

## SPECTRUM SENSING OF WIDE BAND SIGNALS BASED ON ENERGY DETECTION WITH COMPRESSIVE SENSING

\* Ali Mohammad A. AL-Hussain<sup>1</sup>

Maher K. Mahmood<sup>2</sup>

1) MSc. Student, Electrical Engineering Department, Mustansiriya University, Baghdad, Iraq.

2) Prof., Electrical Engineering Department, Mustansiriya University, Baghdad, Iraq.

Received 26/2/2020

Accepted in revised form 21/6/2020

Published 1/11/2020

**Abstract:** Compressive sensing (CS) technique is used to solve the problem of high sampling rate with wide band signal spectrum sensing where high speed analogue to digital converter is needed to do that. This leads to difficult hardware implementation, large time of sensing and detection with high consumptions power. The proposed approach combines energy-based detection, with CS compressive sensing and investigates the probability of detection, and the probability of false alarm as a function of the SNR, showing the effect of compression to spectrum sensing performance of cognitive radio system. The Discrete Cosine Transform (DCT) is used as a sparse representation basis of the received signal, and random matrix as a compressive matrix. The  $\ell_1$  norm algorithm is used to reconstruct the original signal. A closed form of probability of detection and probability of false alarm are derived. Computer simulation shows clearly that the compression ratio, recovery error and SNR level affect the probability of detection.

**Keywords:** *Cognitive radio, Compressive Sensing, Spectrum Sensing, Energy Detection*

### 1. Introduction

The crowded available radio frequency bands due to ineffective use, which is based on Fixed Spectrum Allocation Policy (FSA), creates problems associated with the rapid growth of communications applications to be solved.

Several solutions to overcome this problem are suggested, one of these solutions is Cognitive Radio (CR). Spectrum sensing process is the essential process of CR cognitive radio. There are more several challenges and issues which are associated with this process. The most important of these is the spectrum sensing of wideband signal, with quick acquisition time in particular region in order to meet optimal opportunities to exploit the available radio spectrum. CR Cognitive radio system (CR) includes digital signal processing operation called spectrum sensing, which is used to detect spectrum opportunities, to enable the secondary user to enter the spectrum holes of the primary user frequency bands [1]. The CR is a smart wireless communication system, that is able to adjust its parameters, to improve the usage utilization efficiency of the available spectrum based on the surrounding radio frequency environment. The CR must be able to sense, measure and acknowledge the characteristics of the desired channel [2]. Reconfigurable wireless communication system named Software Defined Radio (SDR) implements CR this function [3, 18]. Many techniques are used to achieve spectrum sensing, such as cyclostationary detection [5,

\* Corresponding Author: [alieng7790@gmail.com](mailto:alieng7790@gmail.com)

17], matching filter detection [6, 16], and energy detection [18]. Energy-based detection method is the most commonly used technique because of its simplicity, which comes from it is non-coherent detection technique, that needs no prior-information of primary user signal, such as carrier frequency and bandwidth, to detect the absence of the Primary User (PU) signal. This method has low cost of implementation, fast time of detection, and it can detect any kind of signal. However, but it cannot distinguish between different types (types of what??) and it is highly affected by noise uncertainty. This means that it requires needs high SNR to achieve good and reliable detection performance. The principle of this technique is based on calculating the amount of energy in the received signal, and simply makes comparison compare it to a with the predefined threshold, which is estimated with more than one strategy, all of which depend on the background noise in the radio channel occupied with noise signal only. The remainder of this paper is constructed as follows, section 2 conventional energy detection is explained, CS compressive sensing energy based detection algorithm is explained in section 3, in section 4 IV the probability of false alarm and probability of detection are derived, simulations results and discussion are drawn and shown in section 5 V. Finally in section 6 VI the conclusion is explained.

## 2. Conventional Energy Detection Technique

Conventional spectrum sensing or non-compressive spectrum sensing techniques, or traditional spectrum sensing techniques can are able to detect narrow band signals, where all of these techniques use Analog to Digital Convertor (ADC) operating at Nyquist sampling rate. One of these methods is energy detection method. Energy of the received signal is calculated in time domain or in frequency domain by squaring the magnitude of the Fast Fourier Transform (FFT) of the received signal (FFT) averaged over the number of the received signal samples  $N$ . The detection of the signal is

done by a binary hypothesis test. The received signal is given by:

$$y(n) = w(n) \quad : H_0 \quad (1)$$

$$y(n) = w(n) + s(n) \quad : H_1 \quad (2)$$

$w(n)$ : noise signal.

$s(n)$ : primary signal.

$H_0$  : Hypothesis denotes the absence of PU signal (noise only).

$H_1$ : Hypothesis denotes the presence of PU signal.

The received signal is assumed to have Gaussian distribution with zero mean and variance  $\sigma_s^2$ , and the noise is Additive White Gaussian Noise (AWGN) with zero mean and variance  $\sigma_w^2$ .

From observation  $y$ , the detection algorithm must determine whether the signal  $s$  is present ( $H_1$ ) or not ( $H_0$ ). A test statistic  $T(x)$  is calculated from the data observed ( $y$ ) and compared to a threshold  $\lambda$ . Let  $P(H_1: H_0)$  state the probability to decide  $H_1$  when  $H_0$  is true. The probability of detecting  $s$  successfully is defined as the probability of detection and is defined by the following expression:

$P_d = P(H_1: H_1) = P(T(x) > \lambda)$  :  $H_1$ ): primary user signal is present

Similarly, the probability of false alarm,  $P_{fa}$ , is the probability that the signal to be detected will be falsely declared and is given by:

$P_{fa} = P(H_1: H_0) = P(T(x) > \lambda)$  :  $H_0$ ): primary user signal is absent

Clearly  $P_d$  and  $P_{fa}$  are mutually dependent. Depending on the Neyman-Pearson (NP) theory, if  $P_{fa}$  is limited to a maximum value, the test statistic  $T(x)$  maximizing  $P_d$  is the probability test takes the form [4]:

$$T(y) = \frac{f_1}{f_0} \underset{H_1}{\overset{H_0}{>}} \lambda \quad (3)$$

Where:

$f_i(x)$  denotes the Probability Density Function (PDF) of  $x$  under hypothesis  $H_i$ . From [7] it is clear that  $T(y)$  calculated from (3) is proportional to

$\|y\|_2^2 = y^T y$ , which is the energy of the received signal.

**2.1. Distribution of the Test Statistic and Detection Performance**

The test statistic  $T(y)$  is a sum of squares  $N$  number of independent Gaussian random variables. As a result,  $T(y)$  has a central Chi-square distribution under hypothesis  $H_0$  and non-central Chi-square distribution under hypothesis  $H_1$ . Based on central limit theorem, with  $N$  samples one can express these distributions as [8]:

$$H_0 : T(y) = (N \sigma_w^2, 2N \sigma_w^4) \quad (4)$$

$$H_1 : T(y) = (N (\sigma_w^2 + \sigma_s^2), 2N (\sigma_w^2 + \sigma_s^2)^2) \quad (5)$$

Using equation (3), (4) and (5)  $P_d$  and  $P_{fa}$  are expressed as [9]:

$$P_d = Q\left(\frac{\bar{\lambda} - N(1+\gamma)}{\sqrt{2N(1+\gamma)^2}}\right) \quad (6)$$

$$P_{fa} = Q\left(\frac{\lambda - N * (\sigma_w)^2}{\sqrt{2N(\sigma_w)^4}}\right) \quad (7)$$

$$\lambda = ((Q^{-1}(P_{fa})\sqrt{2N} * N) * (\sigma_w)^4) \quad (8)$$

Where:

$\bar{\lambda}$ : denotes the average threshold,  $\bar{\lambda} = \lambda / \sigma_w^2$

$\lambda$ : threshold .

$Q(\cdot)$  : Marcum function .

$\sigma_w^2$ : Noise variance.

$N$  : Number of samples.

$\gamma = E_s / N_0$ . Where:

$E_s$  : signal power .

$N_0$  : noise spectral density.

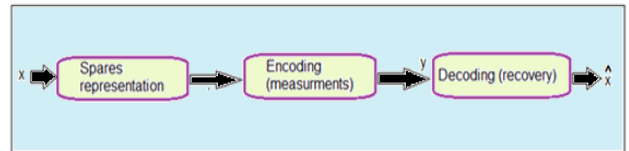
$E_s = \sum_{t=1}^n |s(t)|^2$  : is the signal energy.

For optimum detection, it is necessary to minimize both probability of false alarm and probability of miss-detection ( $1 - P_d$ ).

**3. CS Theory**

In conventional communication systems, it is possible to recover the original signal at the receiver according to Nyquist rate sampling condition based on Nyquist- Shannon theorem (sampling rate  $\geq 2 * \text{maximum signal}$

frequency), while in CS compressive sensing, it is possible to recover the original signal even if its sampling rate is below the Nyquist rate (Sub-Nyquist). CS Compressive Sensing is performed by acquisition process followed by reconstruction process. It consists of three essential cycles, these are: sparse representation, encoding and decoding. Figure (1) shows a block diagram of CS compressive sensing cycles [10].



**Figure 1.** Block diagram of CS Compressive Sensing technique

To clarify sparse representation process let  $x$  be a signal of large dimension consisting of  $N$  number of samples, where this signal is considered sparse signal in some domain, and is denoted by  $k$ -sparse signal, where  $k \ll N$ . This signal can be represented in sparse basis  $\psi$  by number of projections on this basis  $\psi$  [10]. Discrete Cosine Transform (DCT), Fourier transform (FFT) FFT and Hadamard transform are examples of sparse representation basis. Compressible signal can be transformed into sparse signal by using any sparse representation basis [11]. Sparse representation of  $s$  signal is expressed as [9]:

$$x = \psi s \quad (9)$$

$s$  : Projection of the signal in sparse basis  $\psi$ .

$\psi$  : Sparse representation basis matrix.

$\psi = (N * N)$  matrix,  $\|x\|_m = \sqrt[m]{\sum_i |x_i|^m}$  is the  $m$  norm of the signal  $x$  and  $(\|s\|_0 \leq k, \text{ where } k \ll N. \text{ sparse condition}).$

Compression process is achieved by multiplying  $x$  signal by sensing matrix  $\phi$  of dimension  $(M * N)$  as shown in Figure (2). Where  $N$  is the signal samples number and  $M$  is the measurement elements number where  $N \gg M$ .  $M$  measurements should conserve all important

information of the sparse signal, since  $M=O[K\log(N)]$  where random measurements are taken. The measurement vector is expressed as [9].

$$y = \phi s + w \quad (10)$$

Where:

$y$ : (M \* 1) Measurements vector.

$\phi$ : (M \* N) Sensing matrix.

$w$ : Additive noise

$s$ : Sparse signal.

O: order

Therefore:

$$y = \phi x + w = (\phi * \psi)s + w = As + w \quad (11)$$

Figure (2) clarifies CS compressive sensing procedure and matrices [12].

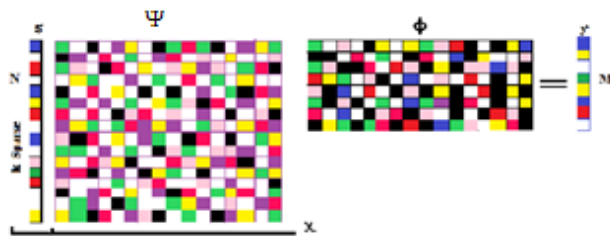


Figure 2. The matrices and vectors of CS process.

The last cycle of CS compressive sensing is reconstruction. For this system, there are infinite solutions due to sparsity of interested signal ( $M < N$ ). So that To solve this underdetermined system, there are more conditions must be added. These conditions are: interested signal must be  $k$ -sparse, sensing matrix must be incoherent and satisfy the condition of Restricted Isometry Property (RIP) [13], that is It is a feature of orthonormal matrices governed with a restricted isometry constant for the restricted isometry property, which is a positive number between 0 and 1. A matrix which meets this property in order  $k$  implies that:

$$\exists \delta \in (0, 1) / (1 - \delta) \|x\|_2^2 \leq \|\phi x\|_2^2 \leq (1 + \delta) \|x\|_2^2$$

Where:  $\delta$  is the restricted isometry constant,  $x$  is the original signal and  $\phi$  is the sensing matrix.

.Both the RIP and coherence conditions are important in establishing true performance of signal recovery algorithms, when these

conditions ensure give a guarantee to achieve accurate reconstruction of to original signal, even if the received signal is embedded with noise, in addition, meet a unique solution of the underdetermined system [13]. Hence signal recovery process is achieved with only  $M$  measurement points of vector  $y$ , by solving the system below [9]:

$$\hat{x} = \operatorname{argmin} \|x\|_1 \text{ S.t. } y = \phi x + w \quad (12)$$

This underdetermined system is an optimization problem, which can be solved by linear program. Several recovery algorithms are suggested to solve this problem such as  $\ell_1$ -norm minimization [14], Orthogonal Matching Pursuit (OMP) [15] ...etc. It is clear that wideband signal can be accurately or almost accurately reconstructed, by adopting small number of acquisition points of the large dimension signal. Namely, much lower rate of sampling than Nyquist rate, which leads to minimize the acquisition and processing time, and increases the sensing performance and user throughput [9].

#### 4. Proposed compressive energy detection

The proposed system combines energy detection with CS compressive sensing in order to reduce the number of received signal samples consequently, reduces sensing time and efficiency of spectrum utilization. The reconstructed signal from the CS compressive sensing process is introduced, to conventional energy detection to achieve spectrum sensing process. Figure (3) shows a block diagram of proposed detection.

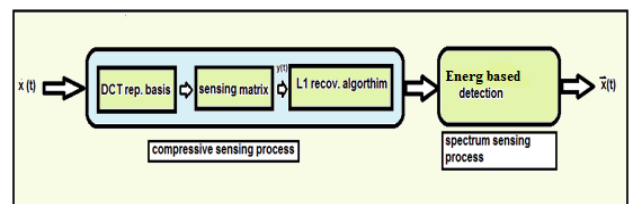


Figure 3. Energy based detection with CS Compressive Sensing

Wideband model signal with Independent and identically distributed (iid) random variables data (iid) plus AWGN noise with zero mean and variance  $\sigma_w^2$  is used. The principle of two hypothesis can be used with different construction of signal where  $(\phi w)$  is the noise signal instead of  $(w)$ , and  $(\phi s(n))$  primary user signal instead of  $s(n)$ .

The DCT sparse representation basis is used, to represent the sparsity of the received signal. The sparse signal is compressed by the Gaussian random matrix which is used as sensing matrix  $\phi$ . To obtain the get measurements vector  $y$ , the then  $\ell_1$ - norm recovery algorithm is used to reconstruct the original signal. The probability of detection is calculated using one of the sensing algorithms, based on Constant False Alarm Rate (CFAR) with  $P_{fa}=0.01$  and low range of SNR.

Several compression ratios (M/N) are used to clarify the effect of M/N, to the detection performance, and show the effect of compression, to reduce the sensing and processing time of the spectrum sensing of a wideband signal.

## 5. Simulation Results and Discussion

### 5.1 Probability of Detection with M/N Ratio

The Monte Carlo computer simulation is used, to simulate the procedure of the proposed system, to calculate the probability of detection  $P_d$  over AWGN channel with constant  $P_{fa}=0.01$ . BFSK signal and low range of SNR is used. The Energy CS compressive sensing detector is performed by reconstructing the original signal from  $y$  vector. The probability of detection is calculated as a function of SNR for several compression ratios M/N. Conventional energy-based detection can be obtained by setting making the compression ratio of the CS compressive sensing (M/N=1), which produces give a results that are similar to the same as no

non CS compressive sensing. Figure (4) shows the CS compressive sensing technique with energy detector for with several compression ratios (M/N), to investigate the effect of CS compressive sensing process on energy performance.

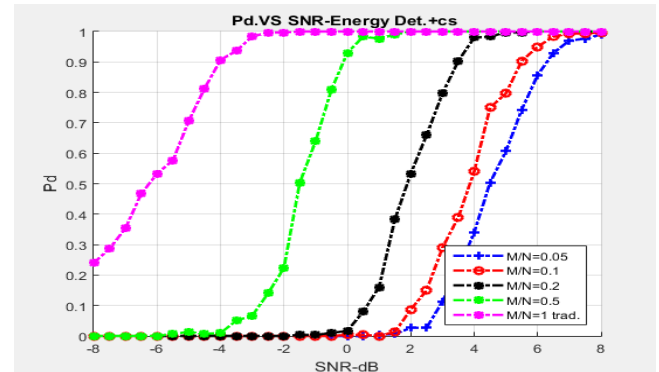


Figure 4. Probability of detection versus SNR for several M/N

Figure (4) shows that the detection performance is highly affected by compression process, it is clear that to keep probability of detection  $P_d=0.9$ , SNR is increased by about 4 dB with compression ratio (M/N) equal to 0.5, this means that when collecting half of received signal samples, that are needed to increase SNR by 4 dB to keep  $P_d$  at the constant desired level ( $P_d=0.9$ ), when M/N is less than 0.5, SNR is required to be increased by more than 4 dB, to keep the target  $P_d$ . This is interpreted as; reducing the received samples leads to reduce the received energy, so that the detection performance is degraded and the need to increase SNR is required to be increased, to compensate the loss of energy. Table (1) shows the required SNR for different M/N ratio to achieve  $P_d=0.9$ , it's clear that to conserve  $P_d=0.9$  with compression process of with M/N=0.5 the SNR it must be increased SNR by 4 dB. Table (2) shows the large effect affection of compression process to  $P_d$  for specific value of SNR, It is shows that  $P_d$  is large effected when M/N less than 0.5.

**Table 1.** M/N VS required SNR for  $P_d=0.9$

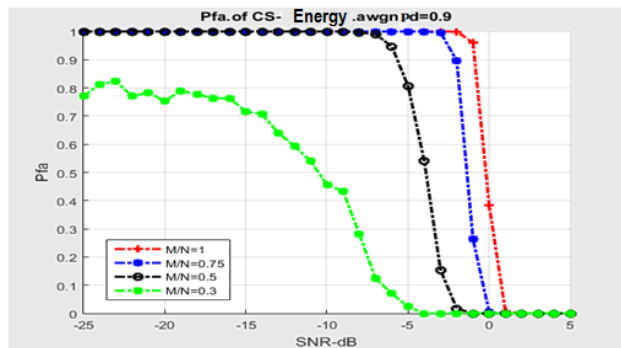
M/N	1	0.5	0.2	0.1	0.05
SNR(dB)	-4.1	-0.1	0.9	5.4	6.2

**Table 2.** M/N VS  $P_d$  for SNR = 2dB

M/N	1	0.5	0.2	0.1	0.05
$P_d$	1	1	0.47	0.05	0.02

**5.2 Probability of False Alarm with M/N ratio**

Figure (5) shows that performance is highly effected affected by the compression process regarding the probability of false alarm  $P_{fa}$ . Where  $P_{fa}$  is decreased with increasing compression process (increasing M/N ratio) but remaining with large value when SNR is less than -5 dB even with M/N is less than 0.3. In addition, it is clear that SNR has high effects on  $P_{fa}$ . Table (3) shows the effect of compression process to  $P_{fa}$  regarding to the level of SNR.



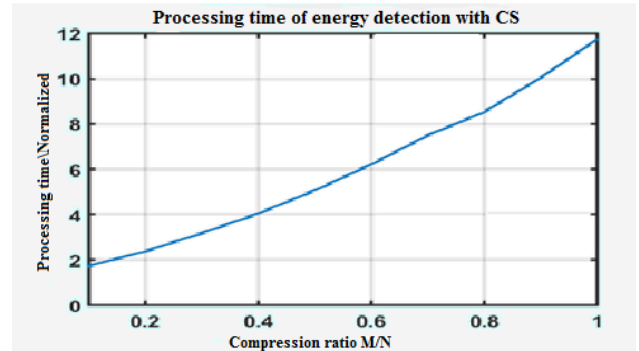
**Figure 5.** Probability of false alarm versus SNR for several M/N

**Table 3.** M/N VS SNR for  $P_d=0.9$

M/N	1	0.75	0.5	0.3
SNR(dB)	0.91	-0.18	-2.2	-5.6

The processing time is related directly to the M/N ratio, since very small values of M/N meet good detection performance, so that the processing time is decreased, with the same range of M/N to a very small value, related to the sensing time required, to operate with the

whole wideband signal. This is shown clearly in Figure (6).



**Figure 6.** Processing time versus compression ratio M/N

**6. Conclusions**

This paper presents the spectrum sensing techniques with wideband signal. Energy detection with CS compressive sensing technique is discussed, where the received signal is sampled at very low rate. DCT basis and random Gaussian sensing matrix are used, to obtain the measurement vector, which has low sampling rate. From the simulation results, it is clear that the energy detector has poor performance when combined with CS compressive sensing to deal with wideband signal, especially when the compression ratio (M/N) is very small (high compression process), but this performance is acceptable when M/N is larger than 0.7. Although this ratio of compression is low, but it reduces the sensing time and reduces the probability of false alarm, where sensing time is very important parameter, that acts approximately at all aspects of spectrum sensing process and CR cognitive radio system functions.

**Acknowledgements**

I wish to thank the staff of electrical engineering dept., college of engineering, Al-Mustansiriyah University for supporting this work.

## Abbreviations

AWGN: Additive White Gaussian Noise

FFT: Fast Fourier Transform

M/N: Compression ratio.

## Conflict of interest

There are not conflicts to declare.

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