# ON A CLOSED AND COMPLETELY CLOSED FILTER WITH RESPECT TO AN ELEMENT OF A BH-ALGEBRA

**Husein Hadi Abbass**
College of Education for girls
Department of Mathematics
Msc_hussien@yahoo.com
**Hayder Abbas Dahham**
College of Dentistry
Hay_ad24@yahoo.com
University of Kufa

Abstract
In this paper, we introduce the notions of a closed filter with respect to an element of a BH-algebra and a completely closed filter with respect to an element of a BH-algebra and study these notions on a BG-algebra. Also We stated and prove some theorems which determine the relationship between these notions and some types of ideals of a BH-algebra.
**Keywords** : CLOSED FILTER, COMPLETELY CLOSED FILTER , BH-ALGEBRA

# حول المرشح المغلق و المغلق التام بالنسبة الى عنصر في جبر- BH

**حسين هادي عباس**
كلية التربية للبنات / قسم الرياضيات
**حيدر عباس دحام**
جامعة الكوفة/كلية طب الاسنان

**الملخص:**
قدمنا في هذا البحث مفهوم المرشح المغلق بالنسبة الى عنصر في جبر-BH و المرشح المغلق التام بالنسبة الى عنصر في جبر-BH, كما درسنا هذا المفهوم في جبر-BG .
كما وضعنا وأثبتنا بعض المبرهنات ذات العلاقة بين هذا المفهوم و بعض انواع المثاليات في جبر BH و جبر BG.

## INTRODUCTION

The notions of a BCK-algebra and a BCI-algebra was formulated first in 1966 by Y.Imai and K.Iseki [8]. In 1983,Q. P. Hu and X. Li introduced the notion of BCH-algebra which are a generalization of BCK/BCI-algebras [10]. In 1991, C. S. Hoo introduced the notions of a filter and closed filter of a BCI–algebra[2].In 1996, M. A. Chaudhry and H. F-Ud-Din studied the concepts of filter and closed filter of a BCH-algebra[9]. In the same year, J.Neggers introduced the notion of d-algebra[7]. In 1998, Y.B.Jun, E.H.Roh and H.S.Kim introduced a new notion, called a BH-algebra[12]. In 2002 ,J.Neggers and H.S.Kim introduced the notion of B-algebra, which is a generalization of a BCK-algebra [6]. In 2008, C.B.Kim and H.S.kim introdeced the notion of BG-algebras, which is a generalization of a B-algebras[1]. In 2011, H.H.Abass and H.M.A.Saeed introduced the notion of a closed ideal with respect to an element of a BCH-algebra[3]. In 2012, H.H.Abass and H.A.Dahham introduced the notions of a completely closed ideal and completely closed ideal with respect to an element of a BH-algebra[4].In the same year, H.H.Abass and H.A.Dahham introduced the notions of a completely closed filter of a BH-algebra[5]. In this paper, we introduce the notions as we mentioned in the abstract.

1.PRELIMINARIES

In this section, we give some basic concepts about a BG-algebra , a BH-algebra, ideal of a BH-algebra , closed ideal of a BH-algebra, a completely closed ideal of a BH-algebra , closed ideal with respect to an element of a BH-algebra, completely closed ideal with respect to an element of a BH-algebra, a normal set , filter, closed filter and completely closed filter with some theorems and propositions which we needed in our work.

Definition ( 1.1 ) [1] :

A BG-algebra is a non-empty set X with a constant 0 and a binary operation " * " satisfying the following axioms:

1) $x * x = 0$,
2) $x * 0 = x$,
3) $(x * y) * (0 * y) = x$, for all $x, y \in X$.

Definition ( 1.2 ) [12] :

A BH-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following conditions:

1) $x * x = 0$, $\forall x \in X$.
2) $x * y = 0$ and $y * x = 0$ imply $x = y$, $\forall x, y \in X$.
3) $x * 0 = x$, $\forall x \in X$.

Proposition (1.3) [1]:

Every BG-algebra is a BH-algebra.

Definition ( 1.4) [11] :

A nonempty subset S of a BH-algebra X is called a BH-Subalgebra or Subalgebra of X if $x * y \in S$ for all $x, y \in S$.

Lemma ( 1.5 ) [1] :

Let $(X, *, 0)$ be a BG-algebra. Then

1) the right cancellation law holds in X, i.e., $x*y = z*y$ implies $x = z$,
2) $0 * (0 * x) = x$ for all $x \in X$,
3) if $x * y = 0$, then $x = y$ for all $x, y \in X$,
4) if $0 * x = 0 * y$, then $x = y$ for all $x, y \in X$,
5) $(x * (0 * x)) * x = x$ for all $x \in X$.

Definition ( 1.6 ) [12] :

Let I be a nonempty subset of a BH-algebra X. Then I is called an ideal of X if it satisfies:

1) $0 \in I$.
2) $x*y \in I$ and $y \in I$ imply $x \in I$

Definition ( 1.7 ) [3] :

An ideal I of a BH-algebra X is called a closed ideal of X if :for every $x \in I$, we have $0*x \in I$.

Definition ( 1.8 )[4]:

An ideal I of a BH-algebras is called a completely closed ideal if $x*y \in I$, $\forall x,y \in I$.

Definition (1.9)[3]:

Let X be a BH-algebra and I be an ideal of X . Then I is called a Closed Ideal with respect to an element $b \in X$ (denoted b-closed ideal) if $b*(0*x) \in I$ , for all $x \in I$.

Definition (1.10)[4]:

Let I be an ideal of a BH-algebras X and $b \in X$. Then I is called a completely closed ideal with respect to b(denoted by b-completely closed ideal) if $b*(x*y) \in I$, $\forall x,y \in I$.

Theorem ( 1.11)[4]:

Let X be a BG-algebra. Then every ideal in X is a completely closed ideal.

Definition (1.12)[5]:

A filter of a BH-algebra X is a non-empty subset F of X such that:

1) If $x \in F$ and $y \in F$, then $y*(y*x) \in F$ and $x*(x*y) \in F$.
2) If $x \in F$ and $x*y=0$, then $y \in F$.

Further F is a closed filter if $0*x \in F$ for all $x \in F$. In the sequel we shall denote $y*(y*x)$ by $x \wedge y$.

Definition (1.13)[5]:

Let X be a BH-algebra, and F be a filter. Then F is called a completely closed filter if $x*y \in F$ $\forall x, y \in F$.

Remark(1.14)[5]:

Every completely closed filter is a closed filter.

Definition ( 1.15 ) [4] :

A non-empty subset N of a BH-algebra X is said to be normal of X if $(x * a) *(y * b) \in N$ for any $x * y, a * b \in N$.

Theorem ( 1.16) [4].

Every normal subset N of a BH-algebra X is a subalgebra of X.

Theorem (1.17)[5]:

Let X be a BG-algebra and N be a normal subalgebra .Then N is a completely closed filter.

Proposition (1.18)[5]:

Let X be a BH-algebra, and F is a completely closed filter. Then $0 \in F$.

Theorem (1.19)[5]:

Let X be a BH-algebra ,if x*y=y*x ∀x, y∈X ,then every completely closed ideal is a completely closed filter.

Proposition (1.20)[5]:

Let X be a BG-algebra. Then every ideal in X is a completely closed filter.

## 2.THE MAIN RESULTS

In this section, we define the notions of a closed filter with respect to an element of a BH-algebra and a completely closed filter with respect to an element of a BH-algebra . For our discussion , we shall link these notions with other notions which mentioned in preliminaries.

Definition (2.1):

Let X be a BH-algebra and b∈X . A filter F is called a closed filter with respect to b(denoted by b- closed filter) if b*(0*x)∈F ∀x∈F.

Example(2.2):

Consider the BH-algebra X = {0,a,b,c} with the following operation table.

| * | 0 | A | b | c |
|---|---|---|---|---|
| 0 | 0 | A | b | c |
| a | a | 0 | c | b |
| b | b | C | 0 | a |
| c | c | B | a | 0 |

The filter F={0,a} is a 0- closed filter, since

0*(0*0)=0∈F , 0*(0*a)=a∈F.

Proposition (2.3):

Let X be a BG-algebra. Then every filter in X is a 0-closed filter

Proof:

Let X be a BG-algebra ,and F be a filter

Since 0*(0*x)=x ∀x∈X [By lemma (1.5)(2)]

⇒ 0*(0*x)=x ∀x∈F [Since F⊆X]

⇒ 0*(0*x)∈F ∀x∈F

⇒ F is a 0-closed filter.■

Proposition (2.4):

Let X be a BH-algebra. If F is a filter and normal subalgebra, then F is a b-closed filter ∀b∈F.

Proof:

Let b,x∈F,

⇒b=b*0∈F . [Since x=x*0. By definition(1.2)(3)]

Also x*x=0=0∈F [Since X is a BH-algebra and F is a subalgebra]

⇒0*x∈F [Since $0, x \in F$ and F is a subalgebra. By definition(1.4)]

⇒(b*0)*(0*x)∈F [Since F is a normal set, By definition(1.15)]

⇒b*(0*x)∈F [Since b*0=b. By definition(1.2)(3)]

⇒ F is a b-closed filter.■

Proposition (2.5):

Let X be a BG-algebra and I be an ideal of X. Then I is a b-closed filter,∀b∈I.

Proof :

Since I is an ideal of a BG-algebra X

⇒I is a completely closed filter [By Proposition(1.20)]

Now, let b∈I,x∈I,

⇒b*(0*x)∈I [Since 0∈I and every ideal in BG is a completely closed ideal. By Proposition(1.11)]

Therefore, I is a b-closed filter ∀b∈I.■

Proposition (2.6):

Let X be a BH-algebra. Such that x*y=y*x ∀x, y∈X and let I be a completely closed ideal of X. Then I is a b-closed filter for all b∈I.

Proof:

Let I be a completely closed ideal,

⇒I is a completely closed filter. [By Theorem(1.19)]

Now, let b∈I, x∈I

⇒b*(0*x)∈I [Since 0∈I and I is a completely closed ideal].

Therefore, I is a b-closed filter.■

Definition (2.7):

Let X be a BH-algebra and b∈X, a filter F is called a completely closed filter with respect to b (denoted by b- completely closed filter) if b*(x*y)∈F ∀x, y∈F.

Example (2.8):

Consider the BH-algebra X = {0,1,2,3} with the following operation table.

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 3 | 2 | 1 |
| 1 | 1 | 0 | 3 | 2 |
| 2 | 2 | 1 | 0 | 3 |
| 3 | 3 | 2 | 1 | 0 |

The set F={2,3} is a 2-completely closed filter, since

1)F is a filter, since

But F is not a 3-completely closed filter, since

$3*(2*3)=3*3=0\notin F$.

Remark(2.9):

1) The b-closed filter of a BH-algebra X may be not a b-completely closed filter of X.The filter F={2,3} in example (2.8) is a 0-closed filter, Since

$0*(0*2)=2\in F$ ,$0*(0*3)=3\in F$ but it is not 0-completely closed filters since $0*(2*2)=0\notin F$.

2) If F is a b-completely closed filter, then it is not necessary to be a b-closed filter, as example (2.8). The filter F={2,3} is a 2-completely closed filter, but it is not 2-closed filter since $3\in F$ but $2*(0*3)=2*1=1\notin F$.

Proposition (2.10):

Let X be a BH-algebra and let F be a filter of X. Then F is not b-completely closed filter, $\forall b\notin F$.

Proof:

Let F be a filter, $b\notin F$ and $x\in F$.

$\Rightarrow b*(x*x)=b*0=b\notin F$.

$\Rightarrow$ F is not a b-completely closed filter.■

Proposition (2.11):

Let X be a BH-algebra. Then every completely closed filter is a b-completely closed filter, $\forall b\in F$.

Proof :

Let F be a completely closed filter, $b\in F$, $x, y\in F$

$\Rightarrow b*(x*y)\in F$ [Since F is a completely closed filter. By definition(1.13)]

$\Rightarrow$ F is a b-completely closed filter.■

Proposition (2.12):

Let X be a BG-algebra. Then every ideal is a b-completely closed filter,$\forall b\in I$.

Proof:

Let X be a BG-algebra, and let I be an ideal

$\Rightarrow$I is a completely closed filter [By Proposition(1.20)]

Now, let b, x, $y\in I$,

$\Rightarrow b*(x*y)\in I$ [Since I is a completely closed filter .By definition(1.13)]

Therefore, I is a b- completely closed filter $\forall b\in I$.■

Proposition (2. 13):

Let X be a BH-algebra. Then every completely closed filter is a b- closed filter, $\forall b\in F$.

Proof:

Let F be completely closed filter, $b\in F$ and $x\in F$.

Since F is a completely closed filter.

Then $0\in F$ [By Proposition(1.18)]

$\Rightarrow b*(0*x)\in F$ [Since F is a completely closed filter By definition(1.13)]

$\therefore$F is a b-closed filter $\forall b\in F$. ■

Proposition (2.14):

Let X be a BH-algebra. Then every completely closed filter is not a b- closed filter, $\forall b\notin F$.

Proof:

Let F be completely closed filter, $b\notin F$,

Since F is a completely closed filter,

Then $0\in F$ [By Proposition(1.18)]

$\Rightarrow b*(0*0)=b\notin F$

$\Rightarrow$ F is not b-closed filter $\forall b\notin F$. ■

Proposition (2.15):

Let X be a BG-algebra and N be a normal subalgebra of X. Then N is a b-closed filter $\forall b\in N$.

Proof:

Let N be a normal subalgebra.

$\Rightarrow$N is a completely closed filter [By theorem(1.17)]

$\Rightarrow$N is a closed filter. [By remark (1.14)]

Now, let b, $x\in N$

⇒b*0∈N, 0*x∈N, [Since F is a closed filter. By definition(1.12)]

⇒(b*0)*(0*x)∈N [Since N is a normal subalgebra. By definition(1.15)]

⇒b*(0*x)∈N

⇒N is a b-closed filter.■

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