of approximation-based adaptive control that is deal with the nonlinear systems of unstructured uncertainties behavior [2]. For this reason this topic has been presented to support the reader with an important concepts and basic headlines depending on different researches to synthesize and viewing them in a simple idea with a sequence steps if necessary. It involves a wide range of researches and details that cannot be encompassed in a few pages, therefore for the additional details returning to the corresponding reference.

2. Adaptive Control

Adaptive control is a type of control that is implemented in systems which have an unknown or a time variant parameters, in other words; systems their behaviors are changed during the operation in unpredictable form. The more active concepts that are required to be explained are

1. Control law: The equation constructed from known parameters to stabilize a controlled system [3].
2. Adaptive law: How the unknown system parameters will be estimated and in which direction will be updated to derive the control law [1].

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This type of control systems require an adaptive law, in which system identification and parameter estimation methods are implemented. From the adaptive law, a control law is constructed, which is the goal of all the adaptive process. By the proper control law, the stability of the controlled system is performed.

On the other hand, if the system is changed in a predictable form; then, its parameters can be calculated and the proper control law is derived in offline. Otherwise, an online adaptive law must be performed.

It is important to point to another crucial note, that is; how rapid the system parameters are changed. Depending on the type of the system, its uncertainties, external disturbances and the required performance will affect the feasibility of designing the adaptive controller for the dedicated case.

3. Adaptive Control Approaches

Depending on the way that the adaptive law is combined with control law [1, 3, 11], there are two adaptive control approaches:

1- Direct adaptive control. In this approach the controller parameters are estimated directly to derive the control law.

2- Indirect adaptive control. In this approach, the unknown parameters of the controlled system are estimated, and then the controller is designed depending on these estimated parameters. The controller parameters are calculated, proposing that the actual system is to be controlled.

In any case, there is an important design principle called certainty equivalent principle, where the controller is designed regarding a real parameters of the plant are presented in the design, but in fact, the plant parameters are estimated [1]. Fig. 1 and fig. 2 shows the direct and indirect adaptive control, where

θ^* : is a plant parameters

θ_c : is a controller parameters

$C(\theta^*)$: is a controller function

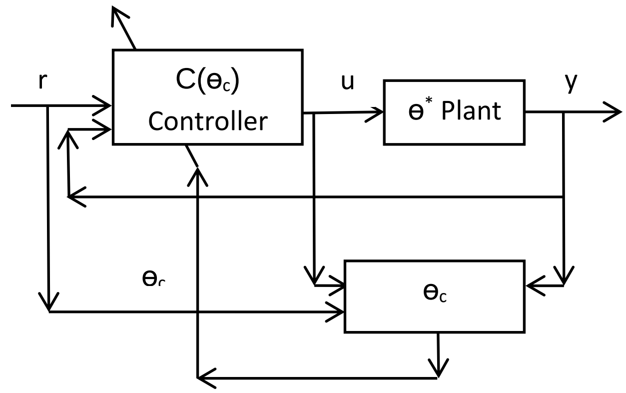

Figure 1. Direct Adaptive Control

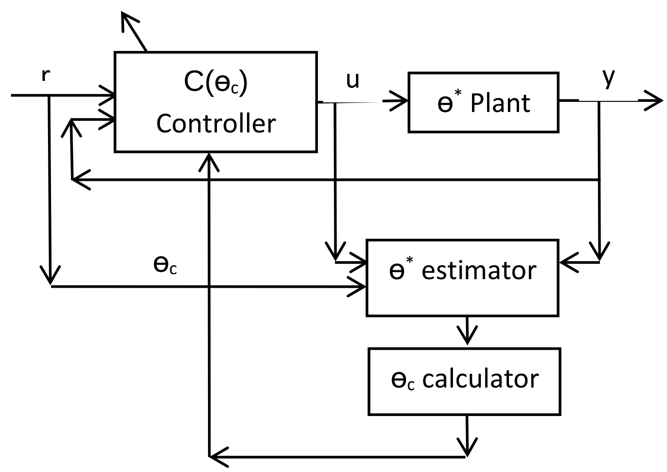

Figure 2. Indirect Adaptive Control

4. Adaptive Control Guide

The adaptive control system required some mechanisms to track the plant to a desired performance, which will be called an "Adaptive Control Guides". Some of these guides are:

1- Adaptive Model Reference Control.
2- Adaptive Pole Placement Control.

4.1. Adaptive Model Reference Control

The adaptive model reference control can be used to track the controlled system to a specific reference performance. As shown in fig. 3 [3], it can be combined with the direct or indirect adaptive controller. It takes the input signal from the same reference input of the adaptive system and its output is compared with the adaptive system output to produce the reference error e_r, then according to the e_r value and sign the control signal will adjusted.

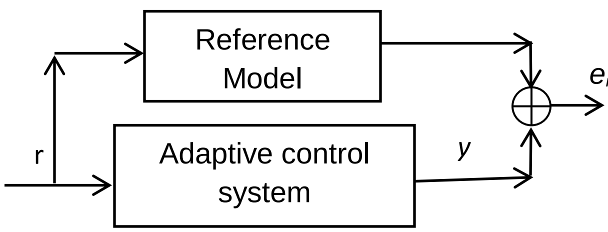

Figure 3. Model Reference Adaptive Control

4.2. Adaptive Pole Placement Control

It is one of the typical control system design methods. As the others, it can be used as an adaptive control guide, where the required performance is mapped to the desired poles location, then; the control law is derived to stabilize the controlled system accordingly. The adaptive mechanism comes from the fact that the plant parameters are unknown, therefor; they will be estimated to build the control law. The same as the adaptive reference model, it may be direct or indirect adaptive control [3].

5. Adaptive Law Approaches

There are different approaches for a parameter derivation, stability consideration and convergence affirmation of an online adaptive law:

5.1. Sensitivity Methods [3]**:**

For the estimated parameters, the partial derivative of the performance function and the error function are used to derive the adaptive law. In these methods, the partial derivative is called the sensitivity function, where it represents the sensitivity of the system to the change of a particular parameter. The drawbacks of these methods are: 1) Mostly, it is online inapplicable because it cannot be generated in online application. 2) Because of approximation of the sensitivity function, it has a weak stability or may be not established. 3) The sensitivity functions may not be implementable when the performance cost function is minimized.

In order to derive a sensitivity function, a scale function which is called a cast function is a dependable. The coast function measures the performance of adaptive law; it is a convex function that has a global minimum at which there is no change in the dedicated parameter. The simplest form of the coast function which is used the error signal is:

$$J(\theta) = \frac{1}{2} e^2(\theta) \quad (1)$$

Where

J(.): is the coast function

e(.): is the error signal

ϴ(.): is the estimated parameter

5.2. Positivity and Lyapunov Methods [1, 3]

In this method the state tracking error and estimation error are contained in the candidate Lyapunov positive function. The adaptive law is constructed based on the derivation of the non-positive Lyapunov derivative function along the error trajectory. The adaptive law in this method is derived using the direct Lyapunov method represented by its positive candidate function.

The solution of the stability condition is used to derive the differential equation of the adaptive law. It is similar to the sensitivity method, but it overcomes the drawbacks of the online problem and stability weakness.

5.3. Error Estimation Methods Represented by Gradient Method and Least Squares Methods [1, 3]

It is represented by gradient method and least squares methods. In these methods, the parameters have an explicit model to derive them other than controller module.

This method avoids the third drawback mentioned above in section 5.1, where it provides a measurable sensitivity function by choosing a cost function based on the estimated error (the difference between actual and estimated parameters). The most cost criteria are the gradient and least square methods, which are used to produce proper sensitivity functions.

6. Hybrid Adaptive Law

The combination of different methods of adaptive control produces a hybrid adaptive control. The type and the sequence of combination depend on the dedicated system and the case to be handled.

7. Control law Techniques:

Some techniques of the control law derivation in nonlinear systems are listed here and will be explained next in section 14

1- Parallel distributed compensation [1, 4].
2- Feedback linearization [1, 5].
3- Back stepping [1].

8. Adaptive Observer

A scheme at which a system states are online estimated, depending on the input and output measurements. At this scheme, the estimated states are compared with the actual states of the system; the difference between them represents an observer error that will be used to derive the estimated states to asymptotically approach the actual states.

9. The Nussbaum Type Gain Functions

In adaptive control system, the control direction, which defines the sign of the control gain; represents a crucial effect of the system stability. Normally, it is assumed to be known a priori in design stage. Practically, most of the nonlinear systems have unknown control direction and different methods are used to deal with. One of these methods is using an even Nussbaum type gain function *N(.)* [2], by which, the control direction can be identified a priory and the sign of the gain can be determined to direct the control signal [6]. Its behavior can be represented as in [2, 7, 8]:

$$\lim_{s\to+\infty} sup \frac{1}{s}\int_0^s N(\zeta)d\zeta = +\infty \tag{2}$$

$$\lim_{s\to-\infty} inf \frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty \tag{3}$$

Such an even Nussbaum gain functions are the following cosines (or sine functions) [7]:

$$N_1(\zeta) = \zeta^2\cos(\zeta), \quad N_2(\zeta) = \zeta\cos\left(\sqrt{|\zeta|}\right), \quad N_3(\zeta) = \cos\left(\frac{\pi}{2}\zeta\right)e^{\zeta^2}$$

10. Nonlinear System Representation

The nonlinear system can be represented in different mathematical models of state space representation depending on system type, the proper accuracy of the system application, the perturbation effect on ad-hoc application and how match it can be mathematically palliated and so on. Some of nonlinear system representations are:

1- Pure Feedback Nonlinear System [9, 6].
2- Strict Feedback Nonlinear System [5].
3- Chua's Chaotic System [10].

For more information, returning to the corresponding references.

On the other hand, if the control signal appears linearly (explicitly) in the nonlinear system, then this system is called (affine system). When the control signal appears implicitly within the nonlinear system, it is defined as non-affine system.

11. Stability Considerations

Consider as an example the pure feedback nonlinear affine system, its details are stated in [9, 6]

$$\dot{x}_i = f_i(\boldsymbol{x}_i) + g_i(\boldsymbol{x}_i)x_{i+1} \qquad 1 \le i \le n-1 \quad (4)$$

$$\dot{x}_n = f_n(\boldsymbol{x}) + g_n(\boldsymbol{x})u \quad (5)$$

$$y = x_1 \quad (6)$$

With the origin (input: $x = 0$, output: $u = 0$) to be asymptotically stable equilibrium point in the closed loop design, the stability considerations of this system are [5]:

$f(x) \in R^n$: is a bounded smooth function (n times continuously differentiable) in D_x.

$f(0) = 0$.

$g(x) \neq 0$: is a bounded smooth function Nonsingular function in D_x.

$g(x) \geq g_0 > 0$

$\dot{x} = 0$ at $x=0$

In most systems the control direction is assumed to be known, otherwise, the Nussbaum gain function can be used to solve the unknown direction.

12. Fuzzy System Identification and Parameter Estimation

In adaptive control, the concept of "parameter estimation" is referred to an "adaptive law" [3]. Mostly, any controlled system has uncertainties. In adaptive control, there may be part of or all the controlled system is undefined. In any case, the undefined object must be identified, and their parameters must be estimated. Different methods of identification are implemented in adaptive control system, from which; the fuzzy system is successful application in this field.

On the other hand, any continuous differentiable nonlinear system can be approximated by a summation of a finite number of linear subsystems. Where, depending on the rule base structure of the fuzzy system, it can be used as a universal approximation of nonlinear system by combining the linear subsystems in each rule base and produce a proper decision depending on the input vector. The universal approximation property of the fuzzy system is used in the adaptive control to identify the unknown system functions, and then the estimated parameters of the identified function can be updated using the recursive least squares (RLS) or gradient method.

Consider the *ith* rule T-S fuzzy identifier discrete time model of the polynomial representation form [11]

$IF\ y(k)\ is\ \Omega_1^i\ and \dots y(k-i+1)\ is\ \Omega_j^i\ and \dots y(k-N+1)\ is\ \Omega_N^i$

$$THEN\ \hat{y}^i(k+1) = \alpha_1^i y(k) + \cdots + \alpha_N^i y(k-N+1) + \beta_1^i u(k) + \cdots + \beta_M^i u(k-M+1) \quad (7)$$

The output result of the singleton fuzzification with product inference and center average defuzzification will be:

$$\hat{y}(k+1) = \frac{\sum_{i=1}^R \hat{y}^i(k+1)\prod_j^N \mu_{\Omega_j^i}\big(y(k-j+1)\big)}{\sum_{i=1}^R \prod_j^N \mu_{\Omega_j^i}\big(y(k-j+1)\big)} \quad (8)$$

$$\hat{y}(k+1) = \frac{\sum_{i=1}^R \hat{y}^i(k+1)\xi^i\big((k-j+1)\big)}{\sum_{i=1}^R \xi^i\big((k-j+1)\big)} \quad (9)$$

The fuzzy identifier then become

$$\hat{y}(k+1) = \boldsymbol{\theta}^T\boldsymbol{\phi} \quad (10)$$

Where

$\xi^i\big(y(k-j+1)\big) = \prod_j^N \mu_{\Omega_j^i}\big(y(k-j+1)\big) \qquad \xi^j \geq 0, \sum_{j=1}^N \xi^j = 1$

$\boldsymbol{\theta} = \left[\alpha_1^1, \dots, \alpha_1^i, \dots, \alpha_1^R, \dots, \beta_M^1, \dots, \beta_M^i, \dots, \beta_M^R\right]^T$

$\boldsymbol{\phi} = \left[y(k)\xi^1, \dots, y(k)\xi^i, y(k)\xi^R, \dots, u(k-M+1)\xi^1, \dots, u(k-M+1)\xi^i, u(k-M+1)\xi^R\right]^T$

y, u : are the output and control signals respectively.

N, M : are the output and control signals dimensions respectively.

α_j^i, β_k^i : are the *jth* and *kth* output and control signals parameters respectively of the *ith* rule.

The same result can be reached if the standard fuzzy is applied. Consider the *ith* rule of the standard fuzzy system [1].

$$IF\ x_1(t)\ is\ \Omega_1^i\ and \dots x_i(t)\ is\ \Omega_i^i\ and \dots and\ x_N(t)\ is\ \Omega_N^i\ THEN\ f(\boldsymbol{x}(t))\ is\ \mho^i \tag{11}$$

Where

i = 1, …, R fuzzy rules.

j = 1, …, N input vector dimension.

$\boldsymbol{x} = [x_1, \dots, x_N] \in \mathbb{R}^N$: is the input vector to the fuzzy system.

Ω^i_j : is the *jth* membership function of the *ith* rule premise part. It represents the compact set of the x_j input signal.

$\mho^i$: is the output membership function of the *ith* rule consequent part.

$f(\boldsymbol{x}(t)) \in \mathbb{R}$: is the output of the fuzzy system.

The output result of the singleton fuzzification with product inference and center average defuzzification will be:

$$f(\boldsymbol{x}(t)) = \frac{\sum_{i=1}^R \theta^i(t) \prod_j^N \mu_{\Omega_j^i}\big(x_j(t)\big)}{\sum_{i=1}^R \prod_j^N \mu_{\Omega_j^i}\big(x_j(t)\big)} \tag{12}$$

$$f(\boldsymbol{x}(t)) = \frac{\sum_{i=1}^R \theta^i(t)\xi^i\big(\boldsymbol{x}(t)\big)}{\sum_{i=1}^R \xi^i\big(\boldsymbol{x}(t)\big)} \tag{13}$$

$$f(\boldsymbol{x}(t)) = \sum_{i=1}^R \theta^i(t)\phi^i(\boldsymbol{x}(t)) \tag{14}$$

$$f(\boldsymbol{x}(t)) = \boldsymbol{\theta}^T(t)\boldsymbol{\phi}(\boldsymbol{x}(t)) \tag{15}$$

Where

$\theta^i(t) = \max(\mu_{\mho^i})$

$\xi^i\big(\boldsymbol{x}(t)\big) = \prod_j^N \mu_{\Omega_j^i}\big(x_j(t)\big) \qquad \boldsymbol{\phi}^i(\boldsymbol{x}(t)) = \frac{\xi^i(\boldsymbol{x}(t))}{\sum_{i=1}^R \xi^i(\boldsymbol{x}(t))} \qquad \xi^j \geq 0, \sum_{j=1}^N \xi^j = 1$

$\boldsymbol{\theta}(t) = [\theta^1(t), \dots, \theta^R(t)]^T$

$\boldsymbol{\phi}(\boldsymbol{x}(t)) = \left[\phi^1(\boldsymbol{x}(t)), \dots, \phi^R(\boldsymbol{x}(t))\right]^T$

Equations (10)and (15) can be represented us

$$\hat{f}(\boldsymbol{x}(t)) = \boldsymbol{\theta}^{*T}(t)\boldsymbol{\phi}(\boldsymbol{x}(t)) + \varepsilon \tag{16}$$

Where

$\hat{f}(\boldsymbol{x}(t))$: is the estimated value of $f(x(t))$.

$\boldsymbol{\theta}^*(t)$: is the optimal parameters.

ε : is the estimated error.

From (16), it can be seen that the nonlinear function can be estimated by a fuzzy system with some estimated error.

13. Adaptive Fuzzy Control System

There are different applications of the fuzzy system in the adaptive control; the main use is to build the controller depending on the global behavior of the system; it is used to identify the unknown system parts, this may be all the plant or may be the actuator or the sensor dead zone or transmission time delay etc. the other application represented by using the fuzzy system as a reference module, where each fuzzy rule represents a linear subsystem reference module.

14. Adaptive Fuzzy Control law Techniques:

As stated in section 7, the three techniques will be explained

14.1. Adaptive Fuzzy Parallel Distributed Compensation

Using the fuzzy system, the nonlinear plant is represented by a combination of linear subsystems. These subsystems are represented by the fuzzy rules. Corresponding to each plant subsystem (rule), there is a controller linear subsystem (rule). This structure is called a parallel distributed compensation. The idea of this structure is exploited in the fuzzy control and then in the adaptive fuzzy design.

Although some references such as [1] deals with this idea as a part of fuzzy controller construction and it is appear implicitly in the design of the other two adaptive fuzzy control law techniques, the authors in [11] simply derive an explicit controller depending on the parallel distributed compensation idea together with pole placement stability criteria as follows:

1. Define the ith rule of the fuzzy system identifier as in (7).
2. The output of the fuzzy system identifier will be as in (16).
3. Defining the *ith* rule of the T-S fuzzy controller as

$$IF\ y(k)\ is\ \Omega_1^i\ and\ \dots y(k-i+1)\ is\ \Omega_i^i\ and\ \dots y(k-N+1)\ is\ \Omega_N^i$$

$$THEN\ u^i(k) = F^i(.) \quad (17)$$

Where

$F^i(.)$: is the *ith* rule linear function, that may be depends on the past values of input, output or reference signals.

4. $F^i(.)$ may be of different forms such as a proportional controller of the form

$$F^i(r,y) = k_0^i r(k) + k_1^i y(k) \quad (18)$$

Where

k^i_0 and k^i_1 : are the controller gains.

5. The unknown system parameters α_j^i, β_j^i can be estimated and updated using RLS or gradient methods.
6. On the other hand, the controller gains $k_0^i\ and\ k_1^i$ can be adjusted using different stability considerations, such as pole placement or lyapunov methods.

14.2. Adaptive Fuzzy Feedback Linearization

Feedback linearization transforms the nonlinear systems to equivalent linear systems, by replacing the control signal with another one in a new coordinates [2]. Different feedback methods are used and some of them will be explained:

14.2.1. Adaptive T–S Fuzzy State Tracking Control Using State Feedback with linear reference model [1]

In this method, the T-S Fuzzy system is used to construct each of the nonlinear (plant, controller and may be the reference system), by dividing each of them to a linear subsystem. The important note is that; each sub-controller with sub-plant in the closed loop feedback system must equivalent to the same reference system. The objective is to design a state feedback controller *u(t)* for the global fuzzy plant (19) with unknown parameters to asymptotically following a reference state x_m. The headlines of this method with indirect adaptive control can be verified as

1- The state space of the T-S fuzzy system is:
$\dot{x}(t) = \sum_{i=1}^{N} \xi^i [A^i x(t) + B^i u(t)]$ (19)
where the canonical matrices are
$$A^i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_1^i & -a_2^i & -a_3^i & \dots & -a_n^i \end{bmatrix} \quad B^i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b^i \end{bmatrix} \quad (20)$$

2- The *ith* rule *nth* state T-S Fuzzy system of the plant module (note (16) the nth state of total fuzzy system) $\dot{x}_n(t) = a^{iT} x(t) + b^i u(t)$ (21)

3- The linear reference model and the canonical form of its matrices are:
$\dot{x}_m(t) = A_m x_m(t) + B_m r(t)$ (22)
$$A_m = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \quad B_m = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_m \end{bmatrix} \quad (23)$$

There are two cases depending on our assumptions

Case1: if the b^i members of the matrix B^i have equal and known signs for all $i \in R^N$, $|b^i| \geq b_0^{*i} > 0, \sum_{i=1}^{N} \xi^i b^i \neq 0$.

Case2: if the b^i members of the matrix B^i have unknown signs, $b^i \in [min_b^i, max_b^i]$, $\sum_{i=1}^{N} \xi^i b^i \neq 0$.

4- In case 1, the control law is:
$u(t) = \sum_{i=1}^{N} \xi^i [K_1^{iT} x(t) + K_2^i r(t)]$ (24)
and the feedback fuzzy system is
$\dot{x}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \xi^i \xi^j [(A^i + B^i K_1^{jT}) x(t) + B^i K_2^j r(t)]$ (25)

5- The matching conditions between the feedback fuzzy system and the linear reference model will be
$$A_m = A^i + B^i K_1^{jT} \qquad B_m = B^i K_2^j \quad (26)$$
which is improper to apply matching for each (A^i,B^i) to every K^j.

6- In case 2, which is used to overcome the matching conditions problem between the system parameters and control parameters,

7- The control law will be
$$u(t) = \frac{\sum_{i=1}^{N} \xi^i [a^T{}_m x(t) - a^{iT} x(t) + b_m r(t)]}{\sum_{i=1}^{N} \xi^i b^i} \quad (27)$$

8- The matching conditions will be
$A_m = A^i + B^i K_1^{iT} \qquad B_m = B^i K_2^i$
which is more relevant than the case in step 5, where the matching condition here is only required with in the same closed loop subsystem and do not need the controller parameters of the other subsystems.

9- In any case, the equivalent system will be
$\dot{x}(t) = A_m x(t) + B_m r(t)$ (28)

10- The system parameters are unknown, therefor they must be estimated and the adaptive control law will be
$$u(t) = \frac{\sum_{i=1}^{N} \xi^i [a^T{}_m x(t) - \hat{a}^{iT} x(t) + b_m r(t)]}{\sum_{i=1}^{N} \xi^i \hat{b}^i} \quad (29)$$

11- Using lyapunov stability criteria to prove and derive the adaptive control and parameter laws.

14.2.2. Adaptive T–S Fuzzy State Tracking Control Using State Feedback with fuzzy reference model [1]

If the fuzzy system is used instead of the linear reference model, the same steps used in the linear reference model with indirect adaptive control are used here with the following differences:

In step 3: the fuzzy reference model is:

$$\dot{x}_m(t) = \sum_{i=1}^{N} \xi^i \left[A_m^i x_m(t) + B_m^i r(t)\right] \quad (30)$$

where the canonical matrices are

$$A_m^i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_{m1}^i & -a_{m2}^i & -a_{m3}^i & \dots & -a_{mn}^i \end{bmatrix} \quad B_m^i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_m^i \end{bmatrix} \quad (31)$$

In step 5: The matching conditions between the feedback fuzzy system and the linear reference model will be

$$A_m^i = A^i + B^i K_1^{jT} \qquad B_m^i = B^i K_2^j \quad (32)$$

which is improper to apply matching for each (A_m^i,B_m^i) to every K^j.

In step 8: The matching conditions between the feedback fuzzy system and the linear reference model will be

$$A_m^i = A^i + B^i K_1^{iT} \qquad B_m^i = B^i K_2^i \quad (33)$$

which is more relevant than the case in modification of step 5, where the matching condition here is only required with in the same closed loop subsystem and do not need the controller parameters of the other subsystems.

14.2.3. Adaptive T–S Fuzzy Output Tracking Control Using State Feedback [1]

The relative degree ρ of the system affects the feedback linearization method in nonlinear T–S fuzzy control system. The causality of global T–S fuzzy system will be ensured, if the premise part vector $\xi(\tau)$ is contained only the values of states such that $\tau \le t - \rho$

1- The state space of the T-S fuzzy system is:
$$x(k+1) = \sum_{i=1}^{N} \xi^i(k)\left[A^i x(k) + B^i u(k)\right] \quad (34)$$
2- Determine the relative degree ρ of the T-S fuzzy system
3- The causal T-S Fuzzy system for $\tau \le t - \rho$ is
$$x(k+1) = \sum_{i=1}^{N} \xi^i(k-\rho)\left[A^i x(k) + B^i u(k)\right] \quad (35)$$
$$y(k) = \sum_{i=1}^{N} \xi^i(k-\rho) C^i x(k) \quad (36)$$

Taking ρ = 1 for T–S Fuzzy System

4- $$x(k+1) = \sum_{i=1}^{N} \xi^i(k-1)\left[A^i x(k) + B^i u(k)\right] \quad (37)$$
5- $$y(k) = \sum_{i=1}^{N} \xi^i(k-1) C^i x(k) \quad (38)$$
6- The T–S Fuzzy Controller
$$y(k+1) = \sum_{i=1}^{N} \xi^i(k) c^i x(k+1) = R(k)x(k) + G(k)u(k) \quad (39)$$
$$R(k) = \sum_{i=1}^{N} \xi^i(k) c^i \sum_{j=1}^{N} \xi^j(k-1) A^i \in R^{1\times n} \quad (40)$$
$$G(k) = \sum_{i=1}^{N} \xi^i(k) c^i \sum_{j=1}^{N} \xi^j(k-1) B^j \in R \quad (41)$$
7- The general relative degree conditions of the T-S Fuzzy system are
$$C^i A^{j1} \dots A^{jk} B^l = 0 \qquad i, j_1, \dots, J_k, l = 1,2,3,\dots N \quad k = 0,1,\dots,\rho-2 \quad (42)$$
$$\sum_{i=1}^{N} \xi^i(k) C^i \sum_{j1=1}^{N} \xi^{j1}(k-1) A^{j1} \dots \sum_{j1=1}^{N} \xi^{j\rho-1}(k-\rho+1) A^{j\rho-1} \sum_{l=1}^{N} \xi^l(k-\rho) B^l \neq 0 \quad (43)$$
8- Assumption 1: $G(k) \neq 0 \quad k \ge 0$
Assumption 2: the fuzzy system (36) is minimum phase condition, that is

$|u(k-d)| \le c_1|y(k)| + c_2 \sum_{\tau=0}^{k-1} \lambda^{k-\tau-1} |y(\tau)|, k \ge d$ (44)

$c_1 > 0, c_2 > 0, \lambda \in (0,1)$

9- From reference output signal:

$$y_m(k+1) = R(k)x(k) + G(k)u(k) \tag{45}$$

$$u(k) = \frac{1}{G(k)}\left(y_m(k+1) - R(k)x(k)\right) \tag{46}$$

Or with error

$$u(k) = \frac{1}{G(k)}(y_m(k+1) - R(k)x(k) - K_1e(k) - K_2e(k-1) - \cdots - K_3e(k-q+1)) \tag{47}$$

10- The parameters $K_1, K_2, \ldots$ are selected to ensure the polynomial $1+K_1z^{-1}+\ldots+K_qz^{-q}$ is inside the unit circle.

11- To design the adaptive control

$$y(k+1) = K_1^T w_1(k) + K_2^T w_2(k) \tag{48}$$

$$K_1 = [c_1A_1, c_1A_2, c_2A_1, c_2A_2]^T \tag{49}$$

$$K_2 = [c_1B_1, c_1B_2, c_2B_1, c_2B_2]^T \tag{50}$$

$$w_1 = [\xi_1(k)\xi_1(k-1)x^T(k), \xi_1(k)\xi_2(k-1)x^T(k), \xi_2(k)\xi_1(k-1)x^T(k), \xi_2(k)\xi_2(k-1)x^T(k)]^T \tag{51}$$

$$w_2 = [\xi_1(k)\xi_1(k-1)u^T(k), \xi_1(k)\xi_2(k-1)u^T(k), \xi_2(k)\xi_1(k-1)u^T(k), \xi_2(k)\xi_2(k-1)u^T(k)]^T \tag{52}$$

12- $y(k+1) = \theta^T \emptyset(k)$ (53)

$\theta = [K_1^T, K_2^T]^T$

$\emptyset(k) = w_1^T(k) + w_2^T(k)$

13- The gradient adaptive law is used to estimate the unknown parameters

14.2.4. Adaptive T–S Fuzzy Control Using Output Feedback [1]

Two approaches will be explained briefly and for more details returning to the reference [1]. In any case, consider the general form of the *ith* rule T-S Fuzzy system in SISO discrete system:

$IF\ I_1(t)\ is\ F_1^i\ and\ \ldots I_j(t)\ is\ F_j^i\ and\ \ldots$

$and\ I_L(t)\ is\ F_L^i\ THEN$

$$y(k) + a_1^i y(k-1) + \cdots + a_n^i y(k-n) = b_0^i u(k-d) + b_1^i u(k-d-1) + \cdots + b_{n-d}^i u(k-n) \qquad b_0^i \ne 0 \tag{54}$$

$i = 1, \ldots R\ Rules \quad j = 1, \ldots L\ Input\ vector\ dimention$

$d: delay \quad n: degree$

14.2.4.1. daptive T–S Fuzzy State Tracking Control Using State Feedback with fuzzy reference model [1]

The prediction models are obtained first and then construct the overall nonlinear prediction model by fuzzy combination of all these predicted models:

1- The *ith* rule, (54) can be rewritten as:

$$A^i(z^{-1})[y](k) = z^{-d}\overline{B}^i(z^{-1})[u](k) \tag{55}$$

and with some mathematics the fuzzy output with *d* delays using T-S Fuzzy system with singleton fuzzification and product inference, using weighted average defuzzification will be:

$$y(k+d) = \sum_{i=1}^{N} \xi^i \alpha^i(z^{-1})[y](k) + \sum_{i=1}^{N} \xi^i \beta^i(z^{-1})[u](k) \tag{56}$$

2- The fuzzy system satisfies the minimum phase condition:

$|u(k-d)| \le c_1|y(k)| +$

$c_2 \sum_{\tau=0}^{k-1} \lambda^{k-\tau-1} |y(\tau)|, k \geq d$ (57)

3- The control signal that is derived according to the reference signal is
$u(k) = \frac{1}{\sum_{i=1}^{N} \xi^i \beta_0^i u(k)} \left(y_m(k+d) - \sum_{i=1}^{N} \xi^i \alpha^i (z^{-1})[y](k) - \sum_{i=1}^{N} \xi^i \overline{\beta}^i (z^{-1})[u](k) \right)$ (58)

4- The gradient adaptive law is used to estimate the unknown parameters $\hat{\alpha}^i = \alpha^i \, \hat{\overline{\beta}}^i = \overline{\beta}^i$ and $\hat{\beta}_0^i = \beta_0^i$.

The system stability with adaptive and control laws are proved using lyapunov criteria.

14.2.4.2. Total Nonlinear Prediction Last (TNPL) [1]

In TNPL, the overall nonlinear fuzzy model (54) is obtained first and then the nonlinear prediction model is directly constructed from it:

1- Taking the local linear system of the form
$y(k) = \bar{A}^i (z^{-1})[y](k) + z^{-d} \bar{B}^i (z^{-1})[u](k)$ (59)
$y(k+d) = \bar{A}^i (z^{-1})[y](k+d) + \bar{B}^i (z^{-1})[u](k)$ (60)
where
$\bar{A}^i (z^{-1}) = -\alpha_1^i z^{-1} + \cdots + \alpha_n^i z^{-n}$

$B^i (z^{-1}) = B_0^i + B_1^i z^{-1} + \cdots + B_{n-d}^i z^{-n+d}$

2- The T-S Fuzzy system with singleton fuzzification and product inference, using weighted average defuzzification will be
$y(k+d) = \sum_{i=1}^{N} \xi^i(k) \bar{A}^i (z^{-1})[y](k+d) + \sum_{i=1}^{N} \xi^i(k) \bar{B}^i (z^{-1})[u](k)$ (61)

3- To predict the T–S fuzzy model using nonlinear prediction and from (61)
$y(k+d) = f_y(\xi^i(.), y(.)) + f_u(\xi^i(.), u(.))$ (62)
$f_y(\xi^i(.), y(.)) = f_y\left(\xi^i(k), \ldots, \xi^i(k-d+1), y(k), \ldots, y(k-n+1)\right)$ (63)
$f_u(\xi^i(.), u(.)) = f_u\left(\xi^i(k), \ldots, \xi^i(k-d+1), u(k), \ldots, u(k-n+1)\right)$ (64)
$f_u(\xi^i(.), u(.)) = \sum_{i=1}^{N} \xi^i(k) b_0^i u(k) + f_{u1}\left(\xi^i(k), \ldots, \xi^i(k-d+1), u(k-1), \ldots, u(k-n+1)\right)$ (65)

4- From (62) and (65) the prediction is independent of the *y(t+d-1), …, y(t+1)*. Where (65) and (62) can be represented as
$y(k+d) = \theta_y^T \emptyset_y(k) + \theta_{u0}^T \emptyset_{u0}(k) + \theta_{u1}^T \emptyset_{u1}(k)$ (66)
with some mathematics
$y(k+d) = \theta^T \emptyset(k)$ (67)
$y(k) = \theta^T \emptyset(k-d)$ (68)
Equations (67) or (63) represents an adaptive Predictor

5- The general form of the *ith* rule T-S Fuzzy controller with estimated parameters is
$IF\ I_1(t)\ is\ F_1^i\ and \ldots I_j(t)\ is\ F_j^i\ and \ldots and\ I_L(t)\ is\ F_L^i\ THEN$
$u(k) = \frac{1}{\sum_{i=1}^{R} \xi^i \hat{b}_0^i} \left(y_m(k+d) - \hat{\theta}_y^T \emptyset_y(k) - \hat{\theta}_{u1}^T \emptyset_{u1}(k) \right)$ (69)
$i = 1, \ldots R\ Rules$

6- The gradient adaptive law is used to estimate the unknown parameters $\hat{\theta}\ of\ \theta$. And the lyapunov criterion is used for stability considerations.

14.3. Adaptive Fuzzy Back Stepping Control [5, 2, 12]

Consider the strict feedback stat space, the back stepping typical control method can be outlined by the following points:

1- Take the first state which is of the form
$$\dot{\boldsymbol{x}} = f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})z_1 \quad (70)$$

2- Treats z_1 as a virtual control signal for (70).

3- Select a smooth state feedback control signal $\phi(x), \quad \phi(0) = 0$.

4- Let $z_1 = \phi(x)$, such that the origin of $\dot{\boldsymbol{x}} = f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})\emptyset(x)$ is asymptotically stable.

5- Select a smooth lyapunov function $V_0(x) > 0$ such that
$$\frac{\partial V_0}{\partial x}[f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})\emptyset(x)] \leq -W_0(x), \quad W_0(x) > 0 \quad (71)$$

6- If such a lyapunov function is defined then (70) is asymptotically stable.

7- Define the error signal as $e = z_1 - \phi(x)$ (72)

8- Solve $\dot{\boldsymbol{x}} = f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})e, \quad \dot{e} = \dot{z}_1 - \dot{\phi}(x)$

9- Select a lyapunov function to stabilize all the previous system and derive the equation of the next virtual control signal:
$V_1(x, e) > 0$, such that
$$\frac{\partial V_1}{\partial x}[f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})e] \leq -W_1(x), \quad W_1(x) > 0 \quad (73)$$

10- Repeat for the *nth* order states of the system, where the final lyapunov function represents the sum of all the previous lyapunov functions to solve the equation of the actual control signal.

In this case, it can be noted that the application of the fuzzy system in back stepping control represented by identifying the unknown functions. Together with the lyapunov stability criteria, the control and adaptive laws are derived.

15. Literature Survey

Different applications in adaptive fuzzy control system are designed and studied. Some of them take the total system to be adaptively controlled, other applications take a special case of the system such as input or output dead zone, sampling problems, predefined control direction problem, etc…. . Here, a dedicated works are selected to view a variety of adaptive fuzzy applications:

In [6], Xu, Q.-Y. and Li, X.-D used adaptive fuzzy iterative learning control ILC for discrete time nonlinear system with unknown input dead zone and control direction. An adaptive mechanism is used to handle the dead zone and a discrete Nussbaum gain technique is used to deal with unknown control direction problem.

Shahriari-kahkeshi and Rahmani in [13], proposed an adaptive dynamic surface control scheme using an interval type-2 fuzzy system to control nonlinear uncertain applications that have unknown non symmetric dead zone. They represent the dead zone as a time varying system that affected by a bounded disturbance. The system is represented by n states, except the last one, each state is affected by a virtual control signal. The actual control signal is affect the last state. The interval tupe-2 fuzzy system is used to estimate the unknown system parameters and an adaptive term is suggested to deal with the disturbance-like effects in the dead zone. Adaptive laws are derived to update two types of parameters: the interval type-2 fuzzy system consequent parameters and the dead zone model. In this work the authors avoided the complexity explosion that introduced by the back stepping calculations which used repeated differentiation of virtual control inputs, where they applying a low pass filter to the virtual and actual control signals.

As in [13], Wang et al. in [10], proposed an adaptive fuzzy system using a command filter to solve the problem of (explosion of complexity) that results from the back stepping control approach. This adaptive fuzzy system is used in Chua's chaotic system with external disturbance. The lyapunov stability criterion is used to ensure the closed loop signals to be bounded and the tracking error is converged to a small region.

In [14], Ketata et al. propose a method that guarantees the robustness and global stability of nonlinear system using an adaptive T-S fuzzy controller with the gradient method for adaption algorithm. The proposed method used an appropriate gradient step to avoid local minima that produced by the parameter saturation.

In [15], Bechlioulis and Rovithakis used a robust adaptive fuzzy system that, guaranties error bound in transient and steady state regions to control non-affine nonlinear systems. They proposed a prescribed system performance with exogenous disturbance, by which; they the maximum overshot and steady state error are predefined using an envelope function *ρ(t)*. Also, the unknown direction of the control signal was solved by using the Nussbaum function *N(.)* and the simplified calculation was implemented using a filtered error *s(t)*.

In [16], Li et al. proposed an indirect adaptive fuzzy control system to deal with the input and output constraints by using a barrier Lyapunov function and an auxiliary system for uncertain nonlinear applications. They used the adaptive back stepping technique with the adaptive fuzzy system to estimate the unknown parameters and reduced them to 2n.

In [17], Phan and Gale presented an adaptive fuzzy control system based on self-structuring direct adaptive approach for affine non-linear system. This system had only one restriction represented by a positive gain g(x), no constraints of knowing the upper bound or the derivative of the gain g(x). The self-structuring is represented by a variable rule base structure, that has been updated and a new rule will be added if necessary with in a predefined rule bound. The Lyapunov criterion was implemented for stability considerations and parameters adaptive laws construction.

To ensure the stability of the self-structuring system, they prove that the Lyapunov function is the same before and after changing the rule structure $V(t_c^-) = V(t_c^+)$, let:

$V(.)$: the Lyapunov function.

t_c: *the time at changing the rule structure*

t_c^-: *the time befor changing the rule structure*

t_c^+: *the time after changing the rule structure*

The control signal is continuous at t_c , that is

$$u(t_c^-) = u(t_c^+)$$

This leads to equal states

$$\boldsymbol{x}(t_c^-) = \boldsymbol{x}(t_c^+)$$

Then

$$\boldsymbol{e}(t_c^-) = \boldsymbol{e}(t_c^+)$$

So, the parameters are equal

$$\boldsymbol{\theta}(t_c^-) = \boldsymbol{\theta}(t_c^+)$$

And leads to

$$V(t_c^-) = V(t_c^+)$$

Boulkroune et al. in [7] investigated a variable structure adaptive fuzzy controller for multi-input multi-output (MIMO) uncertain system with time delay sector nonlinearity and dead-zone. For controller design and stability analysis, they used the decomposition property of the control gain matrix. The lyapunov stability criterion was used for stability analysis and adaptive algorithms investigation. The closed loop signals boundedness are priori determined. There was not priory knowledge of the time varying delay, but the upper bound and time delay derivative must be known.

Liu, X. et al., in [2], proposed a fault tolerant adaptive fuzzy control system for unknown control direction pure feedback nonlinear non-affine systems with sensor failures. They used the mean value theorem to transform the pure feedback system to a strict feedback form. The sensor faults causes a state faults which are solved by separating the unknown failed parameters and regrouping them with the plant parameters. The adaptive fuzzy fault-tolerant overcomes the effects of the control direction problem faults in the multi sensor system. The lyapunov stability criterion is used to prove the exponential stability even with sensor failures and to extract the adaptive laws.

Boulkroune et al, in [18], presented an adaptive fuzzy controller to deal with multi variable uncertain unknown control direction systems that have unknown nonlinear actuator parameters represented by dead-zone and backlash-like hysteresis. The design exhibits two phases; with or without varying time delay. The unknown control direction which is represented by the sign of the control gain matrix was handled using the Nussbaum-type function in the control low.

In [19], Li et al. proposed an adaptive fuzzy control system for SISO strict feedback nonlinear system, where the states are not available and unknown nonlinear functions. The fuzzy system was used in to cases; first, to estimate the unknown nonlinear functions; second, the adaptive fuzzy high gain observer was used to estimate the immeasurable states. The back stepping framework is used to derive the nonlinear system and the corresponding adaptive fuzzy control is designed. The lyapunov stability approach was used to guarantee the semi-global boundedness of the closed loop system.

As in [19], Zhang and Su in [8], proposed an observer based adaptive fuzzy control to estimate the unknown states of nonlinear uncertain system with the lyapunov stability criteria. In this proposition, the system was in MIMO form, and there was also unknown output dead zone. They used the Nussbaum gain function to deal with the unknown control direction.

16. Conclusions

From this work, it can be seen that the typical adaptive control theories and techniques were implemented in fuzzy system field. Combining different approaches, the adaptive fuzzy control system can deal with variety types of applications' problem. The unknown control direction was solved by using the Nussbaum function, the nonlinearity of complicated systems was dealed with by using the parallel distributed compensation, feedback linearization or back stepping techniques, which are consistent with the structure of the fuzzy system. Although it is complicated, the stability considerations and control law derivation was implemented by the Lyapunov criteria, which gives a robust results in mathematical derivation. As a literature survey, more details and investigations are required for each of the adaptive fuzzy techniques for future works.

Conflict of Interest

The authors confirm that the publication of this article causes no conflict of interest.

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